

# Computer algebra independent integration tests

1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.3General/1.1.3.3(a+bx^n)^p(c+dx^n)^q

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# 1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

## 1.1 Listing of CAS systems tested

The following systems were tested at this time.

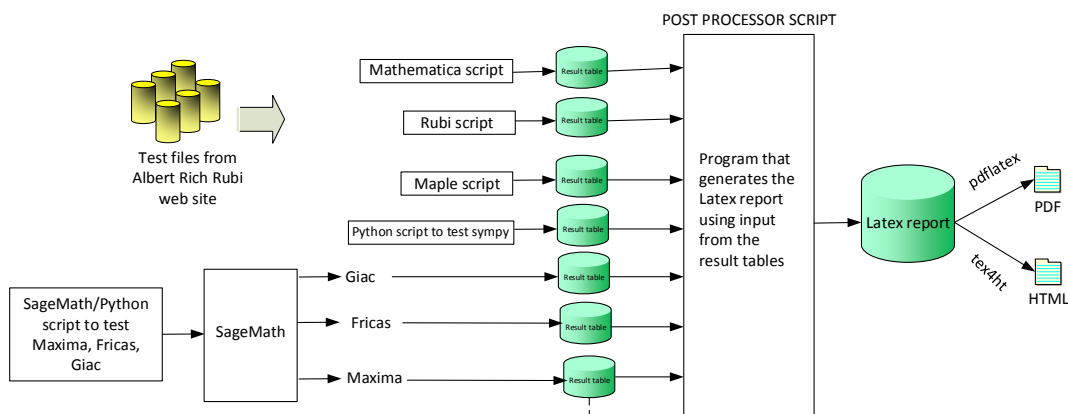
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

## 1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is comma delimited. It contains 12 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

**High level overview of the CAS independent integration test build system**

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## 1.3 Timing

The command `AbsoluteTiming[ ]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int', int(expr, x)), output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the `integrate` command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima `integrate` was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express>

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

    except Exception as ee:
        leafCount =1
```

When these cas systems implement a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. ( 286 )	% 0. ( 0 )
Rubi in Sympy	% 79.72 ( 228 )	% 20.28 ( 58 )
Mathematica	% 97.55 ( 279 )	% 2.45 ( 7 )
Maple	% 64.34 ( 184 )	% 35.66 ( 102 )
Maxima	% 18.18 ( 52 )	% 81.82 ( 234 )
Fricas	% 61.89 ( 177 )	% 38.11 ( 109 )
Sympy	% 37.06 ( 106 )	% 62.94 ( 180 )
Giac	% 45.8 ( 131 )	% 54.2 ( 155 )

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

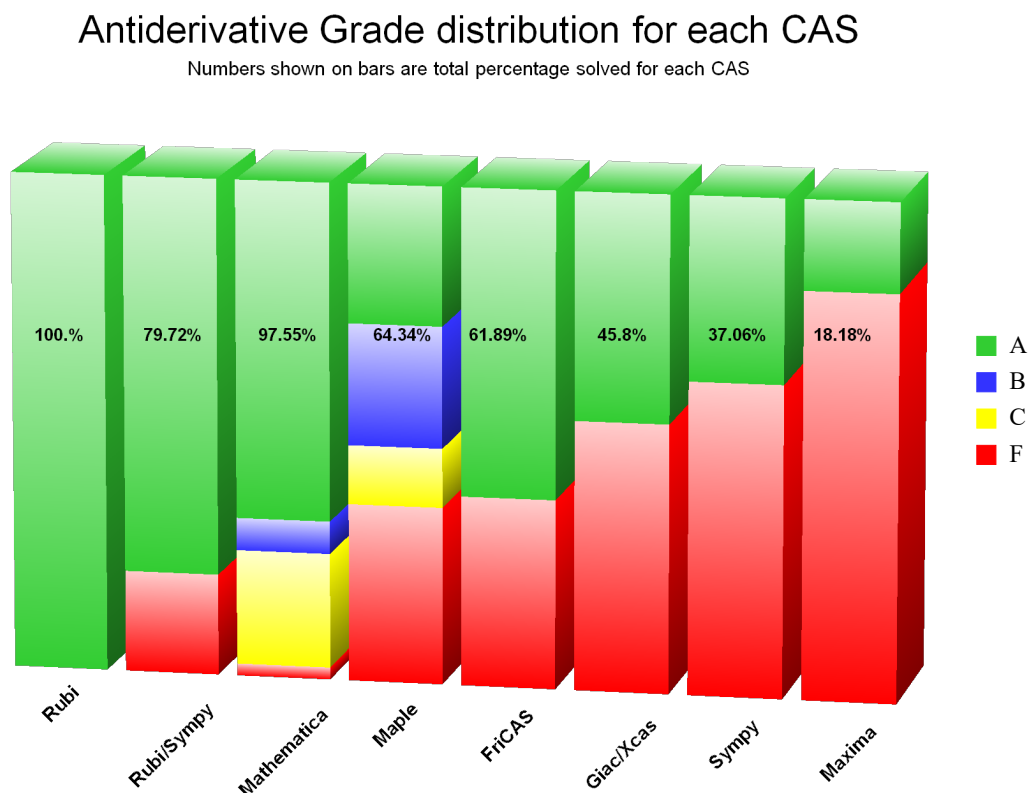
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ul style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ul>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

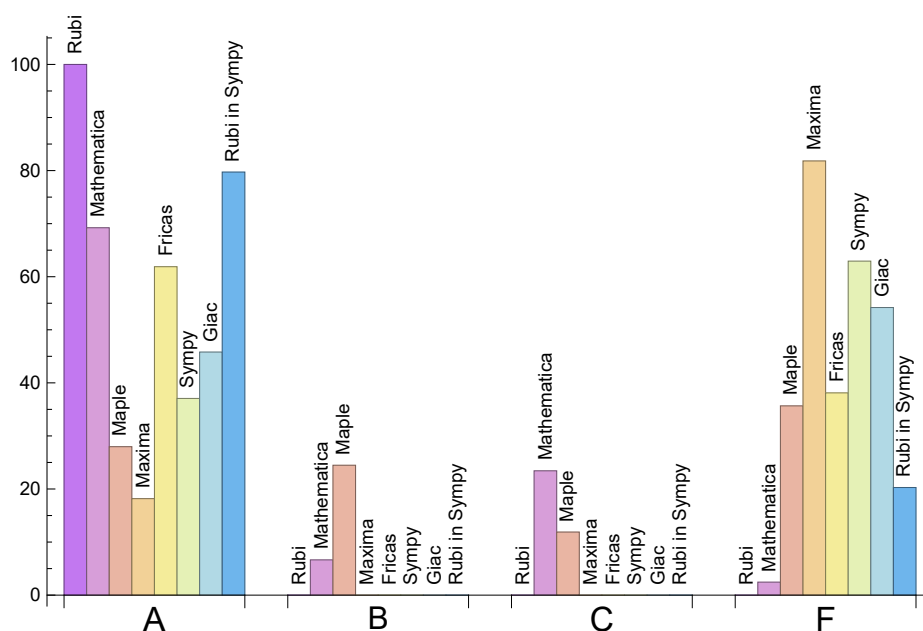
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Rubi in Sympy	79.72	0.	0.	20.28
Mathematica	69.23	6.64	23.43	2.45
Maple	27.97	24.48	11.89	35.66
Maxima	18.18	0.	0.	81.82
Fricas	61.89	0.	0.	38.11
Sympy	37.06	0.	0.	62.94
Giac	45.8	0.	0.	54.2

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



## 1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.42	172.17	1.	124.	1.
Rubi in Sympy	42.81	134.72	0.86	104.	0.87
Mathematica	0.99	194.65	1.5	150.	1.
Maple	0.02	455.77	2.46	233.5	1.42
Maxima	1.44	127.5	1.47	111.	1.38
Fricas	1.25	441.94	2.42	184.	1.98
Sympy	30.54	324.75	2.78	167.5	1.43
Giac	0.24	299.06	1.99	217.	1.75

## 1.8 list of integrals that has no closed form antiderivative

{}

## 1.9 list of integrals not solved by each system

**Not solved by Rubi** {}

**Not solved by Rubi in Sympy** {1, 2, 3, 4, 8, 9, 10, 11, 12, 14, 15, 16, 20, 21, 22, 23, 39, 47, 48, 49, 50, 54, 55, 56, 57, 58, 59, 61, 62, 63, 67, 68, 69, 70, 73, 80, 83, 84, 85, 91, 124, 159, 166, 177, 178, 181, 185, 186, 187, 188, 193, 194, 195, 199, 200, 206, 212, 214}

**Not solved by Mathematica** {228, 237, 281, 282, 283, 284, 286}

**Not solved by Maple** {31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 170, 175, 182, 183, 184, 189, 190, 191, 192, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 281, 282, 283, 284, 285, 286}

**Not solved by Maxima** {5, 6, 7, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 51, 52, 53, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 241, 242, 243, 247, 248, 264, 265, 266, 267, 268, 274, 275, 276, 277, 278, 281, 282, 283, 284, 285, 286}

**Not solved by Fricas** {31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 170, 172, 173, 174, 175, 182, 183, 184, 189, 190, 191, 192, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 234, 235, 236, 237, 283, 284, 285}

**Not solved by Sympy** {19, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 65, 66, 67, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 129, 130, 131, 136, 137, 138, 143, 144, 145, 150, 151, 152, 153, 154, 157, 158, 159, 160, 161, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 182, 183, 184, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 259, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286}

**Not solved by Giac** {27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 65, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 167, 168,

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## 1.10 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Rubi in Sympy** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {80, 81, 82, 83, 84}

**Mathematica** {30, 31, 32, 33, 34, 37, 38, 42, 43, 44, 45, 46, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 122, 123, 124, 170, 175, 213, 218, 219, 220, 229, 230, 235, 236, 280, 285}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Rubi in Sympy** Verification phase not implemented yet.



## 2 detailed summary tables of results

### 2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	97	130	1	104	131	0
normalized size	1	1.	1.	1.03	1.38	0.01	1.11	1.39	0.
time (sec)	N/A	0.149	0.035	0.	1.41	0.18	0.145	0.218	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	95	1	80	100	0
normalized size	1	1.	1.	1.04	1.36	0.01	1.14	1.43	0.
time (sec)	N/A	0.103	0.021	0.001	1.376	0.181	0.123	0.214	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	1	51	68	0
normalized size	1	1.	1.	0.98	1.3	0.02	1.02	1.36	0.
time (sec)	N/A	0.074	0.014	0.001	1.361	0.181	0.109	0.213	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	32	1	26	35	0
normalized size	1	1.	1.	0.89	1.14	0.04	0.93	1.25	0.
time (sec)	N/A	0.039	0.009	0.002	1.353	0.182	0.075	0.211	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	128	195	0	176	71	217	134
normalized size	1	1.	0.89	1.35	0.	1.22	0.49	1.51	0.93
time (sec)	N/A	0.211	0.14	0.007	0.	0.215	1.937	0.217	32.683

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	145	221	0	275	97	246	155
normalized size	1	1.	0.86	1.31	0.	1.63	0.57	1.46	0.92
time (sec)	N/A	0.195	0.17	0.011	0.	0.215	2.62	0.218	34.232

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	175	249	0	414	133	273	180
normalized size	1	1.	0.89	1.26	0.	2.1	0.68	1.39	0.91
time (sec)	N/A	0.229	0.225	0.014	0.	0.214	3.771	0.221	39.124

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	167	1	139	178	0
normalized size	1	1.	1.	1.02	1.37	0.01	1.14	1.46	0.
time (sec)	N/A	0.175	0.034	0.	1.37	0.182	0.166	0.211	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	87	111	1	90	123	0
normalized size	1	1.	1.	1.06	1.35	0.01	1.1	1.5	0.
time (sec)	N/A	0.117	0.023	0.001	1.407	0.181	0.138	0.211	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	1	51	68	0
normalized size	1	1.	1.	0.98	1.3	0.02	1.02	1.36	0.
time (sec)	N/A	0.075	0.014	0.	1.383	0.18	0.102	0.21	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	167	334	0	258	156	336	0
normalized size	1	1.	0.97	1.93	0.	1.49	0.9	1.94	0.
time (sec)	N/A	0.276	0.171	0.004	0.	0.212	2.861	0.219	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	210	367	0	435	189	358	0
normalized size	1	1.	1.03	1.81	0.	2.14	0.93	1.76	0.
time (sec)	N/A	0.495	0.379	0.014	0.	0.218	4.832	0.22	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	234	388	0	637	233	400	243
normalized size	1	1.	0.91	1.5	0.	2.47	0.9	1.55	0.94
time (sec)	N/A	0.477	0.466	0.015	0.	0.219	7.431	0.222	53.106

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	253	661	0	494	369	622	0
normalized size	1	1.	1.	2.62	0.	1.96	1.46	2.47	0.
time (sec)	N/A	0.415	0.199	0.006	0.	0.216	5.404	0.22	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	203	486	0	379	255	473	0
normalized size	1	1.	0.98	2.34	0.	1.82	1.23	2.27	0.
time (sec)	N/A	0.317	0.156	0.004	0.	0.216	4.164	0.22	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	167	334	0	259	156	336	0
normalized size	1	1.	0.97	1.93	0.	1.5	0.9	1.94	0.
time (sec)	N/A	0.266	0.185	0.005	0.	0.212	2.995	0.218	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	129	195	0	184	71	217	134
normalized size	1	1.	0.89	1.34	0.	1.27	0.49	1.5	0.92
time (sec)	N/A	0.182	0.109	0.005	0.	0.216	2.025	0.216	37.706

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	224	222	0	377	447	375	260
normalized size	1	1.	0.78	0.77	0.	1.31	1.55	1.3	0.9
time (sec)	N/A	0.321	0.22	0.009	0.	0.279	104.613	0.228	68.209

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	336	406	0	625	0	598	321
normalized size	1	1.	0.97	1.17	0.	1.81	0.	1.73	0.93
time (sec)	N/A	0.613	0.351	0.018	0.	7.366	0.	0.227	108.441

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	313	905	0	1000	536	822	0
normalized size	1	1.	0.98	2.83	0.	3.12	1.68	2.57	0.
time (sec)	N/A	0.633	0.439	0.017	0.	0.223	25.775	0.222	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	260	708	0	794	403	643	0
normalized size	1	1.	0.97	2.65	0.	2.97	1.51	2.41	0.
time (sec)	N/A	0.498	0.364	0.016	0.	0.223	14.071	0.221	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	227	529	0	606	289	495	0
normalized size	1	1.	0.97	2.26	0.	2.59	1.24	2.12	0.
time (sec)	N/A	0.482	0.261	0.013	0.	0.22	8.056	0.221	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	205	367	0	427	189	350	0
normalized size	1	1.	1.01	1.81	0.	2.1	0.93	1.72	0.
time (sec)	N/A	0.498	0.412	0.013	0.	0.219	5.08	0.218	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	145	221	0	277	97	246	155
normalized size	1	1.	0.86	1.31	0.	1.64	0.57	1.46	0.92
time (sec)	N/A	0.193	0.173	0.012	0.	0.216	2.686	0.218	34.112

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	337	406	0	633	0	598	321
normalized size	1	1.	0.97	1.17	0.	1.83	0.	1.73	0.93
time (sec)	N/A	0.589	0.36	0.017	0.	7.404	0.	0.229	109.102

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	381	606	0	1250	0	896	386
normalized size	1	1.	0.91	1.45	0.	2.98	0.	2.14	0.92
time (sec)	N/A	1.078	1.638	0.023	0.	77.342	0.	0.23	177.573

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	120	134	246	224	0	0	102
normalized size	1	1.	1.1	1.23	2.26	2.06	0.	0.	0.94
time (sec)	N/A	0.116	0.133	0.01	1.52	0.222	0.	0.	17.977

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	73	76	147	139	0	0	71
normalized size	1	1.	0.94	0.97	1.88	1.78	0.	0.	0.91
time (sec)	N/A	0.07	0.084	0.01	1.384	0.219	0.	0.	11.286

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	37	34	69	73	0	0	41
normalized size	1	1.	0.79	0.72	1.47	1.55	0.	0.	0.87
time (sec)	N/A	0.036	0.045	0.005	1.361	0.218	0.	0.	5.491

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	674	15	19	31	29	0	12
normalized size	1	1.	42.12	0.94	1.19	1.94	1.81	0.	0.75
time (sec)	N/A	0.009	2.202	0.003	1.402	0.211	1.952	0.	1.257

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	168	0	0	0	0	0	185
normalized size	1	1.	0.81	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.458	0.344	0.061	0.	0.	0.	0.	34.618

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	198	0	0	0	0	0	216
normalized size	1	1.	0.82	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.385	0.582	0.058	0.	0.	0.	0.	44.998

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	219	0	0	0	0	0	252
normalized size	1	1.	0.79	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.457	0.728	0.059	0.	0.	0.	0.	58.583

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	61
normalized size	1	1.	2.18	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.107	0.355	0.114	0.	0.	0.	0.	21.148

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	176	106	0	0	0	0	0	153
normalized size	1	1.05	0.63	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.278	0.08	0.082	0.	0.	0.	0.	33.094

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	85	75	0	0	0	0	0	73
normalized size	1	1.01	0.89	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.098	0.031	0.048	0.	0.	0.	0.	11.049

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	42
normalized size	1	1.	2.84	0.	0.	0.	0.	0.	0.74
time (sec)	N/A	0.075	0.284	0.069	0.	0.	0.	0.	21.65

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	44
normalized size	1	1.	2.84	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.073	0.296	0.097	0.	0.	0.	0.	20.499

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	137	0	0	0	0	0	0
normalized size	1	1.	0.46	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.69	0.104	0.07	0.	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	106	0	0	0	0	0	153
normalized size	1	1.	0.6	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.282	0.057	0.07	0.	0.	0.	0.	32.217

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	85	75	0	0	0	0	0	73
normalized size	1	0.91	0.81	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.1	0.03	0.055	0.	0.	0.	0.	10.716

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	196	0	0	0	34	0	34
normalized size	1	1.	4.45	0.	0.	0.	0.77	0.	0.77
time (sec)	N/A	0.027	0.287	0.035	0.	0.	54.267	0.	3.922

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	42
normalized size	1	1.	2.84	0.	0.	0.	0.	0.	0.74
time (sec)	N/A	0.071	0.327	0.076	0.	0.	0.	0.	21.223

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	44
normalized size	1	1.	2.84	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.072	0.298	0.095	0.	0.	0.	0.	19.708

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	44
normalized size	1	1.	2.84	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.071	0.392	0.078	0.	0.	0.	0.	19.867

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	594	71	0	123	0	0	44
normalized size	1	1.	11.21	1.34	0.	2.32	0.	0.	0.83
time (sec)	N/A	0.061	2.436	0.006	0.	0.252	0.	0.	10.552

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	97	130	1	107	132	0
normalized size	1	1.	1.	1.03	1.38	0.01	1.14	1.4	0.
time (sec)	N/A	0.145	0.035	0.001	1.362	0.192	0.149	0.213	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	95	1	76	100	0
normalized size	1	1.	1.	1.04	1.36	0.01	1.09	1.43	0.
time (sec)	N/A	0.104	0.031	0.001	1.356	0.192	0.122	0.212	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	1	53	68	0
normalized size	1	1.	1.	0.98	1.3	0.02	1.06	1.36	0.
time (sec)	N/A	0.074	0.02	0.001	1.413	0.19	0.105	0.211	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	32	1	26	35	0
normalized size	1	1.	1.	0.89	1.14	0.04	0.93	1.25	0.
time (sec)	N/A	0.038	0.009	0.001	1.364	0.189	0.071	0.211	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	196	266	0	743	87	331	204
normalized size	1	1.	0.88	1.19	0.	3.33	0.39	1.48	0.91
time (sec)	N/A	0.328	0.228	0.01	0.	0.24	2.235	0.22	57.162

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	212	295	0	838	112	359	226
normalized size	1	1.	0.87	1.2	0.	3.42	0.46	1.47	0.92
time (sec)	N/A	0.312	0.29	0.013	0.	0.241	3.207	0.219	59.134

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	243	314	0	941	151	386	258
normalized size	1	1.	0.89	1.15	0.	3.45	0.55	1.41	0.95
time (sec)	N/A	0.357	0.372	0.016	0.	0.239	6.697	0.222	65.639

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	163	213	1	185	234	0
normalized size	1	1.	1.	1.06	1.38	0.01	1.2	1.52	0.
time (sec)	N/A	0.236	0.062	0.001	1.372	0.189	0.183	0.21	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	167	1	139	178	0
normalized size	1	1.	1.	1.02	1.37	0.01	1.14	1.46	0.
time (sec)	N/A	0.173	0.043	0.002	1.358	0.191	0.156	0.212	0.



Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	87	111	1	97	123	0
normalized size	1	1.	1.	1.06	1.35	0.01	1.18	1.5	0.
time (sec)	N/A	0.119	0.032	0.001	1.378	0.19	0.138	0.209	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	1	53	68	0
normalized size	1	1.	1.	0.98	1.3	0.02	1.06	1.36	0.
time (sec)	N/A	0.073	0.014	0.002	1.395	0.192	0.108	0.215	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	231	436	0	1457	187	477	0
normalized size	1	1.	0.91	1.72	0.	5.76	0.74	1.89	0.
time (sec)	N/A	0.415	0.193	0.002	0.	0.241	3.589	0.221	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	298	475	0	1592	219	508	0
normalized size	1	1.	1.02	1.63	0.	5.47	0.75	1.75	0.
time (sec)	N/A	0.723	0.283	0.002	0.	0.256	6.902	0.222	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	319	499	0	1692	264	549	337
normalized size	1	1.	0.91	1.43	0.	4.85	0.76	1.57	0.97
time (sec)	N/A	0.55	0.321	0.002	0.	0.258	20.563	0.223	82.463

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	322	837	0	2939	430	833	0
normalized size	1	1.	0.97	2.52	0.	8.85	1.3	2.51	0.
time (sec)	N/A	0.581	0.329	0.008	0.	0.259	9.621	0.22	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	271	627	0	2198	301	649	0
normalized size	1	1.	0.94	2.18	0.	7.63	1.05	2.25	0.
time (sec)	N/A	0.45	0.235	0.002	0.	0.251	6.181	0.218	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	231	436	0	1458	187	477	0
normalized size	1	1.	0.91	1.72	0.	5.76	0.74	1.89	0.
time (sec)	N/A	0.401	0.173	0.003	0.	0.243	3.885	0.218	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	196	266	0	743	87	331	204
normalized size	1	1.	0.88	1.19	0.	3.33	0.39	1.48	0.91
time (sec)	N/A	0.299	0.215	0.005	0.	0.238	2.379	0.218	61.206

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	340	320	0	1472	0	0	400
normalized size	1	1.	0.76	0.71	0.	3.28	0.	0.	0.89
time (sec)	N/A	0.567	0.282	0.002	0.	0.395	0.	0.	119.517

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	498	550	0	3665	0	900	474
normalized size	1	1.	0.97	1.07	0.	7.14	0.	1.75	0.92
time (sec)	N/A	0.884	0.562	0.003	0.	21.891	0.	0.228	174.262

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	391	1118	0	3853	0	1077	0
normalized size	1	1.	0.96	2.75	0.	9.47	0.	2.65	0.
time (sec)	N/A	0.786	0.673	0.018	0.	0.268	0.	0.221	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	341	885	0	3083	466	867	0
normalized size	1	1.	0.96	2.48	0.	8.64	1.31	2.43	0.
time (sec)	N/A	0.719	0.501	0.017	0.	0.257	89.201	0.221	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	301	669	0	2306	335	670	0
normalized size	1	1.	0.95	2.11	0.	7.27	1.06	2.11	0.
time (sec)	N/A	0.611	0.4	0.001	0.	0.248	20.583	0.222	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	297	475	0	1592	219	508	0
normalized size	1	1.	1.02	1.63	0.	5.47	0.75	1.75	0.
time (sec)	N/A	0.724	0.3	0.014	0.	0.245	7.239	0.222	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	212	295	0	838	112	359	226
normalized size	1	1.	0.87	1.2	0.	3.42	0.46	1.47	0.92
time (sec)	N/A	0.317	0.304	0.011	0.	0.23	3.409	0.22	62.06

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	499	550	0	3665	0	900	474
normalized size	1	1.	0.97	1.07	0.	7.14	0.	1.75	0.92
time (sec)	N/A	0.903	0.545	0.018	0.	22.335	0.	0.228	168.513

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	561	784	0	0	0	0	0
normalized size	1	1.	0.94	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	1.468	3.493	0.003	0.	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	385	408	0	0	0	0	292
normalized size	1	1.	1.2	1.27	0.	0.	0.	0.	0.91
time (sec)	N/A	0.914	1.729	0.069	0.	0.	0.	0.	151.844

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	419	311	0	0	0	0	248
normalized size	1	1.	1.51	1.12	0.	0.	0.	0.	0.9
time (sec)	N/A	0.679	0.743	0.027	0.	0.	0.	0.	110.42

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	155	259	0	0	0	0	211
normalized size	1	1.	0.65	1.08	0.	0.	0.	0.	0.88
time (sec)	N/A	0.449	0.277	0.025	0.	0.	0.	0.	73.184

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	156	183	0	0	0	0	143
normalized size	1	1.	0.96	1.13	0.	0.	0.	0.	0.88
time (sec)	N/A	0.326	0.235	0.023	0.	0.	0.	0.	52.459

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	329	301	0	0	0	0	241
normalized size	1	1.	1.17	1.07	0.	0.	0.	0.	0.86
time (sec)	N/A	0.609	0.603	0.043	0.	0.	0.	0.	106.733

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	396	361	0	0	0	0	298
normalized size	1	1.	1.19	1.08	0.	0.	0.	0.	0.89
time (sec)	N/A	0.962	1.568	0.047	0.	0.	0.	0.	165.584

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	973	973	435	322	0	0	0	0	0
normalized size	1	1.	0.45	0.33	0.	0.	0.	0.	0.
time (sec)	N/A	3.707	1.147	0.036	0.	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	932	932	161	273	0	0	0	0	814
normalized size	1	1.	0.17	0.29	0.	0.	0.	0.	0.87
time (sec)	N/A	1.766	0.238	0.023	0.	0.	0.	0.	149.324

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	775	775	161	191	0	0	0	0	690
normalized size	1	1.	0.21	0.25	0.	0.	0.	0.	0.89
time (sec)	N/A	1.234	0.081	0.021	0.	0.	0.	0.	100.558

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	969	969	342	313	0	0	0	0	0
normalized size	1	1.	0.35	0.32	0.	0.	0.	0.	0.
time (sec)	N/A	2.406	0.495	0.043	0.	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1029	1029	406	371	0	0	0	0	0
normalized size	1	1.	0.39	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	4.384	1.641	0.047	0.	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	426	426	580	540	0	0	0	0	0
normalized size	1	1.	1.36	1.27	0.	0.	0.	0.	0.
time (sec)	N/A	1.391	1.826	0.041	0.	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	491	412	0	0	0	0	333
normalized size	1	1.	1.35	1.13	0.	0.	0.	0.	0.91
time (sec)	N/A	1.049	1.362	0.04	0.	0.	0.	0.	168.627

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	423	329	0	0	0	0	279
normalized size	1	1.	1.37	1.06	0.	0.	0.	0.	0.9
time (sec)	N/A	0.746	0.594	0.036	0.	0.	0.	0.	107.566

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	310	294	0	0	0	0	243
normalized size	1	1.	1.12	1.07	0.	0.	0.	0.	0.88
time (sec)	N/A	0.621	0.269	0.033	0.	0.	0.	0.	97.147

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	349	322	0	0	0	0	270
normalized size	1	1.	1.13	1.04	0.	0.	0.	0.	0.87
time (sec)	N/A	0.688	0.991	0.035	0.	0.	0.	0.	109.539

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	465	375	0	0	0	0	323
normalized size	1	1.	1.28	1.04	0.	0.	0.	0.	0.89
time (sec)	N/A	1.074	1.186	0.058	0.	0.	0.	0.	175.193

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	617	484	0	0	0	0	0
normalized size	1	1.	1.41	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	1.439	2.184	0.057	0.	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	155	103	0	450	0	0	94
normalized size	1	1.	1.5	1.	0.	4.37	0.	0.	0.91
time (sec)	N/A	0.141	0.242	0.024	0.	0.592	0.	0.	22.737

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	155	158	0	500	0	0	100
normalized size	1	1.	1.34	1.36	0.	4.31	0.	0.	0.86
time (sec)	N/A	0.079	0.272	0.025	0.	0.587	0.	0.	12.654

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	396	0	0	2776	0	0	192
normalized size	1	1.	1.88	0.	0.	13.16	0.	0.	0.91
time (sec)	N/A	0.523	1.806	0.084	0.	2.723	0.	0.	62.95

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	161	0	0	1006	0	0	148
normalized size	1	1.	0.93	0.	0.	5.82	0.	0.	0.86
time (sec)	N/A	0.24	0.255	0.055	0.	0.294	0.	0.	37.997

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	122	0	0	0	0	0	90
normalized size	1	1.	1.16	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.136	0.124	0.05	0.	0.	0.	0.	23.604

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	153	0	0	0	0	0	117
normalized size	1	1.	1.14	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.236	0.368	0.055	0.	0.	0.	0.	34.045

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	184	0	0	0	0	0	158
normalized size	1	1.	1.02	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.49	0.563	0.057	0.	0.	0.	0.	80.616

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	231	0	0	0	0	0	212
normalized size	1	1.	0.99	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.73	0.684	0.061	0.	0.	0.	0.	133.593

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	396	0	0	0	0	0	282
normalized size	1	1.	1.25	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.922	1.725	0.098	0.	0.	0.	0.	112.517

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	435	0	0	0	0	0	235
normalized size	1	1.	1.59	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.493	0.714	0.094	0.	0.	0.	0.	62.65

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	160	0	0	0	0	0	143
normalized size	1	1.	0.96	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.293	0.247	0.055	0.	0.	0.	0.	41.329

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	161	0	0	0	0	0	221
normalized size	1	1.	0.62	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.471	0.087	0.052	0.	0.	0.	0.	62.698

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	343	0	0	0	0	0	265
normalized size	1	1.	1.13	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.698	0.536	0.054	0.	0.	0.	0.	99.62

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	407	0	0	0	0	0	318
normalized size	1	1.	1.14	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	1.08	1.561	0.058	0.	0.	0.	0.	161.555

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	735	0	0	3868	0	0	255
normalized size	1	1.	2.62	0.	0.	13.81	0.	0.	0.91
time (sec)	N/A	0.893	4.048	0.096	0.	19.954	0.	0.	101.332

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	462	0	0	1972	0	0	206
normalized size	1	1.	2.01	0.	0.	8.57	0.	0.	0.9
time (sec)	N/A	0.439	0.819	0.059	0.	1.425	0.	0.	63.515

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	152	0	0	0	0	0	119
normalized size	1	1.	1.13	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.204	0.351	0.056	0.	0.	0.	0.	31.236

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	171	0	0	0	0	0	141
normalized size	1	1.	1.06	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.301	0.285	0.066	0.	0.	0.	0.	36.648

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	204	0	0	0	0	0	180
normalized size	1	1.	1.	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.495	0.449	0.059	0.	0.	0.	0.	77.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	242	0	0	0	0	0	240
normalized size	1	1.	0.91	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.885	0.822	0.064	0.	0.	0.	0.	144.937



Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	506	0	0	0	0	0	316
normalized size	1	1.	1.43	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.987	1.294	0.094	0.	0.	0.	0.	112.913

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	440	0	0	0	0	0	260
normalized size	1	1.	1.48	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.704	0.646	0.059	0.	0.	0.	0.	83.37

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	322	0	0	0	0	0	267
normalized size	1	1.	1.05	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.637	0.288	0.058	0.	0.	0.	0.	85.741

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	341	0	0	0	0	0	292
normalized size	1	1.	1.03	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.753	0.891	0.059	0.	0.	0.	0.	96.992

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	485	0	0	0	0	0	350
normalized size	1	1.	1.24	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	1.164	1.038	0.06	0.	0.	0.	0.	157.932

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	70	0	0	273	0	0	49
normalized size	1	1.	1.32	0.	0.	5.15	0.	0.	0.92
time (sec)	N/A	0.057	0.085	0.038	0.	4.313	0.	0.	4.794

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	76	0	0	0	0	0	49
normalized size	1	1.	1.33	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.069	0.11	0.076	0.	0.	0.	0.	9.04

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	61
normalized size	1	1.	2.18	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.123	0.359	0.115	0.	0.	0.	0.	21.945

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	106	0	0	0	0	0	153
normalized size	1	1.	0.6	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.316	0.079	0.057	0.	0.	0.	0.	32.823

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	85	75	0	0	0	0	0	73
normalized size	1	0.91	0.81	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.111	0.035	0.049	0.	0.	0.	0.	11.371

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	42
normalized size	1	1.	2.84	0.	0.	0.	0.	0.	0.74
time (sec)	N/A	0.084	0.281	0.072	0.	0.	0.	0.	21.903

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	44
normalized size	1	1.	2.84	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.083	0.308	0.064	0.	0.	0.	0.	20.222

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	379	0	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.311	1.626	0.055	0.	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	131	248	0	1	491	0	131
normalized size	1	1.	0.92	1.73	0.	0.01	3.43	0.	0.92
time (sec)	N/A	0.417	0.205	0.02	0.	0.26	15.493	0.	50.01

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	85	191	0	1	156	0	82
normalized size	1	1.	0.86	1.93	0.	0.01	1.58	0.	0.83
time (sec)	N/A	0.218	0.134	0.017	0.	0.257	12.042	0.	20.394

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	163	0	1	121	0	63
normalized size	1	1.	0.85	2.2	0.	0.01	1.64	0.	0.85
time (sec)	N/A	0.146	0.122	0.016	0.	0.259	21.053	0.	12.818

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	50	74	0	1	42	86	31
normalized size	1	1.	1.28	1.9	0.	0.03	1.08	2.21	0.79
time (sec)	N/A	0.057	0.035	0.006	0.	0.257	6.494	0.233	5.903

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	153	286	0	1	0	0	90
normalized size	1	1.	1.47	2.75	0.	0.01	0.	0.	0.87
time (sec)	N/A	0.355	0.359	0.03	0.	0.288	0.	0.	41.254

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	197	939	0	1	0	0	122
normalized size	1	1.	1.34	6.39	0.	0.01	0.	0.	0.83
time (sec)	N/A	0.607	0.599	0.028	0.	0.3	0.	0.	67.723

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	275	1963	0	1	0	4	182
normalized size	1	1.	1.29	9.22	0.	0.	0.	0.02	0.85
time (sec)	N/A	0.968	0.789	0.028	0.	0.402	0.	0.561	111.143

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	169	332	0	1	1862	0	151
normalized size	1	1.	1.03	2.02	0.	0.01	11.35	0.	0.92
time (sec)	N/A	0.448	0.259	0.02	0.	0.255	41.585	0.	48.464

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	115	244	0	1	576	0	107
normalized size	1	1.	0.91	1.94	0.	0.01	4.57	0.	0.85
time (sec)	N/A	0.249	0.239	0.019	0.	0.259	26.834	0.	22.461

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	84	194	0	1	204	0	87
normalized size	1	1.	0.84	1.94	0.	0.01	2.04	0.	0.87
time (sec)	N/A	0.182	0.163	0.016	0.	0.259	32.568	0.	14.757

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	56	94	0	1	92	0	44
normalized size	1	1.	1.04	1.74	0.	0.02	1.7	0.	0.81
time (sec)	N/A	0.077	0.046	0.006	0.	0.264	9.18	0.	7.711

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	156	528	0	1	0	0	92
normalized size	1	1.	1.47	4.98	0.	0.01	0.	0.	0.87
time (sec)	N/A	0.384	0.494	0.02	0.	0.296	0.	0.	37.956

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	231	1202	0	1	0	0	129
normalized size	1	1.	1.48	7.71	0.	0.01	0.	0.	0.83
time (sec)	N/A	0.67	0.479	0.02	0.	0.293	0.	0.	68.384

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	256	2373	0	1	0	4	182
normalized size	1	1.	1.22	11.35	0.	0.	0.	0.02	0.87
time (sec)	N/A	0.984	0.512	0.021	0.	0.328	0.	0.551	106.166

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	212	434	0	1	5557	0	184
normalized size	1	1.	1.07	2.19	0.	0.01	28.07	0.	0.93
time (sec)	N/A	0.5	0.355	0.022	0.	0.261	82.497	0.	54.841

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	156	318	0	1	1884	0	131
normalized size	1	1.	1.03	2.09	0.	0.01	12.39	0.	0.86
time (sec)	N/A	0.299	0.25	0.02	0.	0.252	48.817	0.	26.754

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	105	240	0	1	561	0	110
normalized size	1	1.	0.84	1.92	0.	0.01	4.49	0.	0.88
time (sec)	N/A	0.209	0.252	0.016	0.	0.263	47.699	0.	17.605

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	112	0	1	99	0	60
normalized size	1	1.	1.	1.58	0.	0.01	1.39	0.	0.85
time (sec)	N/A	0.101	0.102	0.005	0.	0.256	13.361	0.	9.588

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	172	859	0	1	0	0	114
normalized size	1	1.	1.28	6.41	0.	0.01	0.	0.	0.85
time (sec)	N/A	0.639	0.228	0.023	0.	0.436	0.	0.	62.059

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	219	2014	0	1	0	0	138
normalized size	1	1.	1.32	12.13	0.	0.01	0.	0.	0.83
time (sec)	N/A	0.669	0.604	0.024	0.	0.373	0.	0.	63.203

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	304	2807	0	1	0	4	204
normalized size	1	1.	1.28	11.84	0.	0.	0.	0.02	0.86
time (sec)	N/A	1.022	0.651	0.022	0.	0.342	0.	0.549	106.304

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	99	540	0	1	377	207	114
normalized size	1	1.	0.79	4.29	0.	0.01	2.99	1.64	0.9
time (sec)	N/A	0.284	0.289	0.021	0.	0.253	15.563	0.257	25.537

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	354	0	1	105	131	60
normalized size	1	1.	1.03	4.85	0.	0.01	1.44	1.79	0.82
time (sec)	N/A	0.181	0.161	0.02	0.	0.248	12.382	0.256	16.515

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	62	179	0	1	73	99	42
normalized size	1	1.	1.22	3.51	0.	0.02	1.43	1.94	0.82
time (sec)	N/A	0.11	0.075	0.017	0.	0.25	25.068	0.251	9.442

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	53	70	0	1	44	96	32
normalized size	1	1.	1.23	1.63	0.	0.02	1.02	2.23	0.74
time (sec)	N/A	0.059	0.047	0.005	0.	0.247	7.839	0.237	5.357

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	157	389	0	1	0	174	92
normalized size	1	1.	1.45	3.6	0.	0.01	0.	1.61	0.85
time (sec)	N/A	0.317	0.467	0.019	0.	0.277	0.	0.253	40.759

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	224	1137	0	1	0	393	141
normalized size	1	1.	1.3	6.61	0.	0.01	0.	2.28	0.82
time (sec)	N/A	0.65	0.841	0.029	0.	0.36	0.	0.257	68.013

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	301	2269	0	1	0	459	218
normalized size	1	1.	1.2	9.08	0.	0.	0.	1.84	0.87
time (sec)	N/A	1.083	0.95	0.025	0.	0.767	0.	0.261	117.385

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	123	976	0	1	0	297	122
normalized size	1	1.	0.93	7.39	0.	0.01	0.	2.25	0.92
time (sec)	N/A	0.327	0.179	0.025	0.	0.257	0.	0.252	27.207

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	90	100	796	0	1	0	217	82
normalized size	1	0.96	1.06	8.47	0.	0.01	0.	2.31	0.87
time (sec)	N/A	0.248	0.188	0.022	0.	0.243	0.	0.252	18.821

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	81	394	0	1	224	165	66
normalized size	1	1.	1.07	5.18	0.	0.01	2.95	2.17	0.87
time (sec)	N/A	0.155	0.117	0.018	0.	0.247	21.432	0.254	12.511

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	67	203	0	1	71	116	51
normalized size	1	1.	1.1	3.33	0.	0.02	1.16	1.9	0.84
time (sec)	N/A	0.083	0.1	0.005	0.	0.251	11.453	0.25	8.025

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	197	1480	0	1	0	261	122
normalized size	1	1.	1.34	10.07	0.	0.01	0.	1.78	0.83
time (sec)	N/A	0.619	1.228	0.026	0.	0.386	0.	0.254	67.665

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	290	3121	0	1	0	560	194
normalized size	1	1.	1.29	13.93	0.	0.	0.	2.5	0.87
time (sec)	N/A	0.962	1.108	0.026	0.	0.812	0.	0.256	110.149

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	385	5164	0	1	0	678	0
normalized size	1	1.	1.2	16.14	0.	0.	0.	2.12	0.
time (sec)	N/A	1.524	1.824	0.027	0.	2.248	0.	0.261	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	145	1157	0	1	0	267	138
normalized size	1	1.	1.01	8.09	0.	0.01	0.	1.87	0.97
time (sec)	N/A	0.429	0.242	0.023	0.	0.255	0.	0.26	27.837

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	118	123	595	0	1	0	217	107
normalized size	1	0.97	1.01	4.88	0.	0.01	0.	1.78	0.88
time (sec)	N/A	0.296	0.189	0.02	0.	0.254	0.	0.258	22.725

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	102	548	0	1	1479	185	90
normalized size	1	1.	0.99	5.32	0.	0.01	14.36	1.8	0.87
time (sec)	N/A	0.199	0.141	0.019	0.	0.242	39.109	0.256	16.362

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	276	0	1	774	132	70
normalized size	1	1.	1.	3.37	0.	0.01	9.44	1.61	0.85
time (sec)	N/A	0.113	0.134	0.006	0.	0.242	17.9	0.254	11.539

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	250	2490	0	1	0	332	173
normalized size	1	1.	1.24	12.39	0.	0.	0.	1.65	0.86
time (sec)	N/A	0.946	0.963	0.024	0.	0.903	0.	0.258	106.153

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	364	4648	0	1	0	779	252
normalized size	1	1.	1.27	16.2	0.	0.	0.	2.71	0.88
time (sec)	N/A	1.409	1.844	0.028	0.	2.542	0.	0.281	160.074

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	465	7306	0	1	0	703	0
normalized size	1	1.	1.14	17.86	0.	0.	0.	1.72	0.
time (sec)	N/A	2.072	3.82	0.031	0.	6.637	0.	0.266	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	168	253	0	1	0	0	104
normalized size	1	1.	1.37	2.06	0.	0.01	0.	0.	0.85
time (sec)	N/A	0.354	0.355	0.033	0.	0.729	0.	0.	32.876



Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	98	155	0	1	0	0	65
normalized size	1	1.	1.21	1.91	0.	0.01	0.	0.	0.8
time (sec)	N/A	0.193	0.195	0.034	0.	0.327	0.	0.	14.874

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	112	280	0	1	0	0	114
normalized size	1	1.	0.92	2.3	0.	0.01	0.	0.	0.93
time (sec)	N/A	0.301	0.214	0.049	0.	0.329	0.	0.	24.182

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	206	0	0	0	0	0	66
normalized size	1	1.	2.15	0.	0.	0.	0.	0.	0.69
time (sec)	N/A	0.181	0.493	0.142	0.	0.	0.	0.	23.234

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	45	0	1	82	45	34
normalized size	1	1.	1.03	1.15	0.	0.03	2.1	1.15	0.87
time (sec)	N/A	0.072	0.044	0.007	0.	0.235	1.665	0.214	11.336

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	205	277	0	0	0	0	202
normalized size	1	1.	0.88	1.19	0.	0.	0.	0.	0.87
time (sec)	N/A	0.634	0.52	0.05	0.	0.	0.	0.	62.946

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	86	94	0	0	0	0	199
normalized size	1	1.	0.37	0.41	0.	0.	0.	0.	0.86
time (sec)	N/A	0.588	0.102	0.029	0.	0.	0.	0.	56.681

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	191	185	0	0	0	0	230
normalized size	1	1.	0.73	0.71	0.	0.	0.	0.	0.88
time (sec)	N/A	0.836	0.377	0.065	0.	0.	0.	0.	80.5

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	252	0	0	0	0	0	63
normalized size	1	1.	3.19	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.234	0.701	0.141	0.	0.	0.	0.	31.42

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	129	195	0	184	71	217	134
normalized size	1	1.	0.89	1.34	0.	1.27	0.49	1.5	0.92
time (sec)	N/A	0.258	0.166	0.007	0.	0.239	2.223	0.22	37.577

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	51	59	63	65	82	66	0
normalized size	1	1.	1.04	1.2	1.29	1.33	1.67	1.35	0.
time (sec)	N/A	0.122	0.044	0.007	1.706	0.24	0.818	0.216	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	27	27	24	27	0
normalized size	1	1.	1.	0.81	1.04	1.04	0.92	1.04	0.
time (sec)	N/A	0.046	0.01	0.004	1.4	0.23	0.472	0.215	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	31	24	31	31	27	32	15
normalized size	1	1.	1.82	1.41	1.82	1.82	1.59	1.88	0.88
time (sec)	N/A	0.05	0.01	0.005	1.391	0.227	0.649	0.217	10.383

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	102	66	90	123	102	0	99
normalized size	1	1.	0.98	0.63	0.87	1.18	0.98	0.	0.95
time (sec)	N/A	0.203	0.05	0.009	1.648	0.242	20.267	0.	19.88

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	30	30	26	31	0
normalized size	1	1.	1.	0.77	1.	1.	0.87	1.03	0.
time (sec)	N/A	0.055	0.011	0.004	1.376	0.237	0.405	0.216	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	151	0	0	0	0	0	66
normalized size	1	1.	1.91	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.083	0.262	0.118	0.	0.	0.	0.	19.689

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	116	0	0	0	0	0	63
normalized size	1	1.	1.53	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.076	0.147	0.068	0.	0.	0.	0.	19.437

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	0	0	0	56
normalized size	1	1.	1.	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.073	0.126	0.242	0.	0.	0.	0.	21.4

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	123	138	0	711	2744	1073	0
normalized size	1	1.	0.93	1.05	0.	5.39	20.79	8.13	0.
time (sec)	N/A	0.251	0.19	0.018	0.	0.263	9.819	0.229	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	92	104	0	431	1540	656	0
normalized size	1	1.	0.93	1.05	0.	4.35	15.56	6.63	0.
time (sec)	N/A	0.167	0.145	0.016	0.	0.248	5.26	0.225	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	65	74	0	236	726	342	0
normalized size	1	1.	0.93	1.06	0.	3.37	10.37	4.89	0.
time (sec)	N/A	0.114	0.109	0.014	0.	0.257	2.792	0.221	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	37	43	0	93	236	126	0
normalized size	1	1.	0.92	1.08	0.	2.32	5.9	3.15	0.
time (sec)	N/A	0.059	0.083	0.012	0.	0.25	1.312	0.214	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	0	0	0	0	0	31
normalized size	1	1.	0.93	0.	0.	0.	0.	0.	0.72
time (sec)	N/A	0.055	0.037	0.083	0.	0.	0.	0.	7.203

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	0	0	0	0	0	53
normalized size	1	1.	0.93	0.	0.	0.	0.	0.	0.73
time (sec)	N/A	0.091	0.079	0.064	0.	0.	0.	0.	9.663

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	96	0	0	0	0	0	60
normalized size	1	1.	1.23	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.092	0.108	0.076	0.	0.	0.	0.	10.156

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	136	0	0	0	0	0	60
normalized size	1	1.	1.74	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.09	0.165	0.093	0.	0.	0.	0.	10.142

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	149	164	0	900	0	1	0
normalized size	1	1.	0.94	1.04	0.	5.7	0.	0.01	0.
time (sec)	N/A	0.291	0.251	0.019	0.	0.253	0.	0.227	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	105	117	0	500	0	782	0
normalized size	1	1.	0.94	1.04	0.	4.46	0.	6.98	0.
time (sec)	N/A	0.187	0.259	0.017	0.	0.256	0.	0.227	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	65	74	0	236	726	342	0
normalized size	1	1.	0.93	1.06	0.	3.37	10.37	4.89	0.
time (sec)	N/A	0.114	0.111	0.014	0.	0.255	2.771	0.22	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	0	0	0	0	0	70
normalized size	1	1.	0.98	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.219	0.069	0.062	0.	0.	0.	0.	22.834

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	95	0	0	0	0	0	95
normalized size	1	1.	0.83	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.245	0.191	0.067	0.	0.	0.	0.	23.097

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	133	0	0	0	0	0	141
normalized size	1	1.	0.83	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.359	0.173	0.079	0.	0.	0.	0.	27.925

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	146	0	0	0	0	0	0
normalized size	1	1.	0.47	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.128	0.185	0.072	0.	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	104	0	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.614	0.507	0.068	0.	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	0	0	0	0	0	70
normalized size	1	1.	0.98	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.236	0.071	0.062	0.	0.	0.	0.	22.161

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	40	0	0	0	0	0	31
normalized size	1	1.	0.95	0.	0.	0.	0.	0.	0.74
time (sec)	N/A	0.053	0.033	0.058	0.	0.	0.	0.	6.355

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	0	0	0	0	53
normalized size	1	1.	0.89	0.	0.	0.	0.	0.	0.74
time (sec)	N/A	0.079	0.055	0.091	0.	0.	0.	0.	10.966

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	121	0	0	0	0	0	100
normalized size	1	1.	0.98	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	0.36	0.217	0.165	0.	0.	0.	0.	48.486

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	210	0	0	0	0	0	218
normalized size	1	1.	1.	0.	0.	0.	0.	0.	1.04
time (sec)	N/A	0.819	0.334	0.175	0.	0.	0.	0.	112.827

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	217	0	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.287	0.337	0.078	0.	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	167	0	0	0	0	0	172
normalized size	1	1.	0.84	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.61	0.458	0.073	0.	0.	0.	0.	65.611

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	96	0	0	0	0	0	97
normalized size	1	1.	0.83	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.237	0.2	0.066	0.	0.	0.	0.	22.598

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	68	0	0	0	0	0	53
normalized size	1	1.	0.94	0.	0.	0.	0.	0.	0.74
time (sec)	N/A	0.084	0.077	0.065	0.	0.	0.	0.	9.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	108	0	0	0	0	0	100
normalized size	1	1.	0.89	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.344	0.255	0.154	0.	0.	0.	0.	48.624

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	147	0	0	0	0	0	162
normalized size	1	1.	0.76	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.695	0.344	0.153	0.	0.	0.	0.	105.36

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	233	0	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.305	0.536	0.187	0.	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	190	0	0	0	0	0	63
normalized size	1	1.	2.35	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.129	0.662	0.207	0.	0.	0.	0.	21.667

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	401	168	0	0	0	0	0	0
normalized size	1	1.	0.42	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.285	0.444	0.159	0.	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	197	140	0	0	0	0	0	173
normalized size	1	0.98	0.69	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.624	0.248	0.137	0.	0.	0.	0.	36.28

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	89	85	0	0	0	0	0	80
normalized size	1	0.91	0.87	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.12	0.103	0.135	0.	0.	0.	0.	11.037

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	0	0	36
normalized size	1	1.	1.	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.032	0.027	0.002	0.	0.	0.	0.	3.704

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	44
normalized size	1	1.	3.05	0.	0.	0.	0.	0.	0.75
time (sec)	N/A	0.084	0.418	0.122	0.	0.	0.	0.	21.433

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	46
normalized size	1	1.	3.05	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.083	0.474	0.103	0.	0.	0.	0.	19.946

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	46
normalized size	1	1.	3.05	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.084	0.692	0.112	0.	0.	0.	0.	20.116

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	0	0	0	0	0	83
normalized size	1	1.	1.	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.073	0.558	0.249	0.	0.	0.	0.	8.191

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	198	0	0	645	0	0	156
normalized size	1	1.	1.11	0.	0.	3.62	0.	0.	0.88
time (sec)	N/A	0.241	0.608	0.203	0.	0.255	0.	0.	35.596

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	139	0	0	312	0	0	100
normalized size	1	1.	1.2	0.	0.	2.69	0.	0.	0.86
time (sec)	N/A	0.113	0.314	0.177	0.	0.257	0.	0.	18.101



Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	82	0	0	115	0	0	48
normalized size	1	1.	1.41	0.	0.	1.98	0.	0.	0.83
time (sec)	N/A	0.048	0.169	0.165	0.	0.267	0.	0.	6.749

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	53	0	42	0	0	12
normalized size	1	1.	1.	2.94	0.	2.33	0.	0.	0.67
time (sec)	N/A	0.012	0.04	0.036	0.	0.249	0.	0.	1.391

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	79	0	0	0	0	0	39
normalized size	1	1.	1.49	0.	0.	0.	0.	0.	0.74
time (sec)	N/A	0.048	0.301	0.12	0.	0.	0.	0.	6.259

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	0	0	0	0	0	42
normalized size	1	1.	0.98	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.05	0.135	0.112	0.	0.	0.	0.	6.155

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	F	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	0	0	0	0	0	0	44
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.79
time (sec)	N/A	0.052	179.999	0.115	0.	0.	0.	0.	6.47

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	193	179	1414	0	0	0	0	0	155
normalized size	1	0.93	7.33	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.206	53.257	0.264	0.	0.	0.	0.	25.243

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	461	0	0	146	0	0	41
normalized size	1	1.	8.09	0.	0.	2.56	0.	0.	0.72
time (sec)	N/A	0.085	1.939	0.306	0.	0.265	0.	0.	13.522

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	153	0	0	540	0	0	286
normalized size	1	1.	0.47	0.	0.	1.65	0.	0.	0.87
time (sec)	N/A	0.548	0.305	0.21	0.	0.26	0.	0.	81.854

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	96	0	0	234	0	0	105
normalized size	1	1.	0.76	0.	0.	1.84	0.	0.	0.83
time (sec)	N/A	0.173	0.139	0.18	0.	0.261	0.	0.	17.83

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	55	0	0	92	0	0	39
normalized size	1	1.	1.1	0.	0.	1.84	0.	0.	0.78
time (sec)	N/A	0.04	0.037	0.131	0.	0.256	0.	0.	3.682

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	153	0	0	0	0	0	70
normalized size	1	1.	1.61	0.	0.	0.	0.	0.	0.74
time (sec)	N/A	0.106	2.236	0.126	0.	0.	0.	0.	13.62

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	1070	0	0	0	0	0	95
normalized size	1	1.	8.43	0.	0.	0.	0.	0.	0.75
time (sec)	N/A	0.151	62.191	0.116	0.	0.	0.	0.	19.246

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	1241	0	0	0	0	0	105
normalized size	1	1.	9.47	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.162	43.768	0.113	0.	0.	0.	0.	19.697

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	F	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	0	0	0	0	0	0	107
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.171	179.999	0.137	0.	0.	0.	0.	20.657

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	164	109	92	240	741	0	309	150
normalized size	1	1.08	0.72	0.61	1.58	4.88	0.	2.03	0.99
time (sec)	N/A	0.407	0.094	0.01	1.392	0.743	0.	0.257	25.853

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	118	88	68	167	582	0	227	105
normalized size	1	1.08	0.81	0.62	1.53	5.34	0.	2.08	0.96
time (sec)	N/A	0.291	0.075	0.009	1.381	0.413	0.	0.231	17.54

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	72	62	44	95	421	0	126	61
normalized size	1	1.07	0.93	0.66	1.42	6.28	0.	1.88	0.91
time (sec)	N/A	0.151	0.058	0.006	1.378	0.28	0.	0.217	9.894

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	75	174	0	383	0	109	65
normalized size	1	1.	0.94	2.17	0.	4.79	0.	1.36	0.81
time (sec)	N/A	0.286	0.143	0.02	0.	0.25	0.	0.225	17.384

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	114	105	182	0	419	0	212	95
normalized size	1	1.19	1.09	1.9	0.	4.36	0.	2.21	0.99
time (sec)	N/A	0.32	0.197	0.02	0.	0.251	0.	0.24	18.789

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	164	132	226	0	539	0	437	141
normalized size	1	1.36	1.09	1.87	0.	4.45	0.	3.61	1.17
time (sec)	N/A	0.399	0.179	0.023	0.	0.263	0.	0.244	22.663

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	146	298	356	976	0	394	187
normalized size	1	1.	0.7	1.43	1.71	4.69	0.	1.89	0.9
time (sec)	N/A	0.476	0.178	0.032	1.411	0.652	0.	0.306	34.333

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	120	240	284	741	0	311	139
normalized size	1	1.	0.75	1.51	1.79	4.66	0.	1.96	0.87
time (sec)	N/A	0.377	0.122	0.018	1.389	0.346	0.	0.29	23.938

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	96	182	205	517	0	204	129
normalized size	1	1.	0.84	1.6	1.8	4.54	0.	1.79	1.13
time (sec)	N/A	0.161	0.083	0.016	1.435	0.26	0.	0.279	19.364

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	81	153	0	379	0	149	90
normalized size	1	1.	0.78	1.47	0.	3.64	0.	1.43	0.87
time (sec)	N/A	0.256	0.107	0.021	0.	0.242	0.	0.23	16.744

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	78	153	0	313	0	231	70
normalized size	1	1.	0.93	1.82	0.	3.73	0.	2.75	0.83
time (sec)	N/A	0.26	0.162	0.022	0.	0.238	0.	0.234	15.525

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	102	191	231	581	216	205	116
normalized size	1	1.	0.82	1.53	1.85	4.65	1.73	1.64	0.93
time (sec)	N/A	0.335	0.137	0.05	1.486	0.27	134.074	0.247	17.229

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	61	57	128	333	216	130	95
normalized size	1	1.	0.59	0.55	1.24	3.23	2.1	1.26	0.92
time (sec)	N/A	0.266	0.063	0.009	1.394	0.254	89.855	0.218	13.39

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	83	147	177	406	212	151	80
normalized size	1	1.	0.95	1.69	2.03	4.67	2.44	1.74	0.92
time (sec)	N/A	0.262	0.094	0.028	1.381	0.244	86.612	0.249	12.753

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	43	38	73	216	202	74	58
normalized size	1	1.	0.66	0.58	1.12	3.32	3.11	1.14	0.89
time (sec)	N/A	0.142	0.049	0.006	1.417	0.243	57.389	0.22	8.283

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	63	103	120	220	182	96	44
normalized size	1	1.	1.34	2.19	2.55	4.68	3.87	2.04	0.94
time (sec)	N/A	0.09	0.054	0.023	1.425	0.242	37.1	0.246	8.934

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	62	42	149	162	61	41
normalized size	1	1.	1.02	1.35	0.91	3.24	3.52	1.33	0.89
time (sec)	N/A	0.211	0.065	0.026	1.707	0.242	36.489	0.218	9.713

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	53	77	68	109	148	78	27
normalized size	1	1.	1.61	2.33	2.06	3.3	4.48	2.36	0.82
time (sec)	N/A	0.183	0.055	0.029	1.694	0.24	42.474	0.223	8.576

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	61	84	66	223	141	154	49
normalized size	1	1.	1.02	1.4	1.1	3.72	2.35	2.57	0.82
time (sec)	N/A	0.216	0.089	0.028	1.544	0.242	56.108	0.229	9.672

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	42	37	73	140	146	157	51
normalized size	1	1.	0.68	0.6	1.18	2.26	2.35	2.53	0.82
time (sec)	N/A	0.217	0.052	0.008	1.613	0.23	84.097	0.225	9.318

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	80	125	120	423	148	362	88
normalized size	1	1.	0.81	1.26	1.21	4.27	1.49	3.66	0.89
time (sec)	N/A	0.286	0.133	0.029	1.587	0.245	124.583	0.229	13.774

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	123	240	289	734	0	246	146
normalized size	1	1.	0.75	1.46	1.76	4.48	0.	1.5	0.89
time (sec)	N/A	0.422	0.171	0.04	1.398	0.361	0.	0.258	26.19

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	74	68	167	420	240	151	105
normalized size	1	1.	0.63	0.58	1.42	3.56	2.03	1.28	0.89
time (sec)	N/A	0.306	0.08	0.009	1.375	0.275	136.219	0.222	17.574

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	96	182	216	508	236	176	104
normalized size	1	1.	0.81	1.54	1.83	4.31	2.	1.49	0.88
time (sec)	N/A	0.318	0.106	0.03	1.386	0.253	117.765	0.255	17.997

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	48	43	93	262	223	82	61
normalized size	1	1.	0.67	0.6	1.29	3.64	3.1	1.14	0.85
time (sec)	N/A	0.159	0.056	0.006	1.384	0.241	76.023	0.218	9.97

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	71	124	140	261	199	107	75
normalized size	1	1.	1.04	1.82	2.06	3.84	2.93	1.57	1.1
time (sec)	N/A	0.116	0.067	0.025	1.396	0.238	43.666	0.256	13.259

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	108	0	176	178	74	44
normalized size	1	1.	0.98	1.93	0.	3.14	3.18	1.32	0.79
time (sec)	N/A	0.242	0.1	0.026	0.	0.241	44.724	0.22	12.883

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	60	97	0	117	165	89	46
normalized size	1	1.	1.05	1.7	0.	2.05	2.89	1.56	0.81
time (sec)	N/A	0.227	0.073	0.029	0.	0.237	51.506	0.225	11.526

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	103	158	0	279	162	190	63
normalized size	1	1.	1.36	2.08	0.	3.67	2.13	2.5	0.83
time (sec)	N/A	0.261	0.139	0.029	0.	0.245	68.558	0.23	13.362

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	56	49	0	162	170	185	63
normalized size	1	1.	0.75	0.65	0.	2.16	2.27	2.47	0.84
time (sec)	N/A	0.254	0.074	0.008	0.	0.238	108.996	0.234	11.989

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	135	227	0	544	172	439	107
normalized size	1	1.	1.1	1.85	0.	4.42	1.4	3.57	0.87
time (sec)	N/A	0.343	0.196	0.03	0.	0.272	157.614	0.24	19.736

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	150	316	289	702	233	290	146
normalized size	1	1.	0.93	1.96	1.8	4.36	1.45	1.8	0.91
time (sec)	N/A	0.418	0.196	0.047	1.435	0.245	102.515	0.243	26.809

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	72	68	166	354	226	194	102
normalized size	1	1.	0.63	0.59	1.44	3.08	1.97	1.69	0.89
time (sec)	N/A	0.316	0.09	0.008	1.404	0.255	82.293	0.236	18.613

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	121	254	211	448	212	208	134
normalized size	1	1.	0.8	1.67	1.39	2.95	1.39	1.37	0.88
time (sec)	N/A	0.381	0.161	0.033	1.414	0.246	80.972	0.241	24.675

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	43	93	196	201	127	65
normalized size	1	1.	0.59	0.57	1.22	2.58	2.64	1.67	0.86
time (sec)	N/A	0.182	0.076	0.005	1.381	0.247	71.83	0.229	11.335

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	100	160	115	153	182	153	94
normalized size	1	1.	1.59	2.54	1.83	2.43	2.89	2.43	1.49
time (sec)	N/A	0.115	0.093	0.027	1.418	0.237	70.116	0.232	21.082

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	96	188	0	223	172	155	53
normalized size	1	1.	1.48	2.89	0.	3.43	2.65	2.38	0.82
time (sec)	N/A	0.266	0.159	0.034	0.	0.242	93.709	0.253	14.22

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	51	48	0	117	165	296	56
normalized size	1	1.	0.76	0.72	0.	1.75	2.46	4.42	0.84
time (sec)	N/A	0.239	0.085	0.007	0.	0.237	152.285	0.278	11.399

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	126	315	0	474	0	285	100
normalized size	1	1.	1.08	2.69	0.	4.05	0.	2.44	0.85
time (sec)	N/A	0.348	0.29	0.036	0.	0.255	0.	0.296	21.452

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	77	73	0	193	0	327	105
normalized size	1	1.	0.65	0.61	0.	1.62	0.	2.75	0.88
time (sec)	N/A	0.324	0.108	0.008	0.	0.236	0.	0.314	17.908

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	157	387	0	707	0	543	150
normalized size	1	1.	0.95	2.33	0.	4.26	0.	3.27	0.9
time (sec)	N/A	0.459	0.382	0.038	0.	0.305	0.	0.345	30.958

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	53	34	127	148	54	34
normalized size	1	1.	1.05	1.32	0.85	3.18	3.7	1.35	0.85
time (sec)	N/A	0.199	0.054	0.024	1.529	0.241	31.06	0.22	9.29



Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	1424	66	107	88	0	0	42
normalized size	1	1.	26.87	1.25	2.02	1.66	0.	0.	0.79
time (sec)	N/A	0.298	4.263	0.014	1.859	0.262	0.	0.	10.087

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F(-2)	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	93	0	0	31
normalized size	1	1.	0.	0.	0.	2.58	0.	0.	0.86
time (sec)	N/A	0.071	0.948	0.062	0.	0.236	0.	0.	8.379

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	0	0	0	68	0	0	65
normalized size	1	1.	0.	0.	0.	0.91	0.	0.	0.87
time (sec)	N/A	0.146	1.963	0.059	0.	0.242	0.	0.	18.21

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	0	0	0	0	0	0	88
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.235	0.37	0.813	0.	0.	0.	0.	37.859

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	0	0	0	0	0	0	70
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.15	0.278	0.806	0.	0.	0.	0.	32.525

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	258	0	0	0	0	0	58
normalized size	1	1.	3.39	0.	0.	0.	0.	0.	0.76
time (sec)	N/A	0.168	0.493	0.139	0.	0.	0.	0.	35.352

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	104	0	0	0	243	0	0	102
normalized size	1	1.08	0.	0.	0.	2.53	0.	0.	1.06
time (sec)	N/A	0.361	0.929	1.044	0.	0.289	0.	0.	51.438

## 2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique

rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [180] had the largest ratio of [ 0.5294 ]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	17	0.059
2	A	2	1	1.	17	0.059
3	A	2	1	1.	17	0.059
4	A	2	1	1.	15	0.067
5	A	7	7	1.	17	0.412
6	A	7	7	1.	17	0.412
7	A	8	8	1.	17	0.471
8	A	2	1	1.	19	0.053
9	A	2	1	1.	19	0.053
10	A	2	1	1.	17	0.059
11	A	8	7	1.	19	0.368
12	A	9	8	1.	19	0.421
13	A	8	8	1.	19	0.421
14	A	8	7	1.	19	0.368
15	A	8	7	1.	19	0.368
16	A	8	7	1.	19	0.368
17	A	7	7	1.	17	0.412
18	A	13	7	1.	19	0.368
19	A	14	8	1.	19	0.421
20	A	9	8	1.	19	0.421
21	A	9	8	1.	19	0.421
22	A	9	8	1.	19	0.421
23	A	9	8	1.	19	0.421
24	A	7	7	1.	17	0.412
25	A	14	8	1.	19	0.421
26	A	15	9	1.	19	0.474
27	A	4	2	1.	21	0.095
28	A	3	2	1.	21	0.095
29	A	2	2	1.	19	0.105
30	A	1	1	1.	11	0.091
31	A	7	7	1.	21	0.333
32	A	8	8	1.	21	0.381
33	A	9	8	1.	21	0.381
34	A	3	2	1.	19	0.105
35	A	4	4	1.05	19	0.21
36	A	3	3	1.01	17	0.176
37	A	2	2	1.	19	0.105
38	A	2	2	1.	19	0.105
39	A	5	5	1.	19	0.263
40	A	4	4	1.	19	0.21
41	A	3	3	0.91	17	0.176
42	A	2	2	1.	9	0.222
43	A	2	2	1.	19	0.105
44	A	2	2	1.	19	0.105
45	A	2	2	1.	19	0.105

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
46	A	1	1	1.	50	0.02
47	A	2	1	1.	17	0.059
48	A	2	1	1.	17	0.059
49	A	2	1	1.	17	0.059
50	A	2	1	1.	15	0.067
51	A	10	7	1.	17	0.412
52	A	10	7	1.	17	0.412
53	A	11	8	1.	17	0.471
54	A	2	1	1.	19	0.053
55	A	2	1	1.	19	0.053
56	A	2	1	1.	19	0.053
57	A	2	1	1.	17	0.059
58	A	11	7	1.	19	0.368
59	A	12	8	1.	19	0.421
60	A	11	8	1.	19	0.421
61	A	11	7	1.	19	0.368
62	A	11	7	1.	19	0.368
63	A	11	7	1.	19	0.368
64	A	10	7	1.	17	0.412
65	A	19	7	1.	19	0.368
66	A	20	8	1.	19	0.421
67	A	12	8	1.	19	0.421
68	A	12	8	1.	19	0.421
69	A	12	8	1.	19	0.421
70	A	12	8	1.	19	0.421
71	A	10	7	1.	17	0.412
72	A	20	8	1.	19	0.421
73	A	21	9	1.	19	0.474
74	A	10	8	1.	23	0.348
75	A	9	7	1.	23	0.304
76	A	8	6	1.	23	0.261
77	A	5	3	1.	23	0.13
78	A	9	7	1.	23	0.304
79	A	10	8	1.	23	0.348
80	A	8	5	1.	21	0.238
81	A	7	4	1.	21	0.19
82	A	5	3	1.	21	0.143
83	A	8	5	1.	21	0.238
84	A	9	6	1.	21	0.286
85	A	11	8	1.	23	0.348
86	A	10	8	1.	23	0.348
87	A	9	7	1.	23	0.304
88	A	9	7	1.	23	0.304
89	A	9	7	1.	23	0.304
90	A	10	8	1.	23	0.348
91	A	11	8	1.	23	0.348
92	A	4	4	1.	25	0.16
93	A	1	1	1.	25	0.04
94	A	10	9	1.	21	0.429

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	9	8	1.	21	0.381
96	A	4	4	1.	21	0.19
97	A	5	5	1.	21	0.238
98	A	7	7	1.	21	0.333
99	A	8	7	1.	21	0.333
100	A	11	10	1.	21	0.476
101	A	10	9	1.	21	0.429
102	A	4	3	1.	21	0.143
103	A	9	8	1.	21	0.381
104	A	10	9	1.	21	0.429
105	A	11	10	1.	21	0.476
106	A	11	10	1.	21	0.476
107	A	10	9	1.	21	0.429
108	A	5	5	1.	21	0.238
109	A	5	5	1.	21	0.238
110	A	7	7	1.	21	0.333
111	A	8	7	1.	21	0.333
112	A	11	10	1.	21	0.476
113	A	10	9	1.	21	0.429
114	A	10	9	1.	21	0.429
115	A	10	9	1.	21	0.429
116	A	11	10	1.	21	0.476
117	A	4	4	1.	17	0.235
118	A	4	4	1.	26	0.154
119	A	3	2	1.	19	0.105
120	A	4	4	1.	19	0.21
121	A	3	3	0.91	17	0.176
122	A	2	2	1.	19	0.105
123	A	2	2	1.	19	0.105
124	A	7	7	1.	21	0.333
125	A	6	6	1.	21	0.286
126	A	6	6	1.	21	0.286
127	A	5	5	1.	19	0.263
128	A	4	4	1.	11	0.364
129	A	7	6	1.	21	0.286
130	A	8	7	1.	21	0.333
131	A	9	7	1.	21	0.333
132	A	7	7	1.	21	0.333
133	A	7	6	1.	21	0.286
134	A	6	5	1.	19	0.263
135	A	5	5	1.	11	0.454
136	A	7	6	1.	21	0.286
137	A	8	7	1.	21	0.333
138	A	9	7	1.	21	0.333
139	A	8	7	1.	21	0.333
140	A	8	6	1.	21	0.286
141	A	7	5	1.	19	0.263
142	A	6	5	1.	11	0.454
143	A	8	7	1.	21	0.333

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	8	7	1.	21	0.333
145	A	9	8	1.	21	0.381
146	A	5	5	1.	21	0.238
147	A	5	5	1.	21	0.238
148	A	4	4	1.	19	0.21
149	A	4	4	1.	11	0.364
150	A	7	6	1.	21	0.286
151	A	8	7	1.	21	0.333
152	A	9	7	1.	21	0.333
153	A	5	5	1.	21	0.238
154	A	5	5	0.96	21	0.238
155	A	5	5	1.	19	0.263
156	A	5	4	1.	11	0.364
157	A	8	7	1.	21	0.333
158	A	9	8	1.	21	0.381
159	A	10	8	1.	21	0.381
160	A	5	5	1.	21	0.238
161	A	6	6	0.97	21	0.286
162	A	6	5	1.	19	0.263
163	A	6	4	1.	11	0.364
164	A	9	7	1.	21	0.333
165	A	10	8	1.	21	0.381
166	A	11	8	1.	21	0.381
167	A	7	6	1.	23	0.261
168	A	4	4	1.	23	0.174
169	A	5	5	1.	23	0.217
170	A	3	3	1.	19	0.158
171	A	3	3	1.	17	0.176
172	A	6	6	1.	23	0.261
173	A	7	7	1.	23	0.304
174	A	7	7	1.	23	0.304
175	A	4	3	1.	19	0.158
176	A	8	8	1.	17	0.471
177	A	3	2	1.	21	0.095
178	A	3	2	1.	17	0.118
179	A	4	4	1.	17	0.235
180	A	9	9	1.	17	0.529
181	A	4	3	1.	17	0.176
182	A	3	3	1.	24	0.125
183	A	3	3	1.	24	0.125
184	A	3	3	1.	20	0.15
185	A	2	1	1.	17	0.059
186	A	2	1	1.	17	0.059
187	A	2	1	1.	17	0.059
188	A	2	1	1.	15	0.067
189	A	2	2	1.	17	0.118
190	A	2	2	1.	17	0.118
191	A	2	2	1.	17	0.118
192	A	2	2	1.	17	0.118

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	2	1	1.	19	0.053
194	A	2	1	1.	19	0.053
195	A	2	1	1.	17	0.059
196	A	3	3	1.	19	0.158
197	A	3	3	1.	19	0.158
198	A	3	3	1.	19	0.158
199	A	5	4	1.	19	0.21
200	A	4	4	1.	19	0.21
201	A	3	3	1.	19	0.158
202	A	2	2	1.	17	0.118
203	A	3	2	1.	19	0.105
204	A	4	3	1.	19	0.158
205	A	5	4	1.	19	0.21
206	A	5	4	1.	19	0.21
207	A	4	4	1.	19	0.21
208	A	3	3	1.	19	0.158
209	A	2	2	1.	17	0.118
210	A	4	3	1.	19	0.158
211	A	5	4	1.	19	0.21
212	A	6	4	1.	19	0.21
213	A	3	2	1.	19	0.105
214	A	5	5	1.	19	0.263
215	A	4	4	0.98	19	0.21
216	A	3	3	0.91	17	0.176
217	A	2	2	1.	9	0.222
218	A	2	2	1.	19	0.105
219	A	2	2	1.	19	0.105
220	A	2	2	1.	19	0.105
221	A	1	1	1.	28	0.036
222	A	4	2	1.	25	0.08
223	A	3	2	1.	25	0.08
224	A	2	2	1.	23	0.087
225	A	1	1	1.	15	0.067
226	A	1	1	1.	23	0.043
227	A	1	1	1.	25	0.04
228	A	1	1	1.	25	0.04
229	A	2	2	0.93	28	0.071
230	A	1	1	1.	69	0.014
231	A	5	3	1.	25	0.12
232	A	3	3	1.	23	0.13
233	A	2	2	1.	15	0.133
234	A	2	2	1.	25	0.08
235	A	2	2	1.	23	0.087
236	A	2	2	1.	25	0.08
237	A	2	2	1.	25	0.08
238	A	6	4	1.08	31	0.129
239	A	4	4	1.08	31	0.129
240	A	2	2	1.07	29	0.069
241	A	5	5	1.	31	0.161

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
242	A	5	5	1.19	31	0.161
243	A	5	4	1.36	31	0.129
244	A	8	7	1.	31	0.226
245	A	6	6	1.	31	0.194
246	A	4	4	1.	28	0.143
247	A	4	4	1.	31	0.129
248	A	5	5	1.	31	0.161
249	A	5	5	1.	29	0.172
250	A	4	4	1.	29	0.138
251	A	3	3	1.	29	0.103
252	A	2	2	1.	27	0.074
253	A	2	2	1.	26	0.077
254	A	3	3	1.	29	0.103
255	A	2	2	1.	29	0.069
256	A	3	3	1.	29	0.103
257	A	2	2	1.	29	0.069
258	A	5	5	1.	29	0.172
259	A	7	6	1.	31	0.194
260	A	4	4	1.	31	0.129
261	A	5	5	1.	31	0.161
262	A	2	2	1.	29	0.069
263	A	3	3	1.	28	0.107
264	A	3	3	1.	31	0.097
265	A	3	3	1.	31	0.097
266	A	3	3	1.	31	0.097
267	A	2	2	1.	31	0.065
268	A	5	5	1.	31	0.161
269	A	7	7	1.	31	0.226
270	A	4	4	1.	31	0.129
271	A	6	6	1.	31	0.194
272	A	2	2	1.	29	0.069
273	A	3	3	1.	28	0.107
274	A	3	3	1.	31	0.097
275	A	2	2	1.	31	0.065
276	A	5	5	1.	31	0.161
277	A	4	4	1.	31	0.129
278	A	7	7	1.	31	0.226
279	A	3	3	1.	31	0.097
280	A	1	1	1.	57	0.018
281	A	3	3	1.	32	0.094
282	A	3	3	1.	41	0.073
283	A	4	3	1.	31	0.097
284	A	4	4	1.	35	0.114
285	A	3	3	1.	31	0.097
286	A	2	2	1.08	76	0.026

### 3 Listing of integrals

#### 3.1 $\int (a + bx^3) (c + dx^3)^4 dx$

**Optimal.** Leaf size=94

$$\frac{1}{4}c^3x^4(4ad + bc) + \frac{2}{7}c^2dx^7(3ad + 2bc) + \frac{1}{13}d^3x^{13}(ad + 4bc) + \frac{1}{5}cd^2x^{10}(2ad + 3bc) + ac^4x + \frac{1}{16}bd^4x^{16}$$

[Out]  $a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c*d^2*(3*b*c + 2*a*d)*x^{10})/5 + (d^3*(4*b*c + a*d)*x^{13})/13 + (b*d^4*x^{16})/16$

**Rubi [A]** time = 0.14945, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{1}{4}c^3x^4(4ad + bc) + \frac{2}{7}c^2dx^7(3ad + 2bc) + \frac{1}{13}d^3x^{13}(ad + 4bc) + \frac{1}{5}cd^2x^{10}(2ad + 3bc) + ac^4x + \frac{1}{16}bd^4x^{16}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)\*(c + d\*x^3)^4, x]

[Out]  $a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c*d^2*(3*b*c + 2*a*d)*x^{10})/5 + (d^3*(4*b*c + a*d)*x^{13})/13 + (b*d^4*x^{16})/16$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^4x^{16}}{16} + c^4 \int a dx + \frac{c^3x^4(4ad + bc)}{4} + \frac{2c^2dx^7(3ad + 2bc)}{7} + \frac{cd^2x^{10}(2ad + 3bc)}{5} + \frac{d^3x^{13}(ad + 4bc)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*(d\*x\*\*3+c)\*\*4, x)

[Out]  $b*d^{**4}*x^{**16}/16 + c^{**4}*Integral(a, x) + c^{**3}*x^{**4}*(4*a*d + b*c)/4 + 2*c^{**2}*d*x^{**7}*(3*a*d + 2*b*c)/7 + c*d^{**2}*x^{**10}*(2*a*d + 3*b*c)/5 + d^{**3}*x^{**13}*(a*d + 4*b*c)/13$

**Mathematica [A]** time = 0.0345473, size = 94, normalized size = 1.

$$\frac{1}{4}c^3x^4(4ad + bc) + \frac{2}{7}c^2dx^7(3ad + 2bc) + \frac{1}{13}d^3x^{13}(ad + 4bc) + \frac{1}{5}cd^2x^{10}(2ad + 3bc) + ac^4x + \frac{1}{16}bd^4x^{16}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)\*(c + d\*x^3)^4, x]

[Out]  $a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c*d^2*(3*b*c + 2*a*d)*x^{10})/5 + (d^3*(4*b*c + a*d)*x^{13})/13 + (b*d^4*x^{16})/16$

**Maple [A]** time = 0., size = 97, normalized size = 1.

$$\frac{bd^4x^{16}}{16} + \frac{(ad^4 + 4bcd^3)x^{13}}{13} + \frac{(4acd^3 + 6c^2d^2b)x^{10}}{10} + \frac{(6ac^2d^2 + 4c^3db)x^7}{7} + \frac{(4ac^3d + bc^4)x^4}{4} + ac^4x$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)*(d*x^3+c)^4,x)`

[Out]  $\frac{1}{16}bd^4x^{16} + \frac{1}{13}(ad^4 + 4b^2cd^3)x^{13} + \frac{1}{10}(4a^2c^2d^3 + 6b^2c^2d^2)x^{10} + \frac{1}{7}(6a^2c^2d^2 + 4b^2c^3d)x^7 + \frac{1}{4}(4a^2c^3d + b^2c^4)x^4 + ac^4x$

**Maxima [A]** time = 1.40995, size = 130, normalized size = 1.38

$$\frac{1}{16}bd^4x^{16} + \frac{1}{13}(4bcd^3 + ad^4)x^{13} + \frac{1}{5}(3bc^2d^2 + 2acd^3)x^{10} + \frac{2}{7}(2bc^3d + 3ac^2d^2)x^7 + ac^4x + \frac{1}{4}(bc^4 + 4ac^3d)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)*(d*x^3 + c)^4,x, algorithm="maxima")`

[Out]  $\frac{1}{16}bd^4x^{16} + \frac{1}{13}(4b^2cd^3 + ad^4)x^{13} + \frac{1}{5}(3b^2c^2d^2 + 2a^2c^2d^3)x^{10} + \frac{2}{7}(2b^2c^3d + 3a^2c^2d^2)x^7 + ac^4x + \frac{1}{4}(b^2c^4 + 4a^2c^3d)x^4$

**Fricas [A]** time = 0.179713, size = 1, normalized size = 0.01

$$\frac{1}{16}x^{16}d^4b + \frac{4}{13}x^{13}d^3cb + \frac{1}{13}x^{13}d^4a + \frac{3}{5}x^{10}d^2c^2b + \frac{2}{5}x^{10}d^3ca + \frac{4}{7}x^7dc^3b + \frac{6}{7}x^7d^2c^2a + \frac{1}{4}x^4c^4b + x^4dc^3a + xc^4a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)*(d*x^3 + c)^4,x, algorithm="fricas")`

[Out]  $\frac{1}{16}x^{16}d^4b + \frac{4}{13}x^{13}d^3c^2b + \frac{1}{13}x^{13}d^4a + \frac{3}{5}x^{10}d^2c^2b + \frac{2}{5}x^{10}d^3ca + \frac{4}{7}x^7dc^3b + \frac{6}{7}x^7d^2c^2a + \frac{1}{4}x^4c^4b + x^4dc^3a + xc^4a$

**Sympy [A]** time = 0.144622, size = 104, normalized size = 1.11

$$ac^4x + \frac{bd^4x^{16}}{16} + x^{13}\left(\frac{ad^4}{13} + \frac{4bcd^3}{13}\right) + x^{10}\left(\frac{2acd^3}{5} + \frac{3bc^2d^2}{5}\right) + x^7\left(\frac{6ac^2d^2}{7} + \frac{4bc^3d}{7}\right) + x^4\left(ac^3d + \frac{bc^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)*(d*x**3+c)**4,x)`

[Out]  $ac^4x + \frac{bd^4x^{16}}{16} + x^{13}\left(\frac{ad^4}{13} + \frac{4b^2cd^3}{13}\right) + x^{10}\left(\frac{2a^2c^2d^3}{5} + \frac{3b^2c^2d^2}{5}\right) + x^7\left(\frac{6a^2c^2d^2}{7} + \frac{4b^2c^3d}{7}\right) + x^4\left(\frac{ac^3d}{4} + \frac{b^2c^4}{4}\right)$

**GIAC/XCAS [A]** time = 0.217729, size = 131, normalized size = 1.39

$$\frac{1}{16}bd^4x^{16} + \frac{4}{13}bcd^3x^{13} + \frac{1}{13}ad^4x^{13} + \frac{3}{5}bc^2d^2x^{10} + \frac{2}{5}acd^3x^{10} + \frac{4}{7}bc^3dx^7 + \frac{6}{7}ac^2d^2x^7 + \frac{1}{4}bc^4x^4 + ac^3dx^4 + ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3 + a)*(d*x^3 + c)^4,x, algorithm="giac")
```

```
[Out] 1/16*b*d^4*x^16 + 4/13*b*c*d^3*x^13 + 1/13*a*d^4*x^13 + 3/5*b*c^2
*d^2*x^10 + 2/5*a*c*d^3*x^10 + 4/7*b*c^3*d*x^7 + 6/7*a*c^2*d^2*x^
7 + 1/4*b*c^4*x^4 + a*c^3*d*x^4 + a*c^4*x
```

### 3.2 $\int (a + bx^3) (c + dx^3)^3 dx$

**Optimal.** Leaf size=70

$$\frac{1}{4}c^2x^4(3ad + bc) + \frac{1}{10}d^2x^{10}(ad + 3bc) + \frac{3}{7}cdx^7(ad + bc) + ac^3x + \frac{1}{13}bd^3x^{13}$$

[Out]  $a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*b*c + a*d)*x^{10})/10 + (b*d^3*x^{13})/13$

**Rubi [A]** time = 0.102641, antiderivative size = 70, normalized size of antiderivative = 1., number of rules used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{1}{4}c^2x^4(3ad + bc) + \frac{1}{10}d^2x^{10}(ad + 3bc) + \frac{3}{7}cdx^7(ad + bc) + ac^3x + \frac{1}{13}bd^3x^{13}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)\*(c + d\*x^3)^3, x]

[Out]  $a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*b*c + a*d)*x^{10})/10 + (b*d^3*x^{13})/13$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^3x^{13}}{13} + c^3 \int a dx + \frac{c^2x^4(3ad + bc)}{4} + \frac{3cdx^7(ad + bc)}{7} + \frac{d^2x^{10}(ad + 3bc)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*(d\*x\*\*3+c)\*\*3, x)

[Out]  $b*d^3*x^{13}/13 + c^3*Integral(a, x) + c^2*x^4*(3*a*d + b*c)/4 + 3*c*d*x^7*(a*d + b*c)/7 + d^2*x^{10}*(a*d + 3*b*c)/10$

**Mathematica [A]** time = 0.0205167, size = 70, normalized size = 1.

$$\frac{1}{4}c^2x^4(3ad + bc) + \frac{1}{10}d^2x^{10}(ad + 3bc) + \frac{3}{7}cdx^7(ad + bc) + ac^3x + \frac{1}{13}bd^3x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)\*(c + d\*x^3)^3, x]

[Out]  $a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*b*c + a*d)*x^{10})/10 + (b*d^3*x^{13})/13$

**Maple [A]** time = 0.001, size = 73, normalized size = 1.

$$\frac{bd^3x^{13}}{13} + \frac{(ad^3 + 3bcd^2)x^{10}}{10} + \frac{(3acd^2 + 3bc^2d)x^7}{7} + \frac{(3ac^2d + bc^3)x^4}{4} + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)\*(d\*x^3+c)^3, x)

[Out]  $\frac{1}{13}bd^3x^{13} + \frac{1}{10}(ad^3 + 3b^2cd^2)x^{10} + \frac{1}{7}(3a^2cd^2 + 3b^2c^2d)x^7 + \frac{1}{4}(3a^2c^2d + b^2c^3)x^4 + a^3c^3x$

**Maxima [A]** time = 1.37586, size = 95, normalized size = 1.36

$$\frac{1}{13}bd^3x^{13} + \frac{1}{10}(3bcd^2 + ad^3)x^{10} + \frac{3}{7}(bc^2d + acd^2)x^7 + ac^3x + \frac{1}{4}(bc^3 + 3ac^2d)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)*(d*x^3 + c)^3,x, algorithm="maxima")`

[Out]  $\frac{1}{13}b^2d^3x^{13} + \frac{1}{10}(3b^2cd^2 + a^2d^3)x^{10} + \frac{3}{7}(b^2c^2d + a^2cd^2)x^7 + a^3c^3x + \frac{1}{4}(b^2c^3 + 3a^2c^2d)x^4$

**Fricas [A]** time = 0.180698, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}d^3b + \frac{3}{10}x^{10}d^2cb + \frac{1}{10}x^{10}d^3a + \frac{3}{7}x^7dc^2b + \frac{3}{7}x^7d^2ca + \frac{1}{4}x^4c^3b + \frac{3}{4}x^4dc^2a + xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)*(d*x^3 + c)^3,x, algorithm="fricas")`

[Out]  $\frac{1}{13}x^{13}d^3b + \frac{3}{10}x^{10}d^2c^2b + \frac{1}{10}x^{10}d^3a + \frac{3}{7}x^7d^2c^2b + \frac{3}{7}x^7d^2ca + \frac{1}{4}x^4c^3b + \frac{3}{4}x^4dc^2a + xc^3a$

**Sympy [A]** time = 0.123204, size = 80, normalized size = 1.14

$$ac^3x + \frac{bd^3x^{13}}{13} + x^{10}\left(\frac{ad^3}{10} + \frac{3bcd^2}{10}\right) + x^7\left(\frac{3acd^2}{7} + \frac{3bc^2d}{7}\right) + x^4\left(\frac{3ac^2d}{4} + \frac{bc^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)*(d*x**3+c)**3,x)`

[Out]  $a^3c^3x + b^2d^3x^{13}/13 + x^{10}(ad^3/10 + 3b^2cd^2/10) + x^7(3acd^2/7 + 3bc^2d/7) + x^4(3ac^2d/4 + b^2c^3/4)$

**GIAC/XCAS [A]** time = 0.213986, size = 100, normalized size = 1.43

$$\frac{1}{13}bd^3x^{13} + \frac{3}{10}bcd^2x^{10} + \frac{1}{10}ad^3x^{10} + \frac{3}{7}bc^2dx^7 + \frac{3}{7}acd^2x^7 + \frac{1}{4}bc^3x^4 + \frac{3}{4}ac^2dx^4 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)*(d*x^3 + c)^3,x, algorithm="giac")`

[Out]  $\frac{1}{13}b^2d^3x^{13} + \frac{3}{10}b^2cd^2x^{10} + \frac{1}{10}a^2d^3x^{10} + \frac{3}{7}b^2c^2d^2x^7 + \frac{3}{7}a^2cd^2x^7 + \frac{1}{4}b^2c^3x^4 + \frac{3}{4}a^2c^2d^2x^4 + a^3c^3x$

### 3.3 $\int (a + bx^3) (c + dx^3)^2 dx$

**Optimal.** Leaf size=50

$$\frac{1}{7}dx^7(ad + 2bc) + \frac{1}{4}cx^4(2ad + bc) + ac^2x + \frac{1}{10}bd^2x^{10}$$

[Out]  $a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^{10})/10$

**Rubi [A]** time = 0.0742223, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{1}{7}dx^7(ad + 2bc) + \frac{1}{4}cx^4(2ad + bc) + ac^2x + \frac{1}{10}bd^2x^{10}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)\*(c + d\*x^3)^2, x]

[Out]  $a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^{10})/10$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^2x^{10}}{10} + c^2 \int a dx + \frac{cx^4(2ad + bc)}{4} + \frac{dx^7(ad + 2bc)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*(d\*x\*\*3+c)\*\*2, x)

[Out]  $b*d^2*x^{10}/10 + c^2*Integral(a, x) + c*x^4*(2*a*d + b*c)/4 + d*x^7*(a*d + 2*b*c)/7$

**Mathematica [A]** time = 0.0141192, size = 50, normalized size = 1.

$$\frac{1}{7}dx^7(ad + 2bc) + \frac{1}{4}cx^4(2ad + bc) + ac^2x + \frac{1}{10}bd^2x^{10}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)\*(c + d\*x^3)^2, x]

[Out]  $a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^{10})/10$

**Maple [A]** time = 0.001, size = 49, normalized size = 1.

$$\frac{bd^2x^{10}}{10} + \frac{(ad^2 + 2bcd)x^7}{7} + \frac{(2acd + bc^2)x^4}{4} + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)\*(d\*x^3+c)^2, x)

[Out]  $\frac{1}{10} b d^2 x^{10} + \frac{1}{7} (a d^2 + 2 b c d) x^7 + \frac{1}{4} (2 a c d + b c^2) x^4 + a c^2 x$

**Maxima [A]** time = 1.36066, size = 65, normalized size = 1.3

$$\frac{1}{10} b d^2 x^{10} + \frac{1}{7} (2 b c d + a d^2) x^7 + \frac{1}{4} (b c^2 + 2 a c d) x^4 + a c^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)*(d*x^3 + c)^2,x, algorithm="maxima")`

[Out]  $\frac{1}{10} b d^2 x^{10} + \frac{1}{7} (2 b c d + a d^2) x^7 + \frac{1}{4} (b c^2 + 2 a c d) x^4 + a c^2 x$

**Fricas [A]** time = 0.180838, size = 1, normalized size = 0.02

$$\frac{1}{10} x^{10} d^2 b + \frac{2}{7} x^7 d c b + \frac{1}{7} x^7 d^2 a + \frac{1}{4} x^4 c^2 b + \frac{1}{2} x^4 d c a + x c^2 a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)*(d*x^3 + c)^2,x, algorithm="fricas")`

[Out]  $\frac{1}{10} x^{10} d^2 b + \frac{2}{7} x^7 d c b + \frac{1}{7} x^7 d^2 a + \frac{1}{4} x^4 c^2 b + \frac{1}{2} x^4 d c a + x c^2 a$

**Sympy [A]** time = 0.108516, size = 51, normalized size = 1.02

$$a c^2 x + \frac{b d^2 x^{10}}{10} + x^7 \left( \frac{a d^2}{7} + \frac{2 b c d}{7} \right) + x^4 \left( \frac{a c d}{2} + \frac{b c^2}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)*(d*x**3+c)**2,x)`

[Out]  $a c^2 x + \frac{b d^2 x^{10}}{10} + x^7 (a d^2 / 7 + 2 b c d / 7) + x^4 (a c d / 2 + b c^2 / 4)$

**GIAC/XCAS [A]** time = 0.213265, size = 68, normalized size = 1.36

$$\frac{1}{10} b d^2 x^{10} + \frac{2}{7} b c d x^7 + \frac{1}{7} a d^2 x^7 + \frac{1}{4} b c^2 x^4 + \frac{1}{2} a c d x^4 + a c^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)*(d*x^3 + c)^2,x, algorithm="giac")`

[Out]  $\frac{1}{10} b d^2 x^{10} + \frac{2}{7} b c d x^7 + \frac{1}{7} a d^2 x^7 + \frac{1}{4} b c^2 x^4 + \frac{1}{2} a c d x^4 + a c^2 x$

### 3.4 $\int (a + bx^3) (c + dx^3) dx$

**Optimal.** Leaf size=28

$$\frac{1}{4}x^4(ad + bc) + acx + \frac{1}{7}bdx^7$$

[Out]  $a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7$

**Rubi [A]** time = 0.0393509, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{1}{4}x^4(ad + bc) + acx + \frac{1}{7}bdx^7$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)\*(c + d\*x^3), x]

[Out]  $a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bdx^7}{7} + c \int a dx + x^4 \left( \frac{ad}{4} + \frac{bc}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*(d\*x\*\*3+c), x)

[Out]  $b*d*x**7/7 + c*Integral(a, x) + x**4*(a*d/4 + b*c/4)$

**Mathematica [A]** time = 0.00892945, size = 28, normalized size = 1.

$$\frac{1}{4}x^4(ad + bc) + acx + \frac{1}{7}bdx^7$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)\*(c + d\*x^3), x]

[Out]  $a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7$

**Maple [A]** time = 0.002, size = 25, normalized size = 0.9

$$acx + \frac{(ad + bc)x^4}{4} + \frac{bdx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)\*(d\*x^3+c), x)

[Out]  $a*c*x+1/4*(a*d+b*c)*x^4+1/7*b*d*x^7$

---

**Maxima [A]** time = 1.35272, size = 32, normalized size = 1.14

$$\frac{1}{7} bdx^7 + \frac{1}{4} (bc + ad)x^4 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)\*(d\*x^3 + c),x, algorithm="maxima")

[Out] 1/7\*b\*d\*x^7 + 1/4\*(b\*c + a\*d)\*x^4 + a\*c\*x

---

**Fricas [A]** time = 0.18203, size = 1, normalized size = 0.04

$$\frac{1}{7}x^7db + \frac{1}{4}x^4cb + \frac{1}{4}x^4da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)\*(d\*x^3 + c),x, algorithm="fricas")

[Out] 1/7\*x^7\*d\*b + 1/4\*x^4\*c\*b + 1/4\*x^4\*d\*a + x\*c\*a

---

**Sympy [A]** time = 0.074719, size = 26, normalized size = 0.93

$$acx + \frac{bdx^7}{7} + x^4 \left( \frac{ad}{4} + \frac{bc}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*(d\*x\*\*3+c),x)

[Out] a\*c\*x + b\*d\*x\*\*7/7 + x\*\*4\*(a\*d/4 + b\*c/4)

---

**GIAC/XCAS [A]** time = 0.211328, size = 35, normalized size = 1.25

$$\frac{1}{7} bdx^7 + \frac{1}{4} bcx^4 + \frac{1}{4} adx^4 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)\*(d\*x^3 + c),x, algorithm="giac")

[Out] 1/7\*b\*d\*x^7 + 1/4\*b\*c\*x^4 + 1/4\*a\*d\*x^4 + a\*c\*x



### 3.5 $\int \frac{a+bx^3}{c+dx^3} dx$

**Optimal.** Leaf size=144

$$\frac{(bc - ad) \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} - \frac{(bc - ad) \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{3c^{2/3}d^{4/3}} + \frac{(bc - ad) \tan^{-1}\left(\frac{\sqrt[3]{c-2}\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}} + \frac{bx}{d}$$

[Out] (b\*x)/d + ((b\*c - a\*d)\*ArcTan[(c^(1/3) - 2\*d^(1/3)\*x)/(Sqrt[3]\*c^(1/3))])/(Sqrt[3]\*c^(2/3)\*d^(4/3)) - ((b\*c - a\*d)\*Log[c^(1/3) + d^(1/3)\*x])/(3\*c^(2/3)\*d^(4/3)) + ((b\*c - a\*d)\*Log[c^(2/3) - c^(1/3)\*d^(1/3)\*x + d^(2/3)\*x^2])/(6\*c^(2/3)\*d^(4/3))

**Rubi [A]** time = 0.211318, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$\frac{(bc - ad) \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} - \frac{(bc - ad) \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{3c^{2/3}d^{4/3}} + \frac{(bc - ad) \tan^{-1}\left(\frac{\sqrt[3]{c-2}\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}} + \frac{bx}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)/(c + d\*x^3), x]

[Out] (b\*x)/d + ((b\*c - a\*d)\*ArcTan[(c^(1/3) - 2\*d^(1/3)\*x)/(Sqrt[3]\*c^(1/3))])/(Sqrt[3]\*c^(2/3)\*d^(4/3)) - ((b\*c - a\*d)\*Log[c^(1/3) + d^(1/3)\*x])/(3\*c^(2/3)\*d^(4/3)) + ((b\*c - a\*d)\*Log[c^(2/3) - c^(1/3)\*d^(1/3)\*x + d^(2/3)\*x^2])/(6\*c^(2/3)\*d^(4/3))

**Rubi in Sympy [A]** time = 32.6833, size = 134, normalized size = 0.93

$$\frac{bx}{d} + \frac{(ad - bc) \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{3c^{2/3}d^{4/3}} - \frac{(ad - bc) \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} - \frac{\sqrt{3}(ad - bc) \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3} - \frac{2\sqrt[3]{dx}}{3}\right)}{\sqrt[3]{c}}\right)}{3c^{2/3}d^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)/(d\*x\*\*3+c), x)

[Out] b\*x/d + (a\*d - b\*c)\*log(c\*\*(1/3) + d\*\*(1/3)\*x)/(3\*c\*\*(2/3)\*d\*\*(4/3)) - (a\*d - b\*c)\*log(c\*\*(2/3) - c\*\*(1/3)\*d\*\*(1/3)\*x + d\*\*(2/3)\*x\*\*2)/(6\*c\*\*(2/3)\*d\*\*(4/3)) - sqrt(3)\*(a\*d - b\*c)\*atan(sqrt(3)\*(c\*\*(1/3)/3 - 2\*d\*\*(1/3)\*x/3)/c\*\*(1/3))/(3\*c\*\*(2/3)\*d\*\*(4/3))

**Mathematica [A]** time = 0.140246, size = 128, normalized size = 0.89

$$\frac{(bc - ad) \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right) - 2(bc - ad) \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right) + 2\sqrt{3}(bc - ad) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right) + 6bc^{2/3}\sqrt[3]{dx}}{6c^{2/3}d^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)/(c + d\*x^3), x]

[Out]  $(6*b*c^{(2/3)}*d^{(1/3)}*x + 2*\sqrt{3}*(b*c - a*d)*\text{ArcTan}[(1 - (2*d^{(1/3)}*x)/c^{(1/3)})/\sqrt{3}] - 2*(b*c - a*d)*\text{Log}[c^{(1/3)} + d^{(1/3)}*x] + (b*c - a*d)*\text{Log}[c^{(2/3)} - c^{(1/3)}*d^{(1/3)}*x + d^{(2/3)}*x^2])/(6*c^{(2/3)}*d^{(4/3)})$

**Maple [A]** time = 0.007, size = 195, normalized size = 1.4

$$\begin{aligned} & \frac{bx}{d} + \frac{a}{3d} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{bc}{3d^2} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & - \frac{a}{6d} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{bc}{6d^2} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & + \frac{\sqrt{3}a}{3d} \arctan\left(\frac{\sqrt{3}}{3} \left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{\sqrt{3}bc}{3d^2} \arctan\left(\frac{\sqrt{3}}{3} \left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)/(d*x^3+c), x)`

[Out]  $b*x/d + 1/3/d/(c/d)^{(2/3)}*\ln(x+(c/d)^{(1/3)})*a - 1/3/d^2/(c/d)^{(2/3)}*1n(x+(c/d)^{(1/3)})*b*c - 1/6/d/(c/d)^{(2/3)}*\ln(x^2-x*(c/d)^{(1/3)}+(c/d)^{(2/3)})*a + 1/6/d^2/(c/d)^{(2/3)}*\ln(x^2-x*(c/d)^{(1/3)}+(c/d)^{(2/3)})*b*c + 1/3/d/(c/d)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)}*x-1))*a - 1/3/d^2/(c/d)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)}*x-1))*b*c$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)/(d*x^3 + c), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.215331, size = 176, normalized size = 1.22

$$\frac{\sqrt{3}\left(6\sqrt{3}(c^2d)^{\frac{1}{3}}bx + \sqrt{3}(bc - ad)\log\left((c^2d)^{\frac{2}{3}}x^2 - (c^2d)^{\frac{1}{3}}cx + c^2\right) - 2\sqrt{3}(bc - ad)\log\left((c^2d)^{\frac{1}{3}}x + c\right) - 6(bc - ad)\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right)\right)}{18(c^2d)^{\frac{1}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)/(d*x^3 + c), x, algorithm="fricas")`

[Out]  $1/18*\sqrt{3}*(6*\sqrt{3}*(c^2*d)^{(1/3)}*b*x + \sqrt{3}*(b*c - a*d)*1og((c^2*d)^{(2/3)}*x^2 - (c^2*d)^{(1/3)}*c*x + c^2) - 2*\sqrt{3}*(b*c - a*d)*log((c^2*d)^{(1/3)}*x + c) - 6*(b*c - a*d)*\arctan(1/3*(2*\sqrt{3}t(3)*(c^2*d)^{(1/3)}*x - \sqrt{3}*(c)/c))/((c^2*d)^{(1/3)}*d)$

**Sympy [A]** time = 1.93706, size = 71, normalized size = 0.49

$$\frac{bx}{d} + \text{RootSum}\left(27t^3c^2d^4 - a^3d^3 + 3a^2bcd^2 - 3ab^2c^2d + b^3c^3, \left(t \mapsto t \log\left(\frac{3tcd}{ad - bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)/(d\*x\*\*3+c),x)

[Out] b\*x/d + RootSum(27\*\_t\*\*3\*c\*\*2\*d\*\*4 - a\*\*3\*d\*\*3 + 3\*a\*\*2\*b\*c\*d\*\*2 - 3\*a\*b\*\*2\*c\*\*2\*d + b\*\*3\*c\*\*3, Lambda(\_t, \_t\*log(3\*\_t\*c\*d/(a\*d - b\*c) + x)))

**GIAC/XCAS [A]** time = 0.21749, size = 217, normalized size = 1.51

$$\frac{bx}{d} + \frac{(bc - ad) \left(-\frac{c}{d}\right)^{\frac{1}{3}} \ln\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3cd} - \frac{\sqrt{3} \left(\left(-cd^2\right)^{\frac{1}{3}} bc - \left(-cd^2\right)^{\frac{1}{3}} ad\right) \arctan\left(\frac{\sqrt{3} \left(2x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3cd^2} - \frac{\left(\left(-cd^2\right)^{\frac{1}{3}} bc - \left(-cd^2\right)^{\frac{1}{3}} ad\right) \ln\left(x^2 + x \left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)/(d\*x^3 + c),x, algorithm="giac")

[Out] b\*x/d + 1/3\*(b\*c - a\*d)\*(-c/d)^(1/3)\*ln(abs(x - (-c/d)^(1/3)))/(c\*d) - 1/3\*sqrt(3)\*((-c\*d^2)^(1/3)\*b\*c - (-c\*d^2)^(1/3)\*a\*d)\*arctan(1/3\*sqrt(3)\*(2\*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(c\*d^2) - 1/6\*((-c\*d^2)^(1/3)\*b\*c - (-c\*d^2)^(1/3)\*a\*d)\*ln(x^2 + x\*(-c/d)^(1/3) + (-c/d)^(2/3))/(c\*d^2)

### 3.6 $\int \frac{a+bx^3}{(c+dx^3)^2} dx$

**Optimal.** Leaf size=169

$$-\frac{(2ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx}+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}+\frac{(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{dx}\right)}{9c^{5/3}d^{4/3}}$$

$$-\frac{(2ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{4/3}}-\frac{x(bc-ad)}{3cd(c+dx^3)}$$

[Out]  $-\left((b*c - a*d)*x\right)/\left(3*c*d*(c + d*x^3)\right) - \left((b*c + 2*a*d)*\text{ArcTan}\left[\left(c^{1/3} - 2*d^{1/3}*x\right)/\left(\text{Sqrt}[3]*c^{1/3}\right)\right]\right)/\left(3*\text{Sqrt}[3]*c^{5/3}*d^{4/3}\right) + \left((b*c + 2*a*d)*\text{Log}\left[c^{1/3} + d^{1/3}*x\right]\right)/\left(9*c^{5/3}*d^{4/3}\right) - \left((b*c + 2*a*d)*\text{Log}\left[c^{2/3} - c^{1/3}*d^{1/3}*x + d^{2/3}*x^2\right]\right)/\left(18*c^{5/3}*d^{4/3}\right)$

**Rubi [A]** time = 0.195134, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$-\frac{(2ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx}+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}+\frac{(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{dx}\right)}{9c^{5/3}d^{4/3}}$$

$$-\frac{(2ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{4/3}}-\frac{x(bc-ad)}{3cd(c+dx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)/(c + d\*x^3)^2, x]

[Out]  $-\left((b*c - a*d)*x\right)/\left(3*c*d*(c + d*x^3)\right) - \left((b*c + 2*a*d)*\text{ArcTan}\left[\left(c^{1/3} - 2*d^{1/3}*x\right)/\left(\text{Sqrt}[3]*c^{1/3}\right)\right]\right)/\left(3*\text{Sqrt}[3]*c^{5/3}*d^{4/3}\right) + \left((b*c + 2*a*d)*\text{Log}\left[c^{1/3} + d^{1/3}*x\right]\right)/\left(9*c^{5/3}*d^{4/3}\right) - \left((b*c + 2*a*d)*\text{Log}\left[c^{2/3} - c^{1/3}*d^{1/3}*x + d^{2/3}*x^2\right]\right)/\left(18*c^{5/3}*d^{4/3}\right)$

**Rubi in Sympy [A]** time = 34.2323, size = 155, normalized size = 0.92

$$\frac{x(ad-bc)}{3cd(c+dx^3)} + \frac{(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{dx}\right)}{9c^{5/3}d^{4/3}} - \frac{(2ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx}+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}$$

$$-\frac{\sqrt{3}(2ad+bc)\text{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3}-\frac{2\sqrt[3]{dx}}{3}\right)}{\sqrt[3]{c}}\right)}{9c^{5/3}d^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)/(d\*x\*\*3+c)\*\*2, x)

[Out]  $x*(a*d - b*c)/(3*c*d*(c + d*x^3)) + (2*a*d + b*c)*\log(c^{1/3} + d^{1/3}*x)/(9*c^{5/3}*d^{4/3}) - (2*a*d + b*c)*\log(c^{2/3} - c^{1/3}*d^{1/3}*x + d^{2/3}*x^2)/(18*c^{5/3}*d^{4/3}) - \text{sqrt}(3)*(2*a*d + b*c)*\text{atan}(\text{sqrt}(3)*(c^{1/3}/3 - 2*d^{1/3}*x/3)/c^{1/3})/(9*c^{5/3}*d^{4/3})$

**Mathematica [A]** time = 0.1699, size = 145, normalized size = 0.86

$$\frac{-(2ad + bc) \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right) - \frac{6c^{2/3}\sqrt[3]{dx}(bc-ad)}{c+dx^3} + 2(2ad + bc) \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right) - 2\sqrt{3}(2ad + bc) \tan^{-1}\left(\frac{1-2\sqrt[3]{dx}}{\sqrt[3]{c}}\right)}{18c^{5/3}d^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)/(c + d\*x^3)^2, x]

[Out] ((-6\*c^(2/3)\*d^(1/3)\*(b\*c - a\*d)\*x)/(c + d\*x^3) - 2\*Sqrt[3]\*(b\*c + 2\*a\*d)\*ArcTan[(1 - (2\*d^(1/3)\*x)/c^(1/3))/Sqrt[3]] + 2\*(b\*c + 2\*a\*d)\*Log[c^(1/3) + d^(1/3)\*x] - (b\*c + 2\*a\*d)\*Log[c^(2/3) - c^(1/3)\*d^(1/3)\*x + d^(2/3)\*x^2])/(18\*c^(5/3)\*d^(4/3))

**Maple [A]** time = 0.011, size = 221, normalized size = 1.3

$$\begin{aligned} & \frac{(ad - bc)x}{3cd(dx^3 + c)} + \frac{2a}{9cd} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{b}{9d^2} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & - \frac{a}{9cd} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{b}{18d^2} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & + \frac{2\sqrt{3}a}{9cd} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}b}{9d^2} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)/(d\*x^3+c)^2, x)

[Out] 1/3\*(a\*d-b\*c)/c/d\*x/(d\*x^3+c)+2/9/c/d/(c/d)^(2/3)\*ln(x+(c/d)^(1/3))^a+1/9/d^2/(c/d)^(2/3)\*ln(x+(c/d)^(1/3))\*b-1/9/c/d/(c/d)^(2/3)\*ln(x^2-x\*(c/d)^(1/3)+(c/d)^(2/3))\*a-1/18/d^2/(c/d)^(2/3)\*ln(x^2-x\*(c/d)^(1/3)+(c/d)^(2/3))\*b+2/9/c/d/(c/d)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(c/d)^(1/3)\*x-1))\*a+1/9/d^2/(c/d)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(c/d)^(1/3)\*x-1))\*b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)/(d\*x^3 + c)^2, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.214528, size = 275, normalized size = 1.63

$$\frac{\sqrt{3}\left(6\sqrt{3}(c^2d)^{\frac{1}{3}}(bc-ad)x + \sqrt{3}((bcd+2ad^2)x^3 + bc^2 + 2acd) \log\left((c^2d)^{\frac{2}{3}}x^2 - (c^2d)^{\frac{1}{3}}cx + c^2\right) - 2\sqrt{3}((bcd+2ad^2)x\right)}{54(cd^2x^3 + c^2d)(c^2d)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)/(d\*x^3 + c)^2,x, algorithm="fricas")

[Out] 
$$-1/54*\sqrt{3}*(6*\sqrt{3}*(c^2*d)^{1/3}*(b*c - a*d)*x + \sqrt{3}*((b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*\log((c^2*d)^{2/3}*x^2 - (c^2*d)^{1/3}*c*x + c^2) - 2*\sqrt{3}*((b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*\log((c^2*d)^{1/3}*x + c) - 6*((b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*\arctan(1/3*(2*\sqrt{3}*(c^2*d)^{1/3}*x - \sqrt{3}*c)/c))/((c*d^2*x^3 + c^2*d)*(c^2*d)^{1/3})$$

**Sympy [A]** time = 2.61964, size = 97, normalized size = 0.57

$$\frac{x(ad - bc)}{3c^2d + 3cd^2x^3} + \text{RootSum}\left(729t^3c^5d^4 - 8a^3d^3 - 12a^2bcd^2 - 6ab^2c^2d - b^3c^3, \left(t \mapsto t \log\left(\frac{9tc^2d}{2ad + bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)/(d\*x\*\*3+c)\*\*2,x)

[Out] 
$$x*(a*d - b*c)/(3*c**2*d + 3*c*d**2*x**3) + \text{RootSum}(729*_t**3*c**5*d**4 - 8*a**3*d**3 - 12*a**2*b*c*d**2 - 6*a*b**2*c**2*d - b**3*c**3, \text{Lambda}(_t, _t*\log(9*_t*c**2*d/(2*a*d + b*c) + x)))$$

**GIAC/XCAS [A]** time = 0.217759, size = 246, normalized size = 1.46

$$-\frac{(bc + 2ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}} \ln\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9c^2d} + \frac{\sqrt{3}\left(\left(-cd^2\right)^{\frac{1}{3}}bc + 2\left(-cd^2\right)^{\frac{1}{3}}ad\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9c^2d^2} - \frac{bcx - adx}{3(dx^3 + c)cd} + \frac{\left(\left(-cd^2\right)^{\frac{1}{3}}bc + 2\left(-cd^2\right)^{\frac{1}{3}}ad\right) \ln\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)/(d\*x^3 + c)^2,x, algorithm="giac")

[Out] 
$$-1/9*(b*c + 2*a*d)*(-c/d)^{1/3}*\ln(\text{abs}(x - (-c/d)^{1/3}))/c^2*d + 1/9*\sqrt{3}*((-c*d^2)^{1/3}*b*c + 2*(-c*d^2)^{1/3}*a*d)*\arctan(1/3*\sqrt{3}*(2*x + (-c/d)^{1/3})/(-c/d)^{1/3})/c^2*d^2 - 1/3*(b*c*x - a*d*x)/((d*x^3 + c)*c*d) + 1/18*((-c*d^2)^{1/3}*b*c + 2*(-c*d^2)^{1/3}*a*d)*\ln(x^2 + x*(-c/d)^{1/3} + (-c/d)^{2/3})/c^2*d^2$$

### 3.7 $\int \frac{a+bx^3}{(c+dx^3)^3} dx$

**Optimal.** Leaf size=197

$$\begin{aligned} & -\frac{(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx}+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}} + \frac{(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{dx}\right)}{27c^{8/3}d^{4/3}} \\ & -\frac{(5ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c-2\sqrt[3]{dx}}}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{4/3}} + \frac{x(5ad+bc)}{18c^2d(c+dx^3)} - \frac{x(bc-ad)}{6cd(c+dx^3)^2} \end{aligned}$$

[Out]  $-\frac{(b*c - a*d)*x}{(6*c*d*(c + d*x^3)^2)} + \frac{(b*c + 5*a*d)*x}{(18*c^2*d*(c + d*x^3))} - \frac{(b*c + 5*a*d)*\text{ArcTan}[(c^{(1/3)} - 2*d^{(1/3)}*x)/(\text{Sqrt}[3]*c^{(1/3)})]}{(9*\text{Sqrt}[3]*c^{(8/3)}*d^{(4/3)})} + \frac{(b*c + 5*a*d)*\text{Log}[c^{(1/3)} + d^{(1/3)}*x]}{(27*c^{(8/3)}*d^{(4/3)})} - \frac{(b*c + 5*a*d)*\text{Log}[c^{(2/3)} - c^{(1/3)}*d^{(1/3)}*x + d^{(2/3)}*x^2]}{(54*c^{(8/3)}*d^{(4/3)})}$

**Rubi [A]** time = 0.22915, antiderivative size = 197, normalized size of antiderivative = 1., number of rules used = 8, number of rules used = 8, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$

$$\begin{aligned} & -\frac{(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx}+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}} + \frac{(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{dx}\right)}{27c^{8/3}d^{4/3}} \\ & -\frac{(5ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c-2\sqrt[3]{dx}}}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{4/3}} + \frac{x(5ad+bc)}{18c^2d(c+dx^3)} - \frac{x(bc-ad)}{6cd(c+dx^3)^2} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^3)/(c + d*x^3)^3, x]$

[Out]  $-\frac{(b*c - a*d)*x}{(6*c*d*(c + d*x^3)^2)} + \frac{(b*c + 5*a*d)*x}{(18*c^2*d*(c + d*x^3))} - \frac{(b*c + 5*a*d)*\text{ArcTan}[(c^{(1/3)} - 2*d^{(1/3)}*x)/(\text{Sqrt}[3]*c^{(1/3)})]}{(9*\text{Sqrt}[3]*c^{(8/3)}*d^{(4/3)})} + \frac{(b*c + 5*a*d)*\text{Log}[c^{(1/3)} + d^{(1/3)}*x]}{(27*c^{(8/3)}*d^{(4/3)})} - \frac{(b*c + 5*a*d)*\text{Log}[c^{(2/3)} - c^{(1/3)}*d^{(1/3)}*x + d^{(2/3)}*x^2]}{(54*c^{(8/3)}*d^{(4/3)})}$

**Rubi in Sympy [A]** time = 39.1236, size = 180, normalized size = 0.91

$$\begin{aligned} & \frac{x(ad-bc)}{6cd(c+dx^3)^2} + \frac{x(5ad+bc)}{18c^2d(c+dx^3)} + \frac{(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{dx}\right)}{27c^{\frac{8}{3}}d^{\frac{4}{3}}} \\ & -\frac{(5ad+bc)\log\left(c^{\frac{2}{3}}-\sqrt[3]{c}\sqrt[3]{dx}+d^{\frac{2}{3}}x^2\right)}{54c^{\frac{8}{3}}d^{\frac{4}{3}}} - \frac{\sqrt{3}(5ad+bc)\text{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3}-\frac{2\sqrt[3]{dx}}{3}\right)}{\sqrt[3]{c}}\right)}{27c^{\frac{8}{3}}d^{\frac{4}{3}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x**3+a)/(d*x**3+c)**3, x)$

[Out]  $x*(a*d - b*c)/(6*c*d*(c + d*x**3)**2) + x*(5*a*d + b*c)/(18*c**2*d*(c + d*x**3)) + (5*a*d + b*c)*\log(c**(1/3) + d**(1/3)*x)/(27*c**(8/3)*d**(4/3)) - (5*a*d + b*c)*\log(c**(2/3) - c**(1/3)*d**(1/3)*x + d**(2/3)*x**2)/(54*c**(8/3)*d**(4/3)) - \text{sqrt}(3)*(5*a*d + b*c)*\text{atan}(\text{sqrt}(3)*(c**(1/3)/3 - 2*d**(1/3)*x/3)/c**(1/3))/(27*c**(8/3)*d**(4/3))$

**Mathematica [A]** time = 0.225062, size = 175, normalized size = 0.89

$$-(5ad + bc) \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right) - \frac{9c^{5/3}\sqrt[3]{dx}(bc-ad)}{(c+dx^3)^2} + \frac{3c^{2/3}\sqrt[3]{dx}(5ad+bc)}{c+dx^3} + 2(5ad + bc) \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right) - 2\sqrt{3}(5ad +$$

$$54c^{8/3}d^{4/3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)/(c + d\*x^3)^3, x]

[Out]  $\left(\frac{-9c^{5/3}d^{1/3}(b^3c - a^3d)x}{(c + dx^3)^2} + \frac{3c^{2/3}d^{1/3}(b^3c + 5a^3d)x}{(c + dx^3)} - 2\sqrt{3}(b^3c + 5a^3d)\operatorname{ArcTan}\left[\frac{1 - (2d^{1/3}x)/c^{1/3}}{\sqrt{3}}\right] + 2(b^3c + 5a^3d)\operatorname{Log}\left[c^{1/3} + d^{1/3}x\right] - (b^3c + 5a^3d)\operatorname{Log}\left[c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2\right]\right)/(54c^{8/3}d^{4/3})$

**Maple [A]** time = 0.014, size = 249, normalized size = 1.3

$$\begin{aligned} & \frac{1}{(dx^3 + c)^2} \left( \frac{(5ad + bc)x^4}{18c^2} + \frac{(4ad - bc)x}{9cd} \right) + \frac{5a}{27c^2d} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & + \frac{b}{27cd^2} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{5a}{54c^2d} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & - \frac{b}{54cd^2} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{5\sqrt{3}a}{27c^2d} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & + \frac{\sqrt{3}b}{27cd^2} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)/(d\*x^3+c)^3, x)

[Out]  $\left(\frac{1}{18} \frac{(5a^3d + b^3c)}{c^2} x^4 + \frac{1}{9} \frac{(4a^3d - b^3c)}{cd} x\right) / (d^2 x^3 + c)^2 + \frac{5}{27} \frac{a}{c^2 d} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{b}{27} \frac{1}{cd^2} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{5}{54} \frac{a}{c^2 d} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{b}{54} \frac{1}{cd^2} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{5\sqrt{3}a}{27c^2d} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}b}{27cd^2} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)/(d\*x^3 + c)^3, x, algorithm="maxima")

[Out] Exception raised: ValueError



**Fricas [A]** time = 0.214195, size = 414, normalized size = 2.1

$$\sqrt{3} \left( \sqrt{3} ((bcd^2 + 5ad^3)x^6 + bc^3 + 5ac^2d + 2(bc^2d + 5acd^2)x^3) \log \left( (c^2d)^{\frac{2}{3}} x^2 - (c^2d)^{\frac{1}{3}} cx + c^2 \right) - 2\sqrt{3} ((bcd^2 + 5ad^3)x^6 + bc^3 + 5ac^2d + 2(bc^2d + 5acd^2)x^3) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)/(d\*x^3 + c)^3,x, algorithm="fricas")

[Out] 
$$-1/162 \sqrt{3} \left( \sqrt{3} \left( (b^2 c^2 d^2 + 5 a^2 d^3) x^6 + b^2 c^3 + 5 a^2 c^2 d + 2 (b^2 c^2 d^2 + 5 a^2 c^2 d^2) x^3 \right) \log \left( (c^2 d)^{2/3} x^2 - (c^2 d)^{1/3} c x + c^2 \right) \right. \\ \left. - 2 \sqrt{3} \left( (b^2 c^2 d^2 + 5 a^2 d^3) x^6 + b^2 c^3 + 5 a^2 c^2 d + 2 (b^2 c^2 d^2 + 5 a^2 c^2 d^2) x^3 \right) \log \left( (c^2 d)^{1/3} x + c \right) \right. \\ \left. - 6 \left( (b^2 c^2 d^2 + 5 a^2 d^3) x^6 + b^2 c^3 + 5 a^2 c^2 d + 2 (b^2 c^2 d^2 + 5 a^2 c^2 d^2) x^3 \right) \arctan \left( \frac{1}{3} \left( 2 \sqrt{3} (c^2 d)^{1/3} x - \sqrt{3} c \right) / c \right) \right. \\ \left. - 3 \sqrt{3} \left( (b^2 c^2 d^2 + 5 a^2 d^3) x^6 + 2 (b^2 c^2 d^2 + 5 a^2 c^2 d^2) x^3 \right) \arctan \left( \frac{1}{3} \left( 2 \sqrt{3} (c^2 d)^{1/3} x - \sqrt{3} c \right) / c \right) \right) / \left( (c^2 d)^3 x^6 + 2 c^3 d^2 x^3 + c^4 d \right) (c^2 d)^{1/3}$$

**Sympy [A]** time = 3.77063, size = 133, normalized size = 0.68

$$\frac{x^4 (5ad^2 + bcd) + x (8acd - 2bc^2)}{18c^4d + 36c^3d^2x^3 + 18c^2d^3x^6} + \text{RootSum} \left( 19683t^3c^8d^4 - 125a^3d^3 - 75a^2bcd^2 - 15ab^2c^2d - b^3c^3, \left( t \mapsto t \log \left( \frac{27tc^3d}{5ad + bc} + x \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)/(d\*x\*\*3+c)\*\*3,x)

[Out] 
$$(x^4(5a^2d^2 + b^2cd) + x(8acd - 2b^2c^2)) / (18c^4d + 36c^3d^2x^3 + 18c^2d^3x^6) + \text{RootSum}(19683t^3c^8d^4 - 125a^3d^3 - 75a^2bcd^2 - 15ab^2c^2d - b^3c^3, \text{Lambda}(t, t \log(27t^3c^3d/(5ad + bc) + x)))$$

**GIAC/XCAS [A]** time = 0.220801, size = 273, normalized size = 1.39

$$-\frac{(bc + 5ad) \left(-\frac{c}{d}\right)^{\frac{1}{3}} \ln \left( \left| x - \left(-\frac{c}{d}\right)^{\frac{1}{3}} \right| \right)}{27c^3d} + \frac{\sqrt{3} \left( (-cd^2)^{\frac{1}{3}} bc + 5(-cd^2)^{\frac{1}{3}} ad \right) \arctan \left( \frac{\sqrt{3} \left( 2x + \left(-\frac{c}{d}\right)^{\frac{1}{3}} \right)}{3 \left(-\frac{c}{d}\right)^{\frac{1}{3}}} \right)}{27c^3d^2} \\ + \frac{\left( (-cd^2)^{\frac{1}{3}} bc + 5(-cd^2)^{\frac{1}{3}} ad \right) \ln \left( x^2 + x \left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}} \right)}{54c^3d^2} + \frac{bcdx^4 + 5ad^2x^4 - 2bc^2x + 8acdx}{18(dx^3 + c)^2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)/(d\*x^3 + c)^3,x, algorithm="giac")

[Out] 
$$-1/27 (b^2c + 5a^2d) (-c/d)^{1/3} \ln(\text{abs}(x - (-c/d)^{1/3})) / (c^3d) \\ + 1/27 \sqrt{3} \left( (-c^2d^2)^{1/3} b^2c + 5(-c^2d^2)^{1/3} a^2d \right) \arctan \left( \frac{1}{3} \sqrt{3} \left( 2x + (-c/d)^{1/3} \right) / (-c/d)^{1/3} \right) / (c^3d^2) + 1/54 \\ \left( (-c^2d^2)^{1/3} b^2c + 5(-c^2d^2)^{1/3} a^2d \right) \ln \left( x^2 + x(-c/d)^{1/3} + (-c/d)^{2/3} \right) / (c^3d^2) + 1/18 (b^2cd^2x^4 + 5a^2d^2x^4 - 2b^2c^2x + 8acdx) / ((d^3x^3 + c)^2c^2d)$$



$$0)/10 + (b^2 d^2 (3 b^2 c + 2 a^2 d) x^{13})/13 + (b^2 d^3 x^{16})/16$$

**Maple [A]** time = 0., size = 125, normalized size = 1.

$$\frac{b^2 d^3 x^{16}}{16} + \frac{(2 a b d^3 + 3 b^2 c d^2) x^{13}}{13} + \frac{(a^2 d^3 + 6 a b c d^2 + 3 b^2 c^2 d) x^{10}}{10} + \frac{(3 a^2 c d^2 + 6 a b c^2 d + b^2 c^3) x^7}{7} + \frac{(3 a^2 c^2 d + 2 a b c^3) x^4}{4} + a^2 c^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^2\*(d\*x^3+c)^3,x)

[Out] 1/16\*b^2\*d^3\*x^16+1/13\*(2\*a\*b\*d^3+3\*b^2\*c\*d^2)\*x^13+1/10\*(a^2\*d^3+6\*a\*b\*c\*d^2+3\*b^2\*c^2\*d)\*x^10+1/7\*(3\*a^2\*c\*d^2+6\*a\*b\*c^2\*d+b^2\*c^3)\*x^7+1/4\*(3\*a^2\*c^2\*d+2\*a\*b\*c^3)\*x^4+a^2\*c^3\*x

**Maxima [A]** time = 1.36992, size = 167, normalized size = 1.37

$$\frac{1}{16} b^2 d^3 x^{16} + \frac{1}{13} (3 b^2 c d^2 + 2 a b d^3) x^{13} + \frac{1}{10} (3 b^2 c^2 d + 6 a b c d^2 + a^2 d^3) x^{10} + \frac{1}{7} (b^2 c^3 + 6 a b c^2 d + 3 a^2 c d^2) x^7 + a^2 c^3 x + \frac{1}{4} (2 a b c^3 + 3 a^2 c^2 d) x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^2\*(d\*x^3 + c)^3,x, algorithm="maxima")

[Out] 1/16\*b^2\*d^3\*x^16 + 1/13\*(3\*b^2\*c\*d^2 + 2\*a\*b\*d^3)\*x^13 + 1/10\*(3\*b^2\*c^2\*d + 6\*a\*b\*c\*d^2 + a^2\*d^3)\*x^10 + 1/7\*(b^2\*c^3 + 6\*a\*b\*c^2\*d + 3\*a^2\*c\*d^2)\*x^7 + a^2\*c^3\*x + 1/4\*(2\*a\*b\*c^3 + 3\*a^2\*c^2\*d)\*x^4

**Fricas [A]** time = 0.182125, size = 1, normalized size = 0.01

$$\frac{1}{16} x^{16} d^3 b^2 + \frac{3}{13} x^{13} d^2 c b^2 + \frac{2}{13} x^{13} d^3 b a + \frac{3}{10} x^{10} d c^2 b^2 + \frac{3}{5} x^{10} d^2 c b a + \frac{1}{10} x^{10} d^3 a^2 + \frac{1}{7} x^7 c^3 b^2 + \frac{6}{7} x^7 d c^2 b a + \frac{3}{7} x^7 d^2 c a^2 + \frac{1}{2} x^4 c^3 b a + \frac{3}{4} x^4 d c^2 a^2 + x c^3 a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^2\*(d\*x^3 + c)^3,x, algorithm="fricas")

[Out] 1/16\*x^16\*d^3\*b^2 + 3/13\*x^13\*d^2\*c\*b^2 + 2/13\*x^13\*d^3\*b\*a + 3/10\*x^10\*d\*c^2\*b^2 + 3/5\*x^10\*d^2\*c\*b\*a + 1/10\*x^10\*d^3\*a^2 + 1/7\*x^7\*c^3\*b^2 + 6/7\*x^7\*d\*c^2\*b\*a + 3/7\*x^7\*d^2\*c\*a^2 + 1/2\*x^4\*c^3\*b\*a + 3/4\*x^4\*d\*c^2\*a^2 + x\*c^3\*a^2

**Sympy [A]** time = 0.165601, size = 139, normalized size = 1.14

$$a^2 c^3 x + \frac{b^2 d^3 x^{16}}{16} + x^{13} \left( \frac{2 a b d^3}{13} + \frac{3 b^2 c d^2}{13} \right) + x^{10} \left( \frac{a^2 d^3}{10} + \frac{3 a b c d^2}{5} + \frac{3 b^2 c^2 d}{10} \right) + x^7 \left( \frac{3 a^2 c d^2}{7} + \frac{6 a b c^2 d}{7} + \frac{b^2 c^3}{7} \right) + x^4 \left( \frac{3 a^2 c^2 d}{4} + \frac{a b c^3}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*2\*(d\*x\*\*3+c)\*\*3,x)

[Out] a\*\*2\*c\*\*3\*x + b\*\*2\*d\*\*3\*x\*\*16/16 + x\*\*13\*(2\*a\*b\*d\*\*3/13 + 3\*b\*\*2\*c\*d\*\*2/13) + x\*\*10\*(a\*\*2\*d\*\*3/10 + 3\*a\*b\*c\*d\*\*2/5 + 3\*b\*\*2\*c\*\*2\*d/10) + x\*\*7\*(3\*a\*\*2\*c\*d\*\*2/7 + 6\*a\*b\*c\*\*2\*d/7 + b\*\*2\*c\*\*3/7) + x\*\*4\*(3\*a\*\*2\*c\*\*2\*d/4 + a\*b\*c\*\*3/2)

**GIAC/XCAS [A]** time = 0.211415, size = 178, normalized size = 1.46

$$\frac{1}{16} b^2 d^3 x^{16} + \frac{3}{13} b^2 c d^2 x^{13} + \frac{2}{13} a b d^3 x^{13} + \frac{3}{10} b^2 c^2 d x^{10} + \frac{3}{5} a b c d^2 x^{10} + \frac{1}{10} a^2 d^3 x^{10} + \frac{1}{7} b^2 c^3 x^7 + \frac{6}{7} a b c^2 d x^7 + \frac{3}{7} a^2 c d^2 x^7 + \frac{1}{2} a b c^3 x^4 + \frac{3}{4} a^2 c^2 d x^4 + a^2 c^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^2\*(d\*x^3 + c)^3,x, algorithm="giac")

[Out] 1/16\*b^2\*d^3\*x^16 + 3/13\*b^2\*c\*d^2\*x^13 + 2/13\*a\*b\*d^3\*x^13 + 3/10\*b^2\*c^2\*d\*x^10 + 3/5\*a\*b\*c\*d^2\*x^10 + 1/10\*a^2\*d^3\*x^10 + 1/7\*b^2\*c^3\*x^7 + 6/7\*a\*b\*c^2\*d\*x^7 + 3/7\*a^2\*c\*d^2\*x^7 + 1/2\*a\*b\*c^3\*x^4 + 3/4\*a^2\*c^2\*d\*x^4 + a^2\*c^3\*x

### 3.9 $\int (a + bx^3)^2 (c + dx^3)^2 dx$

**Optimal.** Leaf size=82

$$\frac{1}{7}x^7(a^2d^2 + 4abcd + b^2c^2) + a^2c^2x + \frac{1}{5}bdx^{10}(ad + bc) + \frac{1}{2}acx^4(ad + bc) + \frac{1}{13}b^2d^2x^{13}$$

[Out]  $a^2c^2x + (a^2c^2x^7 + (a^2c^2 + 4abcd + b^2c^2)x^7)/7 + (bd^2x^{10}(ad + bc) + a^2c^2x^5)/5 + (acx^4(ad + bc) + b^2d^2x^{13})/13$

**Rubi [A]** time = 0.116628, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{1}{7}x^7(a^2d^2 + 4abcd + b^2c^2) + a^2c^2x + \frac{1}{5}bdx^{10}(ad + bc) + \frac{1}{2}acx^4(ad + bc) + \frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^2\*(c + d\*x^3)^2, x]

[Out]  $a^2c^2x + (a^2c^2x^7 + (a^2c^2 + 4abcd + b^2c^2)x^7)/7 + (bd^2x^{10}(ad + bc) + a^2c^2x^5)/5 + (acx^4(ad + bc) + b^2d^2x^{13})/13$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{acx^4(ad + bc)}{2} + \frac{b^2d^2x^{13}}{13} + \frac{bdx^{10}(ad + bc)}{5} + c^2 \int a^2 dx + x^7 \left( \frac{a^2d^2}{7} + \frac{4abcd}{7} + \frac{b^2c^2}{7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*\*2\*(d\*x\*\*3+c)\*\*2, x)

[Out]  $a^2c^2x^4(a^2d^2 + b^2c^2)/2 + b^2d^2x^{13}/13 + b^2d^2x^{10}(ad + bc)/5 + c^2 \int a^2 dx + x^7(a^2d^2/7 + 4abcd/7 + b^2c^2/7)$

**Mathematica [A]** time = 0.0233658, size = 82, normalized size = 1.

$$\frac{1}{7}x^7(a^2d^2 + 4abcd + b^2c^2) + a^2c^2x + \frac{1}{5}bdx^{10}(ad + bc) + \frac{1}{2}acx^4(ad + bc) + \frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^2\*(c + d\*x^3)^2, x]

[Out]  $a^2c^2x + (a^2c^2x^7 + (a^2c^2 + 4abcd + b^2c^2)x^7)/7 + (bd^2x^{10}(ad + bc) + a^2c^2x^5)/5 + (acx^4(ad + bc) + b^2d^2x^{13})/13$

**Maple [A]** time = 0.001, size = 87, normalized size = 1.1

$$\frac{b^2d^2x^{13}}{13} + \frac{(2abd^2 + 2b^2cd)x^{10}}{10} + \frac{(a^2d^2 + 4cabd + b^2c^2)x^7}{7} + \frac{(2a^2cd + 2abc^2)x^4}{4} + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^2*(d*x^3+c)^2,x)`

[Out]  $\frac{1}{13}b^2d^2x^{13} + \frac{1}{10}(2ab^2d^2 + 2b^2c^2d)x^{10} + \frac{1}{7}(a^2d^2 + 4a^2b^2c^2d + b^2c^2d^2)x^7 + \frac{1}{4}(2a^2c^2d + 2a^2b^2c^2)x^4 + a^2c^2x$

**Maxima [A]** time = 1.40654, size = 111, normalized size = 1.35

$$\frac{1}{13}b^2d^2x^{13} + \frac{1}{5}(b^2cd + abd^2)x^{10} + \frac{1}{7}(b^2c^2 + 4abcd + a^2d^2)x^7 + a^2c^2x + \frac{1}{2}(abc^2 + a^2cd)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^2*(d*x^3 + c)^2,x, algorithm="maxima")`

[Out]  $\frac{1}{13}b^2d^2x^{13} + \frac{1}{5}(b^2c^2d + a^2b^2d^2)x^{10} + \frac{1}{7}(b^2c^2 + 4a^2b^2c^2d + a^2d^2)x^7 + \frac{1}{2}(a^2c^2d + a^2b^2c^2d)x^4$

**Fricas [A]** time = 0.180726, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}d^2b^2 + \frac{1}{5}x^{10}dcb^2 + \frac{1}{5}x^{10}d^2ba + \frac{1}{7}x^7c^2b^2 + \frac{4}{7}x^7dcba + \frac{1}{7}x^7d^2a^2 + \frac{1}{2}x^4c^2ba + \frac{1}{2}x^4dca^2 + xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^2*(d*x^3 + c)^2,x, algorithm="fricas")`

[Out]  $\frac{1}{13}x^{13}d^2b^2 + \frac{1}{5}x^{10}d^2c^2b^2 + \frac{1}{5}x^{10}d^2b^2a + \frac{1}{7}x^7c^2b^2 + \frac{4}{7}x^7d^2c^2ba + \frac{1}{7}x^7d^2a^2 + \frac{1}{2}x^4c^2ba + \frac{1}{2}x^4dca^2 + xc^2a^2$

**Sympy [A]** time = 0.138084, size = 90, normalized size = 1.1

$$a^2c^2x + \frac{b^2d^2x^{13}}{13} + x^{10}\left(\frac{abd^2}{5} + \frac{b^2cd}{5}\right) + x^7\left(\frac{a^2d^2}{7} + \frac{4abcd}{7} + \frac{b^2c^2}{7}\right) + x^4\left(\frac{a^2cd}{2} + \frac{abc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**2*(d*x**3+c)**2,x)`

[Out]  $a^2c^2x + b^2d^2x^{13}/13 + x^{10}(ab^2d^2/5 + b^2c^2d/5) + x^7(a^2d^2/7 + 4a^2b^2c^2d/7 + b^2c^2/7) + x^4(a^2cd/2 + abc^2/2)$

**GIAC/XCAS [A]** time = 0.211426, size = 123, normalized size = 1.5

$$\frac{1}{13}b^2d^2x^{13} + \frac{1}{5}b^2cdx^{10} + \frac{1}{5}abd^2x^{10} + \frac{1}{7}b^2c^2x^7 + \frac{4}{7}abcdx^7 + \frac{1}{7}a^2d^2x^7 + \frac{1}{2}abc^2x^4 + \frac{1}{2}a^2cdx^4 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^2*(d*x^3 + c)^2,x, algorithm="giac")`

[Out]  $\frac{1}{13}b^2d^2x^{13} + \frac{1}{5}b^2c^2d^2x^{10} + \frac{1}{5}a^2b^2d^2x^{10} + \frac{1}{7}b^2c^2x^7 + \frac{4}{7}a^2b^2c^2d^2x^7 + \frac{1}{7}a^2d^2x^7 + \frac{1}{2}a^2b^2c^2x^4 + \frac{1}{2}a^2c^2d^2x^4 + a^2c^2x$

### 3.10 $\int (a + bx^3)^2 (c + dx^3) dx$

**Optimal.** Leaf size=50

$$a^2cx + \frac{1}{7}bx^7(2ad + bc) + \frac{1}{4}ax^4(ad + 2bc) + \frac{1}{10}b^2dx^{10}$$

[Out]  $a^2c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^{10})/10$

**Rubi [A]** time = 0.0754555, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$a^2cx + \frac{1}{7}bx^7(2ad + bc) + \frac{1}{4}ax^4(ad + 2bc) + \frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^2\*(c + d\*x^3), x]

[Out]  $a^2c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^{10})/10$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$a^2 \int c dx + \frac{ax^4(ad + 2bc)}{4} + \frac{b^2dx^{10}}{10} + \frac{bx^7(2ad + bc)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*\*2\*(d\*x\*\*3+c), x)

[Out]  $a**2*Integral(c, x) + a*x**4*(a*d + 2*b*c)/4 + b**2*d*x**10/10 + b*x**7*(2*a*d + b*c)/7$

**Mathematica [A]** time = 0.0136847, size = 50, normalized size = 1.

$$a^2cx + \frac{1}{7}bx^7(2ad + bc) + \frac{1}{4}ax^4(ad + 2bc) + \frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^2\*(c + d\*x^3), x]

[Out]  $a^2c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^{10})/10$

**Maple [A]** time = 0., size = 49, normalized size = 1.

$$\frac{b^2dx^{10}}{10} + \frac{(2abd + b^2c)x^7}{7} + \frac{(a^2d + 2abc)x^4}{4} + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^2\*(d\*x^3+c), x)

[Out]  $1/10*b^2*d*x^{10}+1/7*(2*a*b*d+b^2*c)*x^7+1/4*(a^2*d+2*a*b*c)*x^4+a^2*c*x$

**Maxima [A]** time = 1.38326, size = 65, normalized size = 1.3

$$\frac{1}{10}b^2dx^{10} + \frac{1}{7}(b^2c + 2abd)x^7 + \frac{1}{4}(2abc + a^2d)x^4 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^2*(d*x^3 + c),x, algorithm="maxima")`

[Out]  $1/10*b^2*d*x^{10} + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/4*(2*a*b*c + a^2*d)*x^4 + a^2*c*x$

**Fricas [A]** time = 0.179662, size = 1, normalized size = 0.02

$$\frac{1}{10}x^{10}db^2 + \frac{1}{7}x^7cb^2 + \frac{2}{7}x^7dba + \frac{1}{2}x^4cba + \frac{1}{4}x^4da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^2*(d*x^3 + c),x, algorithm="fricas")`

[Out]  $1/10*x^{10}*d*b^2 + 1/7*x^7*c*b^2 + 2/7*x^7*d*b*a + 1/2*x^4*c*b*a + 1/4*x^4*d*a^2 + x*c*a^2$

**Sympy [A]** time = 0.102264, size = 51, normalized size = 1.02

$$a^2cx + \frac{b^2dx^{10}}{10} + x^7\left(\frac{2abd}{7} + \frac{b^2c}{7}\right) + x^4\left(\frac{a^2d}{4} + \frac{abc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**2*(d*x**3+c),x)`

[Out]  $a**2*c*x + b**2*d*x**10/10 + x**7*(2*a*b*d/7 + b**2*c/7) + x**4*(a**2*d/4 + a*b*c/2)$

**GIAC/XCAS [A]** time = 0.209987, size = 68, normalized size = 1.36

$$\frac{1}{10}b^2dx^{10} + \frac{1}{7}b^2cx^7 + \frac{2}{7}abdx^7 + \frac{1}{2}abcx^4 + \frac{1}{4}a^2dx^4 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^2*(d*x^3 + c),x, algorithm="giac")`

[Out]  $1/10*b^2*d*x^{10} + 1/7*b^2*c*x^7 + 2/7*a*b*d*x^7 + 1/2*a*b*c*x^4 + 1/4*a^2*d*x^4 + a^2*c*x$



$$3.11 \quad \int \frac{(a+bx^3)^2}{c+dx^3} dx$$

Optimal. Leaf size=173

$$\begin{aligned} & -\frac{(bc-ad)^2 \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}} + \frac{(bc-ad)^2 \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{3c^{2/3}d^{7/3}} \\ & -\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}} - \frac{bx(bc-2ad)}{d^2} + \frac{b^2x^4}{4d} \end{aligned}$$

[Out]  $-\left(\frac{b(b^2c - 2ad)x}{d^2} + \frac{b^2x^4}{4d} - \frac{(b^2c - a^2d)^2 \text{ArcTan}\left[\frac{c^{1/3} - 2d^{1/3}x}{\sqrt{3}c^{1/3}}\right]}{\sqrt{3}c^{2/3}d^{7/3}} + \frac{(b^2c - a^2d)^2 \text{Log}\left[\frac{c^{1/3} + d^{1/3}x}{3c^{2/3}d^{7/3}}\right]}{\sqrt{3}c^{2/3}d^{7/3}} - \frac{(b^2c - a^2d)^2 \text{Log}\left[\frac{c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2}{6c^{2/3}d^{7/3}}\right]}{\sqrt{3}c^{2/3}d^{7/3}}\right)$

**Rubi [A]** time = 0.27559, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$\begin{aligned} & -\frac{(bc-ad)^2 \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}} + \frac{(bc-ad)^2 \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{3c^{2/3}d^{7/3}} \\ & -\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}} - \frac{bx(bc-2ad)}{d^2} + \frac{b^2x^4}{4d} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^2/(c + d\*x^3), x]

[Out]  $-\left(\frac{b(b^2c - 2ad)x}{d^2} + \frac{b^2x^4}{4d} - \frac{(b^2c - a^2d)^2 \text{ArcTan}\left[\frac{c^{1/3} - 2d^{1/3}x}{\sqrt{3}c^{1/3}}\right]}{\sqrt{3}c^{2/3}d^{7/3}} + \frac{(b^2c - a^2d)^2 \text{Log}\left[\frac{c^{1/3} + d^{1/3}x}{3c^{2/3}d^{7/3}}\right]}{\sqrt{3}c^{2/3}d^{7/3}} - \frac{(b^2c - a^2d)^2 \text{Log}\left[\frac{c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2}{6c^{2/3}d^{7/3}}\right]}{\sqrt{3}c^{2/3}d^{7/3}}\right)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & \frac{b^2x^4}{4d} + \frac{(2ad-bc) \int b dx}{d^2} + \frac{(ad-bc)^2 \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{3c^{2/3}d^{7/3}} \\ & -\frac{(ad-bc)^2 \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}} - \frac{\sqrt{3}(ad-bc)^2 \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3} - \frac{2\sqrt[3]{dx}}{3}\right)}{\sqrt[3]{c}}\right)}{3c^{2/3}d^{7/3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*\*2/(d\*x\*\*3+c), x)

[Out]  $b^2x^4/(4d) + (2ad - bc) \text{Integral}(b, x)/d^2 + (a^2d - b^2c) \log(c^{1/3} + d^{1/3}x)/(3c^{2/3}d^{7/3}) - (a^2d - b^2c) \log(c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2)/(6c^{2/3}d^{7/3}) - \sqrt{3}(a^2d - b^2c) \operatorname{atan}(\sqrt{3}(c^{1/3}/3 - 2d^{1/3}x/3)/c^{1/3})/(3c^{2/3}d^{7/3})$

**Mathematica [A]** time = 0.170788, size = 167, normalized size = 0.97

$$\frac{-2(bc - ad)^2 \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right) - 12bc^{2/3}\sqrt[3]{dx}(bc - 2ad) + 4(bc - ad)^2 \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right) + 4\sqrt{3}(bc - ad)^2 \tan^{-1}\left(\frac{c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2}{12c^{2/3}d^{7/3}}\right)}{12c^{2/3}d^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^2/(c + d\*x^3), x]

[Out]  $(-12*b*c^{2/3}*d^{1/3}*(b*c - 2*a*d)*x + 3*b^2*c^{2/3}*d^{4/3}*x^4 + 4*\sqrt{3}*(b*c - a*d)^2*\text{ArcTan}[-c^{1/3} + 2*d^{1/3}*x]/(\sqrt{3}*c^{1/3})) + 4*(b*c - a*d)^2*\text{Log}[c^{1/3} + d^{1/3}*x] - 2*(b*c - a*d)^2*\text{Log}[c^{2/3} - c^{1/3}*d^{1/3}*x + d^{2/3}*x^2]/(12*c^{2/3}*d^{7/3})$

**Maple [B]** time = 0.004, size = 334, normalized size = 1.9

$$\begin{aligned} & \frac{b^2x^4}{4d} + 2\frac{abx}{d} - \frac{b^2xc}{d^2} + \frac{a^2}{3d} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & - \frac{2abc}{3d^2} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{b^2c^2}{3d^3} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & - \frac{a^2}{6d} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{abc}{3d^2} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & - \frac{b^2c^2}{6d^3} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}a^2}{3d} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & - \frac{2\sqrt{3}cab}{3d^2} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}b^2c^2}{3d^3} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^2/(d\*x^3+c), x)

[Out]  $1/4*b^2*x^4/d + 2*b/d*a*x - b^2/d^2*x*c + 1/3/d/(c/d)^{2/3}*\ln(x+(c/d)^{1/3})*a^2 - 2/3/d^2/(c/d)^{2/3}*\ln(x+(c/d)^{1/3})*c*a*b + 1/3/d^3/(c/d)^{2/3}*\ln(x+(c/d)^{1/3})*b^2*c^2 - 1/6/d/(c/d)^{2/3}*\ln(x^2-x*(c/d)^{1/3}+(c/d)^{2/3})*a^2 + 1/3/d^2/(c/d)^{2/3}*\ln(x^2-x*(c/d)^{1/3}+(c/d)^{2/3})*c*a*b - 1/6/d^3/(c/d)^{2/3}*\ln(x^2-x*(c/d)^{1/3}+(c/d)^{2/3})*b^2*c^2 + 1/3/d/(c/d)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(c/d)^{1/3}*x-1))*a^2 - 2/3/d^2/(c/d)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(c/d)^{1/3}*x-1))*c*a*b + 1/3/d^3/(c/d)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(c/d)^{1/3}*x-1))*b^2*c^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^2/(d\*x^3 + c), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.212159, size = 258, normalized size = 1.49

$$\frac{\sqrt{3} \left( 2 \sqrt{3} (b^2 c^2 - 2 abcd + a^2 d^2) \log \left( (c^2 d)^{\frac{2}{3}} x^2 - (c^2 d)^{\frac{1}{3}} cx + c^2 \right) - 4 \sqrt{3} (b^2 c^2 - 2 abcd + a^2 d^2) \log \left( (c^2 d)^{\frac{1}{3}} x + c \right) - 12 \right)}{36 (c^2 d)^{\frac{1}{3}} d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^2/(d\*x^3 + c),x, algorithm="fricas")

[Out] -1/36\*sqrt(3)\*(2\*sqrt(3)\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*log((c^2\*d)^(2/3)\*x^2 - (c^2\*d)^(1/3)\*c\*x + c^2) - 4\*sqrt(3)\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*log((c^2\*d)^(1/3)\*x + c) - 12\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*arctan(1/3\*(2\*sqrt(3)\*(c^2\*d)^(1/3)\*x - sqrt(3)\*c)/c) - 3\*sqrt(3)\*(b^2\*d\*x^4 - 4\*(b^2\*c - 2\*a\*b\*d)\*x)\*(c^2\*d)^(1/3))/((c^2\*d)^(1/3)\*d^2)

**Sympy [A]** time = 2.86124, size = 156, normalized size = 0.9

$$\frac{b^2 x^4}{4d} + \text{RootSum} \left( 27t^3 c^2 d^7 - a^6 d^6 + 6a^5 b c d^5 - 15a^4 b^2 c^2 d^4 + 20a^3 b^3 c^3 d^3 - 15a^2 b^4 c^4 d^2 + 6ab^5 c^5 d - b^6 c^6, \left( t \mapsto t \log \left( \frac{3tc}{a^2 d^2 - 2ab} \right) \right) \right) + \frac{x(2abd - b^2 c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*2/(d\*x\*\*3+c),x)

[Out] b\*\*2\*x\*\*4/(4\*d) + RootSum(27\*\_t\*\*3\*c\*\*2\*d\*\*7 - a\*\*6\*d\*\*6 + 6\*a\*\*5\*b\*c\*d\*\*5 - 15\*a\*\*4\*b\*\*2\*c\*\*2\*d\*\*4 + 20\*a\*\*3\*b\*\*3\*c\*\*3\*d\*\*3 - 15\*a\*\*2\*b\*\*4\*c\*\*4\*d\*\*2 + 6\*a\*b\*\*5\*c\*\*5\*d - b\*\*6\*c\*\*6, Lambda(\_t, \_t\*log(3\*\_t\*c\*d\*\*2/(a\*\*2\*d\*\*2 - 2\*a\*b\*c\*d + b\*\*2\*c\*\*2) + x))) + x\*(2\*a\*b\*d - b\*\*2\*c)/d\*\*2

**GIAC/XCAS [A]** time = 0.218936, size = 336, normalized size = 1.94

$$\frac{\sqrt{3} \left( (-cd^2)^{\frac{1}{3}} b^2 c^2 - 2 (-cd^2)^{\frac{1}{3}} abcd + (-cd^2)^{\frac{1}{3}} a^2 d^2 \right) \arctan \left( \frac{\sqrt{3} \left( 2x + \left( -\frac{c}{d} \right)^{\frac{1}{3}} \right)}{3 \left( -\frac{c}{d} \right)^{\frac{1}{3}}} \right)}{3cd^3} + \frac{\left( (-cd^2)^{\frac{1}{3}} b^2 c^2 - 2 (-cd^2)^{\frac{1}{3}} abcd + (-cd^2)^{\frac{1}{3}} a^2 d^2 \right) \ln \left( x^2 + x \left( -\frac{c}{d} \right)^{\frac{1}{3}} + \left( -\frac{c}{d} \right)^{\frac{2}{3}} \right)}{6cd^3} - \frac{(b^2 c^2 d^2 - 2 abcd^3 + a^2 d^4) \left( -\frac{c}{d} \right)^{\frac{1}{3}} \ln \left( \left| x - \left( -\frac{c}{d} \right)^{\frac{1}{3}} \right| \right)}{3cd^4} + \frac{b^2 d^3 x^4 - 4 b^2 cd^2 x + 8 abd^3 x}{4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^2/(d\*x^3 + c),x, algorithm="giac")

[Out] 1/3\*sqrt(3)\*((-c\*d^2)^(1/3)\*b^2\*c^2 - 2\*(-c\*d^2)^(1/3)\*a\*b\*c\*d + (-c\*d^2)^(1/3)\*a^2\*d^2)\*arctan(1/3\*sqrt(3)\*(2\*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(c\*d^3) + 1/6\*((-c\*d^2)^(1/3)\*b^2\*c^2 - 2\*(-c\*d^2)^(1/3)\*a\*b\*c\*d + (-c\*d^2)^(1/3)\*a^2\*d^2)\*ln(x^2 + x\*(-c/d)^(1/3) + (-c/d)^(2/3))/(c\*d^3) - 1/3\*(b^2\*c^2\*d^2 - 2\*a\*b\*c\*d^3 + a^2\*d^4)\*(-c/d)^(1/3)\*ln(abs(x - (-c/d)^(1/3)))/(c\*d^4) + 1/4\*(b^2\*d^3\*x^4 - 4\*b^2\*c\*d^2\*x + 8\*abd^3\*x)/d^4

$$3.12 \quad \int \frac{(a+bx^3)^2}{(c+dx^3)^2} dx$$

**Optimal.** Leaf size=203

$$\frac{(bc-ad)(ad+2bc) \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}} - \frac{2(bc-ad)(ad+2bc) \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{9c^{5/3}d^{7/3}}$$

$$+ \frac{2(bc-ad)(ad+2bc) \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{7/3}} + \frac{x(bc-ad)^2}{3cd^2(c+dx^3)} + \frac{b^2x}{d^2}$$

[Out] (b^2\*x)/d^2 + ((b\*c - a\*d)^2\*x)/(3\*c\*d^2\*(c + d\*x^3)) + (2\*(b\*c - a\*d)\*(2\*b\*c + a\*d)\*ArcTan[(c^(1/3) - 2\*d^(1/3)\*x)/(Sqrt[3]\*c^(1/3))])/(3\*Sqrt[3]\*c^(5/3)\*d^(7/3)) - (2\*(b\*c - a\*d)\*(2\*b\*c + a\*d)\*Log[c^(1/3) + d^(1/3)\*x])/(9\*c^(5/3)\*d^(7/3)) + ((b\*c - a\*d)\*(2\*b\*c + a\*d)\*Log[c^(2/3) - c^(1/3)\*d^(1/3)\*x + d^(2/3)\*x^2])/(9\*c^(5/3)\*d^(7/3))

**Rubi [A]** time = 0.495065, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\frac{(bc-ad)(ad+2bc) \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}} - \frac{2(bc-ad)(ad+2bc) \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{9c^{5/3}d^{7/3}}$$

$$+ \frac{2(bc-ad)(ad+2bc) \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{7/3}} + \frac{x(bc-ad)^2}{3cd^2(c+dx^3)} + \frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^2/(c + d\*x^3)^2, x]

[Out] (b^2\*x)/d^2 + ((b\*c - a\*d)^2\*x)/(3\*c\*d^2\*(c + d\*x^3)) + (2\*(b\*c - a\*d)\*(2\*b\*c + a\*d)\*ArcTan[(c^(1/3) - 2\*d^(1/3)\*x)/(Sqrt[3]\*c^(1/3))])/(3\*Sqrt[3]\*c^(5/3)\*d^(7/3)) - (2\*(b\*c - a\*d)\*(2\*b\*c + a\*d)\*Log[c^(1/3) + d^(1/3)\*x])/(9\*c^(5/3)\*d^(7/3)) + ((b\*c - a\*d)\*(2\*b\*c + a\*d)\*Log[c^(2/3) - c^(1/3)\*d^(1/3)\*x + d^(2/3)\*x^2])/(9\*c^(5/3)\*d^(7/3))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int b^2 dx}{d^2} + \frac{x(ad-bc)^2}{3cd^2(c+dx^3)} + \frac{2(ad-bc)(ad+2bc) \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{9c^{5/3}d^{7/3}}$$

$$- \frac{(ad-bc)(ad+2bc) \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}} - \frac{2\sqrt{3}(ad-bc)(ad+2bc) \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3} - 2\frac{\sqrt[3]{dx}}{3}\right)}{\sqrt[3]{c}}\right)}{9c^{5/3}d^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*\*2/(d\*x\*\*3+c)\*\*2, x)

[Out] Integral(b\*\*2, x)/d\*\*2 + x\*(a\*d - b\*c)\*\*2/(3\*c\*d\*\*2\*(c + d\*x\*\*3)) + 2\*(a\*d - b\*c)\*(a\*d + 2\*b\*c)\*log(c\*\*(1/3) + d\*\*(1/3)\*x)/(9\*c\*\*(5/3)\*d\*\*(7/3)) - (a\*d - b\*c)\*(a\*d + 2\*b\*c)\*log(c\*\*(2/3) - c\*\*(1/3)\*d\*\*(1/3)\*x + d\*\*(2/3)\*x\*\*2)/(9\*c\*\*(5/3)\*d\*\*(7/3)) - 2\*sqrt(3)\*(a\*d - b\*c)\*(a\*d + 2\*b\*c)\*atan(sqrt(3)\*(c\*\*(1/3)/3 - 2\*d\*\*(1/3)\*x/3)/c\*\*(1/3))/(9\*c\*\*(5/3)\*d\*\*(7/3))

**Mathematica [A]** time = 0.379356, size = 210, normalized size = 1.03

$$\frac{2(-a^2d^2-abcd+2b^2c^2)\log(\sqrt[3]{c}+\sqrt[3]{d}x)}{c^{5/3}} + \frac{2\sqrt{3}(-a^2d^2-abcd+2b^2c^2)\tan^{-1}\left(\frac{1-2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{c^{5/3}} + \frac{(-a^2d^2-abcd+2b^2c^2)\log(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2)}{c^{5/3}} + \frac{3\sqrt[3]{d}x}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^2/(c + d\*x^3)^2,x]

[Out]  $(9*b^2*d^{1/3}*x + (3*d^{1/3}*(b*c - a*d)^2*x)/(c*(c + d*x^3)) + (2*\sqrt{3}*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*\text{ArcTan}[(1 - (2*d^{1/3})x)/c^{1/3}]/\sqrt{3}]/c^{5/3} - (2*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*\text{Log}[c^{1/3} + d^{1/3}*x])/c^{5/3} + ((2*b^2*c^2 - a*b*c*d - a^2*d^2)*\text{Log}[c^{2/3} - c^{1/3}*d^{1/3}*x + d^{2/3}*x^2])/c^{5/3})/(9*d^{7/3})$

**Maple [B]** time = 0.014, size = 367, normalized size = 1.8

$$\begin{aligned} & \frac{b^2x}{d^2} + \frac{xa^2}{3c(dx^3+c)} - \frac{2axb}{3d(dx^3+c)} + \frac{cxb^2}{3d^2(dx^3+c)} + \frac{2a^2}{9cd} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & + \frac{2ab}{9d^2} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{4b^2c}{9d^3} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & - \frac{a^2}{9cd} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{ab}{9d^2} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & + \frac{2b^2c}{9d^3} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{2\sqrt{3}a^2}{9cd} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & + \frac{2\sqrt{3}ab}{9d^2} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{4c\sqrt{3}b^2}{9d^3} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^2/(d\*x^3+c)^2,x)

[Out]  $b^2*x/d^2+1/3/c*x/(d*x^3+c)*a^2-2/3/d*x/(d*x^3+c)*a*b+1/3/d^2*c*x/(d*x^3+c)*b^2+2/9/d/c/(c/d)^{2/3}*ln(x+(c/d)^{1/3})*a^2+2/9/d^2/(c/d)^{2/3}*ln(x+(c/d)^{1/3})*a*b-4/9/d^3*c/(c/d)^{2/3}*ln(x+(c/d)^{1/3})*b^2-1/9/d/c/(c/d)^{2/3}*ln(x^2-x*(c/d)^{1/3}+(c/d)^{2/3})*a^2-1/9/d^2/(c/d)^{2/3}*ln(x^2-x*(c/d)^{1/3}+(c/d)^{2/3})*a*b+2/9/d^3*c/(c/d)^{2/3}*ln(x^2-x*(c/d)^{1/3}+(c/d)^{2/3})*b^2+2/9/d/c/(c/d)^{2/3}*3^{1/2}*arctan(1/3*3^{1/2}*(2/(c/d)^{1/3}*x-1))*a^2+2/9/d^2/(c/d)^{2/3}*3^{1/2}*arctan(1/3*3^{1/2}*(2/(c/d)^{1/3}*x-1))*a*b-4/9/d^3*c/(c/d)^{2/3}*3^{1/2}*arctan(1/3*3^{1/2}*(2/(c/d)^{1/3}*x-1))*b^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^2/(d\*x^3 + c)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.218018, size = 435, normalized size = 2.14

$$\sqrt{3} \left( \sqrt{3} (2b^2c^3 - abc^2d - a^2cd^2 + (2b^2c^2d - abcd^2 - a^2d^3)x^3) \log \left( (c^2d)^{\frac{2}{3}} x^2 - (c^2d)^{\frac{1}{3}} cx + c^2 \right) - 2\sqrt{3} (2b^2c^3 - abc^2d - a^2cd^2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^2/(d\*x^3 + c)^2,x, algorithm="fricas")

$$\begin{aligned} & [Out] \frac{1}{27} \sqrt{3} \left( \sqrt{3} (2b^2c^3 - a^2cd^2 + (2b^2c^2d - abcd^2 - a^2d^3)x^3) \log \left( (c^2d)^{\frac{2}{3}} x^2 - (c^2d)^{\frac{1}{3}} cx + c^2 \right) \right. \\ & - 2\sqrt{3} (2b^2c^3 - abc^2d - a^2cd^2 + (2b^2c^2d - abcd^2 - a^2d^3)x^3) \log \left( (c^2d)^{\frac{1}{3}} x + c \right) \\ & - 6(2b^2c^3 - a^2cd^2 + (2b^2c^2d - abcd^2 - a^2d^3)x^3) \arctan \left( \frac{1}{3} \sqrt{3} (c^2d)^{\frac{1}{3}} x - \sqrt{3} (3c)/c \right) \\ & \left. + 3\sqrt{3} (3b^2c^2d^2x^4 + (4b^2c^2 - 2ab^2c^2d + a^2d^2)x) (c^2d)^{\frac{1}{3}} \right) / \left( (c^2d^3x^3 + c^2d^2) (c^2d)^{\frac{1}{3}} \right) \end{aligned}$$

**Sympy** [A] time = 4.8321, size = 189, normalized size = 0.93

$$\frac{b^2x}{d^2} + \frac{x(a^2d^2 - 2abcd + b^2c^2)}{3c^2d^2 + 3cd^3x^3} + \text{RootSum} \left( 729t^3c^5d^7 - 8a^6d^6 - 24a^5bcd^5 + 24a^4b^2c^2d^4 + 88a^3b^3c^3d^3 - 48a^2b^4c^4d^2 - 96ab^5c^5d + 64b^6c^6, \left( t \mapsto t \log \left( \frac{2a^2d^2 + 3cd^3x^3}{2a^2d^2 + 3cd^3x^3} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*2/(d\*x\*\*3+c)\*\*2,x)

$$\begin{aligned} & [Out] \frac{b^2x}{d^2} + \frac{x(a^2d^2 - 2abcd + b^2c^2)}{(3c^2d^2 + 3cd^3x^3)} + \text{RootSum} \left( 729\_t^3c^5d^7 - 8a^6d^6 - 24a^5bcd^5 + 24a^4b^2c^2d^4 + 88a^3b^3c^3d^3 - 48a^2b^4c^4d^2 - 96ab^5c^5d + 64b^6c^6, \right. \\ & \text{Lambda} \left( \_t, \_t \log \left( 9\_t^2c^2d^2 / (2a^2d^2 + 2ab^2c^2d - 4b^2c^2) + x \right) \right) \left. \right) \end{aligned}$$

**GIAC/XCAS** [A] time = 0.219868, size = 358, normalized size = 1.76

$$\begin{aligned} & \frac{b^2x}{d^2} + \frac{2(2b^2c^2 - abcd - a^2d^2) \left( -\frac{c}{d} \right)^{\frac{1}{3}} \ln \left( \left| x - \left( -\frac{c}{d} \right)^{\frac{1}{3}} \right| \right)}{9c^2d^2} \\ & - \frac{2\sqrt{3} \left( 2(-cd^2)^{\frac{1}{3}} b^2c^2 - (-cd^2)^{\frac{1}{3}} abcd - (-cd^2)^{\frac{1}{3}} a^2d^2 \right) \arctan \left( \frac{\sqrt{3} \left( 2x + \left( -\frac{c}{d} \right)^{\frac{1}{3}} \right)}{3 \left( -\frac{c}{d} \right)^{\frac{1}{3}}} \right)}{9c^2d^3} \\ & + \frac{b^2c^2x - 2abcdx + a^2d^2x}{3(dx^3 + c)cd^2} \\ & - \frac{\left( 2(-cd^2)^{\frac{1}{3}} b^2c^2 - (-cd^2)^{\frac{1}{3}} abcd - (-cd^2)^{\frac{1}{3}} a^2d^2 \right) \ln \left( x^2 + x \left( -\frac{c}{d} \right)^{\frac{1}{3}} + \left( -\frac{c}{d} \right)^{\frac{2}{3}} \right)}{9c^2d^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^2/(d\*x^3 + c)^2,x, algorithm="giac")

```
[Out] b^2*x/d^2 + 2/9*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*(-c/d)^(1/3)*ln(a
bs(x - (-c/d)^(1/3)))/(c^2*d^2) - 2/9*sqrt(3)*(2*(-c*d^2)^(1/3)*b
^2*c^2 - (-c*d^2)^(1/3)*a*b*c*d - (-c*d^2)^(1/3)*a^2*d^2)*arctan(
1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(c^2*d^3) + 1/3*(b
^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^3 + c)*c*d^2) - 1/9*(2*
(-c*d^2)^(1/3)*b^2*c^2 - (-c*d^2)^(1/3)*a*b*c*d - (-c*d^2)^(1/3)*
a^2*d^2)*ln(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(c^2*d^3)
```

$$3.13 \quad \int \frac{(a+bx^3)^2}{(c+dx^3)^3} dx$$

**Optimal.** Leaf size=258

$$\frac{(5a^2d^2 + 2abcd + 2b^2c^2) \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}} + \frac{(5a^2d^2 + 2abcd + 2b^2c^2) \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{27c^{8/3}d^{7/3}}$$

$$- \frac{(5a^2d^2 + 2abcd + 2b^2c^2) \tan^{-1}\left(\frac{\sqrt[3]{c-2}\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{7/3}} - \frac{x(bc-ad)(5ad+4bc)}{18c^2d^2(c+dx^3)} - \frac{x(a+bx^3)(bc-ad)}{6cd(c+dx^3)^2}$$

[Out]  $-\left((b^*c - a^*d) * x^*(a + b^*x^3)\right) / \left(6^*c^*d^*(c + d^*x^3)^2\right) - \left((b^*c - a^*d) * (4^*b^*c + 5^*a^*d) * x\right) / \left(18^*c^2 * d^2 * (c + d^*x^3)\right) - \left(\left(2^*b^2 * c^2 + 2^*a^*b^*c^*d + 5^*a^2 * d^2\right) * \text{ArcTan}\left[\left(c^{1/3} - 2^*d^{1/3} * x\right) / \left(\text{Sqrt}[3] * c^{1/3}\right)\right]\right) / \left(9^* \text{Sqrt}[3] * c^{8/3} * d^{7/3}\right) + \left(\left(2^*b^2 * c^2 + 2^*a^*b^*c^*d + 5^*a^2 * d^2\right) * \text{Log}\left[c^{1/3} + d^{1/3} * x\right]\right) / \left(27^*c^2 * d^2 * (c + dx^3)\right) - \left(\left(2^*b^2 * c^2 + 2^*a^*b^*c^*d + 5^*a^2 * d^2\right) * \text{Log}\left[c^{2/3} - c^{1/3} * d^{1/3} * x + d^{2/3} * x^2\right]\right) / \left(54^*c^{8/3} * d^{7/3}\right)$

**Rubi [A]** time = 0.477267, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\frac{(5a^2d^2 + 2abcd + 2b^2c^2) \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}} + \frac{(5a^2d^2 + 2abcd + 2b^2c^2) \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{27c^{8/3}d^{7/3}}$$

$$- \frac{(5a^2d^2 + 2abcd + 2b^2c^2) \tan^{-1}\left(\frac{\sqrt[3]{c-2}\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{7/3}} - \frac{x(bc-ad)(5ad+4bc)}{18c^2d^2(c+dx^3)} - \frac{x(a+bx^3)(bc-ad)}{6cd(c+dx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^2/(c + d\*x^3)^3, x]

[Out]  $-\left((b^*c - a^*d) * x^*(a + b^*x^3)\right) / \left(6^*c^*d^*(c + d^*x^3)^2\right) - \left((b^*c - a^*d) * (4^*b^*c + 5^*a^*d) * x\right) / \left(18^*c^2 * d^2 * (c + d^*x^3)\right) - \left(\left(2^*b^2 * c^2 + 2^*a^*b^*c^*d + 5^*a^2 * d^2\right) * \text{ArcTan}\left[\left(c^{1/3} - 2^*d^{1/3} * x\right) / \left(\text{Sqrt}[3] * c^{1/3}\right)\right]\right) / \left(9^* \text{Sqrt}[3] * c^{8/3} * d^{7/3}\right) + \left(\left(2^*b^2 * c^2 + 2^*a^*b^*c^*d + 5^*a^2 * d^2\right) * \text{Log}\left[c^{1/3} + d^{1/3} * x\right]\right) / \left(27^*c^2 * d^2 * (c + dx^3)\right) - \left(\left(2^*b^2 * c^2 + 2^*a^*b^*c^*d + 5^*a^2 * d^2\right) * \text{Log}\left[c^{2/3} - c^{1/3} * d^{1/3} * x + d^{2/3} * x^2\right]\right) / \left(54^*c^{8/3} * d^{7/3}\right)$

**Rubi in Sympy [A]** time = 53.106, size = 243, normalized size = 0.94

$$\frac{x(a+bx^3)(ad-bc)}{6cd(c+dx^3)^2} + \frac{x(ad-bc)(5ad+4bc)}{18c^2d^2(c+dx^3)} + \frac{(ad(5ad+bc) + bc(ad+2bc)) \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{27c^{8/3}d^{7/3}}$$

$$- \frac{(ad(5ad+bc) + bc(ad+2bc)) \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}}$$

$$- \frac{\sqrt{3}(ad(5ad+bc) + bc(ad+2bc)) \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3} - 2\frac{\sqrt[3]{dx}}{3}\right)}{\sqrt[3]{c}}\right)}{27c^{8/3}d^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*\*2/(d\*x\*\*3+c)\*\*3, x)

[Out]  $x^*(a + b^*x^3) * (a^*d - b^*c) / (6^*c^*d^*(c + d^*x^3)^2) + x^*(a^*d - b^*c) * (5^*a^*d + 4^*b^*c) / (18^*c^2 * d^2 * (c + d^*x^3)) + (a^*d * (5^*a^*d + b^*c$



) + b\*c\*(a\*d + 2\*b\*c))\*log(c\*\*(1/3) + d\*\*(1/3)\*x)/(27\*c\*\*(8/3)\*d\*(7/3)) - (a\*d\*(5\*a\*d + b\*c) + b\*c\*(a\*d + 2\*b\*c))\*log(c\*\*(2/3) - c\*\*(1/3)\*d\*\*(1/3)\*x + d\*\*(2/3)\*x\*\*2)/(54\*c\*\*(8/3)\*d\*\*(7/3)) - sqrt(3)\*(a\*d\*(5\*a\*d + b\*c) + b\*c\*(a\*d + 2\*b\*c))\*atan(sqrt(3)\*(c\*\*(1/3)/3 - 2\*d\*\*(1/3)\*x/3)/c\*\*(1/3))/(27\*c\*\*(8/3)\*d\*\*(7/3))

**Mathematica [A]** time = 0.465576, size = 234, normalized size = 0.91

$$2(5a^2d^2 + 2abcd + 2b^2c^2) \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right) - 2\sqrt{3}(5a^2d^2 + 2abcd + 2b^2c^2) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{dx}}{\sqrt[3]{c}}\right) - \frac{3c^{2/3}\sqrt[3]{dx}(-a^2d^2(8c+5dx^3)+2abcd)}{(c+dx^3)^2}$$


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$$54c^{8/3}d^{7/3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^2/(c + d\*x^3)^3, x]

[Out] ((-3\*c^(2/3)\*d^(1/3)\*x\*(2\*a\*b\*c\*d\*(2\*c - d\*x^3) - a^2\*d^2\*(8\*c + 5\*d\*x^3) + b^2\*c^2\*(4\*c + 7\*d\*x^3)))/(c + d\*x^3)^2 - 2\*Sqrt[3]\*(2\*b^2\*c^2 + 2\*a\*b\*c\*d + 5\*a^2\*d^2)\*ArcTan[(1 - (2\*d^(1/3)\*x)/c^(1/3))/Sqrt[3]] + 2\*(2\*b^2\*c^2 + 2\*a\*b\*c\*d + 5\*a^2\*d^2)\*Log[c^(1/3) + d^(1/3)\*x] - (2\*b^2\*c^2 + 2\*a\*b\*c\*d + 5\*a^2\*d^2)\*Log[c^(2/3) - c^(1/3)\*d^(1/3)\*x + d^(2/3)\*x^2]/(54\*c^(8/3)\*d^(7/3))

**Maple [A]** time = 0.015, size = 388, normalized size = 1.5

$$\frac{1}{(dx^3 + c)^2} \left( \frac{(5a^2d^2 + 2cabd - 7b^2c^2)x^4}{18c^2d} + \frac{(4a^2d^2 - 2cabd - 2b^2c^2)x}{9d^2c} \right) + \frac{5a^2}{27c^2d} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{2ab}{27d^2c} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{2b^2}{27d^3} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{5a^2}{54c^2d} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{ab}{27d^2c} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{b^2}{27d^3} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{5\sqrt{3}a^2}{27c^2d} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{2\sqrt{3}ab}{27d^2c} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{2\sqrt{3}b^2}{27d^3} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^2/(d\*x^3+c)^3, x)

[Out] (1/18\*(5\*a^2\*d^2+2\*a\*b\*c\*d-7\*b^2\*c^2)/c^2/d\*x^4+2/9\*(2\*a^2\*d^2-a\*b\*c\*d-b^2\*c^2)/d^2/c\*x)/(d\*x^3+c)^2+5/27/c^2/d/(c/d)^(2/3)\*ln(x+(c/d)^(1/3))\*a^2+2/27/c/d^2/(c/d)^(2/3)\*ln(x+(c/d)^(1/3))\*a\*b+2/27/d^3/(c/d)^(2/3)\*ln(x+(c/d)^(1/3))\*b^2-5/54/c^2/d/(c/d)^(2/3)\*ln(x^2-x\*(c/d)^(1/3)+(c/d)^(2/3))\*a^2-1/27/c/d^2/(c/d)^(2/3)\*ln(x^2-x\*(c/d)^(1/3)+(c/d)^(2/3))\*a\*b-1/27/d^3/(c/d)^(2/3)\*ln(x^2-x\*(c/d)^(1/3)+(c/d)^(2/3))\*b^2+5/27/c^2/d/(c/d)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(c/d)^(1/3)\*x-1))\*a^2+2/27/c/d^2/(c/d)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(c/d)^(1/3)\*x-1))\*a\*b+2/27/d^3/(c/d)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(c/d)^(1/3)\*x-1))\*b^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^2/(d\*x^3 + c)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.218697, size = 637, normalized size = 2.47

$$\sqrt{3} \left( \sqrt{3} \left( (2b^2c^2d^2 + 2abcd^3 + 5a^2d^4)x^6 + 2b^2c^4 + 2abc^3d + 5a^2c^2d^2 + 2(2b^2c^3d + 2abc^2d^2 + 5a^2cd^3)x^3 \right) \log \left( (c^2d)^{\frac{2}{3}} x^3 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^2/(d\*x^3 + c)^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/162 \sqrt{3} \left( \sqrt{3} \left( (2b^2c^2d^2 + 2abc^3d + 5a^2d^4) \right. \right. \\ & \left. \left. x^6 + 2b^2c^4 + 2abc^3d + 5a^2c^2d^2 + 2(2b^2c^3d + 2abc^2d^2 + 5a^2cd^3) \right. \right. \\ & \left. \left. x^3 \right) \log \left( (c^2d)^{2/3} x^2 - (c^2d)^{1/3} c^2x + c^2 \right) - 2 \sqrt{3} \left( (2b^2c^2d^2 + 2abc^3d + 5a^2d^4) \right. \right. \\ & \left. \left. x^6 + 2b^2c^4 + 2abc^3d + 5a^2c^2d^2 + 2(2b^2c^3d + 2abc^2d^2 + 5a^2cd^3) \right. \right. \\ & \left. \left. x^3 \right) \log \left( (c^2d)^{1/3} x + c \right) - 6 \left( (2b^2c^2d^2 + 2abc^3d + 5a^2d^4) \right. \right. \\ & \left. \left. x^6 + 2b^2c^4 + 2abc^3d + 5a^2c^2d^2 + 2(2b^2c^3d + 2abc^2d^2 + 5a^2cd^3) \right. \right. \\ & \left. \left. x^3 \right) \arctan \left( \frac{1}{3} \left( 2 \sqrt{3} (c^2d)^{1/3} x - \sqrt{3} c \right) / c \right) + 3 \sqrt{3} \left( (7b^2c^2d - 2abc^3d - 5a^2d^3) \right. \right. \\ & \left. \left. x^4 + 4(b^2c^3 + abc^2d - 2a^2cd^2) x \right) (c^2d)^{1/3} \right) / \left( (c^2d)^{1/3} \right) \end{aligned}$$

**Sympy** [A] time = 7.43143, size = 233, normalized size = 0.9

$$\frac{x^4(5a^2d^3 + 2abcd^2 - 7b^2c^2d) + x(8a^2cd^2 - 4abc^2d - 4b^2c^3)}{18c^4d^2 + 36c^3d^3x^3 + 18c^2d^4x^6} + \text{RootSum} \left( 19683t^3c^8d^7 - 125a^6d^6 - 150a^5bcd^5 - 210a^4b^2c^2d^4 - 128a^3b^3c^3d^3 - 84a^2b^4c^4d^2 - 24ab^5c^5d - 8b^6c^6, \left( t \mapsto t \log \left( \frac{27t^3c^8d^7 - 125a^6d^6 - 150a^5bcd^5 - 210a^4b^2c^2d^4 - 128a^3b^3c^3d^3 - 84a^2b^4c^4d^2 - 24ab^5c^5d - 8b^6c^6}{(5a^2d^2 + 2abc^2d + 2b^2c^2)^2} \right) + x \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*2/(d\*x\*\*3+c)\*\*3,x)

[Out] 
$$\begin{aligned} & (x^4(5a^2d^3 + 2abc^2d - 7b^2c^2d) + x(8a^2cd^2 - 4abc^2d - 4b^2c^3)) / (18c^4d^2 + 36c^3d^3x^3 + 18c^2d^4x^6) \\ & + \text{RootSum}(19683\_t^3c^8d^7 - 125a^6d^6 - 150a^5bcd^5 - 210a^4b^2c^2d^4 - 128a^3b^3c^3d^3 - 84a^2b^4c^4d^2 - 24ab^5c^5d - 8b^6c^6, \\ & \text{Lambda}(\_t, \_t \log(27\_t^3c^8d^7 / (5a^2d^2 + 2abc^2d + 2b^2c^2)^2) + x)) \end{aligned}$$

**GIAC/XCAS [A]** time = 0.222457, size = 400, normalized size = 1.55

$$\frac{(2b^2c^2 + 2abcd + 5a^2d^2) \left(-\frac{c}{d}\right)^{\frac{1}{3}} \ln\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{27c^3d^2} + \frac{\sqrt{3}\left(2(-cd^2)^{\frac{1}{3}}b^2c^2 + 2(-cd^2)^{\frac{1}{3}}abcd + 5(-cd^2)^{\frac{1}{3}}a^2d^2\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27c^3d^3} + \frac{\left(2(-cd^2)^{\frac{1}{3}}b^2c^2 + 2(-cd^2)^{\frac{1}{3}}abcd + 5(-cd^2)^{\frac{1}{3}}a^2d^2\right) \ln\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54c^3d^3} - \frac{7b^2c^2dx^4 - 2abcd^2x^4 - 5a^2d^3x^4 + 4b^2c^3x + 4abc^2dx - 8a^2cd^2x}{18(dx^3 + c)^2c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^2/(d\*x^3 + c)^3,x, algorithm="giac")

[Out]  $-1/27*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*(-c/d)^{(1/3)}*\ln(\text{abs}(x - (-c/d)^{(1/3)}))/(c^3*d^2) + 1/27*\text{sqrt}(3)*(2*(-c*d^2)^{(1/3)}*b^2*c^2 + 2*(-c*d^2)^{(1/3)}*a*b*c*d + 5*(-c*d^2)^{(1/3)}*a^2*d^2)*\arctan(1/3*\text{sqrt}(3)*(2*x + (-c/d)^{(1/3)})/(-c/d)^{(1/3)})/(c^3*d^3) + 1/54*(2*(-c*d^2)^{(1/3)}*b^2*c^2 + 2*(-c*d^2)^{(1/3)}*a*b*c*d + 5*(-c*d^2)^{(1/3)}*a^2*d^2)*\ln(x^2 + x*(-c/d)^{(1/3)} + (-c/d)^{(2/3)})/(c^3*d^3) - 1/18*(7*b^2*c^2*d*x^4 - 2*a*b*c*d^2*x^4 - 5*a^2*d^3*x^4 + 4*b^2*c^3*x + 4*a*b*c^2*d*x - 8*a^2*c*d^2*x)/((d*x^3 + c)^2*c^2*d^2)$

$$3.14 \quad \int \frac{(c+dx^3)^4}{a+bx^3} dx$$

**Optimal.** Leaf size=252

$$\begin{aligned} & -\frac{(bc-ad)^4 \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}} + \frac{(bc-ad)^4 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{13/3}} \\ & -\frac{(bc-ad)^4 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{13/3}} + \frac{dx(2bc-ad)(a^2d^2-2abcd+2b^2c^2)}{b^4} \\ & + \frac{d^2x^4(a^2d^2-4abcd+6b^2c^2)}{4b^3} + \frac{d^3x^7(4bc-ad)}{7b^2} + \frac{d^4x^{10}}{10b} \end{aligned}$$

[Out]  $(d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^4)/(4*b^3) + (d^3*(4*b*c - a*d)*x^7)/(7*b^2) + (d^4*x^{10})/(10*b) - ((b*c - a*d)^4*ArcTan[(a^{1/3} - 2*b^{1/3}*x)/(Sqrt[3]*a^{1/3})])/(Sqrt[3]*a^{2/3}*b^{13/3}) + ((b*c - a*d)^4*Log[a^{1/3} + b^{1/3}*x])/(3*a^{2/3}*b^{13/3}) - ((b*c - a*d)^4*Log[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(6*a^{2/3}*b^{13/3})$

**Rubi [A]** time = 0.415438, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$\begin{aligned} & -\frac{(bc-ad)^4 \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}} + \frac{(bc-ad)^4 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{13/3}} \\ & -\frac{(bc-ad)^4 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{13/3}} + \frac{dx(2bc-ad)(a^2d^2-2abcd+2b^2c^2)}{b^4} \\ & + \frac{d^2x^4(a^2d^2-4abcd+6b^2c^2)}{4b^3} + \frac{d^3x^7(4bc-ad)}{7b^2} + \frac{d^4x^{10}}{10b} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^4/(a + b\*x^3), x]

[Out]  $(d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^4)/(4*b^3) + (d^3*(4*b*c - a*d)*x^7)/(7*b^2) + (d^4*x^{10})/(10*b) - ((b*c - a*d)^4*ArcTan[(a^{1/3} - 2*b^{1/3}*x)/(Sqrt[3]*a^{1/3})])/(Sqrt[3]*a^{2/3}*b^{13/3}) + ((b*c - a*d)^4*Log[a^{1/3} + b^{1/3}*x])/(3*a^{2/3}*b^{13/3}) - ((b*c - a*d)^4*Log[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(6*a^{2/3}*b^{13/3})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & \frac{d^4x^{10}}{10b} - \frac{d^3x^7(ad-4bc)}{7b^2} + \frac{d^2x^4(a^2d^2-4abcd+6b^2c^2)}{4b^3} \\ & - \frac{(ad-2bc)(a^2d^2-2abcd+2b^2c^2) \int dx}{b^4} + \frac{(ad-bc)^4 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{13/3}} \\ & - \frac{(ad-bc)^4 \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}} - \frac{\sqrt{3}(ad-bc)^4 \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} - 2\frac{\sqrt[3]{bx}}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{2/3}b^{13/3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((d*x**3+c)**4/(b*x**3+a),x)`

[Out]  $d^{*4}x^{*10}/(10*b) - d^{*3}x^{*7}(a*d - 4*b*c)/(7*b^{*2}) + d^{*2}x^{*4}(a^{*2}d^{*2} - 4*a*b*c*d + 6*b^{*2}c^{*2})/(4*b^{*3}) - (a*d - 2*b*c)*(a^{*2}d^{*2} - 2*a*b*c*d + 2*b^{*2}c^{*2})*Integral(d, x)/b^{*4} + (a*d - b*c)^{*4}*\log(a^{*}(1/3) + b^{*}(1/3)*x)/(3*a^{*}(2/3)*b^{*}(13/3)) - (a*d - b*c)^{*4}*\log(a^{*}(2/3) - a^{*}(1/3)*b^{*}(1/3)*x + b^{*}(2/3)*x^{*2})/(6*a^{*}(2/3)*b^{*}(13/3)) - \sqrt{3}*(a*d - b*c)^{*4}*atan(\sqrt{3}*(a^{*}(1/3)/3 - 2*b^{*}(1/3)*x/3)/a^{*}(1/3))/(3*a^{*}(2/3)*b^{*}(13/3))$

**Mathematica [A]** time = 0.199474, size = 253, normalized size = 1.

$$\frac{-\frac{70(bc-ad)^4 \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx+b^{2/3}x^2}\right)}{a^{2/3}} + \frac{140(bc-ad)^4 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{a^{2/3}} + \frac{140\sqrt{3}(bc-ad)^4 \tan^{-1}\left(\frac{\sqrt[3]{bx} - \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}} + 105b^{4/3}d^2x^4(a^2d^2 - 4abcd + 420b^{13/3})}{420b^{13/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c + d*x^3)^4/(a + b*x^3),x]`

[Out]  $(420*b^{1/3}*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x + 105*b^{4/3}*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^4 + 60*b^{7/3}*d^3*(4*b*c - a*d)*x^7 + 42*b^{10/3}*d^4*x^{10} + (140*\sqrt[3]{a}*(b*c - a*d)^4*\text{ArcTan}\left[\frac{-a^{1/3} + 2*b^{1/3}*x}{\sqrt[3]{a} + b^{1/3}*x}\right])/a^{2/3} + (140*(b*c - a*d)^4*\text{Log}[a^{1/3} + b^{1/3}*x])/a^{2/3} - (70*(b*c - a*d)^4*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/a^{2/3})/(420*b^{13/3})$

**Maple [B]** time = 0.006, size = 661, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)^4/(b*x^3+a),x)`

[Out]  $-4/3/b^2/(a/b)^{2/3}*\ln(x+(a/b)^{1/3})*a*c^3*d+2/3/b^4/(a/b)^{2/3}*\ln(x^2-x*(a/b)^{1/3}+(a/b)^{2/3})*a^3*c*d^3+2/3/b^2/(a/b)^{2/3}*\ln(x^2-x*(a/b)^{1/3}+(a/b)^{2/3})*a*c^3*d+1/3/b^5/(a/b)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3})*x-1)*a^4*d^4-1/b^3/(a/b)^{2/3}*\ln(x^2-x*(a/b)^{1/3}+(a/b)^{2/3})*a^2*c^2*d^2-4/3/b^4/(a/b)^{2/3}*\ln(x+(a/b)^{1/3})*a^3*c*d^3+2/b^3/(a/b)^{2/3}*\ln(x+(a/b)^{1/3})*a^2*c^2*d^2+1/3/b^5/(a/b)^{2/3}*\ln(x+(a/b)^{1/3})*a^4*d^4-d^3/b^2*x^4*a*c+4*d^3/b^3*a^2*c*x-6*d^2/b^2*a*c^2*x-4/3/b^4/(a/b)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3})*x-1)*a^3*c*d^3+2/b^3/(a/b)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3})*x-1)*a^2*c^2*d^2-1/6/b^5/(a/b)^{2/3}*\ln(x^2-x*(a/b)^{1/3}+(a/b)^{2/3})*a^4*d^4+1/3/b/(a/b)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3})*x-1)*c^4+4/7*d^3/b*x^7*c-1/7*d^4/b^2*x^7*a+3/2*d^2/b*x^4*c^2-d^4/b^4*a^3*x+4*d/b*c^3*x+1/3/b/(a/b)^{2/3}*\ln(x+(a/b)^{1/3})*c^4-1/6/b/(a/b)^{2/3}*\ln(x^2-x*(a/b)^{1/3}+(a/b)^{2/3})*c^4+1/4*d^4/b^3*x^4*a^2-4/3/b^2/(a/b)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3})*x-1)*a*c^3*d+1/10*d^4*x^{10}/b$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^4/(b\*x^3 + a),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.216267, size = 494, normalized size = 1.96

$$\sqrt{3} \left( 70 \sqrt{3} (b^4 c^4 - 4 a b^3 c^3 d + 6 a^2 b^2 c^2 d^2 - 4 a^3 b c d^3 + a^4 d^4) \log \left( (a^2 b)^{\frac{2}{3}} x^2 - (a^2 b)^{\frac{1}{3}} a x + a^2 \right) - 140 \sqrt{3} (b^4 c^4 - 4 a b^3 c^3 d + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^4/(b\*x^3 + a),x, algorithm="fricas")

[Out] 
$$-1/1260 \cdot \sqrt{3} \cdot (70 \cdot \sqrt{3} \cdot (b^4 \cdot c^4 - 4 \cdot a \cdot b^3 \cdot c^3 \cdot d + 6 \cdot a^2 \cdot b^2 \cdot c^2 \cdot d^2 - 4 \cdot a^3 \cdot b \cdot c \cdot d^3 + a^4 \cdot d^4) \cdot \log((a^2 \cdot b)^{2/3} \cdot x^2 - (a^2 \cdot b)^{1/3} \cdot a \cdot x + a^2) - 140 \cdot \sqrt{3} \cdot (b^4 \cdot c^4 - 4 \cdot a \cdot b^3 \cdot c^3 \cdot d + 6 \cdot a^2 \cdot b^2 \cdot c^2 \cdot d^2 - 4 \cdot a^3 \cdot b \cdot c \cdot d^3 + a^4 \cdot d^4) \cdot \log((a^2 \cdot b)^{1/3} \cdot x + a) - 420 \cdot (b^4 \cdot c^4 - 4 \cdot a \cdot b^3 \cdot c^3 \cdot d + 6 \cdot a^2 \cdot b^2 \cdot c^2 \cdot d^2 - 4 \cdot a^3 \cdot b \cdot c \cdot d^3 + a^4 \cdot d^4) \cdot \arctan(1/3 \cdot (2 \cdot \sqrt{3} \cdot (a^2 \cdot b)^{1/3} \cdot x - \sqrt{3} \cdot a) / a) - 3 \cdot \sqrt{3} \cdot (14 \cdot b^4 \cdot d^4 \cdot x^{10} + 20 \cdot (4 \cdot b^3 \cdot c^3 \cdot d^3 - a \cdot b^2 \cdot d^4) \cdot x^7 + 35 \cdot (6 \cdot b^3 \cdot c^2 \cdot d^2 - 4 \cdot a \cdot b^2 \cdot c \cdot d^3 + a^2 \cdot b \cdot d^4) \cdot x^4 + 140 \cdot (4 \cdot b^3 \cdot c^3 \cdot d - 6 \cdot a \cdot b^2 \cdot c^2 \cdot d^2 + 4 \cdot a^2 \cdot b \cdot c \cdot d^3 - a^3 \cdot d^4) \cdot x) \cdot (a^2 \cdot b)^{1/3}) / ((a^2 \cdot b)^{1/3} \cdot b^4)$$

**Sympy [A]** time = 5.40425, size = 369, normalized size = 1.46

$$\text{RootSum} \left( 27t^3 a^2 b^{13} - a^{12} d^{12} + 12a^{11} b c d^{11} - 66a^{10} b^2 c^2 d^{10} + 220a^9 b^3 c^3 d^9 - 495a^8 b^4 c^4 d^8 + 792a^7 b^5 c^5 d^7 - 924a^6 b^6 c^6 d^6 + 792a^5 b^7 c^7 d^5 - 495a^4 b^8 c^8 d^4 + 220a^3 b^9 c^9 d^3 - 66a^2 b^{10} c^{10} d^2 + 12a b^{11} c^{11} d - b^{12} c^{12}, \text{Lambda}(\_t, \_t \log(3 \cdot \_t \cdot a \cdot b^4 / (a^4 \cdot d^4 - 4 \cdot a^3 \cdot b \cdot c \cdot d^3 + 6 \cdot a^2 \cdot b^2 \cdot c^2 \cdot d^2 - 4 \cdot a \cdot b \cdot c^3 \cdot d + b^4 \cdot c^4) + x)) \right) + \frac{d^4 x^{10}}{10b} - \frac{x^7 (ad^4 - 4bcd^3)}{7b^2} + \frac{x^4 (a^2 d^4 - 4abcd^3 + 6b^2 c^2 d^2)}{4b^3} - \frac{x (a^3 d^4 - 4a^2 b c d^3 + 6ab^2 c^2 d^2 - 4b^3 c^3 d)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*4/(b\*x\*\*3+a),x)

[Out] 
$$\text{RootSum}(27 \cdot \_t^{3 \cdot a^{2 \cdot b^{13}} - a^{12} \cdot d^{12} + 12 \cdot a^{11} \cdot b \cdot c \cdot d^{11} - 66 \cdot a^{10} \cdot b^2 \cdot c^2 \cdot d^{10} + 220 \cdot a^9 \cdot b^3 \cdot c^3 \cdot d^9 - 495 \cdot a^8 \cdot b^4 \cdot c^4 \cdot d^8 + 792 \cdot a^7 \cdot b^5 \cdot c^5 \cdot d^7 - 924 \cdot a^6 \cdot b^6 \cdot c^6 \cdot d^6 + 792 \cdot a^5 \cdot b^7 \cdot c^7 \cdot d^5 - 495 \cdot a^4 \cdot b^8 \cdot c^8 \cdot d^4 + 220 \cdot a^3 \cdot b^9 \cdot c^9 \cdot d^3 - 66 \cdot a^2 \cdot b^{10} \cdot c^{10} \cdot d^2 + 12 \cdot a \cdot b^{11} \cdot c^{11} \cdot d - b^{12} \cdot c^{12}, \text{Lambda}(\_t, \_t \log(3 \cdot \_t \cdot a \cdot b^4 / (a^4 \cdot d^4 - 4 \cdot a^3 \cdot b \cdot c \cdot d^3 + 6 \cdot a^2 \cdot b^2 \cdot c^2 \cdot d^2 - 4 \cdot a \cdot b \cdot c^3 \cdot d + b^4 \cdot c^4) + x)) + d^4 \cdot x^{10} / (10 \cdot b) - x^7 \cdot (a \cdot d^4 - 4 \cdot b \cdot c \cdot d^3) / (7 \cdot b^2) + x^4 \cdot (a^2 \cdot d^4 - 4 \cdot a \cdot b \cdot c \cdot d^3 + 6 \cdot b^2 \cdot c^2 \cdot d^2) / (4 \cdot b^3) - x \cdot (a^3 \cdot d^4 - 4 \cdot a^2 \cdot b \cdot c \cdot d^3 + 6 \cdot a \cdot b^2 \cdot c^2 \cdot d^2 - 4 \cdot b^3 \cdot c^3 \cdot d) / b^4)$$

**GIAC/XCAS [A]** time = 0.220205, size = 622, normalized size = 2.47

$$\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}b^4c^4 - 4(-ab^2)^{\frac{1}{3}}ab^3c^3d + 6(-ab^2)^{\frac{1}{3}}a^2b^2c^2d^2 - 4(-ab^2)^{\frac{1}{3}}a^3bcd^3 + (-ab^2)^{\frac{1}{3}}a^4d^4\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^5} + \frac{\left((-ab^2)^{\frac{1}{3}}b^4c^4 - 4(-ab^2)^{\frac{1}{3}}ab^3c^3d + 6(-ab^2)^{\frac{1}{3}}a^2b^2c^2d^2 - 4(-ab^2)^{\frac{1}{3}}a^3bcd^3 + (-ab^2)^{\frac{1}{3}}a^4d^4\right) \ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)\right)}{6ab^5} - \frac{(b^{10}c^4 - 4ab^9c^3d + 6a^2b^8c^2d^2 - 4a^3b^7cd^3 + a^4b^6d^4) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{10}} + \frac{14b^9d^4x^{10} + 80b^9cd^3x^7 - 20ab^8d^4x^7 + 210b^9c^2d^2x^4 - 140ab^8cd^3x^4 + 35a^2b^7d^4x^4 + 560b^9c^3dx - 840ab^8c^2d^2x + 560a^2}{140b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^4/(b\*x^3 + a),x, algorithm="giac")

[Out] 1/3\*sqrt(3)\*((-a\*b^2)^(1/3)\*b^4\*c^4 - 4\*(-a\*b^2)^(1/3)\*a\*b^3\*c^3\*d + 6\*(-a\*b^2)^(1/3)\*a^2\*b^2\*c^2\*d^2 - 4\*(-a\*b^2)^(1/3)\*a^3\*b\*c\*d^3 + (-a\*b^2)^(1/3)\*a^4\*d^4)\*arctan(1/3\*sqrt(3)\*(2\*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a\*b^5) + 1/6\*((-a\*b^2)^(1/3)\*b^4\*c^4 - 4\*(-a\*b^2)^(1/3)\*a\*b^3\*c^3\*d + 6\*(-a\*b^2)^(1/3)\*a^2\*b^2\*c^2\*d^2 - 4\*(-a\*b^2)^(1/3)\*a^3\*b\*c\*d^3 + (-a\*b^2)^(1/3)\*a^4\*d^4)\*ln(x^2 + x\*(-a/b)^(1/3) + (-a/b)^(2/3))/(a\*b^5) - 1/3\*(b^10\*c^4 - 4\*a\*b^9\*c^3\*d + 6\*a^2\*b^8\*c^2\*d^2 - 4\*a^3\*b^7\*c\*d^3 + a^4\*b^6\*d^4)\*(-a/b)^(1/3)\*ln(abs(x - (-a/b)^(1/3)))/(a\*b^10) + 1/140\*(14\*b^9\*d^4\*x^10 + 80\*b^9\*c\*d^3\*x^7 - 20\*a\*b^8\*d^4\*x^7 + 210\*b^9\*c^2\*d^2\*x^4 - 140\*a\*b^8\*c\*d^3\*x^4 + 35\*a^2\*b^7\*d^4\*x^4 + 560\*b^9\*c^3\*d\*x - 840\*a\*b^8\*c^2\*d^2\*x + 560\*a^2\*b^7\*c\*d^3\*x - 140\*a^3\*b^6\*d^4\*x)/b^10

$$3.15 \quad \int \frac{(c+dx^3)^3}{a+bx^3} dx$$

**Optimal.** Leaf size=208

$$\begin{aligned} & -\frac{(bc-ad)^3 \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}} + \frac{(bc-ad)^3 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{10/3}} \\ & -\frac{(bc-ad)^3 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{10/3}} + \frac{dx(a^2d^2 - 3abcd + 3b^2c^2)}{b^3} + \frac{d^2x^4(3bc-ad)}{4b^2} + \frac{d^3x^7}{7b} \end{aligned}$$

[Out]  $(d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^4)/(4*b^2) + (d^3*x^7)/(7*b) - ((b*c - a*d)^3*ArcTan[(a^{1/3} - 2*b^{1/3}*x)/(Sqrt[3]*a^{1/3})])/(Sqrt[3]*a^{2/3}*b^{10/3}) + ((b*c - a*d)^3*Log[a^{1/3} + b^{1/3}*x])/(3*a^{2/3}*b^{10/3}) - ((b*c - a*d)^3*Log[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(6*a^{2/3}*b^{10/3})$

**Rubi [A]** time = 0.31675, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$\begin{aligned} & -\frac{(bc-ad)^3 \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}} + \frac{(bc-ad)^3 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{10/3}} \\ & -\frac{(bc-ad)^3 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{10/3}} + \frac{dx(a^2d^2 - 3abcd + 3b^2c^2)}{b^3} + \frac{d^2x^4(3bc-ad)}{4b^2} + \frac{d^3x^7}{7b} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^3/(a + b\*x^3), x]

[Out]  $(d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^4)/(4*b^2) + (d^3*x^7)/(7*b) - ((b*c - a*d)^3*ArcTan[(a^{1/3} - 2*b^{1/3}*x)/(Sqrt[3]*a^{1/3})])/(Sqrt[3]*a^{2/3}*b^{10/3}) + ((b*c - a*d)^3*Log[a^{1/3} + b^{1/3}*x])/(3*a^{2/3}*b^{10/3}) - ((b*c - a*d)^3*Log[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(6*a^{2/3}*b^{10/3})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & \frac{d^3x^7}{7b} - \frac{d^2x^4(ad-3bc)}{4b^2} + \frac{(a^2d^2 - 3abcd + 3b^2c^2) \int d dx}{b^3} - \frac{(ad-bc)^3 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{10/3}} \\ & + \frac{(ad-bc)^3 \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}} + \frac{\sqrt{3}(ad-bc)^3 \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} - \frac{2\sqrt[3]{bx}}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{2/3}b^{10/3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((d\*x\*\*3+c)\*\*3/(b\*x\*\*3+a), x)

[Out]  $d**3*x**7/(7*b) - d**2*x**4*(a*d - 3*b*c)/(4*b**2) + (a**2*d**2 - 3*a*b*c*d + 3*b**2*c**2)*Integral(d, x)/b**3 - (a*d - b*c)**3*log(a**(1/3) + b**(1/3)*x)/(3*a**(2/3)*b**(10/3)) + (a*d - b*c)**3*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(2/3)*b**(10/3)) + sqrt(3)*(a*d - b*c)**3*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(2/3)*b**(10/3))$



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**Mathematica [A]** time = 0.155733, size = 203, normalized size = 0.98

$$\frac{14(ad-bc)^3 \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}\right)}{a^{2/3}} + \frac{28(bc-ad)^3 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{a^{2/3}} + \frac{28\sqrt{3}(bc-ad)^3 \tan^{-1}\left(\frac{\sqrt[3]{bx} - \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}} + 84\sqrt[3]{bdx} (a^2d^2 - 3abcd + 3b^2c^2)$$

$$84b^{10/3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)^3/(a + b\*x^3), x]

[Out] (84\*b^(1/3)\*d\*(3\*b^2\*c^2 - 3\*a\*b\*c\*d + a^2\*d^2)\*x + 21\*b^(4/3)\*d^2\*(3\*b\*c - a\*d)\*x^4 + 12\*b^(7/3)\*d^3\*x^7 + (28\*Sqrt[3]\*(b\*c - a\*d)^3\*ArcTan[(-a^(1/3) + 2\*b^(1/3)\*x)/(Sqrt[3]\*a^(1/3))])/a^(2/3) + (28\*(b\*c - a\*d)^3\*Log[a^(1/3) + b^(1/3)\*x])/a^(2/3) + (14\*(-(b\*c) + a\*d)^3\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2])/a^(2/3)/(84\*b^(10/3))

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**Maple [B]** time = 0.004, size = 486, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^3/(b\*x^3+a), x)

[Out] 1/7\*d^3\*x^7/b-1/4\*d^3/b^2\*x^4\*a+3/4\*d^2/b\*x^4\*c+d^3/b^3\*a^2\*x-3\*d^2/b^2\*a\*c\*x+3\*d/b\*c^2\*x-1/3/b^4/(a/b)^(2/3)\*ln(x+(a/b)^(1/3))\*a^3\*d^3+1/b^3/(a/b)^(2/3)\*ln(x+(a/b)^(1/3))\*a^2\*c\*d^2-1/b^2/(a/b)^(2/3)\*ln(x+(a/b)^(1/3))\*a\*c^2\*d+1/3/b/(a/b)^(2/3)\*ln(x+(a/b)^(1/3))\*c^3+1/6/b^4/(a/b)^(2/3)\*ln(x^2-x\*(a/b)^(1/3)+(a/b)^(2/3))\*a^3\*d^3-1/2/b^3/(a/b)^(2/3)\*ln(x^2-x\*(a/b)^(1/3)+(a/b)^(2/3))\*a^2\*c\*d^2+1/2/b^2/(a/b)^(2/3)\*ln(x^2-x\*(a/b)^(1/3)+(a/b)^(2/3))\*a\*c^2\*d-1/6/b/(a/b)^(2/3)\*ln(x^2-x\*(a/b)^(1/3)+(a/b)^(2/3))\*c^3-1/3/b^4/(a/b)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(a/b)^(1/3)\*x-1))\*a^3\*d^3+1/b^3/(a/b)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(a/b)^(1/3)\*x-1))\*a^2\*c\*d^2-1/b^2/(a/b)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(a/b)^(1/3)\*x-1))\*a\*c^2\*d+1/3/b/(a/b)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(a/b)^(1/3)\*x-1))\*c^3

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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^3/(b\*x^3 + a), x, algorithm="maxima")

[Out] Exception raised: ValueError

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**Ericas [A]** time = 0.215725, size = 379, normalized size = 1.82

$$\sqrt{3}\left(14\sqrt{3}(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3) \log\left((-a^2b)^{\frac{2}{3}}x^2 + (-a^2b)^{\frac{1}{3}}ax + a^2\right) - 28\sqrt{3}(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^3/(b\*x^3 + a),x, algorithm="fricas")

[Out]  $\frac{1}{252} \sqrt{3} (14 \sqrt{3} (b^3 c^3 - 3 a b^2 c^2 d + 3 a^2 b c d^2 - a^3 d^3) \log((-a^2 b)^{2/3} x^2 + (-a^2 b)^{1/3} a x + a^2) - 28 \sqrt{3} (b^3 c^3 - 3 a b^2 c^2 d + 3 a^2 b c d^2 - a^3 d^3) \log((-a^2 b)^{1/3} x - a) + 84 (b^3 c^3 - 3 a b^2 c^2 d + 3 a^2 b c d^2 - a^3 d^3) \arctan(1/3 (2 \sqrt{3} (-a^2 b)^{1/3} x + \sqrt{3} a) / a) + 3 \sqrt{3} (4 b^2 d^3 x^7 + 7 (3 b^2 c d^2 - a b d^3) x^4 + 28 (3 b^2 c^2 d - 3 a b c d^2 + a^2 d^3) x) (-a^2 b)^{1/3} / ((-a^2 b)^{1/3} b^3)$

**Sympy [A]** time = 4.16369, size = 255, normalized size = 1.23

$$\text{RootSum}\left(27t^3 a^2 b^{10} + a^9 d^9 - 9a^8 b c d^8 + 36a^7 b^2 c^2 d^7 - 84a^6 b^3 c^3 d^6 + 126a^5 b^4 c^4 d^5 - 126a^4 b^5 c^5 d^4 + 84a^3 b^6 c^6 d^3 - 36a^2 b^7 c^7 d^2 + \frac{d^3 x^7}{7b} - \frac{x^4 (ad^3 - 3bcd^2)}{4b^2} + \frac{x (a^2 d^3 - 3abcd^2 + 3b^2 c^2 d)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*3/(b\*x\*\*3+a),x)

[Out]  $\text{RootSum}(27\_t^{**3} a^{**2} b^{**10} + a^{**9} d^{**9} - 9 a^{**8} b c d^{**8} + 36 a^{**7} b^2 c^2 d^{**7} - 84 a^{**6} b^3 c^3 d^{**6} + 126 a^{**5} b^4 c^4 d^{**5} - 126 a^{**4} b^5 c^5 d^{**4} + 84 a^{**3} b^6 c^6 d^{**3} - 36 a^{**2} b^7 c^7 d^{**2} + 9 a^{**7} b^2 c^2 d^{**7} - 84 a^{**6} b^3 c^3 d^{**6} + 126 a^{**5} b^4 c^4 d^{**5} - 126 a^{**4} b^5 c^5 d^{**4} + 84 a^{**3} b^6 c^6 d^{**3} - 36 a^{**2} b^7 c^7 d^{**2} + 9 a^{**7} b^2 c^2 d^{**7} + 9 a^{**7} b^2 c^2 d^{**7} - b^{**9} c^{**9}, \text{Lambda}(\_t, \_t \log(-3 \_t a^{**3} b^{**3} / (a^{**3} d^{**3} - 3 a^{**2} b c d^{**2} + 3 a b^2 c^2 d - b^{**3} c^{**3}) + x))) + d^{**3} x^{**7} / (7 b) - x^{**4} (a d^{**3} - 3 b c d^{**2}) / (4 b^{**2}) + x (a^{**2} d^{**3} - 3 a b c d^{**2} + 3 b^2 c^2 d) / b^{**3}$

**GIAC/XCAS [A]** time = 0.21963, size = 473, normalized size = 2.27

$$\frac{\sqrt{3} \left( (-ab^2)^{\frac{1}{3}} b^3 c^3 - 3 (-ab^2)^{\frac{1}{3}} ab^2 c^2 d + 3 (-ab^2)^{\frac{1}{3}} a^2 b c d^2 - (-ab^2)^{\frac{1}{3}} a^3 d^3 \right) \arctan \left( \frac{\sqrt{3} \left( 2x + \left( -\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left( -\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3 ab^4} + \frac{\left( (-ab^2)^{\frac{1}{3}} b^3 c^3 - 3 (-ab^2)^{\frac{1}{3}} ab^2 c^2 d + 3 (-ab^2)^{\frac{1}{3}} a^2 b c d^2 - (-ab^2)^{\frac{1}{3}} a^3 d^3 \right) \ln \left( x^2 + x \left( -\frac{a}{b} \right)^{\frac{1}{3}} + \left( -\frac{a}{b} \right)^{\frac{2}{3}} \right)}{6 ab^4} - \frac{(b^7 c^3 - 3 ab^6 c^2 d + 3 a^2 b^5 c d^2 - a^3 b^4 d^3) \left( -\frac{a}{b} \right)^{\frac{1}{3}} \ln \left( \left| x - \left( -\frac{a}{b} \right)^{\frac{1}{3}} \right| \right)}{3 ab^7} + \frac{4 b^6 d^3 x^7 + 21 b^6 c d^2 x^4 - 7 ab^5 d^3 x^4 + 84 b^6 c^2 d x - 84 ab^5 c d^2 x + 28 a^2 b^4 d^3 x}{28 b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^3/(b\*x^3 + a),x, algorithm="giac")

[Out]  $\frac{1}{3} \sqrt{3} \left( (-a^2 b)^{1/3} b^3 c^3 - 3 (-a^2 b)^{1/3} a b^2 c^2 d + 3 (-a^2 b)^{1/3} a^2 b c d^2 - (-a^2 b)^{1/3} a^3 d^3 \right) \arctan \left( \frac{1}{3} \sqrt{3} (2x + (-a/b)^{1/3}) / (-a/b)^{1/3} \right) / (a^2 b^4) + \frac{1}{6} \left( (-a^2 b)^{1/3} b^3 c^3 - 3 (-a^2 b)^{1/3} a b^2 c^2 d + 3 (-a^2 b)^{1/3} a^2 b c d^2 - (-a^2 b)^{1/3} a^3 d^3 \right) \ln(x^2 + x (-a/b)^{1/3} + (-a/b)^{2/3}) / (a^2 b^4) - \frac{1}{3} (b^7 c^3 - 3 a^2 b^6 c^2 d + 3 a^2 b^5 c d^2 - a^3 b^4 d^3) (-a/b)^{1/3} \ln(\text{abs}(x - (-a/b)^{1/3})) / (a^2 b^7) + \frac{1}{28} (4 b^6 d^3 x^7 + 21 b^6 c d^2 x^4 - 7 a b^5 d^3 x^4 + 84 b^6 c^2 d x - 84 a b^5 c d^2 x + 28 a^2 b^4 d^3 x) / b^7$

$$3.16 \quad \int \frac{(c+dx^3)^2}{a+bx^3} dx$$

Optimal. Leaf size=173

$$\begin{aligned} & -\frac{(bc-ad)^2 \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}} + \frac{(bc-ad)^2 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{7/3}} \\ & -\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{7/3}} + \frac{dx(2bc-ad)}{b^2} + \frac{d^2x^4}{4b} \end{aligned}$$

[Out] (d\*(2\*b\*c - a\*d)\*x)/b^2 + (d^2\*x^4)/(4\*b) - ((b\*c - a\*d)^2\*ArcTan[(a^(1/3) - 2\*b^(1/3)\*x)/(Sqrt[3]\*a^(1/3))])/(Sqrt[3]\*a^(2/3)\*b^(7/3)) + ((b\*c - a\*d)^2\*Log[a^(1/3) + b^(1/3)\*x])/(3\*a^(2/3)\*b^(7/3)) - ((b\*c - a\*d)^2\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2])/(6\*a^(2/3)\*b^(7/3))

**Rubi [A]** time = 0.265867, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$\begin{aligned} & -\frac{(bc-ad)^2 \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}} + \frac{(bc-ad)^2 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}b^{7/3}} \\ & -\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{7/3}} + \frac{dx(2bc-ad)}{b^2} + \frac{d^2x^4}{4b} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^2/(a + b\*x^3), x]

[Out] (d\*(2\*b\*c - a\*d)\*x)/b^2 + (d^2\*x^4)/(4\*b) - ((b\*c - a\*d)^2\*ArcTan[(a^(1/3) - 2\*b^(1/3)\*x)/(Sqrt[3]\*a^(1/3))])/(Sqrt[3]\*a^(2/3)\*b^(7/3)) + ((b\*c - a\*d)^2\*Log[a^(1/3) + b^(1/3)\*x])/(3\*a^(2/3)\*b^(7/3)) - ((b\*c - a\*d)^2\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2])/(6\*a^(2/3)\*b^(7/3))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & \frac{d^2x^4}{4b} - \frac{(ad-2bc) \int d dx}{b^2} + \frac{(ad-bc)^2 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{\frac{2}{3}}b^{\frac{7}{3}}} \\ & - \frac{(ad-bc)^2 \log\left(a^{\frac{2}{3}} - \sqrt[3]{a}\sqrt[3]{bx} + b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}b^{\frac{7}{3}}} - \frac{\sqrt{3}(ad-bc)^2 \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} - \frac{2\sqrt[3]{bx}}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}b^{\frac{7}{3}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((d\*x\*\*3+c)\*\*2/(b\*x\*\*3+a), x)

[Out] d\*\*2\*x\*\*4/(4\*b) - (a\*d - 2\*b\*c)\*Integral(d, x)/b\*\*2 + (a\*d - b\*c)\*\*2\*log(a\*\*(1/3) + b\*\*(1/3)\*x)/(3\*a\*\*(2/3)\*b\*\*(7/3)) - (a\*d - b\*c)\*\*2\*log(a\*\*(2/3) - a\*\*(1/3)\*b\*\*(1/3)\*x + b\*\*(2/3)\*x\*\*2)/(6\*a\*\*(2/3)\*b\*\*(7/3)) - sqrt(3)\*(a\*d - b\*c)\*\*2\*atan(sqrt(3)\*(a\*\*(1/3)/3 - 2\*b\*\*(1/3)\*x/3)/a\*\*(1/3))/(3\*a\*\*(2/3)\*b\*\*(7/3))

**Mathematica [A]** time = 0.185366, size = 167, normalized size = 0.97

$$\frac{-2(bc - ad)^2 \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right) + 3a^{2/3}b^{4/3}d^2x^4 - 12a^{2/3}\sqrt[3]{bd}x(ad - 2bc) + 4(bc - ad)^2 \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) + 4\sqrt[3]{3}(l)}{12a^{2/3}b^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)^2/(a + b\*x^3), x]

[Out] (-12\*a^(2/3)\*b^(1/3)\*d\*(-2\*b\*c + a\*d)\*x + 3\*a^(2/3)\*b^(4/3)\*d^2\*x^4 + 4\*sqrt[3]\*(b\*c - a\*d)^2\*ArcTan[(-a^(1/3) + 2\*b^(1/3)\*x)/(sqrt[3]\*a^(1/3))] + 4\*(b\*c - a\*d)^2\*Log[a^(1/3) + b^(1/3)\*x] - 2\*(b\*c - a\*d)^2\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2]/(12\*a^(2/3)\*b^(7/3))

**Maple [B]** time = 0.005, size = 334, normalized size = 1.9

$$\begin{aligned} & \frac{d^2x^4}{4b} - \frac{ad^2x}{b^2} + 2\frac{dxc}{b} + \frac{a^2d^2}{3b^3} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \\ & - \frac{2acd}{3b^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{c^2}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \\ & - \frac{a^2d^2}{6b^3} \ln\left(x^2 - x\sqrt[3]{\frac{a}{b}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{acd}{3b^2} \ln\left(x^2 - x\sqrt[3]{\frac{a}{b}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \\ & - \frac{c^2}{6b} \ln\left(x^2 - x\sqrt[3]{\frac{a}{b}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}a^2d^2}{3b^3} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \\ & - \frac{2\sqrt{3}cad}{3b^2} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}c^2}{3b} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^2/(b\*x^3+a), x)

[Out] 1/4\*d^2\*x^4/b-d^2/b^2\*a\*x+2\*d/b\*x\*c+1/3/b^3/(a/b)^(2/3)\*ln(x+(a/b)^(1/3))\*a^2\*d^2-2/3/b^2/(a/b)^(2/3)\*ln(x+(a/b)^(1/3))\*c\*a\*d+1/3/b/(a/b)^(2/3)\*ln(x+(a/b)^(1/3))\*c^2-1/6/b^3/(a/b)^(2/3)\*ln(x^2-x\*(a/b)^(1/3)+(a/b)^(2/3))\*a^2\*d^2+1/3/b^2/(a/b)^(2/3)\*ln(x^2-x\*(a/b)^(1/3)+(a/b)^(2/3))\*c\*a\*d-1/6/b/(a/b)^(2/3)\*ln(x^2-x\*(a/b)^(1/3)+(a/b)^(2/3))\*c^2+1/3/b^3/(a/b)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(a/b)^(1/3)\*x-1))\*a^2\*d^2-2/3/b^2/(a/b)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(a/b)^(1/3)\*x-1))\*c\*a\*d+1/3/b/(a/b)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(a/b)^(1/3)\*x-1))\*c^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^2/(b\*x^3 + a), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.212444, size = 259, normalized size = 1.5

$$\frac{\sqrt{3} \left( 2 \sqrt{3} (b^2 c^2 - 2 abcd + a^2 d^2) \log \left( (a^2 b)^{\frac{2}{3}} x^2 - (a^2 b)^{\frac{1}{3}} ax + a^2 \right) - 4 \sqrt{3} (b^2 c^2 - 2 abcd + a^2 d^2) \log \left( (a^2 b)^{\frac{1}{3}} x + a \right) - 12 \right)}{36 (a^2 b)^{\frac{1}{3}} b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^2/(b\*x^3 + a), x, algorithm="fricas")

[Out] 
$$-1/36 \sqrt{3} (2 \sqrt{3} (b^2 c^2 - 2 a b c d + a^2 d^2) \log((a^2 b)^{2/3} x^2 - (a^2 b)^{1/3} a x + a^2) - 4 \sqrt{3} (b^2 c^2 - 2 a b c d + a^2 d^2) \log((a^2 b)^{1/3} x + a) - 12 (b^2 c^2 - 2 a b c d + a^2 d^2) \arctan(1/3 (2 \sqrt{3} (a^2 b)^{1/3} x - \sqrt{3} a) / a) - 3 \sqrt{3} (b d^2 x^4 + 4 (2 b c d - a d^2) x) (a^2 b)^{1/3}) / ((a^2 b)^{1/3} b^2)$$

**Sympy [A]** time = 2.99486, size = 156, normalized size = 0.9

$$\text{RootSum} \left( 27 t^3 a^2 b^7 - a^6 d^6 + 6 a^5 b c d^5 - 15 a^4 b^2 c^2 d^4 + 20 a^3 b^3 c^3 d^3 - 15 a^2 b^4 c^4 d^2 + 6 a b^5 c^5 d - b^6 c^6, \left( t \mapsto t \log \left( \frac{3 t a b^5}{a^2 d^2 - 2 a b c d} \right) \right) \right) + \frac{d^2 x^4}{4 b} - \frac{x (a d^2 - 2 b c d)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*2/(b\*x\*\*3+a), x)

[Out] 
$$\text{RootSum}(27 * t^{**3} * a^{**2} * b^{**7} - a^{**6} * d^{**6} + 6 * a^{**5} * b * c * d^{**5} - 15 * a^{**4} * b^{**2} * c^{**2} * d^{**4} + 20 * a^{**3} * b^{**3} * c^{**3} * d^{**3} - 15 * a^{**2} * b^{**4} * c^{**4} * d^{**2} + 6 * a * b^{**5} * c^{**5} * d - b^{**6} * c^{**6}, \text{Lambda}(\_t, \_t * \log(3 * t * a * b^{**2} / (a^{**2} * d^{**2} - 2 * a * b * c * d + b^{**2} * c^{**2}) + x))) + d^{**2} * x^{**4} / (4 * b) - x * (a * d^{**2} - 2 * b * c * d) / b^{**2}$$

**GIAC/XCAS [A]** time = 0.218411, size = 336, normalized size = 1.94

$$\frac{\sqrt{3} \left( (-ab^2)^{\frac{1}{3}} b^2 c^2 - 2 (-ab^2)^{\frac{1}{3}} abcd + (-ab^2)^{\frac{1}{3}} a^2 d^2 \right) \arctan \left( \frac{\sqrt{3} \left( 2x + \left( -\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left( -\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3 ab^3} + \frac{\left( (-ab^2)^{\frac{1}{3}} b^2 c^2 - 2 (-ab^2)^{\frac{1}{3}} abcd + (-ab^2)^{\frac{1}{3}} a^2 d^2 \right) \ln \left( x^2 + x \left( -\frac{a}{b} \right)^{\frac{1}{3}} + \left( -\frac{a}{b} \right)^{\frac{2}{3}} \right)}{6 ab^3} - \frac{(b^4 c^2 - 2 ab^3 cd + a^2 b^2 d^2) \left( -\frac{a}{b} \right)^{\frac{1}{3}} \ln \left( \left| x - \left( -\frac{a}{b} \right)^{\frac{1}{3}} \right| \right)}{3 ab^4} + \frac{b^3 d^2 x^4 + 8 b^3 c d x - 4 ab^2 d^2 x}{4 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^2/(b\*x^3 + a), x, algorithm="giac")

[Out] 
$$1/3 \sqrt{3} ((-a * b^2)^{1/3} * b^2 * c^2 - 2 * (-a * b^2)^{1/3} * a * b * c * d + (-a * b^2)^{1/3} * a^2 * d^2) * \arctan(1/3 * \sqrt{3} * (2 * x + (-a/b)^{1/3}) / ((-a/b)^{1/3}) / (a * b^3) + 1/6 * ((-a * b^2)^{1/3} * b^2 * c^2 - 2 * (-a * b^2)^{1/3} * a * b * c * d + (-a * b^2)^{1/3} * a^2 * d^2) * \ln(x^2 + x * (-a/b)^{1/3} + (-a/b)^{2/3}) / (a * b^3) - 1/3 * (b^4 * c^2 - 2 * a * b^3 * c * d + a^2 * b^2 * d^2) * (-a/b)^{1/3} * \ln(\text{abs}(x - (-a/b)^{1/3})) / (a * b^4) + 1/4 * (b^3 * d^2 * x^4 + 8 * b^3 * c * d * x - 4 * a * b^2 * d^2 * x) / b^4$$

### 3.17 $\int \frac{c+dx^3}{a+bx^3} dx$

**Optimal.** Leaf size=145

$$-\frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{3a^{2/3}b^{4/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}}+\frac{dx}{b}$$

[Out] (d\*x)/b - ((b\*c - a\*d)\*ArcTan[(a^(1/3) - 2\*b^(1/3)\*x)/(Sqrt[3]\*a^(1/3))])/(Sqrt[3]\*a^(2/3)\*b^(4/3)) + ((b\*c - a\*d)\*Log[a^(1/3) + b^(1/3)\*x])/(3\*a^(2/3)\*b^(4/3)) - ((b\*c - a\*d)\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2])/(6\*a^(2/3)\*b^(4/3))

**Rubi [A]** time = 0.182217, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$-\frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{3a^{2/3}b^{4/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)/(a + b\*x^3), x]

[Out] (d\*x)/b - ((b\*c - a\*d)\*ArcTan[(a^(1/3) - 2\*b^(1/3)\*x)/(Sqrt[3]\*a^(1/3))])/(Sqrt[3]\*a^(2/3)\*b^(4/3)) + ((b\*c - a\*d)\*Log[a^(1/3) + b^(1/3)\*x])/(3\*a^(2/3)\*b^(4/3)) - ((b\*c - a\*d)\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2])/(6\*a^(2/3)\*b^(4/3))

**Rubi in Sympy [A]** time = 37.7057, size = 134, normalized size = 0.92

$$\frac{dx}{b} - \frac{(ad-bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{3a^{2/3}b^{4/3}} + \frac{(ad-bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}} + \frac{\sqrt{3}(ad-bc)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{bx}}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{2/3}b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((d\*x\*\*3+c)/(b\*x\*\*3+a), x)

[Out] d\*x/b - (a\*d - b\*c)\*log(a\*\*(1/3) + b\*\*(1/3)\*x)/(3\*a\*\*(2/3)\*b\*\*(4/3)) + (a\*d - b\*c)\*log(a\*\*(2/3) - a\*\*(1/3)\*b\*\*(1/3)\*x + b\*\*(2/3)\*x\*\*2)/(6\*a\*\*(2/3)\*b\*\*(4/3)) + sqrt(3)\*(a\*d - b\*c)\*atan(sqrt(3)\*(a\*\*(1/3)/3 - 2\*b\*\*(1/3)\*x/3)/a\*\*(1/3))/(3\*a\*\*(2/3)\*b\*\*(4/3))

**Mathematica [A]** time = 0.109031, size = 129, normalized size = 0.89

$$-\frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)+6a^{2/3}\sqrt[3]{b}dx+2(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)-2\sqrt{3}(bc-ad)\tan^{-1}\left(\frac{1-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{6a^{2/3}b^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)/(a + b\*x^3), x]

[Out]  $(6 * a^{(2/3)} * b^{(1/3)} * d * x - 2 * \text{Sqrt}[3] * (b * c - a * d) * \text{ArcTan}[(1 - (2 * b^{(1/3)} * x) / a^{(1/3)}) / \text{Sqrt}[3]] + 2 * (b * c - a * d) * \text{Log}[a^{(1/3)} + b^{(1/3)} * x] - (b * c - a * d) * \text{Log}[a^{(2/3)} - a^{(1/3)} * b^{(1/3)} * x + b^{(2/3)} * x^2]) / (6 * a^{(2/3)} * b^{(4/3)})$

**Maple [A]** time = 0.005, size = 195, normalized size = 1.3

$$\begin{aligned} & \frac{dx}{b} - \frac{ad}{3b^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{c}{3b} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \\ & + \frac{ad}{6b^2} \ln\left(x^2 - x\sqrt[3]{\frac{a}{b}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{c}{6b} \ln\left(x^2 - x\sqrt[3]{\frac{a}{b}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \\ & - \frac{\sqrt{3}ad}{3b^2} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}c}{3b} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)/(b*x^3+a), x)`

[Out]  $d * x / b - 1/3 / b^2 / (a/b)^{(2/3)} * \ln(x + (a/b)^{(1/3)}) * a * d + 1/3 / b / (a/b)^{(2/3)} * \ln(x + (a/b)^{(1/3)}) * c + 1/6 / b^2 / (a/b)^{(2/3)} * \ln(x^2 - x * (a/b)^{(1/3)} + (a/b)^{(2/3)}) * a * d - 1/6 / b / (a/b)^{(2/3)} * \ln(x^2 - x * (a/b)^{(1/3)} + (a/b)^{(2/3)}) * c - 1/3 / b^2 / (a/b)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2 / (a/b)^{(1/3)} * x - 1)) * a * d + 1/3 / b / (a/b)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2 / (a/b)^{(1/3)} * x - 1)) * c$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3 + c)/(b*x^3 + a), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.215956, size = 184, normalized size = 1.27

$$\frac{\sqrt{3} \left( 6 \sqrt{3} (-a^2 b)^{\frac{1}{3}} dx + \sqrt{3} (bc - ad) \log \left( (-a^2 b)^{\frac{2}{3}} x^2 + (-a^2 b)^{\frac{1}{3}} ax + a^2 \right) - 2 \sqrt{3} (bc - ad) \log \left( (-a^2 b)^{\frac{1}{3}} x - a \right) + 6 (bc - ad) \right)}{18 (-a^2 b)^{\frac{1}{3}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3 + c)/(b*x^3 + a), x, algorithm="fricas")`

[Out]  $1/18 * \text{sqrt}(3) * (6 * \text{sqrt}(3) * (-a^2 * b)^{(1/3)} * d * x + \text{sqrt}(3) * (b * c - a * d) * \log((-a^2 * b)^{(2/3)} * x^2 + (-a^2 * b)^{(1/3)} * a * x + a^2) - 2 * \text{sqrt}(3) * (b * c - a * d) * \log((-a^2 * b)^{(1/3)} * x - a) + 6 * (b * c - a * d) * \arctan(1/3 * (2 * \text{sqrt}(3) * (-a^2 * b)^{(1/3)} * x + \text{sqrt}(3) * a) / a)) / ((-a^2 * b)^{(1/3)} * b)$

**Sympy [A]** time = 2.02482, size = 71, normalized size = 0.49

$$\text{RootSum}\left(27t^3 a^2 b^4 + a^3 d^3 - 3a^2 b c d^2 + 3ab^2 c^2 d - b^3 c^3, \left(t \mapsto t \log\left(-\frac{3tab}{ad - bc} + x\right)\right)\right) + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)/(b\*x\*\*3+a),x)

[Out] RootSum(27\*\_t\*\*3\*a\*\*2\*b\*\*4 + a\*\*3\*d\*\*3 - 3\*a\*\*2\*b\*c\*d\*\*2 + 3\*a\*b\*\*2\*c\*\*2\*d - b\*\*3\*c\*\*3, Lambda(\_t, \_t\*log(-3\*\_t\*a\*b/(a\*d - b\*c) + x))) + d\*x/b

**GIAC/XCAS [A]** time = 0.215884, size = 217, normalized size = 1.5

$$\frac{dx}{b} - \frac{(bc - ad) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab} + \frac{\sqrt{3} \left(\left(-ab^2\right)^{\frac{1}{3}} bc - \left(-ab^2\right)^{\frac{1}{3}} ad\right) \arctan\left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}} bc - \left(-ab^2\right)^{\frac{1}{3}} ad\right) \ln\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)/(b\*x^3 + a),x, algorithm="giac")

[Out] d\*x/b - 1/3\*(b\*c - a\*d)\*(-a/b)^(1/3)\*ln(abs(x - (-a/b)^(1/3)))/(a\*b) + 1/3\*sqrt(3)\*((-a\*b^2)^(1/3)\*b\*c - (-a\*b^2)^(1/3)\*a\*d)\*arctan(1/3\*sqrt(3)\*(2\*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a\*b^2) + 1/6\*((-a\*b^2)^(1/3)\*b\*c - (-a\*b^2)^(1/3)\*a\*d)\*ln(x^2 + x\*(-a/b)^(1/3) + (-a/b)^(2/3))/(a\*b^2)



### 3.18 $\int \frac{1}{(a+bx^3)(c+dx^3)} dx$

**Optimal.** Leaf size=288

$$\begin{aligned} & -\frac{b^{2/3} \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)} + \frac{b^{2/3} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}(bc-ad)} - \frac{b^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{a-2}\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)} \\ & + \frac{d^{2/3} \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)} - \frac{d^{2/3} \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{3c^{2/3}(bc-ad)} + \frac{d^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{c-2}\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)} \end{aligned}$$

[Out]  $-\left(\frac{b^{2/3} \operatorname{ArcTan}\left[\frac{a^{1/3} - 2b^{1/3}x}{\sqrt{3}a^{1/3}}\right]}{\sqrt{3}a^{2/3}(bc-ad)} + \frac{d^{2/3} \operatorname{ArcTan}\left[\frac{c^{1/3} - 2d^{1/3}x}{\sqrt{3}c^{1/3}}\right]}{\sqrt{3}c^{2/3}(bc-ad)} + \frac{b^{2/3} \operatorname{Log}\left[\frac{a^{1/3} + b^{1/3}x}{3a^{2/3}(bc-ad)}\right]}{\sqrt{3}a^{2/3}(bc-ad)} - \frac{d^{2/3} \operatorname{Log}\left[\frac{c^{1/3} + d^{1/3}x}{3c^{2/3}(bc-ad)}\right]}{\sqrt{3}c^{2/3}(bc-ad)} - \frac{b^{2/3} \operatorname{Log}\left[\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{6a^{2/3}(bc-ad)}\right]}{\sqrt{3}a^{2/3}(bc-ad)} + \frac{d^{2/3} \operatorname{Log}\left[\frac{c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2}{6c^{2/3}(bc-ad)}\right]}{\sqrt{3}c^{2/3}(bc-ad)}\right)$

**Rubi [A]** time = 0.320915, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$\begin{aligned} & -\frac{b^{2/3} \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)} + \frac{b^{2/3} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}(bc-ad)} - \frac{b^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{a-2}\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)} \\ & + \frac{d^{2/3} \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)} - \frac{d^{2/3} \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{3c^{2/3}(bc-ad)} + \frac{d^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{c-2}\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{1}{(a + b*x^3)*(c + d*x^3)}, x\right]$

[Out]  $-\left(\frac{b^{2/3} \operatorname{ArcTan}\left[\frac{a^{1/3} - 2b^{1/3}x}{\sqrt{3}a^{1/3}}\right]}{\sqrt{3}a^{2/3}(bc-ad)} + \frac{d^{2/3} \operatorname{ArcTan}\left[\frac{c^{1/3} - 2d^{1/3}x}{\sqrt{3}c^{1/3}}\right]}{\sqrt{3}c^{2/3}(bc-ad)} + \frac{b^{2/3} \operatorname{Log}\left[\frac{a^{1/3} + b^{1/3}x}{3a^{2/3}(bc-ad)}\right]}{\sqrt{3}a^{2/3}(bc-ad)} - \frac{d^{2/3} \operatorname{Log}\left[\frac{c^{1/3} + d^{1/3}x}{3c^{2/3}(bc-ad)}\right]}{\sqrt{3}c^{2/3}(bc-ad)} - \frac{b^{2/3} \operatorname{Log}\left[\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{6a^{2/3}(bc-ad)}\right]}{\sqrt{3}a^{2/3}(bc-ad)} + \frac{d^{2/3} \operatorname{Log}\left[\frac{c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2}{6c^{2/3}(bc-ad)}\right]}{\sqrt{3}c^{2/3}(bc-ad)}\right)$

**Rubi in Sympy [A]** time = 68.2094, size = 260, normalized size = 0.9

$$\begin{aligned} & \frac{d^{2/3} \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{3c^{2/3}(ad-bc)} - \frac{d^{2/3} \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{6c^{2/3}(ad-bc)} - \frac{\sqrt{3}d^{2/3} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3} - \frac{2\sqrt[3]{dx}}{3}\right)}{\sqrt[3]{c}}\right)}{3c^{2/3}(ad-bc)} \\ & - \frac{b^{2/3} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}(ad-bc)} + \frac{b^{2/3} \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}(ad-bc)} + \frac{\sqrt{3}b^{2/3} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} - \frac{2\sqrt[3]{bx}}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{2/3}(ad-bc)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}\left(\frac{1}{(b*x^3+a)/(d*x^3+c)}, x\right)$

[Out]  $d^{2/3} \log(c^{1/3} + d^{1/3}x) / (3c^{2/3}(ad - bc)) - d^{2/3} \log(c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2) / (6c^{2/3}(ad - bc)) - \sqrt{3}d^{2/3} \operatorname{atan}(\sqrt{3}(c^{1/3}/3 - 2d^{1/3}x/3)/c^{1/3}) / (3c^{2/3}(ad - bc)) - b^{2/3} \log(a^{1/3} + b^{1/3}x) / (3a^{2/3}(ad - bc)) + b^{2/3} \log(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2) / (6a^{2/3}(ad - bc)) + \sqrt{3}b^{2/3} \operatorname{atan}(\sqrt{3}(a^{1/3}/3 - 2b^{1/3}x/3)/a^{1/3}) / (3a^{2/3}(ad - bc))$

**Mathematica [A]** time = 0.219728, size = 224, normalized size = 0.78

$$\frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{a^{2/3}} - \frac{2b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{a^{2/3}} + \frac{2\sqrt{3}b^{2/3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}} - \frac{d^{2/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{c^{2/3}} + \frac{2d^{2/3} \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{c^{2/3}} - \frac{\quad}{6ad - 6bc}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^3)\*(c + d\*x^3)), x]

[Out]  $((2\sqrt{3}b^{2/3} \operatorname{ArcTan}[(1 - (2b^{1/3}x)/a^{1/3})/\sqrt{3}]) / a^{2/3} - (2\sqrt{3}d^{2/3} \operatorname{ArcTan}[(1 - (2d^{1/3}x)/c^{1/3})/\sqrt{3}]) / c^{2/3} - (2b^{2/3} \operatorname{Log}[a^{1/3} + b^{1/3}x] / a^{2/3} + (2d^{2/3} \operatorname{Log}[c^{1/3} + d^{1/3}x] / c^{2/3} + (b^{2/3} \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2] / a^{2/3} - (d^{2/3} \operatorname{Log}[c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2] / c^{2/3}) / (-6b^2c + 6a^2d))$

**Maple [A]** time = 0.009, size = 222, normalized size = 0.8

$$-\frac{1}{3ad - 3bc} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{1}{6ad - 6bc} \ln\left(x^2 - x\sqrt[3]{\frac{a}{b}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{\sqrt{3}}{3ad - 3bc} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{1}{3ad - 3bc} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{1}{6ad - 6bc} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}}{3ad - 3bc} \arctan\left(\frac{\sqrt{3}}{3} \left(2x \frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)/(d\*x^3+c), x)

[Out]  $-1/3/(ad - bc) / (a/b)^{2/3} \ln(x + (a/b)^{1/3}) + 1/6/(ad - bc) / (a/b)^{2/3} \ln(x^2 - x(a/b)^{1/3} + (a/b)^{2/3}) - 1/3/(ad - bc) / (a/b)^{2/3} \operatorname{arctan}(1/3 \operatorname{atan}(2/(a/b)^{1/3}x - 1)) + 1/3/(ad - bc) / (c/d)^{2/3} \ln(x + (c/d)^{1/3}) - 1/6/(ad - bc) / (c/d)^{2/3} \ln(x^2 - x(c/d)^{1/3} + (c/d)^{2/3}) + 1/3/(ad - bc) / (c/d)^{2/3} \operatorname{arctan}(1/3 \operatorname{atan}(2/(c/d)^{1/3}x - 1))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^3 + a)\*(d\*x^3 + c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.279112, size = 377, normalized size = 1.31

$$\sqrt{3} \left( \sqrt{3} \left( -\frac{b^2}{a^2} \right)^{\frac{1}{3}} \log \left( b^2 x^2 + abx \left( -\frac{b^2}{a^2} \right)^{\frac{1}{3}} + a^2 \left( -\frac{b^2}{a^2} \right)^{\frac{2}{3}} \right) + \sqrt{3} \left( \frac{d^2}{c^2} \right)^{\frac{1}{3}} \log \left( d^2 x^2 - cdx \left( \frac{d^2}{c^2} \right)^{\frac{1}{3}} + c^2 \left( \frac{d^2}{c^2} \right)^{\frac{2}{3}} \right) - 2 \sqrt{3} \left( -\frac{b^2}{a^2} \right)^{\frac{1}{3}} \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^3 + a)\*(d\*x^3 + c)),x, algorithm="fricas")

[Out]  $\frac{1}{18} \sqrt{3} \left( \sqrt{3} \left( -\frac{b^2}{a^2} \right)^{\frac{1}{3}} \log \left( b^2 x^2 + a b x \left( -\frac{b^2}{a^2} \right)^{\frac{1}{3}} + a^2 \left( -\frac{b^2}{a^2} \right)^{\frac{2}{3}} \right) + \sqrt{3} \left( \frac{d^2}{c^2} \right)^{\frac{1}{3}} \log \left( d^2 x^2 - c d x \left( \frac{d^2}{c^2} \right)^{\frac{1}{3}} + c^2 \left( \frac{d^2}{c^2} \right)^{\frac{2}{3}} \right) - 2 \sqrt{3} \left( -\frac{b^2}{a^2} \right)^{\frac{1}{3}} \log \left( d^2 x^2 - c d x \left( \frac{d^2}{c^2} \right)^{\frac{1}{3}} + c^2 \left( \frac{d^2}{c^2} \right)^{\frac{2}{3}} \right) - 2 \sqrt{3} \left( -\frac{b^2}{a^2} \right)^{\frac{1}{3}} \log \left( b x - a \left( -\frac{b^2}{a^2} \right)^{\frac{1}{3}} \right) - 2 \sqrt{3} \left( \frac{d^2}{c^2} \right)^{\frac{1}{3}} \log \left( d x + c \left( \frac{d^2}{c^2} \right)^{\frac{1}{3}} \right) + 6 \left( -\frac{b^2}{a^2} \right)^{\frac{1}{3}} \arctan \left( \frac{1}{3} \left( 2 \sqrt{3} b x + \sqrt{3} a \left( -\frac{b^2}{a^2} \right)^{\frac{1}{3}} \right) / \left( a \left( -\frac{b^2}{a^2} \right)^{\frac{1}{3}} \right) \right) + 6 \left( \frac{d^2}{c^2} \right)^{\frac{1}{3}} \arctan \left( \frac{-1}{3} \left( 2 \sqrt{3} d x - \sqrt{3} c \left( \frac{d^2}{c^2} \right)^{\frac{1}{3}} \right) / \left( c \left( \frac{d^2}{c^2} \right)^{\frac{1}{3}} \right) \right) \right) / (b c - a d)$

**Sympy** [A] time = 104.613, size = 447, normalized size = 1.55

$$\text{RootSum} \left( t^3 (27a^5d^3 - 81a^4bcd^2 + 81a^3b^2c^2d - 27a^2b^3c^3) + b^2, \left( t \mapsto t \log \left( x + \frac{81t^4a^7c^2d^5 - 243t^4a^6bc^3d^4 + 162t^4a^5b^2c^4}{t^3(27a^5d^3 - 81a^4bcd^2 + 81a^3b^2c^2d - 27a^2b^3c^3) + b^2} \right) \right) \right) + \text{RootSum} \left( t^3 (27a^3c^2d^3 - 81a^2bc^3d^2 + 81ab^2c^4d - 27b^3c^5) - d^2, \left( t \mapsto t \log \left( x + \frac{81t^4a^7c^2d^5 - 243t^4a^6bc^3d^4 + 162t^4a^5b^2c^4}{t^3(27a^3c^2d^3 - 81a^2bc^3d^2 + 81ab^2c^4d - 27b^3c^5) - d^2} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)/(d\*x\*\*3+c),x)

[Out]  $\text{RootSum}(\_t^{**3} (27*a^{**5}*d^{**3} - 81*a^{**4}*b*c*d^{**2} + 81*a^{**3}*b^{**2}*c^{**2}*d - 27*a^{**2}*b^{**3}*c^{**3}) + b^{**2}, \text{Lambda}(\_t, \_t \log(x + (81*\_t^{**4}*a^{**7}*c^{**2}*d^{**5} - 243*\_t^{**4}*a^{**6}*b*c^{**3}*d^{**4} + 162*\_t^{**4}*a^{**5}*b^{**2}*c^{**4}*d^{**3} + 162*\_t^{**4}*a^{**4}*b^{**3}*c^{**5}*d^{**2} - 243*\_t^{**4}*a^{**3}*b^{**4}*c^{**6}*d + 81*\_t^{**4}*a^{**2}*b^{**5}*c^{**7} - 3*\_t^{**4}*a^{**4}*d^{**4} + 3*\_t^{**4}*a^{**3}*b*c*d^{**3} + 3*\_t^{**4}*a*b^{**3}*c^{**3}*d - 3*\_t^{**4}*b^{**4}*c^{**4}) / (a^{**2}*b*d^{**3} + b^{**3}*c^{**2}*d))) + \text{RootSum}(\_t^{**3} (27*a^{**3}*c^{**2}*d^{**3} - 81*a^{**2}*b*c^{**3}*d^{**2} + 81*a*b^{**2}*c^{**4}*d - 27*b^{**3}*c^{**5}) - d^{**2}, \text{Lambda}(\_t, \_t \log(x + (81*\_t^{**4}*a^{**7}*c^{**2}*d^{**5} - 243*\_t^{**4}*a^{**6}*b*c^{**3}*d^{**4} + 162*\_t^{**4}*a^{**5}*b^{**2}*c^{**4}*d^{**3} + 162*\_t^{**4}*a^{**4}*b^{**3}*c^{**5}*d^{**2} - 243*\_t^{**4}*a^{**3}*b^{**4}*c^{**6}*d + 81*\_t^{**4}*a^{**2}*b^{**5}*c^{**7} - 3*\_t^{**4}*a^{**4}*d^{**4} + 3*\_t^{**4}*a^{**3}*b*c*d^{**3} + 3*\_t^{**4}*a*b^{**3}*c^{**3}*d - 3*\_t^{**4}*b^{**4}*c^{**4}) / (a^{**2}*b*d^{**3} + b^{**3}*c^{**2}*d)))$

**GIAC/XCAS [A]** time = 0.227861, size = 375, normalized size = 1.3

$$\begin{aligned}
 & -\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}} \ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3(abc - a^2d)} + \frac{d\left(-\frac{c}{d}\right)^{\frac{1}{3}} \ln\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3(bc^2 - acd)} \\
 & + \frac{\left(-ab^2\right)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}abc - \sqrt{3}a^2d} - \frac{\left(-cd^2\right)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2 - \sqrt{3}acd} \\
 & + \frac{\left(-ab^2\right)^{\frac{1}{3}} \ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(abc - a^2d)} - \frac{\left(-cd^2\right)^{\frac{1}{3}} \ln\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(bc^2 - acd)}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^3 + a)\*(d\*x^3 + c)),x, algorithm="giac")

[Out]  $-1/3*b*(-a/b)^{(1/3)}*\ln(\text{abs}(x - (-a/b)^{(1/3)}))/(a*b*c - a^2*d) + 1/3*d*(-c/d)^{(1/3)}*\ln(\text{abs}(x - (-c/d)^{(1/3)}))/(b*c^2 - a*c*d) + (-a*b^2)^{(1/3)}*\arctan(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(\text{sqrt}(3)*a*b*c - \text{sqrt}(3)*a^2*d) - (-c*d^2)^{(1/3)}*\arctan(1/3*\text{sqrt}(3)*(2*x + (-c/d)^{(1/3)})/(-c/d)^{(1/3)})/(\text{sqrt}(3)*b*c^2 - \text{sqrt}(3)*a*c*d) + 1/6*(-a*b^2)^{(1/3)}*\ln(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a*b*c - a^2*d) - 1/6*(-c*d^2)^{(1/3)}*\ln(x^2 + x*(-c/d)^{(1/3)} + (-c/d)^{(2/3)})/(b*c^2 - a*c*d)$

$$3.19 \quad \int \frac{1}{(a+bx^3)(c+dx^3)^2} dx$$

**Optimal.** Leaf size=346

$$\begin{aligned} & -\frac{b^{5/3} \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)^2} + \frac{b^{5/3} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}(bc-ad)^2} - \frac{b^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{a-2\sqrt[3]{bx}}}{\sqrt[3]{3}\sqrt[3]{a}}\right)}{\sqrt[3]{3}a^{2/3}(bc-ad)^2} \\ & + \frac{d^{2/3}(5bc-2ad) \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{18c^{5/3}(bc-ad)^2} - \frac{d^{2/3}(5bc-2ad) \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{9c^{5/3}(bc-ad)^2} \\ & + \frac{d^{2/3}(5bc-2ad) \tan^{-1}\left(\frac{\sqrt[3]{c-2\sqrt[3]{dx}}}{\sqrt[3]{3}\sqrt[3]{c}}\right)}{3\sqrt[3]{3}c^{5/3}(bc-ad)^2} - \frac{dx}{3c(c+dx^3)(bc-ad)} \end{aligned}$$

[Out]  $-(d*x)/(3*c*(b*c - a*d)*(c + d*x^3)) - (b^{5/3}*ArcTan[(a^{1/3} - 2*b^{1/3}*x)/(Sqrt[3]*a^{1/3})])/(Sqrt[3]*a^{2/3}*(b*c - a*d)^2) + (d^{2/3}*(5*b*c - 2*a*d)*ArcTan[(c^{1/3} - 2*d^{1/3}*x)/(Sqrt[3]*c^{1/3})])/(3*Sqrt[3]*c^{5/3}*(b*c - a*d)^2) + (b^{5/3}*Log[a^{1/3} + b^{1/3}*x])/(3*a^{2/3}*(b*c - a*d)^2) - (d^{2/3}*(5*b*c - 2*a*d)*Log[c^{1/3} + d^{1/3}*x])/(9*c^{5/3}*(b*c - a*d)^2) - (b^{5/3}*Log[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(6*a^{2/3}*(b*c - a*d)^2) + (d^{2/3}*(5*b*c - 2*a*d)*Log[c^{2/3} - c^{1/3}*d^{1/3}*x + d^{2/3}*x^2])/(18*c^{5/3}*(b*c - a*d)^2)$

**Rubi [A]** time = 0.613004, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\begin{aligned} & -\frac{b^{5/3} \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)^2} + \frac{b^{5/3} \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{3a^{2/3}(bc-ad)^2} - \frac{b^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{a-2\sqrt[3]{bx}}}{\sqrt[3]{3}\sqrt[3]{a}}\right)}{\sqrt[3]{3}a^{2/3}(bc-ad)^2} \\ & + \frac{d^{2/3}(5bc-2ad) \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{18c^{5/3}(bc-ad)^2} - \frac{d^{2/3}(5bc-2ad) \log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{9c^{5/3}(bc-ad)^2} \\ & + \frac{d^{2/3}(5bc-2ad) \tan^{-1}\left(\frac{\sqrt[3]{c-2\sqrt[3]{dx}}}{\sqrt[3]{3}\sqrt[3]{c}}\right)}{3\sqrt[3]{3}c^{5/3}(bc-ad)^2} - \frac{dx}{3c(c+dx^3)(bc-ad)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)\*(c + d\*x^3)^2), x]

[Out]  $-(d*x)/(3*c*(b*c - a*d)*(c + d*x^3)) - (b^{5/3}*ArcTan[(a^{1/3} - 2*b^{1/3}*x)/(Sqrt[3]*a^{1/3})])/(Sqrt[3]*a^{2/3}*(b*c - a*d)^2) + (d^{2/3}*(5*b*c - 2*a*d)*ArcTan[(c^{1/3} - 2*d^{1/3}*x)/(Sqrt[3]*c^{1/3})])/(3*Sqrt[3]*c^{5/3}*(b*c - a*d)^2) + (b^{5/3}*Log[a^{1/3} + b^{1/3}*x])/(3*a^{2/3}*(b*c - a*d)^2) - (d^{2/3}*(5*b*c - 2*a*d)*Log[c^{1/3} + d^{1/3}*x])/(9*c^{5/3}*(b*c - a*d)^2) - (b^{5/3}*Log[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(6*a^{2/3}*(b*c - a*d)^2) + (d^{2/3}*(5*b*c - 2*a*d)*Log[c^{2/3} - c^{1/3}*d^{1/3}*x + d^{2/3}*x^2])/(18*c^{5/3}*(b*c - a*d)^2)$

**Rubi in Sympy [A]** time = 108.441, size = 321, normalized size = 0.93

$$\frac{dx}{3c(c+dx^3)(ad-bc)} + \frac{d^{\frac{2}{3}}(2ad-5bc)\log(\sqrt[3]{c} + \sqrt[3]{dx})}{9c^{\frac{5}{3}}(ad-bc)^2}$$

$$- \frac{d^{\frac{2}{3}}(2ad-5bc)\log\left(c^{\frac{2}{3}} - \sqrt[3]{c}\sqrt[3]{dx} + d^{\frac{2}{3}}x^2\right)}{18c^{\frac{5}{3}}(ad-bc)^2} - \frac{\sqrt{3}d^{\frac{2}{3}}(2ad-5bc)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3} - 2\frac{\sqrt[3]{dx}}{3}\right)}{\sqrt[3]{c}}\right)}{9c^{\frac{5}{3}}(ad-bc)^2}$$

$$+ \frac{b^{\frac{5}{3}}\log(\sqrt[3]{a} + \sqrt[3]{bx})}{3a^{\frac{2}{3}}(ad-bc)^2} - \frac{b^{\frac{5}{3}}\log\left(a^{\frac{2}{3}} - \sqrt[3]{a}\sqrt[3]{bx} + b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}(ad-bc)^2} - \frac{\sqrt{3}b^{\frac{5}{3}}\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} - 2\frac{\sqrt[3]{bx}}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(b*x**3+a)/(d*x**3+c)**2,x)`

[Out]  $d*x/(3*c*(c+d*x**3)*(a*d-b*c)) + d**(2/3)*(2*a*d-5*b*c)*\log(c**(1/3)+d**(1/3)*x)/(9*c**(5/3)*(a*d-b*c)**2) - d**(2/3)*(2*a*d-5*b*c)*\log(c**(2/3)-c**(1/3)*d**(1/3)*x+d**(2/3)*x**2)/(18*c**(5/3)*(a*d-b*c)**2) - \sqrt{3}*d**(2/3)*(2*a*d-5*b*c)*\operatorname{atan}(\sqrt{3}*(c**(1/3)/3-2*d**(1/3)*x/3)/c**(1/3))/(9*c**(5/3)*(a*d-b*c)**2) + b**(5/3)*\log(a**(1/3)+b**(1/3)*x)/(3*a**(2/3)*(a*d-b*c)**2) - b**(5/3)*\log(a**(2/3)-a**(1/3)*b**(1/3)*x+b**(2/3)*x**2)/(6*a**(2/3)*(a*d-b*c)**2) - \sqrt{3}*b**(5/3)*\operatorname{atan}(\sqrt{3}*(a**(1/3)/3-2*b**(1/3)*x/3)/a**(1/3))/(3*a**(2/3)*(a*d-b*c)**2)$

**Mathematica [A]** time = 0.351284, size = 336, normalized size = 0.97

$$-3b^{5/3}c^{5/3}(c+dx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)+a^{2/3}d^{2/3}(c+dx^3)(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx}+d^{2/3}x^2\right)+6a^{2/3}c^{2/3}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((a+b*x^3)*(c+d*x^3)^2),x]`

[Out]  $(6*a^{(2/3)}*c^{(2/3)}*d*(-(b*c)+a*d)*x-6*\sqrt{3}*b^{(5/3)}*c^{(5/3)}*(c+d*x^3)*\operatorname{ArcTan}\left[\frac{1-(2*b^{(1/3)}*x)/a^{(1/3)}}{\sqrt{3}}\right]-2*\sqrt{3}*a^{(2/3)}*d^{(2/3)}*(-5*b*c+2*a*d)*(c+d*x^3)*\operatorname{ArcTan}\left[\frac{1-(2*d^{(1/3)}*x)/c^{(1/3)}}{\sqrt{3}}\right]+6*b^{(5/3)}*c^{(5/3)}*(c+d*x^3)*\log[a^{(1/3)}+b^{(1/3)}*x]+2*a^{(2/3)}*d^{(2/3)}*(-5*b*c+2*a*d)*(c+d*x^3)*\log[c^{(1/3)}+d^{(1/3)}*x]-3*b^{(5/3)}*c^{(5/3)}*(c+d*x^3)*\log[a^{(2/3)}-a^{(1/3)}*b^{(1/3)}*x+b^{(2/3)}*x^2]+a^{(2/3)}*d^{(2/3)}*(5*b*c-2*a*d)*(c+d*x^3)*\log[c^{(2/3)}-c^{(1/3)}*d^{(1/3)}*x+d^{(2/3)}*x^2])/(18*a^{(2/3)}*c^{(5/3)}*(b*c-a*d)^2*(c+d*x^3))$

**Maple [A]** time = 0.018, size = 406, normalized size = 1.2

$$\begin{aligned} & \frac{b}{3(ad-bc)^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{b}{6(ad-bc)^2} \ln\left(x^2 - x\sqrt[3]{\frac{a}{b}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \\ & + \frac{b\sqrt{3}}{3(ad-bc)^2} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{d^2xa}{3(ad-bc)^2c(dx^3+c)} \\ & - \frac{dxb}{3(ad-bc)^2(dx^3+c)} + \frac{2ad}{9(ad-bc)^2c} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & - \frac{5b}{9(ad-bc)^2} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{ad}{9(ad-bc)^2c} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & + \frac{5b}{18(ad-bc)^2} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & + \frac{2d\sqrt{3}a}{9(ad-bc)^2c} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\ & - \frac{5b\sqrt{3}}{9(ad-bc)^2} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)/(d\*x^3+c)^2,x)

[Out] 1/3\*b/(a\*d-b\*c)^2/(a/b)^(2/3)\*ln(x+(a/b)^(1/3))-1/6\*b/(a\*d-b\*c)^2/(a/b)^(2/3)\*ln(x^2-x\*(a/b)^(1/3)+(a/b)^(2/3))+1/3\*b/(a\*d-b\*c)^2/(a/b)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(a/b)^(1/3)\*x-1))+1/3\*d^2/(a\*d-b\*c)^2/c\*x/(d\*x^3+c)\*a-1/3\*d/(a\*d-b\*c)^2\*x/(d\*x^3+c)\*b+2/9\*d/(a\*d-b\*c)^2/c/(c/d)^(2/3)\*ln(x+(c/d)^(1/3))\*a-5/9/(a\*d-b\*c)^2/(c/d)^(2/3)\*ln(x+(c/d)^(1/3))\*b-1/9\*d/(a\*d-b\*c)^2/c/(c/d)^(2/3)\*ln(x^2-x\*(c/d)^(1/3)+(c/d)^(2/3))\*a+5/18/(a\*d-b\*c)^2/(c/d)^(2/3)\*ln(x^2-x\*(c/d)^(1/3)+(c/d)^(2/3))\*b+2/9\*d/(a\*d-b\*c)^2/c/(c/d)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(c/d)^(1/3)\*x-1))\*a-5/9/(a\*d-b\*c)^2/(c/d)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(c/d)^(1/3)\*x-1))\*b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^3 + a)\*(d\*x^3 + c)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 7.36613, size = 625, normalized size = 1.81

$$\sqrt{3}\left(3\sqrt{3}(bcdx^3+bc^2)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-\sqrt{3}((5bcd-2ad^2)x^3+5bc^2-2acd)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(d^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^3 + a)\*(d\*x^3 + c)^2),x, algorithm="fricas")

[Out]  $-1/54 \sqrt{3} (3 \sqrt{3}) (b^2 c^2 d^2 x^3 + b^2 c^2 d^2) (b^2/a^2)^{1/3} \log(b^2 x^2 - a b^2 x (b^2/a^2)^{1/3} + a^2 (b^2/a^2)^{2/3}) - \sqrt{3} ((5 b^2 c^2 d - 2 a^2 d^2) x^3 + 5 b^2 c^2 d - 2 a^2 c^2 d) (d^2/c^2)^{1/3} \log(d^2 x^2 - c^2 d x (d^2/c^2)^{1/3} + c^2 (d^2/c^2)^{2/3}) - 6 \sqrt{3} (3) (b^2 c^2 d^2 x^3 + b^2 c^2 d^2) (b^2/a^2)^{1/3} \log(b x + a (b^2/a^2)^{1/3}) + 2 \sqrt{3} ((5 b^2 c^2 d - 2 a^2 d^2) x^3 + 5 b^2 c^2 d - 2 a^2 c^2 d) (d^2/c^2)^{1/3} \log(d x + c (d^2/c^2)^{1/3}) + 6 \sqrt{3} (b^2 c^2 d - a^2 d^2) x + 18 (b^2 c^2 d^2 x^3 + b^2 c^2 d^2) (b^2/a^2)^{1/3} \arctan(-1/3 (2 \sqrt{3} b^2 c^2 d - \sqrt{3} a^2 d^2) x - \sqrt{3} a (b^2/a^2)^{1/3}) / (a^2 (b^2/a^2)^{1/3}) - 6 ((5 b^2 c^2 d - 2 a^2 d^2) x^3 + 5 b^2 c^2 d - 2 a^2 c^2 d) (d^2/c^2)^{1/3} \arctan(-1/3 (2 \sqrt{3} c^2 d^2 x - \sqrt{3} c^2 (d^2/c^2)^{1/3}) / (c^2 (d^2/c^2)^{1/3})) / (b^2 c^4 - 2 a b^2 c^3 d + a^2 c^2 d^2 + (b^2 c^3 d - 2 a^2 b^2 c^2 d^2 + a^2 c^2 d^3) x^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**3+a)/(d*x**3+c)**2,x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.227324, size = 598, normalized size = 1.73

$$\begin{aligned} & \frac{b^2 \left(-\frac{a}{b}\right)^{\frac{1}{3}} \ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3(ab^2c^2 - 2a^2bcd + a^3d^2)} + \frac{(-ab^2)^{\frac{1}{3}} b \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab^2c^2 - 2\sqrt{3}a^2bcd + \sqrt{3}a^3d^2} \\ & + \frac{(-ab^2)^{\frac{1}{3}} b \ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(ab^2c^2 - 2a^2bcd + a^3d^2)} + \frac{(5bcd - 2ad^2)\left(-\frac{c}{d}\right)^{\frac{1}{3}} \ln\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9(b^2c^4 - 2abc^3d + a^2c^2d^2)} \\ & - \frac{\left(5(-cd^2)^{\frac{1}{3}}bc - 2(-cd^2)^{\frac{1}{3}}ad\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}b^2c^4 - 2\sqrt{3}abc^3d + \sqrt{3}a^2c^2d^2\right)} \\ & - \frac{\left(5(-cd^2)^{\frac{1}{3}}bc - 2(-cd^2)^{\frac{1}{3}}ad\right) \ln\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18(b^2c^4 - 2abc^3d + a^2c^2d^2)} - \frac{dx}{3(dx^3 + c)(bc^2 - acd)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^3 + a)*(d*x^3 + c)^2),x, algorithm="giac")`

[Out]  $-1/3 b^2 (-a/b)^{1/3} \ln(\text{abs}(x - (-a/b)^{1/3})) / (a^2 b^2 c^2 - 2 a^2 b^2 c^2 d + a^3 d^2) + (-a b^2)^{1/3} b \arctan(1/3 \sqrt{3} (2 x + (-a/b)^{1/3}) / (-a/b)^{1/3}) / (\sqrt{3} a^2 b^2 c^2 - 2 \sqrt{3} a^2 b^2 c^2 d + \sqrt{3} a^3 d^2) + 1/6 (-a b^2)^{1/3} b \ln(x^2 + x (-a/b)^{1/3} + (-a/b)^{2/3}) / (a^2 b^2 c^2 - 2 a^2 b^2 c^2 d + a^3 d^2) + 1/9 (5 b^2 c^2 d - 2 a^2 d^2) (-c/d)^{1/3} \ln(\text{abs}(x - (-c/d)^{1/3})) / (b^2 c^4 - 2 a b^2 c^3 d + a^2 c^2 d^2) - 1/3 (5 (-c d^2)^{1/3} b c - 2 (-c d^2)^{1/3} a d) \arctan(1/3 \sqrt{3} (2 x + (-c/d)^{1/3}) / (-c/d)^{1/3}) / (\sqrt{3} b^2 c^4 - 2 \sqrt{3} a b^2 c^3 d + \sqrt{3} a^2 c^2 d^2) - 1/18 (5 (-c d^2)^{1/3} b c - 2 (-c d^2)^{1/3} a d) \ln(x^2 + x (-c/d)^{1/3} + (-c/d)^{2/3}) / (b^2 c^4 - 2 a b^2 c^3 d + a^2 c^2 d^2) - 1/3 d x / ((d x^3 + c) (b^2 c^2 - a^2 c d))$



$$3.20 \quad \int \frac{(c+dx^3)^5}{(a+bx^3)^2} dx$$

**Optimal.** Leaf size=320

$$\begin{aligned} & -\frac{(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}} + \frac{(bc-ad)^4(13ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{5/3}b^{16/3}} \\ & -\frac{(bc-ad)^4(13ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a-2}\sqrt[3]{bx}}{\sqrt[3]{3}\sqrt[3]{a}}\right)}{3\sqrt[3]{3}a^{5/3}b^{16/3}} + \frac{d^3x^4(3a^2d^2-10abcd+10b^2c^2)}{4b^4} \\ & + \frac{d^2x(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)}{b^5} + \frac{x(bc-ad)^5}{3ab^5(a+bx^3)} + \frac{d^4x^7(5bc-2ad)}{7b^3} + \frac{d^5x^{10}}{10b^2} \end{aligned}$$

[Out]  $(d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4)/(4*b^4) + (d^4*(5*b*c - 2*a*d)*x^7)/(7*b^3) + (d^5*x^{10})/(10*b^2) + ((b*c - a*d)^5*x)/(3*a*b^5*(a + b*x^3)) - ((b*c - a*d)^4*(2*b*c + 13*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(16/3)) + ((b*c - a*d)^4*(2*b*c + 13*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(16/3)) - ((b*c - a*d)^4*(2*b*c + 13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(16/3))$

**Rubi [A]** time = 0.63287, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\begin{aligned} & -\frac{(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}} + \frac{(bc-ad)^4(13ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{5/3}b^{16/3}} \\ & -\frac{(bc-ad)^4(13ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a-2}\sqrt[3]{bx}}{\sqrt[3]{3}\sqrt[3]{a}}\right)}{3\sqrt[3]{3}a^{5/3}b^{16/3}} + \frac{d^3x^4(3a^2d^2-10abcd+10b^2c^2)}{4b^4} \\ & + \frac{d^2x(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)}{b^5} + \frac{x(bc-ad)^5}{3ab^5(a+bx^3)} + \frac{d^4x^7(5bc-2ad)}{7b^3} + \frac{d^5x^{10}}{10b^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^5/(a + b\*x^3)^2, x]

[Out]  $(d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4)/(4*b^4) + (d^4*(5*b*c - 2*a*d)*x^7)/(7*b^3) + (d^5*x^{10})/(10*b^2) + ((b*c - a*d)^5*x)/(3*a*b^5*(a + b*x^3)) - ((b*c - a*d)^4*(2*b*c + 13*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(16/3)) + ((b*c - a*d)^4*(2*b*c + 13*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(16/3)) - ((b*c - a*d)^4*(2*b*c + 13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(16/3))$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned}
 & -d^2 (4a^3d^3 - 15a^2bcd^2 + 20ab^2c^2d - 10b^3c^3) \int \frac{1}{b^5} dx + \frac{d^5x^{10}}{10b^2} \\
 & - \frac{d^4x^7(2ad - 5bc)}{7b^3} + \frac{d^3x^4(3a^2d^2 - 10abcd + 10b^2c^2)}{4b^4} - \frac{x(ad - bc)^5}{3ab^5(a + bx^3)} \\
 & + \frac{(ad - bc)^4(13ad + 2bc) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{\frac{5}{3}}b^{\frac{16}{3}}} - \frac{(ad - bc)^4(13ad + 2bc) \log\left(a^{\frac{2}{3}} - \sqrt[3]{a}\sqrt[3]{bx} + b^{\frac{2}{3}}x^2\right)}{18a^{\frac{5}{3}}b^{\frac{16}{3}}} \\
 & - \frac{\sqrt{3}(ad - bc)^4(13ad + 2bc) \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} - \frac{2\sqrt[3]{bx}}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}b^{\frac{16}{3}}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((d*x**3+c)**5/(b*x**3+a)**2,x)`

[Out] `-d**2*(4*a**3*d**3 - 15*a**2*b*c*d**2 + 20*a*b**2*c**2*d - 10*b**3*c**3)*Integral(b**(-5), x) + d**5*x**10/(10*b**2) - d**4*x**7*(2*a*d - 5*b*c)/(7*b**3) + d**3*x**4*(3*a**2*d**2 - 10*a*b*c*d + 10*b**2*c**2)/(4*b**4) - x*(a*d - b*c)**5/(3*a*b**5*(a + b*x**3)) + (a*d - b*c)**4*(13*a*d + 2*b*c)*log(a**(1/3) + b**(1/3)*x)/(9*a**(5/3)*b**(16/3)) - (a*d - b*c)**4*(13*a*d + 2*b*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(18*a**(5/3)*b**(16/3)) - sqrt(3)*(a*d - b*c)**4*(13*a*d + 2*b*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(5/3)*b**(16/3))`

**Mathematica [A]** time = 0.438508, size = 313, normalized size = 0.98

$$\frac{70(bc-ad)^4(13ad+2bc) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{a^{5/3}} + \frac{140(bc-ad)^4(13ad+2bc) \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{a^{5/3}} + \frac{140\sqrt{3}(bc-ad)^4(13ad+2bc) \tan^{-1}\left(\frac{2\sqrt[3]{bx} - \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}} + 3$$

Antiderivative was successfully verified.

[In] `Integrate[(c + d*x^3)^5/(a + b*x^3)^2,x]`

[Out] `(1260*b^(1/3)*d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x + 315*b^(4/3)*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 180*b^(7/3)*d^4*(5*b*c - 2*a*d)*x^7 + 126*b^(10/3)*d^5*x^10 + (420*b^(1/3)*(b*c - a*d)^5*x)/(a*(a + b*x^3)) + (140*sqrt(3)*(b*c - a*d)^4*(2*b*c + 13*a*d)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(sqrt(3)*a^(1/3))]/a^(5/3) + (140*(b*c - a*d)^4*(2*b*c + 13*a*d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (70*(b*c - a*d)^4*(2*b*c + 13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(1260*b^(16/3))`

**Maple [B]** time = 0.017, size = 905, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)^5/(b*x^3+a)^2,x)`

[Out] `13/9/b^6*a^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d^5+5/9/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^4*d+2/9/b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^5-13/18/b^6*a^4/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d^4`

$$\begin{aligned}
& 5 - 5/18/b^2/(a/b)^{(2/3)} \cdot \ln(x^2 - x \cdot (a/b)^{(1/3)} + (a/b)^{(2/3)}) \cdot c^4 \cdot d - 1/ \\
& 9/b/a/(a/b)^{(2/3)} \cdot \ln(x^2 - x \cdot (a/b)^{(1/3)} + (a/b)^{(2/3)}) \cdot c^5 - 20 \cdot d^3/b^3 \cdot a \cdot c^2 \cdot x - 5/2 \cdot d^4/b^3 \cdot x^4 \cdot a \cdot c + 15 \cdot d^4/b^4 \cdot a^2 \cdot c \cdot x - 1/3/b^5 \cdot x \cdot a^4/(b \cdot x^3 + a) \cdot d^5 + 70/9/b^4 \cdot a^2/(a/b)^{(2/3)} \cdot \ln(x + (a/b)^{(1/3)}) \cdot c^2 \cdot d^3 - 40 \\
& /9/b^3 \cdot a/(a/b)^{(2/3)} \cdot \ln(x + (a/b)^{(1/3)}) \cdot c^3 \cdot d^2 + 25/9/b^5 \cdot a^3/(a/b)^{(2/3)} \cdot \ln(x^2 - x \cdot (a/b)^{(1/3)} + (a/b)^{(2/3)}) \cdot c \cdot d^4 - 35/9/b^4 \cdot a^2/(a/b)^{(2/3)} \cdot \ln(x^2 - x \cdot (a/b)^{(1/3)} + (a/b)^{(2/3)}) \cdot c^2 \cdot d^3 + 1/3 \cdot x/a/(b \cdot x^3 + a) \cdot c^5 + 10 \cdot d^2/b^2 \cdot c^3 \cdot x - 2/7 \cdot d^5/b^3 \cdot x^7 \cdot a + 5/7 \cdot d^4/b^2 \cdot x^7 \cdot c + 3/4 \cdot d^5/b^4 \cdot x^4 \cdot a^2 + 5/2 \cdot d^3/b^2 \cdot x^4 \cdot c^2 - 4 \cdot d^5/b^5 \cdot a^3 \cdot x + 20/9/b^3 \cdot a/(a/b)^{(2/3)} \cdot \ln(x^2 - x \cdot (a/b)^{(1/3)} + (a/b)^{(2/3)}) \cdot c^3 \cdot d^2 - 5/3/b \cdot x/(b \cdot x^3 + a) \cdot c^4 \cdot d - 50/9/b^5 \cdot a^3/(a/b)^{(2/3)} \cdot 3^{(1/2)} \cdot \arctan(1/3 \cdot 3^{(1/2)} \cdot (2/(a/b)^{(1/3)} \cdot x - 1)) \cdot c \cdot d^4 + 70/9/b^4 \cdot a^2/(a/b)^{(2/3)} \cdot 3^{(1/2)} \cdot \arctan(1/3 \cdot 3^{(1/2)} \cdot (2/(a/b)^{(1/3)} \cdot x - 1)) \cdot c^2 \cdot d^3 - 40/9/b^3 \cdot a/(a/b)^{(2/3)} \cdot 3^{(1/2)} \cdot \arctan(1/3 \cdot 3^{(1/2)} \cdot (2/(a/b)^{(1/3)} \cdot x - 1)) \cdot c^3 \cdot d^2 + 1/10 \cdot d^5 \cdot x^{10}/b^2 + 2/9/b/a/(a/b)^{(2/3)} \cdot 3^{(1/2)} \cdot \arctan(1/3 \cdot 3^{(1/2)} \cdot (2/(a/b)^{(1/3)} \cdot x - 1)) \cdot c^5 + 13/9/b^6 \cdot a^4/(a/b)^{(2/3)} \cdot 3^{(1/2)} \cdot \arctan(1/3 \cdot 3^{(1/2)} \cdot (2/(a/b)^{(1/3)} \cdot x - 1)) \cdot d^5 + 5/9/b^2/(a/b)^{(2/3)} \cdot 3^{(1/2)} \cdot \arctan(1/3 \cdot 3^{(1/2)} \cdot (2/(a/b)^{(1/3)} \cdot x - 1)) \cdot c^4 \cdot d - 50/9/b^5 \cdot a^3/(a/b)^{(2/3)} \cdot \ln(x + (a/b)^{(1/3)}) \cdot c \cdot d^4 + 5/3/b^4 \cdot x \cdot a^3/(b \cdot x^3 + a) \cdot c \cdot d^4 - 10/3/b^3 \cdot x \cdot a^2/(b \cdot x^3 + a) \cdot c^2 \cdot d^3 + 10/3/b^2 \cdot x \cdot a/(b \cdot x^3 + a) \cdot c^3 \cdot d^2
\end{aligned}$$


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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^5/(b\*x^3 + a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

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**Fricas [A]** time = 0.223337, size = 1000, normalized size = 3.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^5/(b\*x^3 + a)^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned}
& -1/3780 \cdot \sqrt{3} \cdot (70 \cdot \sqrt{3}) \cdot (2 \cdot a \cdot b^5 \cdot c^5 + 5 \cdot a^2 \cdot b^4 \cdot c^4 \cdot d - 40 \cdot a^3 \cdot b^3 \cdot c^3 \cdot d^2 + 70 \cdot a^4 \cdot b^2 \cdot c^2 \cdot d^3 - 50 \cdot a^5 \cdot b \cdot c \cdot d^4 + 13 \cdot a^6 \cdot d^5 \\
& + (2 \cdot b^6 \cdot c^5 + 5 \cdot a \cdot b^5 \cdot c^4 \cdot d - 40 \cdot a^2 \cdot b^4 \cdot c^3 \cdot d^2 + 70 \cdot a^3 \cdot b^3 \cdot c^2 \cdot d^3 - 50 \cdot a^4 \cdot b^2 \cdot c \cdot d^4 + 13 \cdot a^5 \cdot b \cdot d^5) \cdot x^3) \cdot \log((a^2 \cdot b)^{(2/3)} \cdot x^2 - (a^2 \cdot b)^{(1/3)} \cdot a \cdot x + a^2) - 140 \cdot \sqrt{3} \cdot (2 \cdot a \cdot b^5 \cdot c^5 + 5 \cdot a^2 \cdot b^4 \cdot c^4 \cdot d - 40 \cdot a^3 \cdot b^3 \cdot c^3 \cdot d^2 + 70 \cdot a^4 \cdot b^2 \cdot c^2 \cdot d^3 - 50 \cdot a^5 \cdot b \cdot c \cdot d^4 + 13 \cdot a^6 \cdot d^5 + (2 \cdot b^6 \cdot c^5 + 5 \cdot a \cdot b^5 \cdot c^4 \cdot d - 40 \cdot a^2 \cdot b^4 \cdot c^3 \cdot d^2 + 70 \cdot a^3 \cdot b^3 \cdot c^2 \cdot d^3 - 50 \cdot a^4 \cdot b^2 \cdot c \cdot d^4 + 13 \cdot a^5 \cdot b \cdot d^5) \cdot x^3) \cdot \log((a^2 \cdot b)^{(1/3)} \cdot x + a) - 420 \cdot (2 \cdot a \cdot b^5 \cdot c^5 + 5 \cdot a^2 \cdot b^4 \cdot c^4 \cdot d - 40 \cdot a^3 \cdot b^3 \cdot c^3 \cdot d^2 + 70 \cdot a^4 \cdot b^2 \cdot c^2 \cdot d^3 - 50 \cdot a^5 \cdot b \cdot c \cdot d^4 + 13 \cdot a^6 \cdot d^5 + (2 \cdot b^6 \cdot c^5 + 5 \cdot a \cdot b^5 \cdot c^4 \cdot d - 40 \cdot a^2 \cdot b^4 \cdot c^3 \cdot d^2 + 70 \cdot a^3 \cdot b^3 \cdot c^2 \cdot d^3 - 50 \cdot a^4 \cdot b^2 \cdot c \cdot d^4 + 13 \cdot a^5 \cdot b \cdot d^5) \cdot x^3) \cdot \arctan(1/3 \cdot (2 \cdot \sqrt{3}) \cdot (a^2 \cdot b)^{(1/3)} \cdot x - \sqrt{3}) \cdot a/a) - 3 \cdot \sqrt{3} \cdot (42 \cdot a \cdot b^4 \cdot d^5 \cdot x^{13} + 6 \cdot (50 \cdot a \cdot b^4 \cdot c \cdot d^4 - 13 \cdot a^2 \cdot b^3 \cdot d^5) \cdot x^{10} + 15 \cdot (70 \cdot a \cdot b^4 \cdot c^2 \cdot d^3 - 50 \cdot a^2 \cdot b^3 \cdot c \cdot d^4 + 13 \cdot a^3 \cdot b^2 \cdot d^5) \cdot x^7 + 105 \cdot (40 \cdot a \cdot b^4 \cdot c^3 \cdot d^2 - 70 \cdot a^2 \cdot b^3 \cdot c^2 \cdot d^3 + 50 \cdot a^3 \cdot b^2 \cdot c \cdot d^4 - 13 \cdot a^4 \cdot b \cdot d^5) \cdot x^4 + 140 \cdot (b^5 \cdot c^5 - 5 \cdot a \cdot b^4 \cdot c^4 \cdot d + 40 \cdot a^2 \cdot b^3 \cdot c^3 \cdot d^2 - 70 \cdot a^3 \cdot b^2 \cdot c^2 \cdot d^3 + 50 \cdot a^4 \cdot b \cdot c \cdot d^4 - 13 \cdot a^5 \cdot d^5) \cdot x) \cdot (a^2 \cdot b)^{(1/3)}) / ((a \cdot b^6 \cdot x^3 + a^2 \cdot b^5) \cdot (a^2 \cdot b)^{(1/3)})
\end{aligned}$$

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**Sympy [A]** time = 25.775, size = 536, normalized size = 1.68

$$\frac{x(a^5d^5 - 5a^4bcd^4 + 10a^3b^2c^2d^3 - 10a^2b^3c^3d^2 + 5ab^4c^4d - b^5c^5)}{3a^2b^5 + 3ab^6x^3} + \text{RootSum}\left(729t^3a^5b^{16} - 2197a^{15}d^{15} + 25350a^{14}bcd^{14} - 132990a^{13}b^2c^2d^{13} + 418280a^{12}b^3c^3d^{12} - 874635a^{11}b^4c^4d^{11} + 1271886a^{10}b^5c^5d^{10} - 1302400a^9b^6c^6d^9 + 922680a^8b^7c^7d^8 - 422235a^7b^8c^8d^7 + 97570a^6b^9c^9d^6 + 7194a^5b^{10}c^{10}d^5 - 10200a^4b^{11}c^{11}d^4 + 1435a^3b^{12}c^{12}d^3 + 330a^2b^{13}c^{13}d^2 - 60ab^{14}c^{14}d - 8b^{15}c^{15}\right) \frac{d^5x^{10}}{10b^2} - \frac{x^7(2ad^5 - 5bcd^4)}{7b^3} + \frac{x^4(3a^2d^5 - 10abcd^4 + 10b^2c^2d^3)}{4b^4} - \frac{x(4a^3d^5 - 15a^2bcd^4 + 20ab^2c^2d^3 - 10b^3c^3d^2)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*5/(b\*x\*\*3+a)\*\*2,x)

[Out]  $-x*(a^{**5}d^{**5} - 5*a^{**4}b*c*d^{**4} + 10*a^{**3}b^{**2}c^{**2}d^{**3} - 10*a^{**2}b^{**3}c^{**3}d^{**2} + 5*a*b^{**4}c^{**4}d - b^{**5}c^{**5})/(3*a^{**2}b^{**5} + 3*a*b^{**6}x^{**3}) + \text{RootSum}(729\_t^{**3}a^{**5}b^{**16} - 2197*a^{**15}d^{**15} + 25350*a^{**14}b*c*d^{**14} - 132990*a^{**13}b^{**2}c^{**2}d^{**13} + 418280*a^{**12}b^{**3}c^{**3}d^{**12} - 874635*a^{**11}b^{**4}c^{**4}d^{**11} + 1271886*a^{**10}b^{**5}c^{**5}d^{**10} - 1302400*a^{**9}b^{**6}c^{**6}d^{**9} + 922680*a^{**8}b^{**7}c^{**7}d^{**8} - 422235*a^{**7}b^{**8}c^{**8}d^{**7} + 97570*a^{**6}b^{**9}c^{**9}d^{**6} + 7194*a^{**5}b^{**10}c^{**10}d^{**5} - 10200*a^{**4}b^{**11}c^{**11}d^{**4} + 1435*a^{**3}b^{**12}c^{**12}d^{**3} + 330*a^{**2}b^{**13}c^{**13}d^{**2} - 60*a*b^{**14}c^{**14}d - 8*b^{**15}c^{**15}, \text{Lambda}(\_t, \_t*\log(9*\_t*a^{**2}b^{**5}/(13*a^{**5}d^{**5} - 50*a^{**4}b*c*d^{**4} + 70*a^{**3}b^{**2}c^{**2}d^{**3} - 40*a^{**2}b^{**3}c^{**3}d^{**2} + 5*a*b^{**4}c^{**4}d + 2*b^{**5}c^{**5}) + x)) + d^{**5}x^{**2}/(10*b^{**2}) - x^{**7}*(2*a*d^{**5} - 5*b*c*d^{**4})/(7*b^{**3}) + x^{**4}*(3*a^{**2}d^{**5} - 10*a*b*c*d^{**4} + 10*b^{**2}c^{**2}d^{**3})/(4*b^{**4}) - x*(4*a^{**3}d^{**5} - 15*a^{**2}b*c*d^{**4} + 20*a*b^{**2}c^{**2}d^{**3} - 10*b^{**3}c^{**3}d^{**2})/b^{**5}$

**GIAC/XCAS [A]** time = 0.221869, size = 822, normalized size = 2.57

$$\frac{(2b^5c^5 + 5ab^4c^4d - 40a^2b^3c^3d^2 + 70a^3b^2c^2d^3 - 50a^4bcd^4 + 13a^5d^5) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^2b^5} + \frac{\sqrt{3}\left(2(-ab^2)^{\frac{1}{3}}b^5c^5 + 5(-ab^2)^{\frac{1}{3}}ab^4c^4d - 40(-ab^2)^{\frac{1}{3}}a^2b^3c^3d^2 + 70(-ab^2)^{\frac{1}{3}}a^3b^2c^2d^3 - 50(-ab^2)^{\frac{1}{3}}a^4bcd^4 + 13(-ab^2)^{\frac{1}{3}}a^5d^5\right)}{9a^2b^6} + \frac{b^5c^5x - 5ab^4c^4dx + 10a^2b^3c^3d^2x - 10a^3b^2c^2d^3x + 5a^4bcd^4x - a^5d^5x}{3(bx^3 + a)ab^5} + \frac{\left(2(-ab^2)^{\frac{1}{3}}b^5c^5 + 5(-ab^2)^{\frac{1}{3}}ab^4c^4d - 40(-ab^2)^{\frac{1}{3}}a^2b^3c^3d^2 + 70(-ab^2)^{\frac{1}{3}}a^3b^2c^2d^3 - 50(-ab^2)^{\frac{1}{3}}a^4bcd^4 + 13(-ab^2)^{\frac{1}{3}}a^5d^5\right)}{18a^2b^6} + \frac{14b^{18}d^5x^{10} + 100b^{18}cd^4x^7 - 40ab^{17}d^5x^7 + 350b^{18}c^2d^3x^4 - 350ab^{17}cd^4x^4 + 105a^2b^{16}d^5x^4 + 1400b^{18}c^3d^2x - 2800ab^{17}c^2d^2}{140b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^5/(b\*x^3 + a)^2,x, algorithm="giac")

[Out]  $-1/9*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*(-a/b)^{(1/3)}*\ln(\text{abs}(x - (-a/b)^{(1/3)})) / (a^2*b^5) + 1/9*\sqrt{3}*(2*(-a*b^2)^{(1/3)}*b^5*c^5 + 5*(-a*b^2)^{(1/3)}*a*b^4*c^4*d - 40*(-a*b^2)^{(1/3)}*a^2*b^3*c^3*d^2 + 70*(-a*b^2)^{(1/3)}*a^3*b^2*c^2*d^3 - 50*(-a*b^2)^{(1/3)}*a^4*b*c*d^4 + 13*(-a*b^2)^{(1/3)}*a^5*d^5)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)}) / (a^2*b^6) + 1/3*(b^5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*d^3*x + 5*a^4*b*c*d^4*x - a^5*d^5*x) / ((b*x^3 + a)*a*b^5) + 1/18*(2*(-a*b^2)^{(1/3)}*b^5*c^5 + 5*(-a*b^2)^{(1/3)}*a*b^4*c^4*d - 40*(-a*b^2)^{(1/3)}*a^2*b^3*c^3*d^2 + 70*(-a*b^2)^{(1/3)}*a^3*b^2*c^2*d^3 - 50*(-a*b^2)^{(1/3)}*a^4*b*c*d^4 + 13*(-a*b^2)^{(1/3)}*a^5*d^5)$

$$\begin{aligned}
& *c*d^4 + 13*(-a*b^2)^{(1/3)}*a^5*d^5) * \ln(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)}) / (a^2*b^6) + 1/140*(14*b^{18}*d^5*x^{10} + 100*b^{18}*c*d^4*x \\
& ^7 - 40*a*b^{17}*d^5*x^7 + 350*b^{18}*c^2*d^3*x^4 - 350*a*b^{17}*c*d^4* \\
& x^4 + 105*a^2*b^{16}*d^5*x^4 + 1400*b^{18}*c^3*d^2*x - 2800*a*b^{17}*c^2*d^3*x + 2100*a^2*b^{16}*c*d^4*x - 560*a^3*b^{15}*d^5*x) / b^{20}
\end{aligned}$$

$$3.21 \quad \int \frac{(c+dx^3)^4}{(a+bx^3)^2} dx$$

**Optimal.** Leaf size=267

$$\begin{aligned} & \frac{(bc-ad)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}} \\ & + \frac{2(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{5/3}b^{13/3}} - \frac{2(bc-ad)^3(5ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{a-2\sqrt[3]{bx}}}{\sqrt[3]{3}\sqrt[3]{a}}\right)}{3\sqrt[3]{3}a^{5/3}b^{13/3}} \\ & + \frac{d^2x(3a^2d^2-8abcd+6b^2c^2)}{b^4} + \frac{x(bc-ad)^4}{3ab^4(a+bx^3)} + \frac{d^3x^4(2bc-ad)}{2b^3} + \frac{d^4x^7}{7b^2} \end{aligned}$$

[Out]  $(d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (d^3*(2*b*c - a*d)*x^4)/(2*b^3) + (d^4*x^7)/(7*b^2) + ((b*c - a*d)^4*x)/(3*a*b^4*(a + b*x^3)) - (2*(b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(13/3)) + (2*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(13/3)) - ((b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*b^(13/3))$

**Rubi [A]** time = 0.497773, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\begin{aligned} & \frac{(bc-ad)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}} \\ & + \frac{2(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{5/3}b^{13/3}} - \frac{2(bc-ad)^3(5ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{a-2\sqrt[3]{bx}}}{\sqrt[3]{3}\sqrt[3]{a}}\right)}{3\sqrt[3]{3}a^{5/3}b^{13/3}} \\ & + \frac{d^2x(3a^2d^2-8abcd+6b^2c^2)}{b^4} + \frac{x(bc-ad)^4}{3ab^4(a+bx^3)} + \frac{d^3x^4(2bc-ad)}{2b^3} + \frac{d^4x^7}{7b^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^4/(a + b\*x^3)^2, x]

[Out]  $(d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (d^3*(2*b*c - a*d)*x^4)/(2*b^3) + (d^4*x^7)/(7*b^2) + ((b*c - a*d)^4*x)/(3*a*b^4*(a + b*x^3)) - (2*(b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(13/3)) + (2*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(13/3)) - ((b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*b^(13/3))$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & d^2(3a^2d^2-8abcd+6b^2c^2)\int\frac{1}{b^4}dx+\frac{d^4x^7}{7b^2}-\frac{d^3x^4(ad-2bc)}{2b^3}+\frac{x(ad-bc)^4}{3ab^4(a+bx^3)} \\ & - \frac{2(ad-bc)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{\frac{5}{3}}b^{\frac{13}{3}}} + \frac{(ad-bc)^3(5ad+bc)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{bx}+b^{\frac{2}{3}}x^2\right)}{9a^{\frac{5}{3}}b^{\frac{13}{3}}} \\ & + \frac{2\sqrt{3}(ad-bc)^3(5ad+bc)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}b^{\frac{13}{3}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((d*x**3+c)**4/(b*x**3+a)**2,x)`

[Out]  $d^{2*}(3*a^{2*d^2} - 8*a*b*c*d + 6*b^{2*c^2}) * \text{Integral}(b^{(-4)}, x) + d^{4*x^7}/(7*b^{2}) - d^{3*x^4}*(a*d - 2*b*c)/(2*b^{3}) + x*(a*d - b*c)^{4}/(3*a*b^{4}*(a + b*x^{3})) - 2*(a*d - b*c)^{3}*(5*a*d + b*c) * \log(a^{(1/3)} + b^{(1/3)*x})/(9*a^{(5/3)}*b^{(13/3)}) + (a*d - b*c)^{3}*(5*a*d + b*c) * \log(a^{(2/3)} - a^{(1/3)*b^{(1/3)*x}} + b^{(2/3)*x^2})/(9*a^{(5/3)}*b^{(13/3)}) + 2*\text{sqrt}(3)*(a*d - b*c)^{3}*(5*a*d + b*c) * \text{atan}(\text{sqrt}(3)*(a^{(1/3)}/3 - 2*b^{(1/3)*x/3})/a^{(1/3)})/(9*a^{(5/3)}*b^{(13/3)})$

**Mathematica [A]** time = 0.363644, size = 260, normalized size = 0.97

$$\frac{14(ad-bc)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx+b^{2/3}x^2}\right)}{a^{5/3}} + \frac{28(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{a^{5/3}} + \frac{28\sqrt{3}(bc-ad)^3(5ad+bc)\tan^{-1}\left(\frac{\sqrt[2]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}} + 126\sqrt[3]{bd^2x} \Big/ 126b^{13/3}$$

Antiderivative was successfully verified.

[In] `Integrate[(c + d*x^3)^4/(a + b*x^3)^2,x]`

[Out]  $(126*b^{(1/3)}*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 63*b^{(4/3)}*d^3*(2*b*c - a*d)*x^4 + 18*b^{(7/3)}*d^4*x^7 + (42*b^{(1/3)}*(b*c - a*d)^4*x)/(a*(a + b*x^3)) + (28*\text{Sqrt}[3]*(b*c - a*d)^3*(b*c + 5*a*d)*\text{ArcTan}[(-a^{(1/3)} + 2*b^{(1/3)*x})/(\text{Sqrt}[3]*a^{(1/3)})])/a^{(5/3)} + (28*(b*c - a*d)^3*(b*c + 5*a*d)*\text{Log}[a^{(1/3)} + b^{(1/3)*x}])/a^{(5/3)} + (14*(-(b*c) + a*d)^3*(b*c + 5*a*d)*\text{Log}[a^{(2/3)} - a^{(1/3)*b^{(1/3)*x}} + b^{(2/3)*x^2}])/a^{(5/3)})/(126*b^{(13/3)})$

**Maple [B]** time = 0.016, size = 708, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)^4/(b*x^3+a)^2,x)`

[Out]  $-8*d^3/b^3*a*c*x+5/9/b^5*a^3/(a/b)^{(2/3)}*\ln(x^2-x*(a/b)^{(1/3)}+(a/b)^{(2/3)})*d^4-2/9/b^2/(a/b)^{(2/3)}*\ln(x^2-x*(a/b)^{(1/3)}+(a/b)^{(2/3)})*c^3*d+28/9/b^4*a^2/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)*x-1})*c*d^3-8/3/b^3*a/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)*x-1})*c^2*d^2-1/2*d^4/b^3*x^4+a*d^3/b^2*x^4*c+3*d^4/b^4*a^2*x+6*d^2/b^2*c^2*x+1/3*x/a/(b*x^3+a)*c^4-1/9/b/a/(a/b)^{(2/3)}*\ln(x^2-x*(a/b)^{(1/3)}+(a/b)^{(2/3)})*c^4-10/9/b^5*a^3/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*d^4+4/9/b^2/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*c^3*d+2/9/b/a/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*c^4+1/3/b^4*x*a^3/(b*x^3+a)*d^4-4/3/b*x/(b*x^3+a)*c^3*d+1/7*d^4*x^7/b^2+28/9/b^4*a^2/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*c*d^3-8/3/b^3*a/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*c^2*d^2-14/9/b^4*a^2/(a/b)^{(2/3)}*\ln(x^2-x*(a/b)^{(1/3)}+(a/b)^{(2/3)})*c*d^3+4/3/b^3*a/(a/b)^{(2/3)}*\ln(x^2-x*(a/b)^{(1/3)}+(a/b)^{(2/3)})*c^2*d^2-10/9/b^5*a^3/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)*x-1})*d^4+2/9/b/a/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)*x-1})*c^4+2/b^2*x*a/(b*x^3+a)*c^2*d^2+4/9/b^2/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)*x-1})*c^3*d-4/3/b^3*x*a^2/(b*x^3+a)*c*d^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^4/(b\*x^3 + a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.22305, size = 794, normalized size = 2.97

$$\sqrt{3} \left( 14 \sqrt{3} (ab^4c^4 + 2a^2b^3c^3d - 12a^3b^2c^2d^2 + 14a^4bcd^3 - 5a^5d^4 + (b^5c^4 + 2ab^4c^3d - 12a^2b^3c^2d^2 + 14a^3b^2cd^3 - 5a^4bd^4)x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^4/(b\*x^3 + a)^2,x, algorithm="fricas")

[Out]  $\frac{1}{378} \sqrt{3} (14 \sqrt{3} (a^5b^4c^4 + 2a^4b^3c^3d - 12a^3b^2c^2d^2 + 14a^2b^3c^3d - 5a^5d^4 + (b^5c^4 + 2a^4b^3c^3d - 12a^2b^3c^2d^2 + 14a^3b^2cd^3 - 5a^4bd^4)x^3) \log((-a^2b)^{2/3}x^2 + (-a^2b)^{1/3}ax + a^2) - 28 \sqrt{3} (a^5b^4c^4 + 2a^4b^3c^3d - 12a^3b^2c^2d^2 + 14a^2b^3c^3d - 5a^5d^4 + (b^5c^4 + 2a^4b^3c^3d - 12a^2b^3c^2d^2 + 14a^3b^2cd^3 - 5a^4bd^4)x^3) \log((-a^2b)^{1/3}x - a) + 84 (a^5b^4c^4 + 2a^4b^3c^3d - 12a^3b^2c^2d^2 + 14a^2b^3c^3d - 5a^5d^4 + (b^5c^4 + 2a^4b^3c^3d - 12a^2b^3c^2d^2 + 14a^3b^2cd^3 - 5a^4bd^4)x^3) \arctan(1/3 (2 \sqrt{3} (-a^2b)^{1/3}x + \sqrt{3}a)/a) + 3 \sqrt{3} (6a^5b^3d^4x^{10} + 3(14a^4b^3c^3d^3 - 5a^4b^2d^4)x^7 + 21(12a^4b^3c^2d^2 - 14a^4b^2c^3d^3 + 5a^4b^3d^4)x^4 + 14(b^4c^4 - 4a^4b^3c^3d + 24a^4b^2c^2d^2 - 28a^4b^3c^3d + 10a^4d^4)x) (-a^2b)^{1/3} / ((a^5b^5x^3 + a^2b^4) (-a^2b)^{1/3})$

**Sympy** [A] time = 14.071, size = 403, normalized size = 1.51

$$\frac{x(a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4)}{3a^2b^4 + 3ab^5x^3} + \text{RootSum}\left(729t^3a^5b^{13} + 1000a^{12}d^{12} - 8400a^{11}bcd^{11} + 30720a^{10}b^2c^2d^{10} - 63472a^9b^3c^3d^9 + 79848a^8b^4c^4d^8 - 60192a^7b^5c^5d^7 + 22848a^6b^6c^6d^6 + 288a^5b^7c^7d^5 - 3528a^4b^8c^8d^4 + 752a^3b^9c^9d^3 + 192a^2b^{10}c^{10}d^2 - 48ab^{11}c^{11}d - 8b^{12}c^{12}\right) + \frac{d^4x^7}{7b^2} - \frac{x^4(ad^4 - 2bcd^3)}{2b^3} + \frac{x(3a^2d^4 - 8abcd^3 + 6b^2c^2d^2)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*4/(b\*x\*\*3+a)\*\*2,x)

[Out]  $x(a^4d^4 - 4a^3b^3c^3d^3 + 6a^2b^2c^2d^2 - 4a^4b^3c^3d + b^4c^4) / (3a^2b^4 + 3a^4b^5x^3) + \text{RootSum}(729\_t^3a^5b^{13} + 1000a^{12}d^{12} - 8400a^{11}b^3c^3d^{11} + 30720a^{10}b^2c^2d^{10} - 63472a^9b^3c^3d^9 + 79848a^8b^4c^4d^8 - 60192a^7b^5c^5d^7 + 22848a^6b^6c^6d^6 + 288a^5b^7c^7d^5 - 3528a^4b^8c^8d^4 + 752a^3b^9c^9d^3 + 192a^2b^{10}c^{10}d^2 - 48ab^{11}c^{11}d - 8b^{12}c^{12}, \text{Lambda}(\_t, \_t \log(-9\_t a^2b^4 / (10a^4d^4 - 28a^3b^3c^3d^3 + 24a^2b^2c^2d^2 - 4a^4b^3c^3d - 2b^4c^4) + x))) + d^4x^7 / (7b^2) - x^4(ad^4 - 2bcd^3) / (2b^3) + x(3a^2d^4 - 8abcd^3 + 6b^2c^2d^2) / b^4$



**GIAC/XCAS [A]** time = 0.220824, size = 643, normalized size = 2.41

$$\frac{2(b^4c^4 + 2ab^3c^3d - 12a^2b^2c^2d^2 + 14a^3bcd^3 - 5a^4d^4)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^2b^4}$$

$$+ \frac{2\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^4c^4 + 2\left(-ab^2\right)^{\frac{1}{3}}ab^3c^3d - 12\left(-ab^2\right)^{\frac{1}{3}}a^2b^2c^2d^2 + 14\left(-ab^2\right)^{\frac{1}{3}}a^3bcd^3 - 5\left(-ab^2\right)^{\frac{1}{3}}a^4d^4\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b^5}$$

$$+ \frac{b^4c^4x - 4ab^3c^3dx + 6a^2b^2c^2d^2x - 4a^3bcd^3x + a^4d^4x}{3(bx^3 + a)ab^4}$$

$$+ \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^4c^4 + 2\left(-ab^2\right)^{\frac{1}{3}}ab^3c^3d - 12\left(-ab^2\right)^{\frac{1}{3}}a^2b^2c^2d^2 + 14\left(-ab^2\right)^{\frac{1}{3}}a^3bcd^3 - 5\left(-ab^2\right)^{\frac{1}{3}}a^4d^4\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9a^2b^5}$$

$$+ \frac{2b^{12}d^4x^7 + 14b^{12}cd^3x^4 - 7ab^{11}d^4x^4 + 84b^{12}c^2d^2x - 112ab^{11}cd^3x + 42a^2b^{10}d^4x}{14b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^4/(b\*x^3 + a)^2,x, algorithm="giac")

[Out]  $-2/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4)*(-a/b)^{(1/3)}*\ln(\text{abs}(x - (-a/b)^{(1/3)}))/ (a^2*b^4)$   
 $+ 2/9*\text{sqrt}(3)*((-a*b^2)^{(1/3)}*b^4*c^4 + 2*(-a*b^2)^{(1/3)}*a*b^3*c^3*d - 12*(-a*b^2)^{(1/3)}*a^2*b^2*c^2*d^2 + 14*(-a*b^2)^{(1/3)}*a^3*b*c*d^3 - 5*(-a*b^2)^{(1/3)}*a^4*d^4)*\arctan(1/3*\text{sqrt}(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(a^2*b^5) + 1/3*(b^4*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3*x + a^4*d^4*x)/(b*x^3 + a)*a*b^4$   
 $+ 1/9*((-a*b^2)^{(1/3)}*b^4*c^4 + 2*(-a*b^2)^{(1/3)}*a*b^3*c^3*d - 12*(-a*b^2)^{(1/3)}*a^2*b^2*c^2*d^2 + 14*(-a*b^2)^{(1/3)}*a^3*b*c*d^3 - 5*(-a*b^2)^{(1/3)}*a^4*d^4)*\ln(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a^2*b^5) + 1/14*(2*b^{12}*d^4*x^7 + 14*b^{12}*c*d^3*x^4 - 7*a*b^{11}*d^4*x^4 + 84*b^{12}*c^2*d^2*x - 112*a*b^{11}*c*d^3*x + 42*a^2*b^{10}*d^4*x)/b^{14}$

$$3.22 \quad \int \frac{(c+dx^3)^3}{(a+bx^3)^2} dx$$

**Optimal.** Leaf size=234

$$\begin{aligned} & -\frac{(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}} + \frac{(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{5/3}b^{10/3}} \\ & -\frac{(bc-ad)^2(7ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{10/3}} + \frac{d^2x(3bc-2ad)}{b^3} + \frac{x(bc-ad)^3}{3ab^3(a+bx^3)} + \frac{d^3x^4}{4b^2} \end{aligned}$$

[Out]  $(d^2(3bc - 2ad)x)/b^3 + (d^3x^4)/(4b^2) + ((bc - ad)^3x)/(3ab^3(a + bx^3)) - ((bc - ad)^2(2bc + 7ad) \operatorname{ArcTan}[(a^{1/3} - 2b^{1/3}x)/(\sqrt{3}a^{1/3})])/(3\sqrt{3}a^{5/3}b^{10/3}) + ((bc - ad)^2(2bc + 7ad) \operatorname{Log}[a^{1/3} + b^{1/3}x])/(9a^{5/3}b^{10/3}) - ((bc - ad)^2(2bc + 7ad) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2])/(18a^{5/3}b^{10/3})$

**Rubi [A]** time = 0.482122, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\begin{aligned} & -\frac{(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}} + \frac{(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{5/3}b^{10/3}} \\ & -\frac{(bc-ad)^2(7ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{10/3}} + \frac{d^2x(3bc-2ad)}{b^3} + \frac{x(bc-ad)^3}{3ab^3(a+bx^3)} + \frac{d^3x^4}{4b^2} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x^3)^3/(a + b*x^3)^2, x]$

[Out]  $(d^2(3bc - 2ad)x)/b^3 + (d^3x^4)/(4b^2) + ((bc - ad)^3x)/(3ab^3(a + bx^3)) - ((bc - ad)^2(2bc + 7ad) \operatorname{ArcTan}[(a^{1/3} - 2b^{1/3}x)/(\sqrt{3}a^{1/3})])/(3\sqrt{3}a^{5/3}b^{10/3}) + ((bc - ad)^2(2bc + 7ad) \operatorname{Log}[a^{1/3} + b^{1/3}x])/(9a^{5/3}b^{10/3}) - ((bc - ad)^2(2bc + 7ad) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2])/(18a^{5/3}b^{10/3})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & -d^2(2ad-3bc) \int \frac{1}{b^3} dx + \frac{d^3x^4}{4b^2} - \frac{x(ad-bc)^3}{3ab^3(a+bx^3)} + \frac{(ad-bc)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{5/3}b^{10/3}} \\ & -\frac{(ad-bc)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}} \\ & -\frac{\sqrt{3}(ad-bc)^2(7ad+2bc)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{bx}}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{5/3}b^{10/3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}((d*x**3+c)**3/(b*x**3+a)**2, x)$

[Out]  $-d^2(2ad - 3bc) \operatorname{Integral}(b^{(-3)}, x) + d^3x^4/(4b^2) - x(ad - bc)^3/(3ab^3(a + bx^3)) + (ad - bc)^2(7ad + 2bc) \operatorname{Log}(a^{1/3} + b^{1/3}x)/(9a^{5/3}b^{10/3}) - (ad - bc)^2(7ad + 2bc) \operatorname{Log}(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)/(18a^{5/3}b^{10/3}) - \sqrt{3}(ad - bc)^2 \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} - \frac{2\sqrt[3]{bx}}{3}\right)}{\sqrt[3]{a}}\right)/(9a^{5/3}b^{10/3})$

$$\frac{d - b^*c)^{**2}*(7*a*d + 2*b*c)*\log(a^{** (2/3)} - a^{** (1/3)}*b^{** (1/3)}*x + b^{** (2/3)}*x^{**2})/(18*a^{** (5/3)}*b^{** (10/3)}) - \sqrt{3}*(a*d - b*c)^{**2}*(7*a*d + 2*b*c)*\operatorname{atan}(\sqrt{3}*(a^{** (1/3)}/3 - 2*b^{** (1/3)}*x/3)/a^{** (1/3)})/(9*a^{** (5/3)}*b^{** (10/3)})$$

**Mathematica [A]** time = 0.261119, size = 227, normalized size = 0.97

$$\frac{-\frac{2(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx+b^{2/3}x^2}\right)}{a^{5/3}}+\frac{4(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{a^{5/3}}+\frac{4\sqrt{3}(bc-ad)^2(7ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{bx}-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}+36\sqrt[3]{bd^2x}}{36b^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)^3/(a + b\*x^3)^2,x]

[Out] (36\*b^(1/3)\*d^2\*(3\*b\*c - 2\*a\*d)\*x + 9\*b^(4/3)\*d^3\*x^4 + (12\*b^(1/3)\*(b\*c - a\*d)^3\*x)/(a\*(a + b\*x^3)) + (4\*Sqrt[3]\*(b\*c - a\*d)^2\*(2\*b\*c + 7\*a\*d)\*ArcTan[(-a^(1/3) + 2\*b^(1/3)\*x)/(Sqrt[3]\*a^(1/3))])/a^(5/3) + (4\*(b\*c - a\*d)^2\*(2\*b\*c + 7\*a\*d)\*Log[a^(1/3) + b^(1/3)\*x])/a^(5/3) - (2\*(b\*c - a\*d)^2\*(2\*b\*c + 7\*a\*d)\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2])/a^(5/3))/(36\*b^(10/3))

**Maple [B]** time = 0.013, size = 529, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^3/(b\*x^3+a)^2,x)

[Out] 1/4\*d^3\*x^4/b^2-2\*d^3/b^3\*a\*x+3\*d^2/b^2\*x\*c-1/3/b^3\*x\*a^2/(b\*x^3+a)\*d^3+1/b^2\*x\*a/(b\*x^3+a)\*c\*d^2-1/b\*x/(b\*x^3+a)\*c^2\*d+1/3\*x/a/(b\*x^3+a)\*c^3+7/9/b^4\*a^2/(a/b)^(2/3)\*ln(x+(a/b)^(1/3))\*d^3-4/3/b^3\*a/(a/b)^(2/3)\*ln(x+(a/b)^(1/3))\*c\*d^2+1/3/b^2/(a/b)^(2/3)\*ln(x+(a/b)^(1/3))\*c^2\*d+2/9/b/a/(a/b)^(2/3)\*ln(x+(a/b)^(1/3))\*c^3-7/18/b^4\*a^2/(a/b)^(2/3)\*ln(x^2-x\*(a/b)^(1/3)+(a/b)^(2/3))\*d^3+2/3/b^3\*a/(a/b)^(2/3)\*ln(x^2-x\*(a/b)^(1/3)+(a/b)^(2/3))\*c\*d^2-1/6/b^2/(a/b)^(2/3)\*ln(x^2-x\*(a/b)^(1/3)+(a/b)^(2/3))\*c^2\*d-1/9/b/a/(a/b)^(2/3)\*ln(x^2-x\*(a/b)^(1/3)+(a/b)^(2/3))\*c^3+7/9/b^4\*a^2/(a/b)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(a/b)^(1/3)\*x-1))\*d^3-4/3/b^3\*a/(a/b)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(a/b)^(1/3)\*x-1))\*c\*d^2+1/3/b^2/(a/b)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(a/b)^(1/3)\*x-1))\*c^2\*d+2/9/b/a/(a/b)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(a/b)^(1/3)\*x-1))\*c^3

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^3/(b\*x^3 + a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.219973, size = 606, normalized size = 2.59

$$\sqrt{3} \left( 2\sqrt{3}(2ab^3c^3 + 3a^2b^2c^2d - 12a^3bcd^2 + 7a^4d^3 + (2b^4c^3 + 3ab^3c^2d - 12a^2b^2cd^2 + 7a^3bd^3)x^3) \log \left( (a^2b)^{\frac{2}{3}}x^2 - (a^2b)^{\frac{1}{3}}x + a \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^3/(b\*x^3 + a)^2,x, algorithm="fricas")

[Out] 
$$-1/108 \sqrt{3} (2 \sqrt{3} (2 a^2 b^3 c^3 + 3 a^2 b^2 c^2 d - 12 a^3 b c d^2 + 7 a^4 d^3 + (2 b^4 c^3 + 3 a b^3 c^2 d - 12 a^2 b^2 c d^2 + 7 a^3 b d^3) x^3) \log((a^2 b)^{2/3} x^2 - (a^2 b)^{1/3} a x + a^2) - 4 \sqrt{3} (2 a^2 b^3 c^3 + 3 a^2 b^2 c^2 d - 12 a^3 b c d^2 + 7 a^4 d^3 + (2 b^4 c^3 + 3 a b^3 c^2 d - 12 a^2 b^2 c d^2 + 7 a^3 b d^3) x^3) \log((a^2 b)^{1/3} x + a) - 12 (2 a^2 b^3 c^3 + 3 a^2 b^2 c^2 d - 12 a^3 b c d^2 + 7 a^4 d^3 + (2 b^4 c^3 + 3 a b^3 c^2 d - 12 a^2 b^2 c d^2 + 7 a^3 b d^3) x^3) \arctan(1/3 (2 \sqrt{3} (a^2 b)^{1/3} x - \sqrt{3} a) / a) - 3 \sqrt{3} (3 a^2 b^2 d^3 x^7 + 3 (12 a^2 b^2 c^2 d^2 - 7 a^2 b^2 d^3) x^4 + 4 (b^3 c^3 - 3 a b^2 c^2 d + 12 a^2 b^2 c d^2 - 7 a^3 d^3) x) (a^2 b)^{1/3} / ((a b^4 x^3 + a^2 b^3) (a^2 b)^{1/3})$$

**Sympy [A]** time = 8.05575, size = 289, normalized size = 1.24

$$\frac{x(a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3)}{3a^2b^3 + 3ab^4x^3} + \text{RootSum} \left( 729t^3a^5b^{10} - 343a^9d^9 + 1764a^8bcd^8 - 3465a^7b^2c^2d^7 + 2946a^6b^3c^3d^6 - 477a^5b^4c^4d^5 - 792a^4b^5c^5d^4 + 321a^3b^6c^6 \right) + \frac{d^3x^4}{4b^2} - \frac{x(2ad^3 - 3bcd^2)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*3/(b\*x\*\*3+a)\*\*2,x)

[Out] 
$$-x(a^3d^3 - 3a^2b^2c^2d + 3ab^2c^2d - b^3c^3)/(3a^2b^3 + 3ab^4x^3) + \text{RootSum}(729t^3a^5b^{10} - 343a^9d^9 + 1764a^8bcd^8 - 3465a^7b^2c^2d^7 + 2946a^6b^3c^3d^6 - 477a^5b^4c^4d^5 - 792a^4b^5c^5d^4 + 321a^3b^6c^6) + \frac{d^3x^4}{4b^2} - \frac{x(2ad^3 - 3bcd^2)}{b^3}$$

**GIAC/XCAS [A]** time = 0.220806, size = 495, normalized size = 2.12

$$\frac{(2b^3c^3 + 3ab^2c^2d - 12a^2bcd^2 + 7a^3d^3) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \ln \left( \left| x - \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right| \right)}{9a^2b^3} + \frac{\sqrt{3} \left( 2(-ab^2)^{\frac{1}{3}} b^3c^3 + 3(-ab^2)^{\frac{1}{3}} ab^2c^2d - 12(-ab^2)^{\frac{1}{3}} a^2bcd^2 + 7(-ab^2)^{\frac{1}{3}} a^3d^3 \right) \arctan \left( \frac{\sqrt{3} \left( 2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{9a^2b^4} + \frac{b^3c^3x - 3ab^2c^2dx + 3a^2bcd^2x - a^3d^3x}{3(bx^3 + a)ab^3} + \frac{\left( 2(-ab^2)^{\frac{1}{3}} b^3c^3 + 3(-ab^2)^{\frac{1}{3}} ab^2c^2d - 12(-ab^2)^{\frac{1}{3}} a^2bcd^2 + 7(-ab^2)^{\frac{1}{3}} a^3d^3 \right) \ln \left( x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}} \right)}{18a^2b^4} + \frac{b^6d^3x^4 + 12b^6cd^2x - 8ab^5d^3x}{4b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^3/(b\*x^3 + a)^2,x, algorithm="giac")

[Out] 
$$-1/9*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*(-a/b)^{1/3}*\ln(\text{abs}(x - (-a/b)^{1/3}))/a^2*b^3 + 1/9*\sqrt{3}*(2*(-a*b^2)^{1/3}*b^3*c^3 + 3*(-a*b^2)^{1/3}*a*b^2*c^2*d - 12*(-a*b^2)^{1/3}*a^2*b*c*d^2 + 7*(-a*b^2)^{1/3}*a^3*d^3)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{1/3})/(-a/b)^{1/3})/a^2*b^4 + 1/3*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((b*x^3 + a)*a*b^3) + 1/18*(2*(-a*b^2)^{1/3}*b^3*c^3 + 3*(-a*b^2)^{1/3}*a*b^2*c^2*d - 12*(-a*b^2)^{1/3}*a^2*b*c*d^2 + 7*(-a*b^2)^{1/3}*a^3*d^3)*\ln(x^2 + x*(-a/b)^{1/3} + (-a/b)^{2/3})/a^2*b^4 + 1/4*(b^6*d^3*x^4 + 12*b^6*c*d^2*x - 8*a*b^5*d^3*x)/b^8$$

$$3.23 \quad \int \frac{(c+dx^3)^2}{(a+bx^3)^2} dx$$

**Optimal.** Leaf size=203

$$\begin{aligned} & -\frac{(bc-ad)(2ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}} + \frac{2(bc-ad)(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{5/3}b^{7/3}} \\ & -\frac{2(bc-ad)(2ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{7/3}} + \frac{x(bc-ad)^2}{3ab^2(a+bx^3)} + \frac{d^2x}{b^2} \end{aligned}$$

[Out]  $(d^2x)/b^2 + ((b^3c - a^3d)^2x)/(3a^3b^2(a + b^3x^3)) - (2(b^3c - a^3d)(b^3c + 2a^3d) \operatorname{ArcTan}[(a^{1/3} - 2b^{1/3}x)/(\sqrt{3}a^{1/3})])/(3\sqrt{3}a^{5/3}b^{7/3}) + (2(b^3c - a^3d)(b^3c + 2a^3d) \operatorname{Log}[a^{1/3} + b^{1/3}x])/(9a^{5/3}b^{7/3}) - ((b^3c - a^3d)(b^3c + 2a^3d) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2])/(9a^{5/3}b^{7/3})$

**Rubi [A]** time = 0.497695, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\begin{aligned} & -\frac{(bc-ad)(2ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}} + \frac{2(bc-ad)(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{5/3}b^{7/3}} \\ & -\frac{2(bc-ad)(2ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{7/3}} + \frac{x(bc-ad)^2}{3ab^2(a+bx^3)} + \frac{d^2x}{b^2} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d^2x^3)^2/(a + b^3x^3)^2, x]$

[Out]  $(d^2x)/b^2 + ((b^3c - a^3d)^2x)/(3a^3b^2(a + b^3x^3)) - (2(b^3c - a^3d)(b^3c + 2a^3d) \operatorname{ArcTan}[(a^{1/3} - 2b^{1/3}x)/(\sqrt{3}a^{1/3})])/(3\sqrt{3}a^{5/3}b^{7/3}) + (2(b^3c - a^3d)(b^3c + 2a^3d) \operatorname{Log}[a^{1/3} + b^{1/3}x])/(9a^{5/3}b^{7/3}) - ((b^3c - a^3d)(b^3c + 2a^3d) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2])/(9a^{5/3}b^{7/3})$

**Rubi in SymPy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & d^2 \int \frac{1}{b^2} dx + \frac{x(ad-bc)^2}{3ab^2(a+bx^3)} - \frac{2(ad-bc)(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{5/3}b^{7/3}} \\ & + \frac{(ad-bc)(2ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}} + \frac{2\sqrt{3}(ad-bc)(2ad+bc)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{bx}}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{5/3}b^{7/3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}((d^2x^3+c)^2/(b^3x^3+a)^2, x)$

[Out]  $d^2 \operatorname{Integral}(b^{(-2)}, x) + x^*(a^3d - b^3c)^2/(3a^3b^2(a + b^3x^3)) - 2*(a^3d - b^3c)*(2a^3d + b^3c)*\log(a^{1/3} + b^{1/3}x)/(9a^{5/3}b^{7/3}) + (a^3d - b^3c)*(2a^3d + b^3c)*\log(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)/(9a^{5/3}b^{7/3}) + 2*\sqrt{3}*(a^3d - b^3c)*(2a^3d + b^3c)*\operatorname{atan}(\sqrt{3}*(a^{1/3}/3 - 2*b^{1/3}x/3)/a^{1/3})/(9a^{5/3}b^{7/3})$

**Mathematica [A]** time = 0.412179, size = 205, normalized size = 1.01

$$\frac{2(-2a^2d^2+abcd+b^2c^2)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{a^{5/3}} - \frac{2\sqrt{3}(-2a^2d^2+abcd+b^2c^2)\tan^{-1}\left(\frac{1-2\sqrt[3]{bx}}{\sqrt[3]{a}}\right)}{a^{5/3}} - \frac{(-2a^2d^2+abcd+b^2c^2)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{a^{5/3}} + \frac{3\sqrt[3]{bx}(b^{2/3}x^2+a)}{a^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)^2/(a + b\*x^3)^2,x]

[Out]  $(9*b^{1/3}*d^2*x + (3*b^{1/3}*(b*c - a*d)^2*x)/(a*(a + b*x^3)) - (2*\text{Sqrt}[3]*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*\text{ArcTan}[(1 - (2*b^{1/3}*x)/a^{1/3})/\text{Sqrt}[3]])/a^{5/3} + (2*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*\text{Log}[a^{1/3} + b^{1/3}*x])/a^{5/3} - ((b^2*c^2 + a*b*c*d - 2*a^2*d^2)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/a^{5/3})/(9*b^{7/3})$

**Maple [B]** time = 0.013, size = 367, normalized size = 1.8

$$\begin{aligned} & \frac{d^2x}{b^2} + \frac{axd^2}{3b^2(bx^3+a)} - \frac{2cxd}{3b(bx^3+a)} + \frac{xc^2}{3a(bx^3+a)} - \frac{4ad^2}{9b^3} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \\ & + \frac{2cd}{9b^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{2c^2}{9ab} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \\ & + \frac{2ad^2}{9b^3} \ln\left(x^2 - x\sqrt[3]{\frac{a}{b}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{cd}{9b^2} \ln\left(x^2 - x\sqrt[3]{\frac{a}{b}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \\ & - \frac{c^2}{9ab} \ln\left(x^2 - x\sqrt[3]{\frac{a}{b}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{4a\sqrt{3}d^2}{9b^3} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \\ & + \frac{2\sqrt{3}cd}{9b^2} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{2\sqrt{3}c^2}{9ab} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^2/(b\*x^3+a)^2,x)

[Out]  $d^2*x/b^2+1/3/b^2*x*a/(b*x^3+a)*d^2-2/3/b*x/(b*x^3+a)*c*d+1/3*x/a/(b*x^3+a)*c^2-4/9/b^3*a/(a/b)^{2/3}*ln(x+(a/b)^{1/3})*d^2+2/9/b^2/(a/b)^{2/3}*ln(x+(a/b)^{1/3})*c*d+2/9/b/a/(a/b)^{2/3}*ln(x+(a/b)^{1/3})*c^2+2/9/b^3*a/(a/b)^{2/3}*ln(x^2-x*(a/b)^{1/3}+(a/b)^{2/3})*d^2-1/9/b^2/(a/b)^{2/3}*ln(x^2-x*(a/b)^{1/3}+(a/b)^{2/3})*c*d-1/9/b/a/(a/b)^{2/3}*ln(x^2-x*(a/b)^{1/3}+(a/b)^{2/3})*c^2-4/9/b^3*a/(a/b)^{2/3}*3^{1/2}*arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1))*d^2+2/9/b^2/(a/b)^{2/3}*3^{1/2}*arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1))*c*d+2/9/b/a/(a/b)^{2/3}*3^{1/2}*arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1))*c^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^2/(b\*x^3 + a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.218815, size = 427, normalized size = 2.1

$$\sqrt{3} \left( \sqrt{3} (ab^2c^2 + a^2bcd - 2a^3d^2 + (b^3c^2 + ab^2cd - 2a^2bd^2)x^3) \log \left( (-a^2b)^{\frac{2}{3}}x^2 + (-a^2b)^{\frac{1}{3}}ax + a^2 \right) - 2\sqrt{3}(ab^2c^2 + a^2bcd - 2a^3d^2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^2/(b\*x^3 + a)^2,x, algorithm="fricas")

[Out] 1/27\*sqrt(3)\*(sqrt(3)\*(a\*b^2\*c^2 + a^2\*b\*c\*d - 2\*a^3\*d^2 + (b^3\*c^2 + a\*b^2\*c\*d - 2\*a^2\*b\*d^2)\*x^3)\*log((-a^2\*b)^(2/3)\*x^2 + (-a^2\*b)^(1/3)\*a\*x + a^2) - 2\*sqrt(3)\*(a\*b^2\*c^2 + a^2\*b\*c\*d - 2\*a^3\*d^2 + (b^3\*c^2 + a\*b^2\*c\*d - 2\*a^2\*b\*d^2)\*x^3)\*log((-a^2\*b)^(1/3)\*x - a) + 6\*(a\*b^2\*c^2 + a^2\*b\*c\*d - 2\*a^3\*d^2 + (b^3\*c^2 + a\*b^2\*c\*d - 2\*a^2\*b\*d^2)\*x^3)\*arctan(1/3\*(2\*sqrt(3)\*(-a^2\*b)^(1/3)\*x + sqrt(3)\*a)/a) + 3\*sqrt(3)\*(3\*a\*b\*d^2\*x^4 + (b^2\*c^2 - 2\*a\*b\*c\*d + 4\*a^2\*d^2)\*x)\*(-a^2\*b)^(1/3))/((a\*b^3\*x^3 + a^2\*b^2)\*(-a^2\*b)^(1/3))

**Sympy [A]** time = 5.08011, size = 189, normalized size = 0.93

$$\frac{x(a^2d^2 - 2abcd + b^2c^2)}{3a^2b^2 + 3ab^3x^3} + \text{RootSum} \left( 729t^3a^5b^7 + 64a^6d^6 - 96a^5bcd^5 - 48a^4b^2c^2d^4 + 88a^3b^3c^3d^3 + 24a^2b^4c^4d^2 - 24ab^5c^5d - 8b^6c^6, \left( t \mapsto t \log \left( -\frac{d^2x}{b^2} + \frac{d^2x}{b^2} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*2/(b\*x\*\*3+a)\*\*2,x)

[Out] x\*(a\*\*2\*d\*\*2 - 2\*a\*b\*c\*d + b\*\*2\*c\*\*2)/(3\*a\*\*2\*b\*\*2 + 3\*a\*b\*\*3\*x\*\*3) + RootSum(729\*\_t\*\*3\*a\*\*5\*b\*\*7 + 64\*a\*\*6\*d\*\*6 - 96\*a\*\*5\*b\*c\*d\*\*5 - 48\*a\*\*4\*b\*\*2\*c\*\*2\*d\*\*4 + 88\*a\*\*3\*b\*\*3\*c\*\*3\*d\*\*3 + 24\*a\*\*2\*b\*\*4\*c\*\*4\*d\*\*2 - 24\*a\*b\*\*5\*c\*\*5\*d - 8\*b\*\*6\*c\*\*6, Lambda(\_t, \_t\*log(-9\*\_t\*a\*\*2\*b\*\*2/(4\*a\*\*2\*d\*\*2 - 2\*a\*b\*c\*d - 2\*b\*\*2\*c\*\*2) + x))) + d\*\*2\*x/b\*\*2

**GIAC/XCAS [A]** time = 0.218488, size = 350, normalized size = 1.72

$$\frac{d^2x}{b^2} - \frac{2(b^2c^2 + abcd - 2a^2d^2) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \ln \left( \left| x - \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right| \right)}{9a^2b^2} + \frac{2\sqrt{3} \left( (-ab^2)^{\frac{1}{3}} b^2c^2 + (-ab^2)^{\frac{1}{3}} abcd - 2(-ab^2)^{\frac{1}{3}} a^2d^2 \right) \arctan \left( \frac{\sqrt{3} \left( 2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{9a^2b^3} + \frac{b^2c^2x - 2abcdx + a^2d^2x}{3(bx^3 + a)ab^2} + \frac{\left( (-ab^2)^{\frac{1}{3}} b^2c^2 + (-ab^2)^{\frac{1}{3}} abcd - 2(-ab^2)^{\frac{1}{3}} a^2d^2 \right) \ln \left( x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}} \right)}{9a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((d\*x^3 + c)^2/(b\*x^3 + a)^2,x, algorithm="giac")

[Out] 
$$\begin{aligned} & d^2 x / b^2 - 2/9 (b^2 c^2 + a b c d - 2 a^2 d^2) (-a/b)^{1/3} \ln\left(\frac{a b s(x - (-a/b)^{1/3})}{a^2 b^2} + \frac{2/9 \sqrt{3} ((-a b^2)^{1/3} b^2 c^2 + (-a b^2)^{1/3} a b c d - 2 (-a b^2)^{1/3} a^2 d^2) \arctan\left(\frac{1/3 \sqrt{3} (2 x + (-a/b)^{1/3})}{(-a/b)^{1/3}}\right)}{a^2 b^3} + \frac{1/3 (b^2 c^2 x - 2 a b c d x + a^2 d^2 x)}{(b x^3 + a) a b^2} + \frac{1/9 ((-a b^2)^{1/3} b^2 c^2 + (-a b^2)^{1/3} a b c d - 2 (-a b^2)^{1/3} a^2 d^2) \ln(x^2 + x (-a/b)^{1/3} + (-a/b)^{2/3})}{a^2 b^3}\right) \end{aligned}$$

$$3.24 \quad \int \frac{c+dx^3}{(a+bx^3)^2} dx$$

**Optimal.** Leaf size=169

$$\begin{aligned} & -\frac{(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}} + \frac{(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{5/3}b^{4/3}} \\ & -\frac{(ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}} + \frac{x(bc-ad)}{3ab(a+bx^3)} \end{aligned}$$

[Out] ((b\*c - a\*d)\*x)/(3\*a\*b\*(a + b\*x^3)) - ((2\*b\*c + a\*d)\*ArcTan[(a^(1/3) - 2\*b^(1/3)\*x)/(Sqrt[3]\*a^(1/3))])/(3\*Sqrt[3]\*a^(5/3)\*b^(4/3)) + ((2\*b\*c + a\*d)\*Log[a^(1/3) + b^(1/3)\*x])/(9\*a^(5/3)\*b^(4/3)) - ((2\*b\*c + a\*d)\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2])/(18\*a^(5/3)\*b^(4/3))

**Rubi [A]** time = 0.193477, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$\begin{aligned} & -\frac{(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}} + \frac{(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{5/3}b^{4/3}} \\ & -\frac{(ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}} + \frac{x(bc-ad)}{3ab(a+bx^3)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)/(a + b\*x^3)^2, x]

[Out] ((b\*c - a\*d)\*x)/(3\*a\*b\*(a + b\*x^3)) - ((2\*b\*c + a\*d)\*ArcTan[(a^(1/3) - 2\*b^(1/3)\*x)/(Sqrt[3]\*a^(1/3))])/(3\*Sqrt[3]\*a^(5/3)\*b^(4/3)) + ((2\*b\*c + a\*d)\*Log[a^(1/3) + b^(1/3)\*x])/(9\*a^(5/3)\*b^(4/3)) - ((2\*b\*c + a\*d)\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2])/(18\*a^(5/3)\*b^(4/3))

**Rubi in Sympy [A]** time = 34.112, size = 155, normalized size = 0.92

$$\begin{aligned} & -\frac{x(ad-bc)}{3ab(a+bx^3)} + \frac{(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{5/3}b^{4/3}} \\ & -\frac{(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}} - \frac{\sqrt{3}(ad+2bc)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{bx}}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{5/3}b^{4/3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((d\*x\*\*3+c)/(b\*x\*\*3+a)\*\*2, x)

[Out] -x\*(a\*d - b\*c)/(3\*a\*b\*(a + b\*x\*\*3)) + (a\*d + 2\*b\*c)\*log(a\*\*(1/3) + b\*\*(1/3)\*x)/(9\*a\*\*(5/3)\*b\*\*(4/3)) - (a\*d + 2\*b\*c)\*log(a\*\*(2/3) - a\*\*(1/3)\*b\*\*(1/3)\*x + b\*\*(2/3)\*x\*\*2)/(18\*a\*\*(5/3)\*b\*\*(4/3)) - sqrt(3)\*(a\*d + 2\*b\*c)\*atan(sqrt(3)\*(a\*\*(1/3)/3 - 2\*b\*\*(1/3)\*x/3)/a\*\*(1/3))/(9\*a\*\*(5/3)\*b\*\*(4/3))

**Mathematica [A]** time = 0.173095, size = 145, normalized size = 0.86

$$-(ad + 2bc) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right) - \frac{6a^{2/3}\sqrt[3]{bx}(ad-bc)}{a+bx^3} + 2(ad + 2bc) \log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right) - 2\sqrt{3}(ad + 2bc) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx}}{\sqrt[3]{a}}\right)$$


---


$$18a^{5/3}b^{4/3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)/(a + b\*x^3)^2, x]

[Out]  $((-6*a^{2/3}*b^{1/3})*(-(b*c) + a*d)*x)/(a + b*x^3) - 2*\text{Sqrt}[3]*(2*b*c + a*d)*\text{ArcTan}[(1 - (2*b^{1/3}*x)/a^{1/3})/\text{Sqrt}[3]] + 2*(2*b*c + a*d)*\text{Log}[a^{1/3} + b^{1/3}*x] - (2*b*c + a*d)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2]/(18*a^{5/3}*b^{4/3})$

**Maple [A]** time = 0.012, size = 221, normalized size = 1.3

$$-\frac{(ad-bc)x}{3ab(bx^3+a)} + \frac{d}{9b^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{2c}{9ab} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{d}{18b^2} \ln\left(x^2 - x\sqrt[3]{\frac{a}{b}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{c}{9ab} \ln\left(x^2 - x\sqrt[3]{\frac{a}{b}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}d}{9b^2} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{2\sqrt{3}c}{9ab} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)/(b\*x^3+a)^2, x)

[Out]  $-1/3*(a*d-b*c)/a/b*x/(b*x^3+a)+1/9/b^2/(a/b)^{2/3}*ln(x+(a/b)^{1/3})^2*d+2/9/b/a/(a/b)^{2/3}*ln(x+(a/b)^{1/3})^2*c-1/18/b^2/(a/b)^{2/3}*ln(x^2-x*(a/b)^{1/3}+(a/b)^{2/3})^2*d-1/9/b/a/(a/b)^{2/3}*ln(x^2-x*(a/b)^{1/3}+(a/b)^{2/3})^2*c+1/9/b^2/(a/b)^{2/3}*3^{1/2}*arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1))^2*d+2/9/b/a/(a/b)^{2/3}*3^{1/2}*arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1))^2*c$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)/(b\*x^3 + a)^2, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.216441, size = 277, normalized size = 1.64

$$\sqrt{3}\left(6\sqrt{3}(a^2b)^{\frac{1}{3}}(bc-ad)x - \sqrt{3}((2b^2c+abd)x^3 + 2abc + a^2d) \log\left((a^2b)^{\frac{2}{3}}x^2 - (a^2b)^{\frac{1}{3}}ax + a^2\right) + 2\sqrt{3}((2b^2c+abd)x^3 + 2abc + a^2d)\right)$$


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$$54(ab^2x^3 + a^2b)(a^2b)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)/(b\*x^3 + a)^2,x, algorithm="fricas")

[Out]  $\frac{1}{54}\sqrt{3} \cdot (6\sqrt{3} \cdot (a^2b)^{1/3} \cdot (b^2c - a^2d)x - \sqrt{3} \cdot ((2b^2c + a^2d)x^3 + 2ab^2c + a^2d) \cdot \log((a^2b)^{2/3}x^2 - (a^2b)^{1/3}ax + a^2) + 2\sqrt{3} \cdot ((2b^2c + a^2d)x^3 + 2ab^2c + a^2d) \cdot \log((a^2b)^{1/3}x + a) + 6 \cdot ((2b^2c + a^2d)x^3 + 2ab^2c + a^2d) \cdot \arctan(1/3 \cdot (2\sqrt{3} \cdot (a^2b)^{1/3}x - \sqrt{3} \cdot a)/a)) / ((a^2bx^3 + a^2b) \cdot (a^2b)^{1/3})$

**Sympy [A]** time = 2.68576, size = 97, normalized size = 0.57

$$-\frac{x(ad-bc)}{3a^2b+3ab^2x^3} + \text{RootSum}\left(729t^3a^5b^4 - a^3d^3 - 6a^2bcd^2 - 12ab^2c^2d - 8b^3c^3, \left(t \mapsto t \log\left(\frac{9ta^2b}{ad+2bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)/(b\*x\*\*3+a)\*\*2,x)

[Out]  $-x \cdot (a^2d - b^2c) / (3a^2b + 3ab^2x^3) + \text{RootSum}(729\_t^3a^5b^4 - a^3d^3 - 6a^2bcd^2 - 12ab^2c^2d - 8b^3c^3, \text{Lambda}(\_t, \_t \cdot \log(9\_t a^2b / (a^2d + 2b^2c) + x)))$

**GIAC/XCAS [A]** time = 0.217503, size = 246, normalized size = 1.46

$$\frac{(2bc + ad) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^2b} + \frac{\sqrt{3} \left(2 \left(-ab^2\right)^{\frac{1}{3}} bc + \left(-ab^2\right)^{\frac{1}{3}} ad\right) \arctan\left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b^2} + \frac{bcx - adx}{3(bx^3 + a)ab} + \frac{\left(2 \left(-ab^2\right)^{\frac{1}{3}} bc + \left(-ab^2\right)^{\frac{1}{3}} ad\right) \ln\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)/(b\*x^3 + a)^2,x, algorithm="giac")

[Out]  $-\frac{1}{9} \cdot (2b^2c + a^2d) \cdot \left(-\frac{a}{b}\right)^{1/3} \cdot \ln(\text{abs}(x - \left(-\frac{a}{b}\right)^{1/3})) / (a^2b) + \frac{1}{9} \cdot \sqrt{3} \cdot (2 \cdot \left(-a^2b\right)^{1/3} \cdot b^2c + \left(-a^2b\right)^{1/3} \cdot a^2d) \cdot \arctan\left(\frac{1/3 \cdot \sqrt{3} \cdot (2x + \left(-\frac{a}{b}\right)^{1/3})}{\left(-\frac{a}{b}\right)^{1/3}}\right) / (a^2b^2) + \frac{1}{3} \cdot (b^2cx - a^2dx) / ((b^2x^3 + a) \cdot a^2b) + \frac{1}{18} \cdot (2 \cdot \left(-a^2b\right)^{1/3} \cdot b^2c + \left(-a^2b\right)^{1/3} \cdot a^2d) \cdot \ln(x^2 + x \cdot \left(-\frac{a}{b}\right)^{1/3} + \left(-\frac{a}{b}\right)^{2/3}) / (a^2b^2)$

$$3.25 \quad \int \frac{1}{(a+bx^3)^2(c+dx^3)} dx$$

**Optimal.** Leaf size=346

$$\begin{aligned} & -\frac{b^{2/3}(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{18a^{5/3}(bc-ad)^2} + \frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{5/3}(bc-ad)^2} \\ & -\frac{b^{2/3}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt[3]{a-2\sqrt[3]{bx}}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^2} - \frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx}+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)^2} \\ & + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{dx}\right)}{3c^{2/3}(bc-ad)^2} - \frac{d^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c-2\sqrt[3]{dx}}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^2} + \frac{bx}{3a(a+bx^3)(bc-ad)} \end{aligned}$$

[Out] (b\*x)/(3\*a\*(b\*c - a\*d)\*(a + b\*x^3)) - (b^(2/3)\*(2\*b\*c - 5\*a\*d)\*ArcTan[(a^(1/3) - 2\*b^(1/3)\*x)/(Sqrt[3]\*a^(1/3))]/(3\*Sqrt[3]\*a^(5/3)\*(b\*c - a\*d)^2) - (d^(5/3)\*ArcTan[(c^(1/3) - 2\*d^(1/3)\*x)/(Sqrt[3]\*c^(1/3))]/(Sqrt[3]\*c^(2/3)\*(b\*c - a\*d)^2) + (b^(2/3)\*(2\*b\*c - 5\*a\*d)\*Log[a^(1/3) + b^(1/3)\*x]/(9\*a^(5/3)\*(b\*c - a\*d)^2) + (d^(5/3)\*Log[c^(1/3) + d^(1/3)\*x]/(3\*c^(2/3)\*(b\*c - a\*d)^2) - (b^(2/3)\*(2\*b\*c - 5\*a\*d)\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2]/(18\*a^(5/3)\*(b\*c - a\*d)^2) - (d^(5/3)\*Log[c^(2/3) - c^(1/3)\*d^(1/3)\*x + d^(2/3)\*x^2]/(6\*c^(2/3)\*(b\*c - a\*d)^2)

**Rubi [A]** time = 0.589269, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\begin{aligned} & -\frac{b^{2/3}(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{18a^{5/3}(bc-ad)^2} + \frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{5/3}(bc-ad)^2} \\ & -\frac{b^{2/3}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt[3]{a-2\sqrt[3]{bx}}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^2} - \frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx}+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)^2} \\ & + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{dx}\right)}{3c^{2/3}(bc-ad)^2} - \frac{d^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c-2\sqrt[3]{dx}}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^2} + \frac{bx}{3a(a+bx^3)(bc-ad)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)^2\*(c + d\*x^3)), x]

[Out] (b\*x)/(3\*a\*(b\*c - a\*d)\*(a + b\*x^3)) - (b^(2/3)\*(2\*b\*c - 5\*a\*d)\*ArcTan[(a^(1/3) - 2\*b^(1/3)\*x)/(Sqrt[3]\*a^(1/3))]/(3\*Sqrt[3]\*a^(5/3)\*(b\*c - a\*d)^2) - (d^(5/3)\*ArcTan[(c^(1/3) - 2\*d^(1/3)\*x)/(Sqrt[3]\*c^(1/3))]/(Sqrt[3]\*c^(2/3)\*(b\*c - a\*d)^2) + (b^(2/3)\*(2\*b\*c - 5\*a\*d)\*Log[a^(1/3) + b^(1/3)\*x]/(9\*a^(5/3)\*(b\*c - a\*d)^2) + (d^(5/3)\*Log[c^(1/3) + d^(1/3)\*x]/(3\*c^(2/3)\*(b\*c - a\*d)^2) - (b^(2/3)\*(2\*b\*c - 5\*a\*d)\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2]/(18\*a^(5/3)\*(b\*c - a\*d)^2) - (d^(5/3)\*Log[c^(2/3) - c^(1/3)\*d^(1/3)\*x + d^(2/3)\*x^2]/(6\*c^(2/3)\*(b\*c - a\*d)^2)

**Rubi in Sympy [A]** time = 109.102, size = 321, normalized size = 0.93

$$\begin{aligned} & \frac{d^{\frac{5}{3}} \log(\sqrt[3]{c} + \sqrt[3]{dx})}{3c^{\frac{2}{3}}(ad - bc)^2} - \frac{d^{\frac{5}{3}} \log(c^{\frac{2}{3}} - \sqrt[3]{c}\sqrt[3]{dx} + d^{\frac{2}{3}}x^2)}{6c^{\frac{2}{3}}(ad - bc)^2} - \frac{\sqrt{3}d^{\frac{5}{3}} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3} - 2\frac{\sqrt[3]{dx}}{3}\right)}{\sqrt[3]{c}}\right)}{3c^{\frac{2}{3}}(ad - bc)^2} \\ & - \frac{bx}{3a(a + bx^3)(ad - bc)} - \frac{b^{\frac{2}{3}}(5ad - 2bc) \log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{\frac{5}{3}}(ad - bc)^2} \\ & + \frac{b^{\frac{2}{3}}(5ad - 2bc) \log\left(a^{\frac{2}{3}} - \sqrt[3]{a}\sqrt[3]{bx} + b^{\frac{2}{3}}x^2\right)}{18a^{\frac{5}{3}}(ad - bc)^2} + \frac{\sqrt{3}b^{\frac{2}{3}}(5ad - 2bc) \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} - 2\frac{\sqrt[3]{bx}}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}(ad - bc)^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(b*x**3+a)**2/(d*x**3+c), x)`

[Out]  $d^{5/3} \log(c^{1/3} + d^{1/3}x)/(3c^{2/3}(ad - bc)^2) - d^{5/3} \log(c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2)/(6c^{2/3}(ad - bc)^2) - \sqrt{3}d^{5/3} \operatorname{atan}(\sqrt{3}(c^{1/3}/3 - 2d^{1/3}x/3)/c^{1/3})/(3c^{2/3}(ad - bc)^2) - bx/(3a(a + bx^3)(ad - bc)) - b^{2/3}(5ad - 2bc) \log(a^{1/3} + b^{1/3}x)/(9a^{5/3}(ad - bc)^2) + b^{2/3}(5ad - 2bc) \log(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2)/(18a^{5/3}(ad - bc)^2) + \sqrt{3}b^{2/3}(5ad - 2bc) \operatorname{atan}(\sqrt{3}(a^{1/3}/3 - 2b^{1/3}x/3)/a^{1/3})/(9a^{5/3}(ad - bc)^2)$

**Mathematica [A]** time = 0.360038, size = 337, normalized size = 0.97

$$-b^{2/3}c^{2/3}(a + bx^3)(2bc - 5ad) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right) - 3a^{5/3}d^{5/3}(a + bx^3) \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right) + 6a^{2/3}bc$$

Antiderivative was successfully verified.

[In] `Integrate[1/((a + b*x^3)^2*(c + d*x^3)), x]`

[Out]  $(6a^{2/3}b^2c^{2/3}(bc - ad)x - 2\sqrt{3}b^{2/3}c^{2/3}(2bc - 5ad)(a + bx^3) \operatorname{ArcTan}\left[\frac{1 - (2b^{1/3}x)/a^{1/3}}{\sqrt{3}}\right] - 6\sqrt{3}a^{5/3}d^{5/3}(a + bx^3) \operatorname{ArcTan}\left[\frac{1 - (2d^{1/3}x)/c^{1/3}}{\sqrt{3}}\right] + 2b^{2/3}c^{2/3}(2bc - 5ad)(a + bx^3) \operatorname{Log}[a^{1/3} + b^{1/3}x] + 6a^{5/3}d^{5/3}(a + bx^3) \operatorname{Log}[c^{1/3} + d^{1/3}x] - b^{2/3}c^{2/3}(2bc - 5ad)(a + bx^3) \operatorname{Log}[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2] - 3a^{5/3}d^{5/3}(a + bx^3) \operatorname{Log}[c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2])/(18a^{5/3}c^{2/3}b^2(bc - ad)^2(a + bx^3))$

**Maple [A]** time = 0.017, size = 406, normalized size = 1.2

$$\begin{aligned}
& -\frac{bx d}{3(ad-bc)^2(bx^3+a)} + \frac{b^2xc}{3(ad-bc)^2a(bx^3+a)} - \frac{5d}{9(ad-bc)^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \\
& + \frac{2bc}{9(ad-bc)^2a} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{5d}{18(ad-bc)^2} \ln\left(x^2 - x\sqrt[3]{\frac{a}{b}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \\
& - \frac{bc}{9(ad-bc)^2a} \ln\left(x^2 - x\sqrt[3]{\frac{a}{b}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \\
& - \frac{5\sqrt{3}d}{9(ad-bc)^2} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \\
& + \frac{2b\sqrt{3}c}{9(ad-bc)^2a} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}} - 1\right)\right) \left(\frac{a}{b}\right)^{-\frac{2}{3}} \\
& + \frac{d}{3(ad-bc)^2} \ln\left(x + \sqrt[3]{\frac{c}{d}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{d}{6(ad-bc)^2} \ln\left(x^2 - x\sqrt[3]{\frac{c}{d}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}} \\
& + \frac{\sqrt{3}d}{3(ad-bc)^2} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}} - 1\right)\right) \left(\frac{c}{d}\right)^{-\frac{2}{3}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^2/(d\*x^3+c), x)

[Out] -1/3\*b/(a\*d-b\*c)^2\*x/(b\*x^3+a)\*d+1/3\*b^2/(a\*d-b\*c)^2\*x/a/(b\*x^3+a)\*c-5/9/(a\*d-b\*c)^2/(a/b)^(2/3)\*ln(x+(a/b)^(1/3))\*d+2/9\*b/(a\*d-b\*c)^2/a/(a/b)^(2/3)\*ln(x+(a/b)^(1/3))\*c+5/18/(a\*d-b\*c)^2/(a/b)^(2/3)\*ln(x^2-x\*(a/b)^(1/3)+(a/b)^(2/3))\*d-1/9\*b/(a\*d-b\*c)^2/a/(a/b)^(2/3)\*ln(x^2-x\*(a/b)^(1/3)+(a/b)^(2/3))\*c-5/9/(a\*d-b\*c)^2/(a/b)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(a/b)^(1/3)\*x-1))\*d+2/9\*b/(a\*d-b\*c)^2/a/(a/b)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(a/b)^(1/3)\*x-1))\*c+1/3\*d/(a\*d-b\*c)^2/(c/d)^(2/3)\*ln(x+(c/d)^(1/3))-1/6\*d/(a\*d-b\*c)^2/(c/d)^(2/3)\*ln(x^2-x\*(c/d)^(1/3)+(c/d)^(2/3))+1/3\*d/(a\*d-b\*c)^2/(c/d)^(2/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(2/(c/d)^(1/3)\*x-1))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^3 + a)^2\*(d\*x^3 + c)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 7.40408, size = 633, normalized size = 1.83

$$\sqrt{3} \left( \sqrt{3} \left( (2b^2c - 5abd)x^3 + 2abc - 5a^2d \right) \left( -\frac{b^2}{a^2} \right)^{\frac{1}{3}} \log \left( b^2x^2 + abx \left( -\frac{b^2}{a^2} \right)^{\frac{1}{3}} + a^2 \left( -\frac{b^2}{a^2} \right)^{\frac{2}{3}} \right) - 3\sqrt{3}(abdx^3 + a^2d) \left( \frac{d^2}{c^2} \right)^{\frac{1}{3}} \log \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^3 + a)^2\*(d\*x^3 + c)),x, algorithm="fricas")

[Out]  $\frac{1}{54} \sqrt{3} \left( \sqrt{3} \left( (2b^2c - 5abd)x^3 + 2abc - 5a^2d \right) \left( -\frac{b^2}{a^2} \right)^{1/3} \log \left( \frac{b^2x^2 + abx \left( -\frac{b^2}{a^2} \right)^{1/3} + a^2 \left( -\frac{b^2}{a^2} \right)^{2/3}}{d^2x^2 - cd^2x \left( \frac{d^2}{c^2} \right)^{1/3} + c^2 \left( \frac{d^2}{c^2} \right)^{2/3}} \right) - 3 \sqrt{3} \left( ab^2d^2x^3 + a^2d^2 \right) \left( \frac{d^2}{c^2} \right)^{1/3} \log \left( \frac{d^2x^2 - cd^2x \left( \frac{d^2}{c^2} \right)^{1/3} + c^2 \left( \frac{d^2}{c^2} \right)^{2/3}}{d^2x^2 - cd^2x \left( \frac{d^2}{c^2} \right)^{1/3} + c^2 \left( \frac{d^2}{c^2} \right)^{2/3}} \right) - 2 \sqrt{3} \left( (2b^2c - 5abd)x^3 + 2abc - 5a^2d \right) \left( -\frac{b^2}{a^2} \right)^{1/3} \log \left( \frac{bx - a \left( -\frac{b^2}{a^2} \right)^{1/3}}{d^2x^2 - cd^2x \left( \frac{d^2}{c^2} \right)^{1/3} + c^2 \left( \frac{d^2}{c^2} \right)^{2/3}} \right) + 6 \sqrt{3} \left( ab^2d^2x^3 + a^2d^2 \right) \left( \frac{d^2}{c^2} \right)^{1/3} \log \left( \frac{dx + c \left( \frac{d^2}{c^2} \right)^{1/3}}{d^2x^2 - cd^2x \left( \frac{d^2}{c^2} \right)^{1/3} + c^2 \left( \frac{d^2}{c^2} \right)^{2/3}} \right) + 6 \sqrt{3} \left( b^2c - ab^2d \right) x + 6 \left( (2b^2c - 5abd)x^3 + 2abc - 5a^2d \right) \left( -\frac{b^2}{a^2} \right)^{1/3} \arctan \left( \frac{1}{3} \left( 2 \sqrt{3} b^2x + \sqrt{3} a \left( -\frac{b^2}{a^2} \right)^{1/3} \right) \right) / \left( a \left( -\frac{b^2}{a^2} \right)^{1/3} \right) - 18 \left( ab^2d^2x^3 + a^2d^2 \right) \left( \frac{d^2}{c^2} \right)^{1/3} \arctan \left( \frac{-1}{3} \left( 2 \sqrt{3} d^2x - \sqrt{3} c \left( \frac{d^2}{c^2} \right)^{1/3} \right) \right) / \left( c \left( \frac{d^2}{c^2} \right)^{1/3} \right) \right) / \left( a^2b^2c^2 - 2a^3b^2cd + a^4d^2 + (ab^3c^2 - 2a^2b^2c^2d + a^3b^2d^2)x^3 \right)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)\*\*2/(d\*x\*\*3+c),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.228681, size = 598, normalized size = 1.73

$$\begin{aligned} & \frac{d^2 \left( -\frac{c}{d} \right)^{1/3} \ln \left( \left| x - \left( -\frac{c}{d} \right)^{1/3} \right| \right)}{3(b^2c^3 - 2abc^2d + a^2cd^2)} + \frac{(-cd^2)^{1/3} d \arctan \left( \frac{\sqrt{3} \left( 2x + \left( -\frac{c}{d} \right)^{1/3} \right)}{3 \left( -\frac{c}{d} \right)^{1/3}} \right)}{\sqrt{3}b^2c^3 - 2\sqrt{3}abc^2d + \sqrt{3}a^2cd^2} \\ & + \frac{(-cd^2)^{1/3} d \ln \left( x^2 + x \left( -\frac{c}{d} \right)^{1/3} + \left( -\frac{c}{d} \right)^{2/3} \right)}{6(b^2c^3 - 2abc^2d + a^2cd^2)} - \frac{(2b^2c - 5abd) \left( -\frac{a}{b} \right)^{1/3} \ln \left( \left| x - \left( -\frac{a}{b} \right)^{1/3} \right| \right)}{9(a^2b^2c^2 - 2a^3bcd + a^4d^2)} \\ & + \frac{\left( 2(-ab^2)^{1/3} bc - 5(-ab^2)^{1/3} ad \right) \arctan \left( \frac{\sqrt{3} \left( 2x + \left( -\frac{a}{b} \right)^{1/3} \right)}{3 \left( -\frac{a}{b} \right)^{1/3}} \right)}{3 \left( \sqrt{3}a^2b^2c^2 - 2\sqrt{3}a^3bcd + \sqrt{3}a^4d^2 \right)} \\ & + \frac{\left( 2(-ab^2)^{1/3} bc - 5(-ab^2)^{1/3} ad \right) \ln \left( x^2 + x \left( -\frac{a}{b} \right)^{1/3} + \left( -\frac{a}{b} \right)^{2/3} \right)}{18(a^2b^2c^2 - 2a^3bcd + a^4d^2)} + \frac{bx}{3(bx^3 + a)(abc - a^2d)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^3 + a)^2\*(d\*x^3 + c)),x, algorithm="giac")

[Out]  $-\frac{1}{3} d^2 \left( -\frac{c}{d} \right)^{1/3} \ln \left( \text{abs} \left( x - \left( -\frac{c}{d} \right)^{1/3} \right) \right) / \left( b^2c^3 - 2a^2b^2c^2d + a^2c^2d^2 \right) + \left( -c^2d^2 \right)^{1/3} d \arctan \left( \frac{1}{3} \sqrt{3} \left( 2x + \left( -\frac{c}{d} \right)^{1/3} \right) / \left( -\frac{c}{d} \right)^{1/3} \right) / \left( \sqrt{3} b^2c^3 - 2\sqrt{3} abc^2d + \sqrt{3} a^2cd^2 \right) + \sqrt{3} a^2c^2d^2 + \frac{1}{6} \left( -c^2d^2 \right)^{1/3} d \ln \left( x^2 + x \left( -\frac{c}{d} \right)^{1/3} + \left( -\frac{c}{d} \right)^{2/3} \right) / \left( b^2c^3 - 2a^2b^2c^2d + a^2c^2d^2 \right) - \frac{1}{9} \left( 2b^2c - 5abd \right) \left( -\frac{a}{b} \right)^{1/3} \ln \left( \text{abs} \left( x - \left( -\frac{a}{b} \right)^{1/3} \right) \right) / \left( a^2b^2c^2 - 2a^3bcd + a^4d^2 \right) + \frac{1}{3} \left( 2 \left( -a^2b^2 \right)^{1/3} bc - 5 \left( -a^2b^2 \right)^{1/3} ad \right) \arctan \left( \frac{1}{3} \sqrt{3} \left( 2x + \left( -\frac{a}{b} \right)^{1/3} \right) / \left( -\frac{a}{b} \right)^{1/3} \right) / \left( \sqrt{3} a^2b^2c^2 - 2\sqrt{3} abc^2d + \sqrt{3} a^4d^2 \right) + \frac{1}{18} \left( 2 \left( -a^2b^2 \right)^{1/3} bc - 5 \left( -a^2b^2 \right)^{1/3} ad \right) \ln \left( x^2 + x \left( -\frac{a}{b} \right)^{1/3} + \left( -\frac{a}{b} \right)^{2/3} \right) / \left( a^2b^2c^2 - 2a^3bcd + a^4d^2 \right) + \frac{1}{3} b^2x / \left( (b^2x^3 + a) \left( ab^2c - a^2d \right) \right)$



$$3.26 \quad \int \frac{1}{(a+bx^3)^2(c+dx^3)^2} dx$$

**Optimal.** Leaf size=419

$$\begin{aligned} & -\frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{9a^{5/3}(bc-ad)^3} + \frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{5/3}(bc-ad)^3} \\ & -\frac{2b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^3} - \frac{d^{5/3}(4bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx}+d^{2/3}x^2\right)}{9c^{5/3}(bc-ad)^3} \\ & + \frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{dx}\right)}{9c^{5/3}(bc-ad)^3} - \frac{2d^{5/3}(4bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^3} \\ & + \frac{bx}{3a(a+bx^3)(c+dx^3)(bc-ad)} + \frac{dx(ad+bc)}{3ac(c+dx^3)(bc-ad)^2} \end{aligned}$$

[Out]  $(d*(b*c + a*d)*x)/(3*a*c*(b*c - a*d)^2*(c + d*x^3)) + (b*x)/(3*a*(b*c - a*d)*(a + b*x^3)*(c + d*x^3)) - (2*b^(5/3)*(b*c - 4*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*(b*c - a*d)^3) - (2*d^(5/3)*(4*b*c - a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]/(3*Sqrt[3]*c^(5/3)*(b*c - a*d)^3) + (2*b^(5/3)*(b*c - 4*a*d)*Log[a^(1/3) + b^(1/3)*x]/(9*a^(5/3)*(b*c - a*d)^3) + (2*d^(5/3)*(4*b*c - a*d)*Log[c^(1/3) + d^(1/3)*x]/(9*c^(5/3)*(b*c - a*d)^3) - (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(9*a^(5/3)*(b*c - a*d)^3) - (d^(5/3)*(4*b*c - a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(9*c^(5/3)*(b*c - a*d)^3)$

**Rubi [A]** time = 1.07782, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$

$$\begin{aligned} & -\frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2\right)}{9a^{5/3}(bc-ad)^3} + \frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{9a^{5/3}(bc-ad)^3} \\ & -\frac{2b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^3} - \frac{d^{5/3}(4bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{dx}+d^{2/3}x^2\right)}{9c^{5/3}(bc-ad)^3} \\ & + \frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{dx}\right)}{9c^{5/3}(bc-ad)^3} - \frac{2d^{5/3}(4bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^3} \\ & + \frac{bx}{3a(a+bx^3)(c+dx^3)(bc-ad)} + \frac{dx(ad+bc)}{3ac(c+dx^3)(bc-ad)^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)^2\*(c + d\*x^3)^2), x]

[Out]  $(d*(b*c + a*d)*x)/(3*a*c*(b*c - a*d)^2*(c + d*x^3)) + (b*x)/(3*a*(b*c - a*d)*(a + b*x^3)*(c + d*x^3)) - (2*b^(5/3)*(b*c - 4*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*(b*c - a*d)^3) - (2*d^(5/3)*(4*b*c - a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]/(3*Sqrt[3]*c^(5/3)*(b*c - a*d)^3) + (2*b^(5/3)*(b*c - 4*a*d)*Log[a^(1/3) + b^(1/3)*x]/(9*a^(5/3)*(b*c - a*d)^3) + (2*d^(5/3)*(4*b*c - a*d)*Log[c^(1/3) + d^(1/3)*x]/(9*c^(5/3)*(b*c - a*d)^3) - (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(9*a^(5/3)*(b*c - a*d)^3) - (d^(5/3)*(4*b*c - a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(9*c^(5/3)*(b*c - a*d)^3)$

**Rubi in Sympy [A]** time = 177.573, size = 386, normalized size = 0.92

$$\frac{dx}{3c(a+bx^3)(c+dx^3)(ad-bc)} + \frac{2d^{5/3}(ad-4bc)\log(\sqrt[3]{c} + \sqrt[3]{dx})}{9c^{5/3}(ad-bc)^3}$$

$$- \frac{d^{5/3}(ad-4bc)\log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{9c^{5/3}(ad-bc)^3} - \frac{2\sqrt{3}d^{5/3}(ad-4bc)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3} - \frac{2\sqrt[3]{dx}}{3}\right)}{\sqrt[3]{c}}\right)}{9c^{5/3}(ad-bc)^3}$$

$$+ \frac{bx(ad+bc)}{3ac(a+bx^3)(ad-bc)^2} + \frac{2b^{5/3}(4ad-bc)\log(\sqrt[3]{a} + \sqrt[3]{bx})}{9a^{5/3}(ad-bc)^3}$$

$$- \frac{b^{5/3}(4ad-bc)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{9a^{5/3}(ad-bc)^3} - \frac{2\sqrt{3}b^{5/3}(4ad-bc)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} - \frac{2\sqrt[3]{bx}}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{5/3}(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(b*x**3+a)**2/(d*x**3+c)**2,x)`

[Out]  $d*x/(3*c*(a+b*x**3)*(c+d*x**3)*(a*d-b*c)) + 2*d**(5/3)*(a*d - 4*b*c)*\log(c**(1/3) + d**(1/3)*x)/(9*c**(5/3)*(a*d - b*c)**3) - d**(5/3)*(a*d - 4*b*c)*\log(c**(2/3) - c**(1/3)*d**(1/3)*x + d**(2/3)*x**2)/(9*c**(5/3)*(a*d - b*c)**3) - 2*\sqrt{3}*d**(5/3)*(a*d - 4*b*c)*\operatorname{atan}(\sqrt{3}*(c**(1/3)/3 - 2*d**(1/3)*x/3)/c**(1/3))/(9*c**(5/3)*(a*d - b*c)**3) + b*x*(a*d + b*c)/(3*a*c*(a + b*x**3)*(a*d - b*c)**2) + 2*b**(5/3)*(4*a*d - b*c)*\log(a**(1/3) + b**(1/3)*x)/(9*a**(5/3)*(a*d - b*c)**3) - b**(5/3)*(4*a*d - b*c)*\log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(9*a**(5/3)*(a*d - b*c)**3) - 2*\sqrt{3}*b**(5/3)*(4*a*d - b*c)*\operatorname{atan}(\sqrt{3}*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(5/3)*(a*d - b*c)**3)$

**Mathematica [A]** time = 1.63819, size = 381, normalized size = 0.91

$$\frac{1}{9} \left( \frac{b^{5/3}(bc-4ad)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2\right)}{a^{5/3}(ad-bc)^3} + \frac{2b^{5/3}(4ad-bc)\log\left(\sqrt[3]{a} + \sqrt[3]{bx}\right)}{a^{5/3}(ad-bc)^3} \right.$$

$$+ \frac{2\sqrt{3}b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}(ad-bc)^3} + \frac{3b^2x}{a(a+bx^3)(bc-ad)^2}$$

$$+ \frac{d^{5/3}(ad-4bc)\log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{dx} + d^{2/3}x^2\right)}{c^{5/3}(bc-ad)^3} + \frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c} + \sqrt[3]{dx}\right)}{c^{5/3}(bc-ad)^3}$$

$$\left. + \frac{2\sqrt{3}d^{5/3}(ad-4bc)\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{dx}}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}(bc-ad)^3} + \frac{3d^2x}{c(c+dx^3)(bc-ad)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^3)^2\*(c + d\*x^3)^2),x]

[Out] 
$$\begin{aligned} & ((3*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^3)) + (3*d^2*x)/(c*(b*c - a*d)^2*(c + d*x^3)) + (2*\sqrt{3}*b^{5/3}*(b*c - 4*a*d)*\text{ArcTan}[(1 - (2*b^{1/3}*x)/a^{1/3})/\sqrt{3}])/(a^{5/3}*(-(b*c) + a*d)^3) + (2*\sqrt{3}*d^{5/3}*(-4*b*c + a*d)*\text{ArcTan}[(1 - (2*d^{1/3}*x)/c^{1/3})/\sqrt{3}])/(c^{5/3}*(b*c - a*d)^3) + (2*b^{5/3}*(-(b*c) + 4*a*d)*\text{Log}[a^{1/3} + b^{1/3}*x])/(a^{5/3}*(-(b*c) + a*d)^3) + (2*d^{5/3}*(4*b*c - a*d)*\text{Log}[c^{1/3} + d^{1/3}*x])/(c^{5/3}*(b*c - a*d)^3) + (b^{5/3}*(b*c - 4*a*d)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/(a^{5/3}*(-(b*c) + a*d)^3) + (d^{5/3}*(-4*b*c + a*d)*\text{Log}[c^{2/3} - c^{1/3}*d^{1/3}*x + d^{2/3}*x^2])/(c^{5/3}*(b*c - a*d)^3) \end{aligned} / 9$$

**Maple [A]** time = 0.023, size = 606, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^2/(d\*x^3+c)^2,x)

[Out] 
$$\begin{aligned} & 1/3*b^2/(a*d-b*c)^3*x/(b*x^3+a)*d-1/3*b^3/(a*d-b*c)^3*x/a/(b*x^3+a)*c+8/9*b/(a*d-b*c)^3/(a/b)^{2/3}*\ln(x+(a/b)^{1/3})*d-2/9*b^2/(a*d-b*c)^3/a/(a/b)^{2/3}*\ln(x+(a/b)^{1/3})*c-4/9*b/(a*d-b*c)^3/(a/b)^{2/3}*\ln(x^2-x*(a/b)^{1/3}+(a/b)^{2/3})*d+1/9*b^2/(a*d-b*c)^3/a/(a/b)^{2/3}*\ln(x^2-x*(a/b)^{1/3}+(a/b)^{2/3})*c+8/9*b/(a*d-b*c)^3/(a/b)^{2/3}*\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1))*d-2/9*b^2/(a*d-b*c)^3/a/(a/b)^{2/3}*\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1))*c+1/3*d^3/(a*d-b*c)^3*c*x/(d*x^3+c)*a-1/3*d^2/(a*d-b*c)^3*x/(d*x^3+c)*b+2/9*d^2/(a*d-b*c)^3/c/(c/d)^{2/3}*\ln(x+(c/d)^{1/3})*a-8/9*d/(a*d-b*c)^3/c/(c/d)^{2/3}*\ln(x+(c/d)^{1/3})*b-1/9*d^2/(a*d-b*c)^3/c/(c/d)^{2/3}*\ln(x^2-x*(c/d)^{1/3}+(c/d)^{2/3})*a+4/9*d/(a*d-b*c)^3/c/(c/d)^{2/3}*\ln(x^2-x*(c/d)^{1/3}+(c/d)^{2/3})*b+2/9*d^2/(a*d-b*c)^3/c/(c/d)^{2/3}*\arctan(1/3*3^{1/2}*(2/(c/d)^{1/3}*x-1))*a-8/9*d/(a*d-b*c)^3/c/(c/d)^{2/3}*\arctan(1/3*3^{1/2}*(2/(c/d)^{1/3}*x-1))*b \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^3 + a)^2\*(d\*x^3 + c)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 77.3424, size = 1250, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^3 + a)^2\*(d\*x^3 + c)^2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/27*\sqrt{3}*(\sqrt{3}*((b^3*c^2*d - 4*a*b^2*c*d^2)*x^6 + a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2*b*c*d^2)*x^3) * (b^2/a^2)^{1/3}*\log(b^2*x^2 - a*b*x*(b^2/a^2)^{1/3} + a^2*(b^2/a^2)^{2/3}) + \sqrt{3}*((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b \end{aligned}$$

$$\begin{aligned}
& c^2 d - a^3 c d^2 + (4 a^2 b^2 c^2 d + 3 a^2 b^2 c d^2 - a^3 d^3) x^3 \\
& \left( \frac{d^2}{c^2} \right)^{1/3} \log(d^2 x^2 - c d x \left( \frac{d^2}{c^2} \right)^{1/3} + c^2 \left( \frac{d^2}{c^2} \right)^{2/3}) - 2 \sqrt{3} \left( (b^3 c^2 d - 4 a^2 b^2 c d^2) x^6 + a^2 b^2 c^3 \right. \\
& \left. - 4 a^2 b^2 c^2 d + (b^3 c^3 - 3 a^2 b^2 c^2 d - 4 a^2 b^2 c d^2) x^3 \right) \left( \frac{b^2}{a^2} \right)^{1/3} \log(b x + a \left( \frac{b^2}{a^2} \right)^{1/3}) - 2 \sqrt{3} \left( (4 a^2 b^2 c d^2 - a^2 b^2 d^3) x^6 \right. \\
& \left. + 4 a^2 b^2 c^2 d - a^3 c d^2 + (4 a^2 b^2 c^2 d + 3 a^2 b^2 c d^2 - a^3 d^3) x^3 \right) \left( \frac{d^2}{c^2} \right)^{1/3} \log(d x + c \left( \frac{d^2}{c^2} \right)^{1/3}) \\
& + 6 \left( (b^3 c^2 d - 4 a^2 b^2 c d^2) x^6 + a^2 b^2 c^3 - 4 a^2 b^2 c^2 d + (b^3 c^3 - 3 a^2 b^2 c^2 d - 4 a^2 b^2 c d^2) x^3 \right) \left( \frac{b^2}{a^2} \right)^{1/3} \arctan\left( \frac{-1/3 (2 \sqrt{3} b x - \sqrt{3} a \left( \frac{b^2}{a^2} \right)^{1/3})}{a \left( \frac{b^2}{a^2} \right)^{1/3}} \right) \\
& + 6 \left( (4 a^2 b^2 c d^2 - a^2 b^2 d^3) x^6 + 4 a^2 b^2 c^2 d - a^3 c d^2 + (4 a^2 b^2 c^2 d + 3 a^2 b^2 c d^2 - a^3 d^3) x^3 \right) \left( \frac{d^2}{c^2} \right)^{1/3} \arctan\left( \frac{-1/3 (2 \sqrt{3} d x - \sqrt{3} c \left( \frac{d^2}{c^2} \right)^{1/3})}{c \left( \frac{d^2}{c^2} \right)^{1/3}} \right) \\
& - 3 \sqrt{3} \left( (b^3 c^2 d - a^2 b^2 d^3) x^4 + (b^3 c^3 - a^2 b^2 c^2 d + a^2 b^2 c d^2 - a^3 d^3) x \right) / \left( a^2 b^3 c^5 - 3 a^3 b^2 c^4 d + 3 a^4 b^2 c^3 d^2 - a^5 c^2 d^3 \right. \\
& \left. + (a^2 b^4 c^4 d - 3 a^2 b^3 c^3 d^2 + 3 a^3 b^2 c^2 d^3 - a^4 b^2 c d^4) x^6 + (a^2 b^4 c^5 - 2 a^2 b^3 c^4 d + 2 a^4 b^2 c^2 d^3 - a^5 c d^4) x^3 \right)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)\*\*2/(d\*x\*\*3+c)\*\*2,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.230403, size = 896, normalized size = 2.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^3 + a)^2\*(d\*x^3 + c)^2),x, algorithm="giac")

[Out] 
$$\begin{aligned}
& -2/9 (b^3 c - 4 a^2 b^2 d) (-a/b)^{1/3} \ln(\text{abs}(x - (-a/b)^{1/3})) / (a^2 b^3 c^3 - 3 a^3 b^2 c^2 d + 3 a^4 b^2 c d^2 - a^5 d^3) - 2/9 (4 b^2 c d^2 - a^2 d^3) (-c/d)^{1/3} \ln(\text{abs}(x - (-c/d)^{1/3})) / (b^3 c^5 - 3 a^2 b^2 c^4 d + 3 a^2 b^2 c^3 d^2 - a^3 c^2 d^3) \\
& + 2/3 ((-a^2 b^2)^{1/3} b^2 c - 4 (-a^2 b^2)^{1/3} a b^2 d) \arctan(1/3 \sqrt{3} (2 x + (-a/b)^{1/3}) / (-a/b)^{1/3}) / (\sqrt{3} a^2 b^3 c^3 - 3 \sqrt{3} a^3 b^2 c^2 d + 3 \sqrt{3} a^4 b^2 c d^2 - \sqrt{3} a^5 d^3) \\
& + 2/3 (4 (-c^2 d^2)^{1/3} b^2 c d - (-c^2 d^2)^{1/3} a^2 d^2) \arctan(1/3 \sqrt{3} (2 x + (-c/d)^{1/3}) / (-c/d)^{1/3}) / (\sqrt{3} b^3 c^5 - 3 \sqrt{3} a^2 b^2 c^4 d + 3 \sqrt{3} a^2 b^2 c^3 d^2 - \sqrt{3} a^3 c^2 d^3) \\
& + 1/9 ((-a^2 b^2)^{1/3} b^2 c - 4 (-a^2 b^2)^{1/3} a b^2 d) \ln(x^2 + x (-a/b)^{1/3} + (-a/b)^{2/3}) / (a^2 b^3 c^3 - 3 a^3 b^2 c^2 d + 3 a^4 b^2 c d^2 - a^5 d^3) \\
& + 1/9 (4 (-c^2 d^2)^{1/3} b^2 c d - (-c^2 d^2)^{1/3} a^2 d^2) \ln(x^2 + x (-c/d)^{1/3} + (-c/d)^{2/3}) / (b^3 c^5 - 3 a^2 b^2 c^4 d + 3 a^2 b^2 c^3 d^2 - a^3 c^2 d^3) \\
& + 1/3 (b^2 c^2 d x^4 + a^2 b^2 d^2 x^4 + b^2 c^2 d x + a^2 d^2 x) / ((b^2 d x^6 + b^2 c x^3 + a^2 d x^3 + a^2 c) (a^2 b^2 c^3 - 2 a^2 b^2 c^2 d + a^3 c d^2))
\end{aligned}$$

$$3.27 \quad \int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}} dx$$

**Optimal.** Leaf size=109

$$\frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}} + \frac{27a^2x(a+bx^3)}{140c^3(c+dx^3)^{4/3}} + \frac{9ax(a+bx^3)^2}{70c^2(c+dx^3)^{7/3}} + \frac{x(a+bx^3)^3}{10c(c+dx^3)^{10/3}}$$

[Out]  $(x*(a+b*x^3)^3)/(10*c*(c+d*x^3)^(10/3)) + (9*a*x*(a+b*x^3)^2)/(70*c^2*(c+d*x^3)^(7/3)) + (27*a^2*x*(a+b*x^3))/(140*c^3*(c+d*x^3)^(4/3)) + (81*a^3*x)/(140*c^4*(c+d*x^3)^(1/3))$

**Rubi [A]** time = 0.115874, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}} + \frac{27a^2x(a+bx^3)}{140c^3(c+dx^3)^{4/3}} + \frac{9ax(a+bx^3)^2}{70c^2(c+dx^3)^{7/3}} + \frac{x(a+bx^3)^3}{10c(c+dx^3)^{10/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^3/(c + d\*x^3)^(13/3), x]

[Out]  $(x*(a+b*x^3)^3)/(10*c*(c+d*x^3)^(10/3)) + (9*a*x*(a+b*x^3)^2)/(70*c^2*(c+d*x^3)^(7/3)) + (27*a^2*x*(a+b*x^3))/(140*c^3*(c+d*x^3)^(4/3)) + (81*a^3*x)/(140*c^4*(c+d*x^3)^(1/3))$

**Rubi in Sympy [A]** time = 17.977, size = 102, normalized size = 0.94

$$\frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}} + \frac{27a^2x(a+bx^3)}{140c^3(c+dx^3)^{4/3}} + \frac{9ax(a+bx^3)^2}{70c^2(c+dx^3)^{7/3}} + \frac{x(a+bx^3)^3}{10c(c+dx^3)^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*\*3/(d\*x\*\*3+c)\*\*(13/3), x)

[Out]  $81*a^3*x/(140*c^4*(c+d*x^3)^(1/3)) + 27*a^2*x*(a+b*x^3)/(140*c^3*(c+d*x^3)^(4/3)) + 9*a*x*(a+b*x^3)**2/(70*c^2*(c+d*x^3)^(7/3)) + x*(a+b*x^3)**3/(10*c*(c+d*x^3)^(10/3))$

**Mathematica [A]** time = 0.132955, size = 120, normalized size = 1.1

$$\frac{x(a^3(140c^3 + 315c^2dx^3 + 270cd^2x^6 + 81d^3x^9) + 3a^2bcx^3(35c^2 + 30cdx^3 + 9d^2x^6) + 6ab^2c^2x^6(10c + 3dx^3) + 14b^3c^3x^9)}{140c^4(c+dx^3)^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^3/(c + d\*x^3)^(13/3), x]

[Out]  $(x*(14*b^3*c^3*x^9 + 6*a*b^2*c^2*x^6*(10*c + 3*d*x^3) + 3*a^2*b*c*x^3*(35*c^2 + 30*c*d*x^3 + 9*d^2*x^6) + a^3*(140*c^3 + 315*c^2*d*x^3 + 270*c*d^2*x^6 + 81*d^3*x^9)))/(140*c^4*(c+d*x^3)^(10/3))$

**Maple [A]** time = 0.01, size = 134, normalized size = 1.2

$$\frac{x(81a^3d^3x^9 + 27a^2bcd^2x^9 + 18ab^2c^2dx^9 + 14b^3c^3x^9 + 270a^3cd^2x^6 + 90a^2bc^2dx^6 + 60ab^2c^3x^6 + 315a^3c^2dx^3 + 105a^2bc^3)}{140c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^3/(d*x^3+c)^(13/3), x)`

[Out]  $\frac{1}{140}x(81a^3d^3x^9 + 27a^2b^2c^2d^2x^9 + 18a^2b^2c^2d^2x^9 + 14b^3c^3x^9 + 270a^3cd^2x^6 + 90a^2b^2c^2d^2x^6 + 60a^2b^2c^3x^6 + 315a^3c^2dx^3 + 105a^2bc^3)/(d^2x^3+c)^{10/3}/c^4$

**Maxima [A]** time = 1.52004, size = 246, normalized size = 2.26

$$\frac{b^3x^{10}}{10(dx^3+c)^{\frac{10}{3}}c} - \frac{3ab^2\left(7d - \frac{10(dx^3+c)}{x^3}\right)x^{10}}{70(dx^3+c)^{\frac{10}{3}}c^2} + \frac{3\left(14d^2 - \frac{40(dx^3+c)d}{x^3} + \frac{35(dx^3+c)^2}{x^6}\right)a^2bx^{10}}{140(dx^3+c)^{\frac{10}{3}}c^3} - \frac{\left(14d^3 - \frac{60(dx^3+c)d^2}{x^3} + \frac{105(dx^3+c)^2d}{x^6} - \frac{140(dx^3+c)^3}{x^9}\right)a^3x^{10}}{140(dx^3+c)^{\frac{10}{3}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^3/(d*x^3 + c)^(13/3), x, algorithm="maxima")`

[Out]  $\frac{1}{10}b^3x^{10}/((d^2x^3+c)^{(10/3)}c) - \frac{3}{70}a^2b^2(7d - 10(d^2x^3+c)/x^3)x^{10}/((d^2x^3+c)^{(10/3)}c^2) + \frac{3}{140}(14d^2 - 40(d^2x^3+c)d/x^3 + 35(d^2x^3+c)^2/x^6)a^2bx^{10}/((d^2x^3+c)^{(10/3)}c^3) - \frac{1}{140}(14d^3 - 60(d^2x^3+c)d^2/x^3 + 105(d^2x^3+c)^2d/x^6 - 140(d^2x^3+c)^3/x^9)a^3x^{10}/((d^2x^3+c)^{(10/3)}c^4)$

**Fricas [A]** time = 0.222484, size = 224, normalized size = 2.06

$$\frac{((14b^3c^3 + 18ab^2c^2d + 27a^2bcd^2 + 81a^3d^3)x^{10} + 30(2ab^2c^3 + 3a^2bc^2d + 9a^3cd^2)x^7 + 140a^3c^3x + 105(a^2bc^3 + 3a^3c^2d)x)}{140(c^4d^4x^{12} + 4c^5d^3x^9 + 6c^6d^2x^6 + 4c^7dx^3 + c^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^3/(d*x^3 + c)^(13/3), x, algorithm="fricas")`

[Out]  $\frac{1}{140}((14b^3c^3 + 18a^2b^2c^2d + 27a^2b^2c^2d + 81a^3d^3)x^{10} + 30(2a^2b^2c^3 + 3a^2b^2c^2d + 9a^3c^2d^2)x^7 + 140a^3c^3x + 105(a^2b^2c^3 + 3a^3c^2d)x^4)(d^2x^3+c)^{(2/3)}/(c^4d^4x^{12} + 4c^5d^3x^9 + 6c^6d^2x^6 + 4c^7d^2x^3 + c^8)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**3/(d*x**3+c)**(13/3), x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + a)^3}{(dx^3 + c)^{\frac{13}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^3/(d*x^3 + c)^(13/3),x, algorithm="giac")`

[Out] `integrate((b*x^3 + a)^3/(d*x^3 + c)^(13/3), x)`

$$3.28 \quad \int \frac{(a+bx^3)^2}{(c+dx^3)^{10/3}} dx$$

**Optimal.** Leaf size=78

$$\frac{9a^2x}{14c^3\sqrt[3]{c+dx^3}} + \frac{3ax(a+bx^3)}{14c^2(c+dx^3)^{4/3}} + \frac{x(a+bx^3)^2}{7c(c+dx^3)^{7/3}}$$

[Out]  $(x*(a+b*x^3)^2)/(7*c*(c+d*x^3)^{(7/3)}) + (3*a*x*(a+b*x^3))/(14*c^2*(c+d*x^3)^{(4/3)}) + (9*a^2*x)/(14*c^3*(c+d*x^3)^{(1/3)})$

**Rubi [A]** time = 0.0701422, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{9a^2x}{14c^3\sqrt[3]{c+dx^3}} + \frac{3ax(a+bx^3)}{14c^2(c+dx^3)^{4/3}} + \frac{x(a+bx^3)^2}{7c(c+dx^3)^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^2/(c + d\*x^3)^(10/3), x]

[Out]  $(x*(a+b*x^3)^2)/(7*c*(c+d*x^3)^{(7/3)}) + (3*a*x*(a+b*x^3))/(14*c^2*(c+d*x^3)^{(4/3)}) + (9*a^2*x)/(14*c^3*(c+d*x^3)^{(1/3)})$

**Rubi in Sympy [A]** time = 11.2865, size = 71, normalized size = 0.91

$$\frac{9a^2x}{14c^3\sqrt[3]{c+dx^3}} + \frac{3ax(a+bx^3)}{14c^2(c+dx^3)^{4/3}} + \frac{x(a+bx^3)^2}{7c(c+dx^3)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*\*2/(d\*x\*\*3+c)\*\*(10/3), x)

[Out]  $9*a**2*x/(14*c**3*(c+d*x**3)**(1/3)) + 3*a*x*(a+b*x**3)/(14*c**2*(c+d*x**3)**(4/3)) + x*(a+b*x**3)**2/(7*c*(c+d*x**3)**(7/3))$

**Mathematica [A]** time = 0.0835213, size = 73, normalized size = 0.94

$$\frac{a^2(14c^2x + 21cdx^4 + 9d^2x^7) + abcx^4(7c + 3dx^3) + 2b^2c^2x^7}{14c^3(c+dx^3)^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^2/(c + d\*x^3)^(10/3), x]

[Out]  $(2*b^2*c^2*x^7 + a*b*c*x^4*(7*c + 3*d*x^3) + a^2*(14*c^2*x + 21*c*d*x^4 + 9*d^2*x^7))/(14*c^3*(c+d*x^3)^{(7/3)})$

**Maple [A]** time = 0.01, size = 76, normalized size = 1.

$$\frac{x(9a^2d^2x^6 + 3abcdx^6 + 2b^2c^2x^6 + 21a^2cdx^3 + 7abc^2x^3 + 14a^2c^2)}{14c^3} (dx^3 + c)^{-7/3}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^2/(d*x^3+c)^(10/3),x)`

[Out]  $\frac{1}{14}x^*(9*a^2*d^2*x^6+3*a*b*c*d*x^6+2*b^2*c^2*x^6+21*a^2*c*d*x^3+7*a*b*c^2*x^3+14*a^2*c^2)/(d*x^3+c)^(7/3)/c^3$

**Maxima [A]** time = 1.38355, size = 147, normalized size = 1.88

$$\frac{b^2x^7}{7(dx^3+c)^{\frac{7}{3}}c} - \frac{ab\left(4d - \frac{7(dx^3+c)}{x^3}\right)x^7}{14(dx^3+c)^{\frac{7}{3}}c^2} + \frac{\left(2d^2 - \frac{7(dx^3+c)d}{x^3} + \frac{14(dx^3+c)^2}{x^6}\right)a^2x^7}{14(dx^3+c)^{\frac{7}{3}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^2/(d*x^3 + c)^(10/3),x, algorithm="maxima")`

[Out]  $\frac{1}{7}b^2x^7/((d*x^3 + c)^(7/3)*c) - \frac{1}{14}a*b*(4*d - 7*(d*x^3 + c)/x^3)*x^7/((d*x^3 + c)^(7/3)*c^2) + \frac{1}{14}*(2*d^2 - 7*(d*x^3 + c)*d/x^3 + 14*(d*x^3 + c)^2/x^6)*a^2*x^7/((d*x^3 + c)^(7/3)*c^3)$

**Fricas [A]** time = 0.21906, size = 139, normalized size = 1.78

$$\frac{((2b^2c^2 + 3abcd + 9a^2d^2)x^7 + 14a^2c^2x + 7(abc^2 + 3a^2cd)x^4)(dx^3 + c)^{\frac{2}{3}}}{14(c^3d^3x^9 + 3c^4d^2x^6 + 3c^5dx^3 + c^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^2/(d*x^3 + c)^(10/3),x, algorithm="fricas")`

[Out]  $\frac{1}{14}*((2*b^2*c^2 + 3*a*b*c*d + 9*a^2*d^2)*x^7 + 14*a^2*c^2*x + 7*(a*b*c^2 + 3*a^2*c*d)*x^4)*(d*x^3 + c)^(2/3)/(c^3*d^3*x^9 + 3*c^4*d^2*x^6 + 3*c^5*d*x^3 + c^6)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**2/(d*x**3+c)**(10/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + a)^2}{(dx^3 + c)^{\frac{10}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^2/(d*x^3 + c)^(10/3),x, algorithm="giac")`

[Out] `integrate((b*x^3 + a)^2/(d*x^3 + c)^(10/3), x)`

$$3.29 \quad \int \frac{a+bx^3}{(c+dx^3)^{7/3}} dx$$

**Optimal.** Leaf size=47

$$\frac{x(a+bx^3)}{4c(c+dx^3)^{4/3}} + \frac{3ax}{4c^2\sqrt[3]{c+dx^3}}$$

[Out] (x\*(a + b\*x^3))/(4\*c\*(c + d\*x^3)^(4/3)) + (3\*a\*x)/(4\*c^2\*(c + d\*x^3)^(1/3))

**Rubi [A]** time = 0.0364384, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{x(a+bx^3)}{4c(c+dx^3)^{4/3}} + \frac{3ax}{4c^2\sqrt[3]{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)/(c + d\*x^3)^(7/3), x]

[Out] (x\*(a + b\*x^3))/(4\*c\*(c + d\*x^3)^(4/3)) + (3\*a\*x)/(4\*c^2\*(c + d\*x^3)^(1/3))

**Rubi in Sympy [A]** time = 5.49093, size = 41, normalized size = 0.87

$$\frac{3ax}{4c^2\sqrt[3]{c+dx^3}} + \frac{x(a+bx^3)}{4c(c+dx^3)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)/(d\*x\*\*3+c)\*\*(7/3), x)

[Out] 3\*a\*x/(4\*c\*\*2\*(c + d\*x\*\*3)\*\*(1/3)) + x\*(a + b\*x\*\*3)/(4\*c\*(c + d\*x\*\*3)\*\*(4/3))

**Mathematica [A]** time = 0.044828, size = 37, normalized size = 0.79

$$\frac{x(4ac + 3adx^3 + bcx^3)}{4c^2(c+dx^3)^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)/(c + d\*x^3)^(7/3), x]

[Out] (x\*(4\*a\*c + b\*c\*x^3 + 3\*a\*d\*x^3))/(4\*c^2\*(c + d\*x^3)^(4/3))

**Maple [A]** time = 0.005, size = 34, normalized size = 0.7

$$\frac{x(3adx^3 + bcx^3 + 4ac)}{4c^2} (dx^3 + c)^{-\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)/(d*x^3+c)^(7/3),x)`

[Out]  $\frac{1}{4}x(3ad^2x^3+b^2c^2x^3+4a^2c)/(d^2x^3+c)^{4/3}/c^2$

**Maxima [A]** time = 1.36086, size = 69, normalized size = 1.47

$$\frac{bx^4}{4(dx^3+c)^{4/3}c} - \frac{a\left(d - \frac{4(dx^3+c)}{x^3}\right)x^4}{4(dx^3+c)^{4/3}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)/(d*x^3 + c)^(7/3),x, algorithm="maxima")`

[Out]  $\frac{1}{4}b^2x^4/((d^2x^3 + c)^{4/3}c) - \frac{1}{4}a^2(d - 4(d^2x^3 + c)/x^3)^2x^4/((d^2x^3 + c)^{4/3}c^2)$

**Fricas [A]** time = 0.217738, size = 73, normalized size = 1.55

$$\frac{((bc + 3ad)x^4 + 4acx)(dx^3 + c)^{2/3}}{4(c^2d^2x^6 + 2c^3dx^3 + c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)/(d*x^3 + c)^(7/3),x, algorithm="fricas")`

[Out]  $\frac{1}{4}((b^2c + 3a^2d)x^4 + 4a^2cx)(d^2x^3 + c)^{2/3}/(c^2d^2x^6 + 2c^3d^2x^3 + c^4)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)/(d*x**3+c)**(7/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^3 + a}{(dx^3 + c)^{7/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)/(d*x^3 + c)^(7/3),x, algorithm="giac")`

[Out] `integrate((b*x^3 + a)/(d*x^3 + c)^(7/3), x)`

$$3.30 \quad \int \frac{1}{(c+dx^3)^{4/3}} dx$$

**Optimal.** Leaf size=16

$$\frac{x}{c\sqrt[3]{c+dx^3}}$$

[Out]  $x/(c*(c+d*x^3)^(1/3))$

**Rubi [A]** time = 0.00856531, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{x}{c\sqrt[3]{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^(-4/3), x]

[Out]  $x/(c*(c+d*x^3)^(1/3))$

**Rubi in Sympy [A]** time = 1.25739, size = 12, normalized size = 0.75

$$\frac{x}{c\sqrt[3]{c+dx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(d\*x\*\*3+c)\*\*(4/3), x)

[Out]  $x/(c*(c+d*x**3)**(1/3))$

**Mathematica [C]** time = 2.2023, size = 674, normalized size = 42.12

$$i\sqrt{\frac{\pi}{3}}\left(\frac{1}{3}\right)\left(\frac{(-1)^{2/3}\sqrt[3]{c}}{\sqrt[3]{d}}+x\right)\left(\frac{\sqrt[3]{c+(-1)^{2/3}\sqrt[3]{dx}}}{(1+\sqrt[3]{-1})\sqrt[3]{c}}\right)^{4/3}\left(\frac{\sqrt[3]{dx}}{\sqrt[3]{c}}+1\right)\left(-36i\left((\sqrt{3}-i)\sqrt[3]{c}-\left(\sqrt{3}+i\right)\sqrt[3]{dx}\right)\left(\sqrt[3]{c}+\sqrt[3]{dx}\right)^2{}_3F_2\left(2,2,\frac{7}{3};1,\right.\right.$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^3)^(-4/3), x]

[Out]  $((I/40)*\text{Sqrt}[\text{Pi}/3]*((( -1)^{(2/3)}*c^{(1/3)})/d^{(1/3)}+x)*((c^{(1/3)}+(-1)^{(2/3)}*d^{(1/3)}*x)/((1+(-1)^{(1/3)})*c^{(1/3)}))^{(4/3)}*(1+(d^{(1/3)}*x)/c^{(1/3)})*\text{Gamma}[1/3]*(48*(4*c+2*(2-I*\text{Sqrt}[3])*c^{(2/3)}*d^{(1/3)}*x+2*(3+I*\text{Sqrt}[3])*c^{(1/3)}*d^{(2/3)}*x^2+3*(1+I*\text{Sqrt}[3])*d*x^3)*\text{Hypergeometric2F1}[1,4/3,8/3,(6*((1+I*\text{Sqrt}[3])*c^{(1/3)}+(1-I*\text{Sqrt}[3])*d^{(1/3)}*x))/((3*I+\text{Sqrt}[3])*((3*I+\text{Sqrt}[3])*c^{(1/3)}-2*\text{Sqrt}[3]*d^{(1/3)}*x))]- (12*I)*(c^{(1/3)}+d^{(1/3)}*x)*((-3*I+7*\text{Sqrt}[3])*c^{(2/3)}+2*(-9*I+2*\text{Sqrt}[3])*c^{(1/3)}*d^{(1/3)}*x-9*(I+\text{Sqrt}[3])*d^{(2/3)}*x^2)*\text{Hypergeometric2F1}[2,7/3,11/3,(6*((1+I*\text{Sqrt}[3])*c^{(1/3)}+(1-I*\text{Sqrt}[3])*d^{(1/3)}*x))/((3*I+\text{Sqrt}[3])*((3*I+\text{Sqrt}[3])*c^{(1/3)}-2*\text{Sqrt}[3]*d^{(1/3)}*x))]- (36*I)*(c^{(1/3)}+d^{(1/3)}*x)^2*(-I+\text{Sqrt}[3])*c^{(1/3)}-(I+$

$\text{Sqrt}[3]) * d^{(1/3)} * x) * \text{HypergeometricPFQ}[\{2, 2, 7/3\}, \{1, 11/3\}, (6 * ((1 + I * \text{Sqrt}[3]) * c^{(1/3)} + (1 - I * \text{Sqrt}[3]) * d^{(1/3)} * x)) / ((3 * I + \text{Sqrt}[3]) * ((3 * I + \text{Sqrt}[3]) * c^{(1/3)} - 2 * \text{Sqrt}[3] * d^{(1/3)} * x)))] / (2^{(1/3)} * (3 * I + \text{Sqrt}[3]) * c^{(2/3)} * ((3 * I + \text{Sqrt}[3]) * c^{(1/3)} - 2 * \text{Sqrt}[3] * d^{(1/3)} * x) * (c + d * x^3)^{(4/3)} * (1 + (I * ((-1)^{(2/3)} * c^{(1/3)} + d^{(1/3)} * x)) / (\text{Sqrt}[3] * c^{(1/3)}))^{(4/3)} * \text{Gamma}[2/3] * \text{Gamma}[7/6])$

**Maple [A]** time = 0.003, size = 15, normalized size = 0.9

$$\frac{x}{c \sqrt[3]{dx^3 + c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x^3+c)^(4/3), x)

[Out] x/c/(d\*x^3+c)^(1/3)

**Maxima [A]** time = 1.40159, size = 19, normalized size = 1.19

$$\frac{x}{(dx^3 + c)^{\frac{1}{3}} c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^(-4/3), x, algorithm="maxima")

[Out] x/((d\*x^3 + c)^(1/3) \* c)

**Fricas [A]** time = 0.211339, size = 31, normalized size = 1.94

$$\frac{(dx^3 + c)^{\frac{2}{3}} x}{cdx^3 + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^(-4/3), x, algorithm="fricas")

[Out] (d\*x^3 + c)^(2/3) \* x / (c\*d\*x^3 + c^2)

**Sympy [A]** time = 1.95163, size = 29, normalized size = 1.81

$$\frac{x^{(\frac{1}{3})}}{3c^{\frac{4}{3}} \sqrt[3]{1 + \frac{dx^3}{c}}^{(\frac{4}{3})}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x\*\*3+c)\*\*(4/3), x)

[Out] x\*gamma(1/3)/(3\*c\*\*(4/3)\*(1 + d\*x\*\*3/c)\*\*(1/3)\*gamma(4/3))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx^3 + c)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3 + c)^(-4/3), x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(-4/3), x)
```

$$3.31 \quad \int \frac{1}{(a+bx^3)\sqrt[3]{c+dx^3}} dx$$

**Optimal.** Leaf size=208

$$\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}} + \sqrt[3]{a}\right)}{3a^{2/3}\sqrt[3]{bc-ad}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a-2x\sqrt[3]{bc-ad}}}{\sqrt[3]{c+dx^3}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(a^{2/3} - \frac{\sqrt[3]{ax}\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}} + \frac{x^2(bc-ad)^{2/3}}{(c+dx^3)^{2/3}}\right)}{6a^{2/3}\sqrt[3]{bc-ad}}$$

[Out]  $-(\text{ArcTan}[(a^{1/3} - (2*(b*c - a*d)^{1/3}*x)/(c + d*x^3)^{1/3})]/(\text{Sqrt}[3]*a^{1/3}))/(\text{Sqrt}[3]*a^{2/3}*(b*c - a*d)^{1/3}) + \text{Log}[a^{1/3} + ((b*c - a*d)^{1/3}*x)/(c + d*x^3)^{1/3}]/(3*a^{2/3}*(b*c - a*d)^{1/3}) - \text{Log}[a^{2/3} + ((b*c - a*d)^{2/3}*x^2)/(c + d*x^3)^{2/3} - (a^{1/3}*(b*c - a*d)^{1/3}*x)/(c + d*x^3)^{1/3}]/(6*a^{2/3}*(b*c - a*d)^{1/3})$

**Rubi [A]** time = 0.457942, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}} + \sqrt[3]{a}\right)}{3a^{2/3}\sqrt[3]{bc-ad}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a-2x\sqrt[3]{bc-ad}}}{\sqrt[3]{c+dx^3}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(a^{2/3} - \frac{\sqrt[3]{ax}\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}} + \frac{x^2(bc-ad)^{2/3}}{(c+dx^3)^{2/3}}\right)}{6a^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/((a + b*x^3)*(c + d*x^3)^{1/3}), x]$

[Out]  $-(\text{ArcTan}[(a^{1/3} - (2*(b*c - a*d)^{1/3}*x)/(c + d*x^3)^{1/3})]/(\text{Sqrt}[3]*a^{1/3}))/(\text{Sqrt}[3]*a^{2/3}*(b*c - a*d)^{1/3}) + \text{Log}[a^{1/3} + ((b*c - a*d)^{1/3}*x)/(c + d*x^3)^{1/3}]/(3*a^{2/3}*(b*c - a*d)^{1/3}) - \text{Log}[a^{2/3} + ((b*c - a*d)^{2/3}*x^2)/(c + d*x^3)^{2/3} - (a^{1/3}*(b*c - a*d)^{1/3}*x)/(c + d*x^3)^{1/3}]/(6*a^{2/3}*(b*c - a*d)^{1/3})$

**Rubi in Sympy [A]** time = 34.6177, size = 185, normalized size = 0.89

$$-\frac{\log\left(\sqrt[3]{a} - \frac{x\sqrt[3]{ad-bc}}{\sqrt[3]{c+dx^3}}\right)}{3a^{2/3}\sqrt[3]{ad-bc}} + \frac{\log\left(a^{2/3} + \frac{\sqrt[3]{ax}\sqrt[3]{ad-bc}}{\sqrt[3]{c+dx^3}} + \frac{x^2(ad-bc)^{2/3}}{(c+dx^3)^{2/3}}\right)}{6a^{2/3}\sqrt[3]{ad-bc}} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + \frac{2x\sqrt[3]{ad-bc}}{3\sqrt[3]{c+dx^3}}\right)}{\sqrt[3]{a}}\right)}{3a^{2/3}\sqrt[3]{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/(b*x^3+a)/(d*x^3+c)^{(1/3}), x)$

[Out]  $-\log(a^{1/3} - x*(a*d - b*c)^{1/3}/(c + d*x^3)^{1/3})/(3*a^{2/3}*(a*d - b*c)^{1/3}) + \log(a^{2/3} + a^{1/3}*x*(a*d - b*c)^{1/3}/(c + d*x^3)^{1/3} + x^2*(a*d - b*c)^{2/3}/(c + d*x^3)^{2/3})/(6*a^{2/3}*(a*d - b*c)^{1/3}) + \text{sqrt}(3)*\operatorname{atan}(\text{sqrt}(3)*(a^{1/3}/3 + 2*x*(a*d - b*c)^{1/3}/(3*(c + d*x^3)^{1/3}))/a^{1/3})/(3*a^{2/3}*(a*d - b*c)^{1/3})$

**Mathematica [A]** time = 0.344175, size = 168, normalized size = 0.81

$$\frac{\log\left(a^{2/3} + \frac{\sqrt[3]{ax}\sqrt[3]{ad-bc}}{\sqrt[3]{cx^3+d}} + \frac{x^2(ad-bc)^{2/3}}{(cx^3+d)^{2/3}}\right) - 2\log\left(\sqrt[3]{a} - \frac{x\sqrt[3]{ad-bc}}{\sqrt[3]{cx^3+d}}\right) + 2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{ad-bc}}{\sqrt[3]{a}\sqrt[3]{cx^3+d}}+1}{\sqrt{3}}\right)}{6a^{2/3}\sqrt[3]{ad-bc}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)\*(c + d\*x^3)^(1/3)),x]

[Out] (2\*Sqrt[3]\*ArcTan[(1 + (2\*(-(b\*c) + a\*d)^(1/3)\*x)/(a^(1/3)\*(d + c\*x^3)^(1/3)))/Sqrt[3]] - 2\*Log[a^(1/3) - ((-(b\*c) + a\*d)^(1/3)\*x)/(d + c\*x^3)^(1/3)] + Log[a^(2/3) + ((-(b\*c) + a\*d)^(2/3)\*x^2)/(d + c\*x^3)^(2/3) + (a^(1/3)\*(-(b\*c) + a\*d)^(1/3)\*x)/(d + c\*x^3)^(1/3)])/(6\*a^(2/3)\*(-(b\*c) + a\*d)^(1/3))

**Maple [F]** time = 0.061, size = 0, normalized size = 0.

$$\int \frac{1}{bx^3 + a} \frac{1}{\sqrt[3]{dx^3 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)/(d\*x^3+c)^(1/3),x)

[Out] int(1/(b\*x^3+a)/(d\*x^3+c)^(1/3),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + a)(dx^3 + c)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^3 + a)\*(d\*x^3 + c)^(1/3)),x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)\*(d\*x^3 + c)^(1/3)), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^3 + a)\*(d\*x^3 + c)^(1/3)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^3)\sqrt[3]{c + dx^3}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**3+a)/(d*x**3+c)**(1/3),x)`

[Out] `Integral(1/((a + b*x**3)*(c + d*x**3)**(1/3)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + a)(dx^3 + c)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^3 + a)*(d*x^3 + c)^(1/3)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^3 + a)*(d*x^3 + c)^(1/3)), x)`

$$3.32 \quad \int \frac{(c+dx^3)^{2/3}}{(a+bx^3)^2} dx$$

**Optimal.** Leaf size=241

$$\frac{2c \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}} + \sqrt[3]{a}\right)}{9a^{5/3}\sqrt[3]{bc-ad}} - \frac{2c \tan^{-1}\left(\frac{\sqrt[3]{a-2x\sqrt[3]{bc-ad}}}{\sqrt[3]{c+dx^3}}\right)}{3\sqrt[3]{3}a^{5/3}\sqrt[3]{bc-ad}} - \frac{c \log\left(a^{2/3} - \frac{\sqrt[3]{ax}\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}} + \frac{x^2(bc-ad)^{2/3}}{(c+dx^3)^{2/3}}\right)}{9a^{5/3}\sqrt[3]{bc-ad}} + \frac{x(c+dx^3)^{2/3}}{3a(a+bx^3)}$$

[Out] (x\*(c + d\*x^3)^(2/3))/(3\*a\*(a + b\*x^3)) - (2\*c\*ArcTan[(a^(1/3) - (2\*(b\*c - a\*d)^(1/3)\*x)/(c + d\*x^3)^(1/3))/(Sqrt[3]\*a^(1/3))])/(3\*Sqrt[3]\*a^(5/3)\*(b\*c - a\*d)^(1/3)) + (2\*c\*Log[a^(1/3) + ((b\*c - a\*d)^(1/3)\*x)/(c + d\*x^3)^(1/3)])/(9\*a^(5/3)\*(b\*c - a\*d)^(1/3)) - (c\*Log[a^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(c + d\*x^3)^(2/3) - (a^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(c + d\*x^3)^(1/3)])/(9\*a^(5/3)\*(b\*c - a\*d)^(1/3))

**Rubi [A]** time = 0.384953, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$\frac{2c \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}} + \sqrt[3]{a}\right)}{9a^{5/3}\sqrt[3]{bc-ad}} - \frac{2c \tan^{-1}\left(\frac{\sqrt[3]{a-2x\sqrt[3]{bc-ad}}}{\sqrt[3]{c+dx^3}}\right)}{3\sqrt[3]{3}a^{5/3}\sqrt[3]{bc-ad}} - \frac{c \log\left(a^{2/3} - \frac{\sqrt[3]{ax}\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}} + \frac{x^2(bc-ad)^{2/3}}{(c+dx^3)^{2/3}}\right)}{9a^{5/3}\sqrt[3]{bc-ad}} + \frac{x(c+dx^3)^{2/3}}{3a(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^(2/3)/(a + b\*x^3)^2, x]

[Out] (x\*(c + d\*x^3)^(2/3))/(3\*a\*(a + b\*x^3)) - (2\*c\*ArcTan[(a^(1/3) - (2\*(b\*c - a\*d)^(1/3)\*x)/(c + d\*x^3)^(1/3))/(Sqrt[3]\*a^(1/3))])/(3\*Sqrt[3]\*a^(5/3)\*(b\*c - a\*d)^(1/3)) + (2\*c\*Log[a^(1/3) + ((b\*c - a\*d)^(1/3)\*x)/(c + d\*x^3)^(1/3)])/(9\*a^(5/3)\*(b\*c - a\*d)^(1/3)) - (c\*Log[a^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(c + d\*x^3)^(2/3) - (a^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(c + d\*x^3)^(1/3)])/(9\*a^(5/3)\*(b\*c - a\*d)^(1/3))

**Rubi in Sympy [A]** time = 44.9982, size = 216, normalized size = 0.9

$$\frac{x(c+dx^3)^{2/3}}{3a(a+bx^3)} - \frac{2c \log\left(\sqrt[3]{a} - \frac{x\sqrt[3]{ad-bc}}{\sqrt[3]{c+dx^3}}\right)}{9a^{5/3}\sqrt[3]{ad-bc}} + \frac{c \log\left(a^{2/3} + \frac{\sqrt[3]{ax}\sqrt[3]{ad-bc}}{\sqrt[3]{c+dx^3}} + \frac{x^2(ad-bc)^{2/3}}{(c+dx^3)^{2/3}}\right)}{9a^{5/3}\sqrt[3]{ad-bc}} + \frac{2\sqrt[3]{3}c \operatorname{atan}\left(\frac{\sqrt[3]{\sqrt[3]{a} + \frac{2x\sqrt[3]{ad-bc}}{3}}}{\sqrt[3]{a}}\right)}{9a^{5/3}\sqrt[3]{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((d\*x\*\*3+c)\*\*(2/3)/(b\*x\*\*3+a)\*\*2, x)

[Out]  $x^*(c + d*x^3)^{(2/3)}/(3*a*(a + b*x^3)) - 2*c*\log(a^{(1/3)} - x*(a*d - b*c)^{(1/3)}/(c + d*x^3)^{(1/3)})/(9*a^{(5/3)}*(a*d - b*c)^{(1/3)}) + c*\log(a^{(2/3)} + a^{(1/3)}*x*(a*d - b*c)^{(1/3)}/(c + d*x^3)^{(1/3)}) + x^{2/3}*(a*d - b*c)^{(2/3)}/(c + d*x^3)^{(2/3)})/(9*a^{(5/3)}*(a*d - b*c)^{(1/3)}) + 2*sqrt(3)*c*atan(sqrt(3)*(a^{(1/3)}/3 + 2*x*(a*d - b*c)^{(1/3)}/(3*(c + d*x^3)^{(1/3)}))/a^{(1/3)})/(9*a^{(5/3)}*(a*d - b*c)^{(1/3)})$

**Mathematica [A]** time = 0.5817, size = 198, normalized size = 0.82

$$\frac{c \left( \log \left( a^{2/3} + \frac{\sqrt[3]{ax^3 \sqrt{ad-bc}}}{\sqrt[3]{cx^3+d}} + \frac{x^2(ad-bc)^{2/3}}{(cx^3+d)^{2/3}} \right) - 2 \log \left( \sqrt[3]{a} - \frac{x \sqrt[3]{ad-bc}}{\sqrt[3]{cx^3+d}} \right) + 2\sqrt{3} \tan^{-1} \left( \frac{\frac{2x \sqrt[3]{ad-bc} + 1}{\sqrt[3]{a} \sqrt[3]{cx^3+d}}}{\sqrt{3}} \right) \right)}{9a^{5/3} \sqrt[3]{ad-bc}} + \frac{x(c+dx^3)^{2/3}}{3a(a+bx^3)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^3)^(2/3)/(a + b\*x^3)^2, x]

[Out]  $(x^*(c + d*x^3)^{(2/3)})/(3*a*(a + b*x^3)) + (c*(2*sqrt(3)*ArcTan[(1 + (2*(-b*c) + a*d)^{(1/3)}*x)/(a^{(1/3)}*(d + c*x^3)^{(1/3)})]/sqrt(3)] - 2*Log[a^{(1/3)} - ((-b*c) + a*d)^{(1/3)}*x/(d + c*x^3)^{(1/3)}] + Log[a^{(2/3)} + ((-b*c) + a*d)^{(2/3)}*x^2/(d + c*x^3)^{(2/3)} + (a^{(1/3)}*(-b*c) + a*d)^{(1/3)}*x/(d + c*x^3)^{(1/3)}])/(9*a^{(5/3)}*(-b*c) + a*d)^{(1/3)})$

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + a)^2} (dx^3 + c)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^(2/3)/(b\*x^3+a)^2, x)

[Out] int((d\*x^3+c)^(2/3)/(b\*x^3+a)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3 + c)^{\frac{2}{3}}}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^(2/3)/(b\*x^3 + a)^2, x, algorithm="maxima")

[Out] integrate((d\*x^3 + c)^(2/3)/(b\*x^3 + a)^2, x)

**Ericas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^(2/3)/(b\*x^3 + a)^2,x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*(2/3)/(b\*x\*\*3+a)\*\*2,x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3 + c)^{\frac{2}{3}}}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^(2/3)/(b\*x^3 + a)^2,x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^(2/3)/(b\*x^3 + a)^2, x)

$$3.33 \quad \int \frac{(c+dx^3)^{5/3}}{(a+bx^3)^3} dx$$

**Optimal.** Leaf size=276

$$\frac{5c^2 \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}} + \sqrt[3]{a}\right)}{27a^{8/3}\sqrt[3]{bc-ad}} - \frac{5c^2 \tan^{-1}\left(\frac{\sqrt[3]{a-2x\sqrt[3]{bc-ad}}}{\sqrt[3]{c+dx^3}}\right)}{9\sqrt[3]{3}a^{8/3}\sqrt[3]{bc-ad}} - \frac{5c^2 \log\left(a^{2/3} - \frac{\sqrt[3]{ax}\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}} + \frac{x^2(bc-ad)^{2/3}}{(c+dx^3)^{2/3}}\right)}{54a^{8/3}\sqrt[3]{bc-ad}} + \frac{5cx(c+dx^3)^{2/3}}{18a^2(a+bx^3)} + \frac{x(c+dx^3)^{5/3}}{6a(a+bx^3)^2}$$

[Out]  $(5*c*x*(c+d*x^3)^(2/3))/(18*a^2*(a+b*x^3)) + (x*(c+d*x^3)^(5/3))/(6*a*(a+b*x^3)^2) - (5*c^2*ArcTan[(a^(1/3) - (2*(b*c - a*d)^(1/3)*x)/(c+d*x^3)^(1/3))]/(9*sqrt[3]*a^(8/3)*(b*c - a*d)^(1/3)) + (5*c^2*Log[a^(1/3) + ((b*c - a*d)^(1/3)*x)/(c+d*x^3)^(1/3)])/ (27*a^(8/3)*(b*c - a*d)^(1/3)) - (5*c^2*Log[a^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(c+d*x^3)^(2/3) - (a^(1/3)*(b*c - a*d)^(1/3)*x)/(c+d*x^3)^(1/3)])/ (54*a^(8/3)*(b*c - a*d)^(1/3))$

**Rubi [A]** time = 0.457373, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$\frac{5c^2 \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}} + \sqrt[3]{a}\right)}{27a^{8/3}\sqrt[3]{bc-ad}} - \frac{5c^2 \tan^{-1}\left(\frac{\sqrt[3]{a-2x\sqrt[3]{bc-ad}}}{\sqrt[3]{c+dx^3}}\right)}{9\sqrt[3]{3}a^{8/3}\sqrt[3]{bc-ad}} - \frac{5c^2 \log\left(a^{2/3} - \frac{\sqrt[3]{ax}\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}} + \frac{x^2(bc-ad)^{2/3}}{(c+dx^3)^{2/3}}\right)}{54a^{8/3}\sqrt[3]{bc-ad}} + \frac{5cx(c+dx^3)^{2/3}}{18a^2(a+bx^3)} + \frac{x(c+dx^3)^{5/3}}{6a(a+bx^3)^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^(5/3)/(a + b\*x^3)^3, x]

[Out]  $(5*c*x*(c+d*x^3)^(2/3))/(18*a^2*(a+b*x^3)) + (x*(c+d*x^3)^(5/3))/(6*a*(a+b*x^3)^2) - (5*c^2*ArcTan[(a^(1/3) - (2*(b*c - a*d)^(1/3)*x)/(c+d*x^3)^(1/3))]/(9*sqrt[3]*a^(8/3)*(b*c - a*d)^(1/3)) + (5*c^2*Log[a^(1/3) + ((b*c - a*d)^(1/3)*x)/(c+d*x^3)^(1/3)])/ (27*a^(8/3)*(b*c - a*d)^(1/3)) - (5*c^2*Log[a^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(c+d*x^3)^(2/3) - (a^(1/3)*(b*c - a*d)^(1/3)*x)/(c+d*x^3)^(1/3)])/ (54*a^(8/3)*(b*c - a*d)^(1/3))$

**Rubi in Sympy [A]** time = 58.5826, size = 252, normalized size = 0.91

$$\frac{x(c+dx^3)^{5/3}}{6a(a+bx^3)^2} + \frac{5cx(c+dx^3)^{2/3}}{18a^2(a+bx^3)} - \frac{5c^2 \log\left(\sqrt[3]{a} - \frac{x\sqrt[3]{ad-bc}}{\sqrt[3]{c+dx^3}}\right)}{27a^{8/3}\sqrt[3]{ad-bc}} + \frac{5c^2 \log\left(a^{2/3} + \frac{\sqrt[3]{ax}\sqrt[3]{ad-bc}}{\sqrt[3]{c+dx^3}} + \frac{x^2(ad-bc)^{2/3}}{(c+dx^3)^{2/3}}\right)}{54a^{8/3}\sqrt[3]{ad-bc}} + \frac{5\sqrt[3]{3}c^2 \operatorname{atan}\left(\frac{\sqrt[3]{\sqrt[3]{a} + \frac{2x\sqrt[3]{ad-bc}}{\sqrt[3]{c+dx^3}}}}{\sqrt[3]{a}}\right)}{27a^{8/3}\sqrt[3]{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((d*x**3+c)**(5/3)/(b*x**3+a)**3,x)`

[Out]  $x*(c + d*x**3)**(5/3)/(6*a*(a + b*x**3)**2) + 5*c*x*(c + d*x**3)**(2/3)/(18*a**2*(a + b*x**3)) - 5*c**2*\log(a**(1/3) - x*(a*d - b*c)**(1/3)/(c + d*x**3)**(1/3))/(27*a**(8/3)*(a*d - b*c)**(1/3)) + 5*c**2*\log(a**(2/3) + a**(1/3)*x*(a*d - b*c)**(1/3)/(c + d*x**3)**(1/3) + x**2*(a*d - b*c)**(2/3)/(c + d*x**3)**(2/3))/(54*a**(8/3)*(a*d - b*c)**(1/3)) + 5*\sqrt{3}*c**2*\operatorname{atan}(\sqrt{3}*a**(1/3)/3 + 2*x*(a*d - b*c)**(1/3)/(3*(c + d*x**3)**(1/3)))/a**(1/3)/(27*a**(8/3)*(a*d - b*c)**(1/3))$

**Mathematica [A]** time = 0.727875, size = 219, normalized size = 0.79

$$5c^2 \left( \log \left( a^{2/3} + \frac{\sqrt[3]{ax} \sqrt[3]{ad-bc}}{\sqrt[3]{cx^3+d}} + \frac{x^2(ad-bc)^{2/3}}{(cx^3+d)^{2/3}} \right) - 2 \log \left( \sqrt[3]{a} - \frac{x \sqrt[3]{ad-bc}}{\sqrt[3]{cx^3+d}} \right) + 2\sqrt{3} \tan^{-1} \left( \frac{\frac{2x \sqrt[3]{ad-bc} + 1}{\sqrt[3]{a} \sqrt[3]{cx^3+d}}}{\sqrt{3}} \right) \right) \\ + \frac{x(c+dx^3)^{2/3}(8ac+3adx^3+5bcx^3)}{18a^2(a+bx^3)^2}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(c + d*x^3)^(5/3)/(a + b*x^3)^3,x]`

[Out]  $(x*(c + d*x^3)^{(2/3)}*(8*a*c + 5*b*c*x^3 + 3*a*d*x^3))/(18*a^2*(a + b*x^3)^2) + (5*c^2*(2*\sqrt{3}*\operatorname{ArcTan}[(1 + (2*(-b*c) + a*d)^{(1/3)}*x)/(a^{(1/3)}*(d + c*x^3)^{(1/3)})]/\sqrt{3}] - 2*\operatorname{Log}[a^{(1/3)} - ((-b*c) + a*d)^{(1/3)}*x]/(d + c*x^3)^{(1/3)}] + \operatorname{Log}[a^{(2/3)} + ((-b*c) + a*d)^{(2/3)}*x^2]/(d + c*x^3)^{(2/3)} + (a^{(1/3)}*(-b*c) + a*d)^{(1/3)}*x]/(d + c*x^3)^{(1/3)})]/(54*a^{(8/3)}*(-b*c) + a*d)^{(1/3))$

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + a)^3} (dx^3 + c)^{\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)^(5/3)/(b*x^3+a)^3,x)`

[Out] `int((d*x^3+c)^(5/3)/(b*x^3+a)^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3 + c)^{\frac{5}{3}}}{(bx^3 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3 + c)^(5/3)/(b*x^3 + a)^3,x, algorithm="maxima")`

[Out] `integrate((d*x^3 + c)^(5/3)/(b*x^3 + a)^3, x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3 + c)^(5/3)/(b*x^3 + a)^3,x, algorithm="fricas")`

[Out] Timed out

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)**(5/3)/(b*x**3+a)**3,x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3 + c)^{\frac{5}{3}}}{(bx^3 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3 + c)^(5/3)/(b*x^3 + a)^3,x, algorithm="giac")`

[Out] `integrate((d*x^3 + c)^(5/3)/(b*x^3 + a)^3, x)`

### 3.34 $\int (a + bx^3)^m (c + dx^3)^p dx$

**Optimal.** Leaf size=79

$$x (a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} (c + dx^3)^p \left(\frac{dx^3}{c} + 1\right)^{-p} F_1\left(\frac{1}{3}; -m, -p; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)$$

[Out]  $(x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -((d*x^3)/c)]/((1 + (b*x^3)/a)^m*(1 + (d*x^3)/c)^p)$

**Rubi [A]** time = 0.106559, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$x (a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} (c + dx^3)^p \left(\frac{dx^3}{c} + 1\right)^{-p} F_1\left(\frac{1}{3}; -m, -p; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^m\*(c + d\*x^3)^p,x]

[Out]  $(x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -((d*x^3)/c)]/((1 + (b*x^3)/a)^m*(1 + (d*x^3)/c)^p)$

**Rubi in Sympy [A]** time = 21.1481, size = 61, normalized size = 0.77

$$x \left(1 + \frac{bx^3}{a}\right)^{-m} \left(1 + \frac{dx^3}{c}\right)^{-p} (a + bx^3)^m (c + dx^3)^p \text{appellf1}\left(\frac{1}{3}, -m, -p, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*\*m\*(d\*x\*\*3+c)\*\*p,x)

[Out]  $x*(1 + b*x**3/a)**(-m)*(1 + d*x**3/c)**(-p)*(a + b*x**3)**m*(c + d*x**3)**p*appellf1(1/3, -m, -p, 4/3, -b*x**3/a, -d*x**3/c)$

**Mathematica [B]** time = 0.355429, size = 172, normalized size = 2.18

$$\frac{4acx (a + bx^3)^m (c + dx^3)^p F_1\left(\frac{1}{3}; -m, -p; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{3x^3 \left( bcmF_1\left(\frac{4}{3}; 1 - m, -p; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) + adpF_1\left(\frac{4}{3}; -m, 1 - p; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) \right) + 4acF_1\left(\frac{1}{3}; -m, -p; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^m\*(c + d\*x^3)^p,x]

[Out]  $(4*a*c*x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -((d*x^3)/c)]/(4*a*c*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, -p, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + a*d*p*AppellF1[4/3, -m, 1 - p, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))$

**Maple [F]** time = 0.114, size = 0, normalized size = 0.

$$\int (bx^3 + a)^m (dx^3 + c)^p dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^m*(d*x^3+c)^p,x)`

[Out] `int((b*x^3+a)^m*(d*x^3+c)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + a)^m (dx^3 + c)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^m*(d*x^3 + c)^p,x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a)^m*(d*x^3 + c)^p, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^3 + a)^m (dx^3 + c)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^m*(d*x^3 + c)^p,x, algorithm="fricas")`

[Out] `integral((b*x^3 + a)^m*(d*x^3 + c)^p, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**m*(d*x**3+c)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + a)^m (dx^3 + c)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^m*(d*x^3 + c)^p,x, algorithm="giac")`

[Out] `integrate((b*x^3 + a)^m*(d*x^3 + c)^p, x)`

### 3.35 $\int (a + bx^3)^2 (c + dx^3)^q dx$

**Optimal.** Leaf size=167

$$\frac{x(c + dx^3)^{q+1} (a^2 d^2 (9q^2 + 33q + 28) - 2abcd(3q + 7) + 4b^2 c^2) {}_2F_1\left(1, q + \frac{4}{3}; \frac{4}{3}; -\frac{dx^3}{c}\right)}{cd^2(3q + 4)(3q + 7)} - \frac{bx(c + dx^3)^{q+1} (4bc - ad(3q + 10))}{d^2(3q + 4)(3q + 7)} + \frac{bx(a + bx^3)(c + dx^3)^{q+1}}{d(3q + 7)}$$

[Out]  $-\left(\left(b^*(4*b*c - a*d*(10 + 3*q))*x*(c + d*x^3)^{(1 + q)}\right)/\left(d^{2*(4 + 3*q)}*(7 + 3*q)\right)\right) + \left(b*x*(a + b*x^3)*(c + d*x^3)^{(1 + q)}\right)/\left(d*(7 + 3*q)\right) + \left(\left(4*b^2*c^2 - 2*a*b*c*d*(7 + 3*q) + a^2*d^2*(28 + 33*q + 9*q^2)\right)*x*(c + d*x^3)^{(1 + q)}*\text{Hypergeometric2F1}\left[1, 4/3 + q, 4/3, -\left(d*x^3/c\right)\right]\right)/\left(c*d^{2*(4 + 3*q)}*(7 + 3*q)\right)$

**Rubi [A]** time = 0.278101, antiderivative size = 176, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{x(c + dx^3)^q \left(\frac{dx^3}{c} + 1\right)^{-q} (a^2 d^2 (9q^2 + 33q + 28) - 2abcd(3q + 7) + 4b^2 c^2) {}_2F_1\left(\frac{1}{3}, -q; \frac{4}{3}; -\frac{dx^3}{c}\right)}{d^2(3q + 4)(3q + 7)} - \frac{bx(c + dx^3)^{q+1} (4bc - ad(3q + 10))}{d^2(3q + 4)(3q + 7)} + \frac{bx(a + bx^3)(c + dx^3)^{q+1}}{d(3q + 7)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^2\*(c + d\*x^3)^q, x]

[Out]  $-\left(\left(b^*(4*b*c - a*d*(10 + 3*q))*x*(c + d*x^3)^{(1 + q)}\right)/\left(d^{2*(4 + 3*q)}*(7 + 3*q)\right)\right) + \left(b*x*(a + b*x^3)*(c + d*x^3)^{(1 + q)}\right)/\left(d*(7 + 3*q)\right) + \left(\left(4*b^2*c^2 - 2*a*b*c*d*(7 + 3*q) + a^2*d^2*(28 + 33*q + 9*q^2)\right)*x*(c + d*x^3)^q*\text{Hypergeometric2F1}\left[1/3, -q, 4/3, -\left(d*x^3/c\right)\right]\right)/\left(d^{2*(4 + 3*q)}*(7 + 3*q)*(1 + (d*x^3)/c)^q\right)$

**Rubi in Sympy [A]** time = 33.0944, size = 153, normalized size = 0.92

$$\frac{bx(a + bx^3)(c + dx^3)^{q+1}}{d(3q + 7)} - \frac{bx(c + dx^3)^{q+1}(-ad(3q + 10) + 4bc)}{d^2(3q + 4)(3q + 7)} + \frac{x\left(1 + \frac{dx^3}{c}\right)^{-q}(c + dx^3)^q(-ad(3q + 4)(-ad(3q + 7) + bc) + bc(-ad(3q + 10) + 4bc)) {}_2F_1\left(-q, \frac{1}{3}; \frac{4}{3}; -\frac{dx^3}{c}\right)}{d^2(3q + 4)(3q + 7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*\*2\*(d\*x\*\*3+c)\*\*q, x)

[Out]  $b*x*(a + b*x^3)*(c + d*x^3)**(q + 1)/(d*(3*q + 7)) - b*x*(c + d*x^3)**(q + 1)*(-a*d*(3*q + 10) + 4*b*c)/(d**2*(3*q + 4)*(3*q + 7)) + x*(1 + d*x^3/c)**(-q)*(c + d*x^3)**q*(-a*d*(3*q + 4)*(-a*d*(3*q + 7) + b*c) + b*c*(-a*d*(3*q + 10) + 4*b*c))*\text{hyper}((-q, 1/3), (4/3, ), -d*x^3/c)/(d**2*(3*q + 4)*(3*q + 7))$

**Mathematica [A]** time = 0.0799186, size = 106, normalized size = 0.63

$$\frac{1}{14}x(c + dx^3)^q \left(\frac{dx^3}{c} + 1\right)^{-q} \left(14a^2 {}_2F_1\left(\frac{1}{3}, -q; \frac{4}{3}; -\frac{dx^3}{c}\right) + bx^3 \left(7a {}_2F_1\left(\frac{4}{3}, -q; \frac{7}{3}; -\frac{dx^3}{c}\right) + 2bx^3 {}_2F_1\left(\frac{7}{3}, -q; \frac{10}{3}; -\frac{dx^3}{c}\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^2\*(c + d\*x^3)^q,x]

[Out] (x\*(c + d\*x^3)^q\*(14\*a^2\*Hypergeometric2F1[1/3, -q, 4/3, -((d\*x^3)/c)] + b\*x^3\*(7\*a\*Hypergeometric2F1[4/3, -q, 7/3, -((d\*x^3)/c)] + 2\*b\*x^3\*Hypergeometric2F1[7/3, -q, 10/3, -((d\*x^3)/c)]))/((14\*(1 + (d\*x^3)/c)^q)

**Maple [F]** time = 0.082, size = 0, normalized size = 0.

$$\int (bx^3 + a)^2 (dx^3 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^2\*(d\*x^3+c)^q,x)

[Out] int((b\*x^3+a)^2\*(d\*x^3+c)^q,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + a)^2 (dx^3 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^2\*(d\*x^3 + c)^q,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^2\*(d\*x^3 + c)^q, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^6 + 2abx^3 + a^2\right)(dx^3 + c)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^2\*(d\*x^3 + c)^q,x, algorithm="fricas")

[Out] integral((b^2\*x^6 + 2\*a\*b\*x^3 + a^2)\*(d\*x^3 + c)^q, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*2\*(d\*x\*\*3+c)\*\*q,x)

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + a)^2 (dx^3 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3 + a)^2*(d*x^3 + c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^2*(d*x^3 + c)^q, x)
```

### 3.36 $\int (a + bx^3) (c + dx^3)^q dx$

**Optimal.** Leaf size=84

$$\frac{bx(c+dx^3)^{q+1}}{d(3q+4)} - \frac{x(c+dx^3)^{q+1}(bc-ad(3q+4)) {}_2F_1\left(1, q + \frac{4}{3}; \frac{4}{3}; -\frac{dx^3}{c}\right)}{cd(3q+4)}$$

[Out] (b\*x\*(c + d\*x^3)^(1 + q))/(d\*(4 + 3\*q)) - ((b\*c - a\*d\*(4 + 3\*q))\*x\*(c + d\*x^3)^(1 + q)\*Hypergeometric2F1[1, 4/3 + q, 4/3, -((d\*x^3)/c)]/(c\*d\*(4 + 3\*q)))

**Rubi [A]** time = 0.0984101, antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$x(c+dx^3)^q \left(\frac{dx^3}{c} + 1\right)^{-q} \left(a - \frac{bc}{3dq+4d}\right) {}_2F_1\left(\frac{1}{3}, -q; \frac{4}{3}; -\frac{dx^3}{c}\right) + \frac{bx(c+dx^3)^{q+1}}{d(3q+4)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)\*(c + d\*x^3)^q, x]

[Out] (b\*x\*(c + d\*x^3)^(1 + q))/(d\*(4 + 3\*q)) + ((a - (b\*c)/(4\*d + 3\*d\*q))\*x\*(c + d\*x^3)^q\*Hypergeometric2F1[1/3, -q, 4/3, -((d\*x^3)/c)])/(1 + (d\*x^3)/c)^q

**Rubi in Sympy [A]** time = 11.049, size = 73, normalized size = 0.87

$$\frac{bx(c+dx^3)^{q+1}}{d(3q+4)} - \frac{x\left(1 + \frac{dx^3}{c}\right)^{-q}(c+dx^3)^q(-ad(3q+4) + bc) {}_2F_1\left(-q, \frac{1}{3}; \frac{4}{3}; -\frac{dx^3}{c}\right)}{d(3q+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*(d\*x\*\*3+c)\*\*q, x)

[Out] b\*x\*(c + d\*x\*\*3)\*\*(q + 1)/(d\*(3\*q + 4)) - x\*(1 + d\*x\*\*3/c)\*\*(-q)\*(c + d\*x\*\*3)\*\*q\*(-a\*d\*(3\*q + 4) + b\*c)\*hyper((-q, 1/3), (4/3, ), -d\*x\*\*3/c)/(d\*(3\*q + 4))

**Mathematica [A]** time = 0.0312364, size = 75, normalized size = 0.89

$$\frac{1}{4}x(c+dx^3)^q \left(\frac{dx^3}{c} + 1\right)^{-q} \left(4a {}_2F_1\left(\frac{1}{3}, -q; \frac{4}{3}; -\frac{dx^3}{c}\right) + bx^3 {}_2F_1\left(\frac{4}{3}, -q; \frac{7}{3}; -\frac{dx^3}{c}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)\*(c + d\*x^3)^q, x]

[Out] (x\*(c + d\*x^3)^q\*(4\*a\*Hypergeometric2F1[1/3, -q, 4/3, -((d\*x^3)/c)] + b\*x^3\*Hypergeometric2F1[4/3, -q, 7/3, -((d\*x^3)/c)]))/(4\*(1 + (d\*x^3)/c)^q)

**Maple [F]** time = 0.048, size = 0, normalized size = 0.

$$\int (bx^3 + a)(dx^3 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)*(d*x^3+c)^q,x)`

[Out] `int((b*x^3+a)*(d*x^3+c)^q,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + a)(dx^3 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)*(d*x^3 + c)^q,x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a)*(d*x^3 + c)^q, x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^3 + a)(dx^3 + c)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)*(d*x^3 + c)^q,x, algorithm="fricas")`

[Out] `integral((b*x^3 + a)*(d*x^3 + c)^q, x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)*(d*x**3+c)**q,x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + a)(dx^3 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)*(d*x^3 + c)^q,x, algorithm="giac")`

[Out] `integrate((b*x^3 + a)*(d*x^3 + c)^q, x)`

$$3.37 \quad \int \frac{(c+dx^3)^q}{a+bx^3} dx$$

**Optimal.** Leaf size=57

$$\frac{x (c + dx^3)^q \left(\frac{dx^3}{c} + 1\right)^{-q} F_1\left(\frac{1}{3}; 1, -q; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{a}$$

[Out] (x\*(c + d\*x^3)^q\*AppellF1[1/3, 1, -q, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(a\*(1 + (d\*x^3)/c)^q)

**Rubi [A]** time = 0.0747915, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{x (c + dx^3)^q \left(\frac{dx^3}{c} + 1\right)^{-q} F_1\left(\frac{1}{3}; 1, -q; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^q/(a + b\*x^3), x]

[Out] (x\*(c + d\*x^3)^q\*AppellF1[1/3, 1, -q, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(a\*(1 + (d\*x^3)/c)^q)

**Rubi in Sympy [A]** time = 21.6503, size = 42, normalized size = 0.74

$$\frac{x \left(1 + \frac{dx^3}{c}\right)^{-q} (c + dx^3)^q \text{appellf1}\left(\frac{1}{3}, 1, -q, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((d\*x\*\*3+c)\*\*q/(b\*x\*\*3+a), x)

[Out] x\*(1 + d\*x\*\*3/c)\*\*(-q)\*(c + d\*x\*\*3)\*\*q\*appellf1(1/3, 1, -q, 4/3, -b\*x\*\*3/a, -d\*x\*\*3/c)/a

**Mathematica [B]** time = 0.284151, size = 162, normalized size = 2.84

$$\frac{4acx (c + dx^3)^q F_1\left(\frac{1}{3}; -q, 1; \frac{4}{3}; -\frac{dx^3}{c}, -\frac{bx^3}{a}\right)}{(a + bx^3) \left(3x^3 \left(adqF_1\left(\frac{4}{3}; 1 - q, 1; \frac{7}{3}; -\frac{dx^3}{c}, -\frac{bx^3}{a}\right) - bcF_1\left(\frac{4}{3}; -q, 2; \frac{7}{3}; -\frac{dx^3}{c}, -\frac{bx^3}{a}\right)\right) + 4acF_1\left(\frac{1}{3}; -q, 1; \frac{4}{3}; -\frac{dx^3}{c}, -\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^3)^q/(a + b\*x^3), x]

[Out] (4\*a\*c\*x\*(c + d\*x^3)^q\*AppellF1[1/3, -q, 1, 4/3, -((d\*x^3)/c), -((b\*x^3)/a)]/((a + b\*x^3)\*(4\*a\*c\*AppellF1[1/3, -q, 1, 4/3, -((d\*x^3)/c), -((b\*x^3)/a)] + 3\*x^3\*(a\*d\*q\*AppellF1[4/3, 1 - q, 1, 7/3, -((d\*x^3)/c), -((b\*x^3)/a)] - b\*c\*AppellF1[4/3, -q, 2, 7/3, -((d\*x^3)/c), -((b\*x^3)/a)]))

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int \frac{(dx^3 + c)^q}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)^q/(b*x^3+a),x)`

[Out] `int((d*x^3+c)^q/(b*x^3+a),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3 + c)^q}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3 + c)^q/(b*x^3 + a),x, algorithm="maxima")`

[Out] `integrate((d*x^3 + c)^q/(b*x^3 + a), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^3 + c)^q}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3 + c)^q/(b*x^3 + a),x, algorithm="fricas")`

[Out] `integral((d*x^3 + c)^q/(b*x^3 + a), x)`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)**q/(b*x**3+a),x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3 + c)^q}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3 + c)^q/(b*x^3 + a),x, algorithm="giac")`

[Out] `integrate((d*x^3 + c)^q/(b*x^3 + a), x)`



$$3.38 \quad \int \frac{(c+dx^3)^q}{(a+bx^3)^2} dx$$

**Optimal.** Leaf size=57

$$\frac{x(c+dx^3)^q \left(\frac{dx^3}{c} + 1\right)^{-q} F_1\left(\frac{1}{3}; 2, -q; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{a^2}$$

[Out]  $(x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a^2*(1 + (d*x^3)/c)^q)$

**Rubi [A]** time = 0.0734547, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{x(c+dx^3)^q \left(\frac{dx^3}{c} + 1\right)^{-q} F_1\left(\frac{1}{3}; 2, -q; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^q/(a + b\*x^3)^2, x]

[Out]  $(x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a^2*(1 + (d*x^3)/c)^q)$

**Rubi in Sympy [A]** time = 20.4988, size = 44, normalized size = 0.77

$$\frac{x \left(1 + \frac{dx^3}{c}\right)^{-q} (c + dx^3)^q \text{appellf1}\left(\frac{1}{3}, 2, -q, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((d\*x\*\*3+c)\*\*q/(b\*x\*\*3+a)\*\*2, x)

[Out]  $x*(1 + d*x**3/c)**(-q)*(c + d*x**3)**q*appellf1(1/3, 2, -q, 4/3, -b*x**3/a, -d*x**3/c)/a**2$

**Mathematica [B]** time = 0.296378, size = 162, normalized size = 2.84

$$\frac{4acx(c+dx^3)^q F_1\left(\frac{1}{3}; 2, -q; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{(a+bx^3)^2 \left(3x^3 \left(adqF_1\left(\frac{4}{3}; 2, 1-q; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) - 2bcF_1\left(\frac{4}{3}; 3, -q; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)\right) + 4acF_1\left(\frac{1}{3}; 2, -q; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^3)^q/(a + b\*x^3)^2, x]

[Out]  $(4*a*c*x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)]/((a + b*x^3)^2*(4*a*c*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + 3*x^3*(a*d*q*AppellF1[4/3, 2, 1 - q, 7/3, -((b*x^3)/a), -((d*x^3)/c)] - 2*b*c*AppellF1[4/3, 3, -q, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))$

**Maple [F]** time = 0.097, size = 0, normalized size = 0.

$$\int \frac{(dx^3 + c)^q}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^q/(b\*x^3+a)^2, x)

[Out] int((d\*x^3+c)^q/(b\*x^3+a)^2, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3 + c)^q}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^q/(b\*x^3 + a)^2, x, algorithm="maxima")

[Out] integrate((d\*x^3 + c)^q/(b\*x^3 + a)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^3 + c)^q}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^q/(b\*x^3 + a)^2, x, algorithm="fricas")

[Out] integral((d\*x^3 + c)^q/(b^2\*x^6 + 2\*a\*b\*x^3 + a^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*q/(b\*x\*\*3+a)\*\*2, x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3 + c)^q}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^q/(b\*x^3 + a)^2, x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^q/(b\*x^3 + a)^2, x)

### 3.39 $\int (a + bx^3)^m (c + dx^3)^3 dx$

**Optimal.** Leaf size=298

$$\frac{dx (a + bx^3)^{m+1} (28a^2d^2 - abcd(15m + 92) + b^2c^2 (9m^2 + 60m + 118))}{b^3(3m + 4)(3m + 7)(3m + 10)}$$

$$\frac{x (a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} (28a^3d^3 - 12a^2bcd^2(3m + 10) + 3ab^2c^2d (9m^2 + 51m + 70) - b^3c^3 (27m^3 + 189m^2 + 414m + 28))}{b^3(3m + 4)(3m + 7)(3m + 10)}$$

$$-\frac{dx (c + dx^3) (a + bx^3)^{m+1} (7ad - bc(3m + 16))}{b^2(3m + 7)(3m + 10)} + \frac{dx (c + dx^3)^2 (a + bx^3)^{m+1}}{b(3m + 10)}$$

[Out]  $(d*(28*a^2*d^2 - a*b*c*d*(92 + 15*m) + b^2*c^2*(118 + 60*m + 9*m^2)) * x*(a + b*x^3)^(1 + m))/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)) - (d*(7*a*d - b*c*(16 + 3*m)) * x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b^2*(7 + 3*m)*(10 + 3*m)) + (d*x*(a + b*x^3)^(1 + m)*(c + d*x^3)^2)/(b*(10 + 3*m)) - ((28*a^3*d^3 - 12*a^2*b*c*d^2*(10 + 3*m) + 3*a*b^2*c^2*d*(70 + 51*m + 9*m^2) - b^3*c^3*(280 + 414*m + 189*m^2 + 27*m^3)) * x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -(b*x^3)/a])/ (b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)*(1 + (b*x^3)/a)^m)$

**Rubi [A]** time = 0.689545, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{dx (a + bx^3)^{m+1} (28a^2d^2 - abcd(15m + 92) + b^2c^2 (9m^2 + 60m + 118))}{b^3(3m + 4)(3m + 7)(3m + 10)}$$

$$\frac{x (a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} (28a^3d^3 - 12a^2bcd^2(3m + 10) + 3ab^2c^2d (9m^2 + 51m + 70) - b^3c^3 (27m^3 + 189m^2 + 414m + 28))}{b^3(3m + 4)(3m + 7)(3m + 10)}$$

$$-\frac{dx (c + dx^3) (a + bx^3)^{m+1} (7ad - bc(3m + 16))}{b^2(3m + 7)(3m + 10)} + \frac{dx (c + dx^3)^2 (a + bx^3)^{m+1}}{b(3m + 10)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^m\*(c + d\*x^3)^3, x]

[Out]  $(d*(28*a^2*d^2 - a*b*c*d*(92 + 15*m) + b^2*c^2*(118 + 60*m + 9*m^2)) * x*(a + b*x^3)^(1 + m))/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)) - (d*(7*a*d - b*c*(16 + 3*m)) * x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b^2*(7 + 3*m)*(10 + 3*m)) + (d*x*(a + b*x^3)^(1 + m)*(c + d*x^3)^2)/(b*(10 + 3*m)) - ((28*a^3*d^3 - 12*a^2*b*c*d^2*(10 + 3*m) + 3*a*b^2*c^2*d*(70 + 51*m + 9*m^2) - b^3*c^3*(280 + 414*m + 189*m^2 + 27*m^3)) * x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -(b*x^3)/a])/ (b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)*(1 + (b*x^3)/a)^m)$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*\*m\*(d\*x\*\*3+c)\*\*3, x)

[Out] Timed out

**Mathematica [A]** time = 0.103983, size = 137, normalized size = 0.46

$$\frac{1}{140}x(a+bx^3)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(140c^3{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)+dx^3\left(105c^2{}_2F_1\left(\frac{4}{3},-m;\frac{7}{3};-\frac{bx^3}{a}\right)+2dx^3\left(30c{}_2F_1\left(\frac{7}{3},-m;\frac{10}{3};-\frac{bx^3}{a}\right)+7dx^3{}_2F_1\left(\frac{10}{3},-m;\frac{13}{3};-\frac{bx^3}{a}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^m\*(c + d\*x^3)^3,x]

[Out] (x\*(a + b\*x^3)^m\*(140\*c^3\*Hypergeometric2F1[1/3, -m, 4/3, -((b\*x^3)/a)] + d\*x^3\*(105\*c^2\*Hypergeometric2F1[4/3, -m, 7/3, -((b\*x^3)/a)] + 2\*d\*x^3\*(30\*c\*Hypergeometric2F1[7/3, -m, 10/3, -((b\*x^3)/a)] + 7\*d\*x^3\*Hypergeometric2F1[10/3, -m, 13/3, -((b\*x^3)/a)])))/(140\*(1 + (b\*x^3)/a)^m)

**Maple [F]** time = 0.07, size = 0, normalized size = 0.

$$\int (bx^3 + a)^m (dx^3 + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^m\*(d\*x^3+c)^3,x)

[Out] int((b\*x^3+a)^m\*(d\*x^3+c)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^3 + c)^3 (bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^3\*(b\*x^3 + a)^m,x, algorithm="maxima")

[Out] integrate((d\*x^3 + c)^3\*(b\*x^3 + a)^m, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^3x^9 + 3cd^2x^6 + 3c^2dx^3 + c^3\right)(bx^3 + a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^3\*(b\*x^3 + a)^m,x, algorithm="fricas")

[Out] integral((d^3\*x^9 + 3\*c\*d^2\*x^6 + 3\*c^2\*d\*x^3 + c^3)\*(b\*x^3 + a)^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**m*(d*x**3+c)**3,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^3 + c)^3 (bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3 + c)^3*(b*x^3 + a)^m,x, algorithm="giac")`

[Out] `integrate((d*x^3 + c)^3*(b*x^3 + a)^m, x)`

### 3.40 $\int (a + bx^3)^m (c + dx^3)^2 dx$

**Optimal.** Leaf size=176

$$\frac{x(a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} (4a^2d^2 - 2abcd(3m + 7) + b^2c^2(9m^2 + 33m + 28)) {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right)}{b^2(3m + 4)(3m + 7)} - \frac{dx(a + bx^3)^{m+1}(4ad - bc(3m + 10))}{b^2(3m + 4)(3m + 7)} + \frac{dx(c + dx^3)(a + bx^3)^{m+1}}{b(3m + 7)}$$

[Out]  $-\left(\left(d(4a^2d - b^2c(10 + 3m))x^3(a + b^2x^3)^{(1+m)}\right)/(b^2(4 + 3m)^7(7 + 3m))\right) + \left(d^2x^3(a + b^2x^3)^{(1+m)}(c + d^2x^3)\right)/(b^2(7 + 3m)^7) + \left(\left(4a^2d^2 - 2abcd(7 + 3m) + b^2c^2(28 + 33m + 9m^2)\right)x^3(a + b^2x^3)^m \text{Hypergeometric2F1}\left[\frac{1}{3}, -m, \frac{4}{3}, -\left(\frac{bx^3}{a}\right)\right]\right)/(b^2(4 + 3m)^7(7 + 3m)(1 + (b^2x^3)/a)^m)$

**Rubi [A]** time = 0.28185, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{x(a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} (4a^2d^2 - 2abcd(3m + 7) + b^2c^2(9m^2 + 33m + 28)) {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right)}{b^2(3m + 4)(3m + 7)} - \frac{dx(a + bx^3)^{m+1}(4ad - bc(3m + 10))}{b^2(3m + 4)(3m + 7)} + \frac{dx(c + dx^3)(a + bx^3)^{m+1}}{b(3m + 7)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^m\*(c + d\*x^3)^2, x]

[Out]  $-\left(\left(d(4a^2d - b^2c(10 + 3m))x^3(a + b^2x^3)^{(1+m)}\right)/(b^2(4 + 3m)^7(7 + 3m))\right) + \left(d^2x^3(a + b^2x^3)^{(1+m)}(c + d^2x^3)\right)/(b^2(7 + 3m)^7) + \left(\left(4a^2d^2 - 2abcd(7 + 3m) + b^2c^2(28 + 33m + 9m^2)\right)x^3(a + b^2x^3)^m \text{Hypergeometric2F1}\left[\frac{1}{3}, -m, \frac{4}{3}, -\left(\frac{bx^3}{a}\right)\right]\right)/(b^2(4 + 3m)^7(7 + 3m)(1 + (b^2x^3)/a)^m)$

**Rubi in Sympy [A]** time = 32.2166, size = 153, normalized size = 0.87

$$\frac{dx(a + bx^3)^{m+1}(c + dx^3)}{b(3m + 7)} - \frac{dx(a + bx^3)^{m+1}(4ad - bc(3m + 10))}{b^2(3m + 4)(3m + 7)} + \frac{x\left(1 + \frac{bx^3}{a}\right)^{-m}(a + bx^3)^m(ad(4ad - bc(3m + 10)) - bc(3m + 4)(ad - bc(3m + 7))) {}_2F_1\left(-m, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{b^2(3m + 4)(3m + 7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*\*m\*(d\*x\*\*3+c)\*\*2, x)

[Out]  $d^2x^3(a + b^2x^3)^{(m+1)}(c + d^2x^3)/(b^2(3m + 7)) - d^2x^3(a + b^2x^3)^{(m+1)}(4a^2d - b^2c(3m + 10))/(b^2(3m + 4)(3m + 7)) + x^3(1 + b^2x^3/a)^{-m}(a + b^2x^3)^m(a^2d(4a^2d - b^2c(3m + 10)) - b^2c(3m + 4)(a^2d - b^2c(3m + 7))) \text{hyper}((-m, 1/3), (4/3, ), -b^2x^3/a)/(b^2(3m + 4)(3m + 7))$

**Mathematica [A]** time = 0.0568683, size = 106, normalized size = 0.6

$$\frac{1}{14}x(a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} \left(14c^2 {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right) + dx^3 \left(7c {}_2F_1\left(\frac{4}{3}, -m; \frac{7}{3}; -\frac{bx^3}{a}\right) + 2dx^3 {}_2F_1\left(\frac{7}{3}, -m; \frac{10}{3}; -\frac{bx^3}{a}\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^m\*(c + d\*x^3)^2,x]

[Out] (x\*(a + b\*x^3)^m\*(14\*c^2\*Hypergeometric2F1[1/3, -m, 4/3, -((b\*x^3)/a)] + d\*x^3\*(7\*c\*Hypergeometric2F1[4/3, -m, 7/3, -((b\*x^3)/a)] + 2\*d\*x^3\*Hypergeometric2F1[7/3, -m, 10/3, -((b\*x^3)/a)]))/(14\*(1 + (b\*x^3)/a)^m)

**Maple [F]** time = 0.07, size = 0, normalized size = 0.

$$\int (bx^3 + a)^m (dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^m\*(d\*x^3+c)^2,x)

[Out] int((b\*x^3+a)^m\*(d\*x^3+c)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^3 + c)^2 (bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^2\*(b\*x^3 + a)^m,x, algorithm="maxima")

[Out] integrate((d\*x^3 + c)^2\*(b\*x^3 + a)^m, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2x^6 + 2cdx^3 + c^2\right)(bx^3 + a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3 + c)^2\*(b\*x^3 + a)^m,x, algorithm="fricas")

[Out] integral((d^2\*x^6 + 2\*c\*d\*x^3 + c^2)\*(b\*x^3 + a)^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*m\*(d\*x\*\*3+c)\*\*2,x)

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (dx^3 + c)^2 (bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3 + c)^2*(b*x^3 + a)^m,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2*(b*x^3 + a)^m, x)
```



### 3.41 $\int (a + bx^3)^m (c + dx^3) dx$

**Optimal.** Leaf size=93

$$\frac{dx (a + bx^3)^{m+1}}{b(3m+4)} - \frac{x (a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} (ad - bc(3m+4)) {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right)}{b(3m+4)}$$

[Out] (d\*x\*(a + b\*x^3)^(1 + m))/(b\*(4 + 3\*m)) - ((a\*d - b\*c\*(4 + 3\*m))\*x\*(a + b\*x^3)^m\*Hypergeometric2F1[1/3, -m, 4/3, -(b\*x^3)/a])/(b\*(4 + 3\*m)\*(1 + (b\*x^3)/a)^m)

**Rubi [A]** time = 0.099899, antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$x (a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} \left(c - \frac{ad}{3bm + 4b}\right) {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right) + \frac{dx (a + bx^3)^{m+1}}{b(3m+4)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^m\*(c + d\*x^3), x]

[Out] (d\*x\*(a + b\*x^3)^(1 + m))/(b\*(4 + 3\*m)) + ((c - (a\*d)/(4\*b + 3\*b\*m))\*x\*(a + b\*x^3)^m\*Hypergeometric2F1[1/3, -m, 4/3, -(b\*x^3)/a])/(1 + (b\*x^3)/a)^m

**Rubi in Sympy [A]** time = 10.7156, size = 73, normalized size = 0.78

$$\frac{dx (a + bx^3)^{m+1}}{b(3m+4)} - \frac{x \left(1 + \frac{bx^3}{a}\right)^{-m} (a + bx^3)^m (ad - bc(3m+4)) {}_2F_1\left(-m, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{b(3m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*\*m\*(d\*x\*\*3+c), x)

[Out] d\*x\*(a + b\*x\*\*3)\*\*(m + 1)/(b\*(3\*m + 4)) - x\*(1 + b\*x\*\*3/a)\*\*(-m)\*(a + b\*x\*\*3)\*\*m\*(a\*d - b\*c\*(3\*m + 4))\*hyper((-m, 1/3), (4/3, ), -b\*x\*\*3/a)/(b\*(3\*m + 4))

**Mathematica [A]** time = 0.0298675, size = 75, normalized size = 0.81

$$\frac{1}{4}x (a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} \left(4c {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right) + dx^3 {}_2F_1\left(\frac{4}{3}, -m; \frac{7}{3}; -\frac{bx^3}{a}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^m\*(c + d\*x^3), x]

[Out] (x\*(a + b\*x^3)^m\*(4\*c\*Hypergeometric2F1[1/3, -m, 4/3, -(b\*x^3)/a]) + d\*x^3\*Hypergeometric2F1[4/3, -m, 7/3, -(b\*x^3)/a])/(4\*(1 + (b\*x^3)/a)^m)

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int (bx^3 + a)^m (dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^m*(d*x^3+c), x)`

[Out] `int((b*x^3+a)^m*(d*x^3+c), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^3 + c)(bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3 + c)*(b*x^3 + a)^m, x, algorithm="maxima")`

[Out] `integrate((d*x^3 + c)*(b*x^3 + a)^m, x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx^3 + c)(bx^3 + a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3 + c)*(b*x^3 + a)^m, x, algorithm="fricas")`

[Out] `integral((d*x^3 + c)*(b*x^3 + a)^m, x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**m*(d*x**3+c), x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^3 + c)(bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3 + c)*(b*x^3 + a)^m, x, algorithm="giac")`

[Out] `integrate((d*x^3 + c)*(b*x^3 + a)^m, x)`

### 3.42 $\int (a + bx^3)^m dx$

**Optimal.** Leaf size=44

$$x (a + bx^3)^m \left( \frac{bx^3}{a} + 1 \right)^{-m} {}_2F_1 \left( \frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a} \right)$$

[Out]  $(x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -(b*x^3)/a])/ (1 + (b*x^3)/a)^m$

**Rubi [A]** time = 0.0267679, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$x (a + bx^3)^m \left( \frac{bx^3}{a} + 1 \right)^{-m} {}_2F_1 \left( \frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a} \right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^m, x]

[Out]  $(x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -(b*x^3)/a])/ (1 + (b*x^3)/a)^m$

**Rubi in Sympy [A]** time = 3.92162, size = 34, normalized size = 0.77

$$x \left( 1 + \frac{bx^3}{a} \right)^{-m} (a + bx^3)^m {}_2F_1 \left( -m, \frac{1}{3} \middle| -\frac{bx^3}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*\*m, x)

[Out]  $x*(1 + b*x**3/a)**(-m)*(a + b*x**3)**m*hyper((-m, 1/3), (4/3, ), -b*x**3/a)$

**Mathematica [C]** time = 0.287412, size = 196, normalized size = 4.45

$$2^{-m} \left( (-1)^{2/3} \sqrt[3]{a} + \sqrt[3]{bx} \right) \left( \frac{\sqrt[3]{a+(-1)^{2/3}\sqrt[3]{bx}}}{(1+\sqrt[3]{-1})\sqrt[3]{a}} \right)^{-m} \left( \frac{i \left( \sqrt[3]{\frac{bx}{a}+1} \right)}{\sqrt[3]{3+3i}} \right)^{-m} (a + bx^3)^m F_1 \left( m + 1; -m, -m; m + 2; -\frac{i \left( \sqrt[3]{bx+(-1)^{2/3}\sqrt[3]{a}} \right)}{\sqrt[3]{3}\sqrt[3]{a}}, \frac{-2i}{\sqrt[3]{3}} \right)$$

$\sqrt[3]{b}(m+1)$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^m, x]

[Out]  $(((-1)^{(2/3)}*a^{(1/3)} + b^{(1/3)}*x)*(a + b*x^3)^m*AppellF1[1 + m, -m, -m, 2 + m, ((-1)^{(2/3)}*a^{(1/3)} + b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)}), (I + Sqrt[3] - ((2*I)*b^{(1/3)}*x)/a^{(1/3)})/(3*I + Sqrt[3])])/(2^m*b^{(1/3)}*(1 + m)*((a^{(1/3)} + (-1)^{(2/3)}*b^{(1/3)}*x)/((1 + (-1)^{(1/3)})*a^{(1/3)}))^m*(I*(1 + (b^{(1/3)}*x)/a^{(1/3)}))/(3*I + Sqrt[3]))^m$

**Maple [F]** time = 0.035, size = 0, normalized size = 0.

$$\int (bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^m, x)

[Out] int((b\*x^3+a)^m, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^m, x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^m, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^3 + a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^m, x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^m, x)

---

**Sympy [A]** time = 54.2667, size = 34, normalized size = 0.77

$$\frac{a^m x \left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, -m \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3 \left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*m, x)

[Out] a\*\*m\*x\*gamma(1/3)\*hyper((1/3, -m), (4/3, ), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(4/3))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^m, x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^m, x)

$$3.43 \quad \int \frac{(a+bx^3)^m}{c+dx^3} dx$$

**Optimal.** Leaf size=57

$$\frac{x (a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} F_1\left(\frac{1}{3}; -m, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c}$$

[Out]  $(x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*(1 + (b*x^3)/a)^m)$

**Rubi [A]** time = 0.070779, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{x (a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} F_1\left(\frac{1}{3}; -m, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^m/(c + d\*x^3), x]

[Out]  $(x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*(1 + (b*x^3)/a)^m)$

**Rubi in Sympy [A]** time = 21.2229, size = 42, normalized size = 0.74

$$\frac{x \left(1 + \frac{bx^3}{a}\right)^{-m} (a + bx^3)^m \text{appellf1}\left(\frac{1}{3}, 1, -m, \frac{4}{3}, -\frac{dx^3}{c}, -\frac{bx^3}{a}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*\*m/(d\*x\*\*3+c), x)

[Out]  $x*(1 + b*x**3/a)**(-m)*(a + b*x**3)**m*appellf1(1/3, 1, -m, 4/3, -d*x**3/c, -b*x**3/a)/c$

**Mathematica [B]** time = 0.327331, size = 162, normalized size = 2.84

$$\frac{4acx (a + bx^3)^m F_1\left(\frac{1}{3}; -m, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{(c + dx^3) \left(3x^3 \left(adF_1\left(\frac{4}{3}; -m, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) - bcmF_1\left(\frac{4}{3}; 1 - m, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)\right) - 4acF_1\left(\frac{1}{3}; -m, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^m/(c + d\*x^3), x]

[Out]  $(-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)]/((c + d*x^3)*(-4*a*c*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + 3*x^3*(-(b*c*m*AppellF1[4/3, 1 - m, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]) + a*d*AppellF1[4/3, -m, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))$

**Maple [F]** time = 0.076, size = 0, normalized size = 0.

$$\int \frac{(bx^3 + a)^m}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^m/(d*x^3+c), x)`

[Out] `int((b*x^3+a)^m/(d*x^3+c), x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + a)^m}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^m/(d*x^3 + c), x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a)^m/(d*x^3 + c), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^3 + a)^m}{dx^3 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^m/(d*x^3 + c), x, algorithm="fricas")`

[Out] `integral((b*x^3 + a)^m/(d*x^3 + c), x)`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**m/(d*x**3+c), x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + a)^m}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3 + a)^m/(d*x^3 + c), x, algorithm="giac")`

[Out] `integrate((b*x^3 + a)^m/(d*x^3 + c), x)`

$$3.44 \quad \int \frac{(a+bx^3)^m}{(c+dx^3)^2} dx$$

**Optimal.** Leaf size=57

$$\frac{x (a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} F_1\left(\frac{1}{3}; -m, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^2}$$

[Out]  $(x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c^2*(1 + (b*x^3)/a)^m)$

**Rubi [A]** time = 0.0716967, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{x (a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} F_1\left(\frac{1}{3}; -m, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^m/(c + d\*x^3)^2, x]

[Out]  $(x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c^2*(1 + (b*x^3)/a)^m)$

**Rubi in Sympy [A]** time = 19.708, size = 44, normalized size = 0.77

$$\frac{x \left(1 + \frac{bx^3}{a}\right)^{-m} (a + bx^3)^m \text{appellf1}\left(\frac{1}{3}, 2, -m, \frac{4}{3}, -\frac{dx^3}{c}, -\frac{bx^3}{a}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*\*m/(d\*x\*\*3+c)\*\*2, x)

[Out]  $x*(1 + b*x**3/a)**(-m)*(a + b*x**3)**m*appellf1(1/3, 2, -m, 4/3, -d*x**3/c, -b*x**3/a)/c**2$

**Mathematica [B]** time = 0.297776, size = 162, normalized size = 2.84

$$\frac{4acx (a + bx^3)^m F_1\left(\frac{1}{3}; -m, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{(c + dx^3)^2 \left(-3x^3 \left(bcmF_1\left(\frac{4}{3}; 1 - m, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) - 2adF_1\left(\frac{4}{3}; -m, 3; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)\right) - 4acF_1\left(\frac{1}{3}; -m, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^m/(c + d\*x^3)^2, x]

[Out]  $(-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)]/((c + d*x^3)^2*(-4*a*c*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] - 2*a*d*AppellF1[4/3, -m, 3, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))$

**Maple [F]** time = 0.095, size = 0, normalized size = 0.

$$\int \frac{(bx^3 + a)^m}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^m/(d\*x^3+c)^2, x)

[Out] int((b\*x^3+a)^m/(d\*x^3+c)^2, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + a)^m}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^m/(d\*x^3 + c)^2, x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^m/(d\*x^3 + c)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^3 + a)^m}{d^2x^6 + 2cdx^3 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^m/(d\*x^3 + c)^2, x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^m/(d^2\*x^6 + 2\*c\*d\*x^3 + c^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*m/(d\*x\*\*3+c)\*\*2, x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + a)^m}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^m/(d\*x^3 + c)^2, x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^m/(d\*x^3 + c)^2, x)



$$3.45 \quad \int \frac{(a+bx^3)^m}{(c+dx^3)^3} dx$$

**Optimal.** Leaf size=57

$$\frac{x (a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} F_1\left(\frac{1}{3}; -m, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^3}$$

[Out]  $(x^*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c^3*(1 + (b*x^3)/a)^m)$

**Rubi [A]** time = 0.0714109, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{x (a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} F_1\left(\frac{1}{3}; -m, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^m/(c + d\*x^3)^3, x]

[Out]  $(x^*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c^3*(1 + (b*x^3)/a)^m)$

**Rubi in Sympy [A]** time = 19.8666, size = 44, normalized size = 0.77

$$\frac{x \left(1 + \frac{bx^3}{a}\right)^{-m} (a + bx^3)^m \text{appellf1}\left(\frac{1}{3}, 3, -m, \frac{4}{3}, -\frac{dx^3}{c}, -\frac{bx^3}{a}\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*\*m/(d\*x\*\*3+c)\*\*3, x)

[Out]  $x*(1 + b*x**3/a)**(-m)*(a + b*x**3)**m*appellf1(1/3, 3, -m, 4/3, -d*x**3/c, -b*x**3/a)/c**3$

**Mathematica [B]** time = 0.392116, size = 162, normalized size = 2.84

$$\frac{4acx (a + bx^3)^m F_1\left(\frac{1}{3}; -m, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{(c + dx^3)^3 \left(-3x^3 \left(bcmF_1\left(\frac{4}{3}; 1 - m, 3; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) - 3adF_1\left(\frac{4}{3}; -m, 4; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)\right) - 4acF_1\left(\frac{1}{3}; -m, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^m/(c + d\*x^3)^3, x]

[Out]  $(-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)]/((c + d*x^3)^3*(-4*a*c*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, 3, 7/3, -((b*x^3)/a), -((d*x^3)/c)] - 3*a*d*AppellF1[4/3, -m, 4, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))$

**Maple [F]** time = 0.078, size = 0, normalized size = 0.

$$\int \frac{(bx^3 + a)^m}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^m/(d\*x^3+c)^3,x)

[Out] int((b\*x^3+a)^m/(d\*x^3+c)^3,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + a)^m}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^m/(d\*x^3 + c)^3,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^m/(d\*x^3 + c)^3, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^3 + a)^m}{d^3x^9 + 3cd^2x^6 + 3c^2dx^3 + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^m/(d\*x^3 + c)^3,x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^m/(d^3\*x^9 + 3\*c\*d^2\*x^6 + 3\*c^2\*d\*x^3 + c^3), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*m/(d\*x\*\*3+c)\*\*3,x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + a)^m}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^m/(d\*x^3 + c)^3,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^m/(d\*x^3 + c)^3, x)

$$3.46 \quad \int (a + bx^3)^{-1 - \frac{bc}{3bc-3ad}} (c + dx^3)^{-1 + \frac{ad}{3bc-3ad}} dx$$

**Optimal.** Leaf size=53

$$\frac{x (a + bx^3)^{-\frac{bc}{3bc-3ad}} (c + dx^3)^{\frac{ad}{3bc-3ad}}}{ac}$$

[Out]  $(x*(c + d*x^3)^{((a*d)/(3*b*c - 3*a*d))})/(a*c*(a + b*x^3)^{((b*c)/(3*b*c - 3*a*d))})$

**Rubi [A]** time = 0.0608956, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 50,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.02$

$$\frac{x (a + bx^3)^{-\frac{bc}{3bc-3ad}} (c + dx^3)^{\frac{ad}{3bc-3ad}}}{ac}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(-1 - (b\*c)/(3\*b\*c - 3\*a\*d))\*(c + d\*x^3)^(-1 + (a\*d)/(3\*b\*c - 3\*a\*d)), x]

[Out]  $(x*(c + d*x^3)^{((a*d)/(3*b*c - 3*a*d))})/(a*c*(a + b*x^3)^{((b*c)/(3*b*c - 3*a*d))})$

**Rubi in Sympy [A]** time = 10.5525, size = 44, normalized size = 0.83

$$\frac{x (a + bx^3)^{\frac{bc}{3(ad-bc)}} (c + dx^3)^{-\frac{ad}{3(ad-bc)}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*3+a)\*\*(-1-b\*c/(-3\*a\*d+3\*b\*c))\*(d\*x\*\*3+c)\*\*(-1+a\*d/(-3\*a\*d+3\*b\*c)), x)

[Out]  $x*(a + b*x**3)**(b*c/(3*(a*d - b*c)))*(c + d*x**3)**(-a*d/(3*(a*d - b*c)))/(a*c)$

**Mathematica [C]** time = 2.43632, size = 594, normalized size = 11.21

$$4acx (a + bx^3)^{\frac{bc}{3ad-3bc}} (c + dx^3)^{\frac{ad}{3bc-3ad}} \left( \frac{bF_1\left(\frac{1}{3}; \frac{bc}{3bc-3ad} + 1, \frac{ad}{3ad-3bc}; \frac{4}{3}, -\frac{bx^3}{a}\right)}{(a + bx^3) \left( x^3 \left( a^2 d^2 F_1\left(\frac{4}{3}; \frac{bc}{3bc-3ad} + 1, \frac{ad}{3ad-3bc} + 1; \frac{7}{3}, -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) + bc(3ad - 4bc)F_1\left(\frac{4}{3}; \frac{bc}{3bc-3ad} + 2, \frac{ad}{3ad-3bc}\right) \right) + \frac{dF_1\left(\frac{1}{3}; \frac{bc}{3bc-3ad}, \frac{ad}{3ad-3bc} + 1; \frac{4}{3}, -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{(c + dx^3) \left( x^3 \left( b^2 c^2 F_1\left(\frac{4}{3}; \frac{bc}{3bc-3ad} + 1, \frac{ad}{3ad-3bc} + 1; \frac{7}{3}, -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) + ad(3bc - 4ad)F_1\left(\frac{4}{3}; \frac{bc}{3bc-3ad}, \frac{ad}{3ad-3bc} + 2; \frac{7}{3}, -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) \right)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(-1 - (b\*c)/(3\*b\*c - 3\*a\*d))\*(c + d\*x^3)^(-1 + (a\*d)/(3\*b\*c - 3\*a\*d)), x]

[Out]  $4*a*c*x*(a + b*x^3)^{((b*c)/(-3*b*c + 3*a*d))}*(c + d*x^3)^{((a*d)/(3*b*c - 3*a*d))}*((d*AppellF1[1/3, (b*c)/(3*b*c - 3*a*d), 1 + (a*d)$

)/(-3\*b\*c + 3\*a\*d), 4/3, -((b\*x^3)/a), -((d\*x^3)/c)]/((c + d\*x^3) \* (4\*a\*c\*(-b\*c) + a\*d)\*AppellF1[1/3, (b\*c)/(3\*b\*c - 3\*a\*d), 1 + (a\*d)/(-3\*b\*c + 3\*a\*d), 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] + x^3\*(a\*d\*(3\*b\*c - 4\*a\*d)\*AppellF1[4/3, (b\*c)/(3\*b\*c - 3\*a\*d), 2 + (a\*d)/(-3\*b\*c + 3\*a\*d), 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + b^2\*c^2\*AppellF1[4/3, 1 + (b\*c)/(3\*b\*c - 3\*a\*d), 1 + (a\*d)/(-3\*b\*c + 3\*a\*d), 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])) + (b\*AppellF1[1/3, 1 + (b\*c)/(3\*b\*c - 3\*a\*d), (a\*d)/(-3\*b\*c + 3\*a\*d), 4/3, -((b\*x^3)/a), -((d\*x^3)/c)]/((a + b\*x^3)\*(4\*a\*c\*(b\*c - a\*d)\*AppellF1[1/3, 1 + (b\*c)/(3\*b\*c - 3\*a\*d), (a\*d)/(-3\*b\*c + 3\*a\*d), 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] + x^3\*(a^2\*d^2\*AppellF1[4/3, 1 + (b\*c)/(3\*b\*c - 3\*a\*d), 1 + (a\*d)/(-3\*b\*c + 3\*a\*d), 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + b\*c\*(-4\*b\*c + 3\*a\*d)\*AppellF1[4/3, 2 + (b\*c)/(3\*b\*c - 3\*a\*d), (a\*d)/(-3\*b\*c + 3\*a\*d), 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]))

**Maple [A]** time = 0.006, size = 71, normalized size = 1.3

$$\frac{x}{ac} (bx^3 + a)^{1 - \frac{3ad-4bc}{3ad-3bc}} (dx^3 + c)^{1 - \frac{4ad-3bc}{3ad-3bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(-1-b\*c/(-3\*a\*d+3\*b\*c))\*(d\*x^3+c)^(-1+a\*d/(-3\*a\*d+3\*b\*c)), x)

[Out] (b\*x^3+a)^(1-1/3\*(3\*a\*d-4\*b\*c)/(a\*d-b\*c))\*(d\*x^3+c)^(1-1/3\*(4\*a\*d-3\*b\*c)/(a\*d-b\*c))/a/c\*x

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + a)^{-\frac{bc}{3(bc-ad)}-1} (dx^3 + c)^{\frac{ad}{3(bc-ad)}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^(-1/3\*b\*c/(b\*c - a\*d) - 1)\*(d\*x^3 + c)^(1/3\*a\*d/(b\*c - a\*d)

[Out] integrate((b\*x^3 + a)^(-1/3\*b\*c/(b\*c - a\*d) - 1)\*(d\*x^3 + c)^(1/3\*a\*d/(b\*c - a\*d) - 1), x)

**Fricas [A]** time = 0.252313, size = 123, normalized size = 2.32

$$\frac{bdx^7 + (bc + ad)x^4 + acx}{(bx^3 + a)^{\frac{4bc-3ad}{3(bc-ad)}}(dx^3 + c)^{\frac{3bc-4ad}{3(bc-ad)}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^(-1/3\*b\*c/(b\*c - a\*d) - 1)\*(d\*x^3 + c)^(1/3\*a\*d/(b\*c - a\*d)

[Out] (b\*d\*x^7 + (b\*c + a\*d)\*x^4 + a\*c\*x)/((b\*x^3 + a)^(1/3\*(4\*b\*c - 3\*a\*d)/(b\*c - a\*d))\*(d\*x^3 + c)^(1/3\*(3\*b\*c - 4\*a\*d)/(b\*c - a\*d))\*a\*c)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(-1-b\*c/(-3\*a\*d+3\*b\*c))\*(d\*x\*\*3+c)\*\*(-1+a\*d/(-3\*a\*d+3\*b\*c)),x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + a)^{-\frac{bc}{3(bc-ad)}-1} (dx^3 + c)^{\frac{ad}{3(bc-ad)}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3 + a)^(-1/3\*b\*c/(b\*c - a\*d) - 1)\*(d\*x^3 + c)^(1/3\*a\*d/(b\*c - a\*d)

[Out] integrate((b\*x^3 + a)^(-1/3\*b\*c/(b\*c - a\*d) - 1)\*(d\*x^3 + c)^(1/3\*a\*d/(b\*c - a\*d) - 1), x)

$$3.47 \quad \int (a + bx^4) (c + dx^4)^4 dx$$

**Optimal.** Leaf size=94

$$\frac{1}{5}c^3x^5(4ad + bc) + \frac{2}{9}c^2dx^9(3ad + 2bc) + \frac{1}{17}d^3x^{17}(ad + 4bc) + \frac{2}{13}cd^2x^{13}(2ad + 3bc) + ac^4x + \frac{1}{21}bd^4x^{21}$$

[Out]  $a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2*c*d^2*(3*b*c + 2*a*d)*x^{13})/13 + (d^3*(4*b*c + a*d)*x^{17})/17 + (b*d^4*x^{21})/21$

**Rubi [A]** time = 0.145413, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{1}{5}c^3x^5(4ad + bc) + \frac{2}{9}c^2dx^9(3ad + 2bc) + \frac{1}{17}d^3x^{17}(ad + 4bc) + \frac{2}{13}cd^2x^{13}(2ad + 3bc) + ac^4x + \frac{1}{21}bd^4x^{21}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)\*(c + d\*x^4)^4, x]

[Out]  $a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2*c*d^2*(3*b*c + 2*a*d)*x^{13})/13 + (d^3*(4*b*c + a*d)*x^{17})/17 + (b*d^4*x^{21})/21$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^4x^{21}}{21} + c^4 \int a dx + \frac{c^3x^5(4ad + bc)}{5} + \frac{2c^2dx^9(3ad + 2bc)}{9} + \frac{2cd^2x^{13}(2ad + 3bc)}{13} + \frac{d^3x^{17}(ad + 4bc)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)\*(d\*x\*\*4+c)\*\*4, x)

[Out]  $b*d^4*x^{21}/21 + c^4*Integral(a, x) + c^3*x^5*(4*a*d + b*c)/5 + 2*c^2*d*x^9*(3*a*d + 2*b*c)/9 + 2*c*d^2*x^{13}*(2*a*d + 3*b*c)/13 + d^3*x^{17}*(a*d + 4*b*c)/17$

**Mathematica [A]** time = 0.0351537, size = 94, normalized size = 1.

$$\frac{1}{5}c^3x^5(4ad + bc) + \frac{2}{9}c^2dx^9(3ad + 2bc) + \frac{1}{17}d^3x^{17}(ad + 4bc) + \frac{2}{13}cd^2x^{13}(2ad + 3bc) + ac^4x + \frac{1}{21}bd^4x^{21}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)\*(c + d\*x^4)^4, x]

[Out]  $a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2*c*d^2*(3*b*c + 2*a*d)*x^{13})/13 + (d^3*(4*b*c + a*d)*x^{17})/17 + (b*d^4*x^{21})/21$

**Maple [A]** time = 0.001, size = 97, normalized size = 1.

$$\frac{bd^4x^{21}}{21} + \frac{(ad^4 + 4bcd^3)x^{17}}{17} + \frac{(4acd^3 + 6c^2d^2b)x^{13}}{13} + \frac{(6ac^2d^2 + 4c^3db)x^9}{9} + \frac{(4ac^3d + bc^4)x^5}{5} + ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+a)*(d*x^4+c)^4,x)`

[Out]  $\frac{1}{21}bd^4x^{21} + \frac{1}{17}(ad^4 + 4b^2cd^3)x^{17} + \frac{1}{13}(4a^2c^2d^3 + 6b^2cd^2)x^{13} + \frac{1}{9}(6a^2c^2d^2 + 4b^2cd^3)x^9 + \frac{1}{5}(4a^2c^3d + b^2c^4)x^5 + ac^4x$

**Maxima [A]** time = 1.36215, size = 130, normalized size = 1.38

$$\frac{1}{21}bd^4x^{21} + \frac{1}{17}(4bcd^3 + ad^4)x^{17} + \frac{2}{13}(3bc^2d^2 + 2acd^3)x^{13} + \frac{2}{9}(2bc^3d + 3ac^2d^2)x^9 + ac^4x + \frac{1}{5}(bc^4 + 4ac^3d)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)*(d*x^4 + c)^4,x, algorithm="maxima")`

[Out]  $\frac{1}{21}bd^4x^{21} + \frac{1}{17}(4b^2cd^3 + a^2d^4)x^{17} + \frac{2}{13}(3b^2c^2d^2 + 2a^2c^2d^3)x^{13} + \frac{2}{9}(2b^2c^3d + 3a^2c^2d^2)x^9 + a^2c^4x^5 + ac^4x$

**Fricas [A]** time = 0.191658, size = 1, normalized size = 0.01

$$\frac{1}{21}x^{21}d^4b + \frac{4}{17}x^{17}d^3cb + \frac{1}{17}x^{17}d^4a + \frac{6}{13}x^{13}d^2c^2b + \frac{4}{13}x^{13}d^3ca + \frac{4}{9}x^9dc^3b + \frac{2}{3}x^9d^2c^2a + \frac{1}{5}x^5c^4b + \frac{4}{5}x^5dc^3a + xc^4a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)*(d*x^4 + c)^4,x, algorithm="fricas")`

[Out]  $\frac{1}{21}x^{21}d^4b + \frac{4}{17}x^{17}d^3c^2b + \frac{1}{17}x^{17}d^4a + \frac{6}{13}x^{13}d^2c^2b + \frac{4}{13}x^{13}d^3ca + \frac{4}{9}x^9d^2c^2a + \frac{2}{3}x^9dc^3b + \frac{1}{5}x^5c^4b + \frac{4}{5}x^5dc^3a + xc^4a$

**Sympy [A]** time = 0.149023, size = 107, normalized size = 1.14

$$ac^4x + \frac{bd^4x^{21}}{21} + x^{17}\left(\frac{ad^4}{17} + \frac{4bcd^3}{17}\right) + x^{13}\left(\frac{4acd^3}{13} + \frac{6bc^2d^2}{13}\right) + x^9\left(\frac{2ac^2d^2}{3} + \frac{4bc^3d}{9}\right) + x^5\left(\frac{4ac^3d}{5} + \frac{bc^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**4+a)*(d*x**4+c)**4,x)`

[Out]  $a^2c^4x + b^2d^4x^{21}/21 + x^{17}(ad^4/17 + 4b^2cd^3/17) + x^{13}(4a^2c^2d^3/13 + 6b^2cd^2/13) + x^9(2a^2c^2d^2/3 + 4b^2cd^3/9) + x^5(4a^2c^3d/5 + b^2c^4/5)$

**GIAC/XCAS [A]** time = 0.212669, size = 132, normalized size = 1.4

$$\frac{1}{21}bd^4x^{21} + \frac{4}{17}bcd^3x^{17} + \frac{1}{17}ad^4x^{17} + \frac{6}{13}bc^2d^2x^{13} + \frac{4}{13}acd^3x^{13} + \frac{4}{9}bc^3dx^9 + \frac{2}{3}ac^2d^2x^9 + \frac{1}{5}bc^4x^5 + \frac{4}{5}ac^3dx^5 + ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)*(d*x^4 + c)^4,x, algorithm="giac")
```

```
[Out] 1/21*b*d^4*x^21 + 4/17*b*c*d^3*x^17 + 1/17*a*d^4*x^17 + 6/13*b*c^2*d^2*x^13 + 4/13*a*c*d^3*x^13 + 4/9*b*c^3*d*x^9 + 2/3*a*c^2*d^2*x^9 + 1/5*b*c^4*x^5 + 4/5*a*c^3*d*x^5 + a*c^4*x
```



### 3.48 $\int (a + bx^4)(c + dx^4)^3 dx$

**Optimal.** Leaf size=70

$$\frac{1}{5}c^2x^5(3ad + bc) + \frac{1}{13}d^2x^{13}(ad + 3bc) + \frac{1}{3}cdx^9(ad + bc) + ac^3x + \frac{1}{17}bd^3x^{17}$$

[Out]  $a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*c + a*d)*x^{13})/13 + (b*d^3*x^{17})/17$

**Rubi [A]** time = 0.104301, antiderivative size = 70, normalized size of antiderivative = 1., number of rules used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{1}{5}c^2x^5(3ad + bc) + \frac{1}{13}d^2x^{13}(ad + 3bc) + \frac{1}{3}cdx^9(ad + bc) + ac^3x + \frac{1}{17}bd^3x^{17}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)\*(c + d\*x^4)^3, x]

[Out]  $a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*c + a*d)*x^{13})/13 + (b*d^3*x^{17})/17$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^3x^{17}}{17} + c^3 \int a dx + \frac{c^2x^5(3ad + bc)}{5} + \frac{cdx^9(ad + bc)}{3} + \frac{d^2x^{13}(ad + 3bc)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)\*(d\*x\*\*4+c)\*\*3, x)

[Out]  $b*d^3*x^{17}/17 + c^3*Integral(a, x) + c^2*x^5*(3*a*d + b*c)/5 + c*d*x^9*(a*d + b*c)/3 + d^2*x^{13}*(a*d + 3*b*c)/13$

**Mathematica [A]** time = 0.0311638, size = 70, normalized size = 1.

$$\frac{1}{5}c^2x^5(3ad + bc) + \frac{1}{13}d^2x^{13}(ad + 3bc) + \frac{1}{3}cdx^9(ad + bc) + ac^3x + \frac{1}{17}bd^3x^{17}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)\*(c + d\*x^4)^3, x]

[Out]  $a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*c + a*d)*x^{13})/13 + (b*d^3*x^{17})/17$

**Maple [A]** time = 0.001, size = 73, normalized size = 1.

$$\frac{bd^3x^{17}}{17} + \frac{(ad^3 + 3bcd^2)x^{13}}{13} + \frac{(3acd^2 + 3bc^2d)x^9}{9} + \frac{(3ac^2d + bc^3)x^5}{5} + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)\*(d\*x^4+c)^3, x)

[Out]  $\frac{1}{17}bd^3x^{17} + \frac{1}{13}(ad^3 + 3b^2cd^2)x^{13} + \frac{1}{9}(3a^2cd^2 + 3b^2c^2d)x^9 + \frac{1}{5}(3a^2c^2d + b^2c^3)x^5 + a^2c^3x$

**Maxima [A]** time = 1.3555, size = 95, normalized size = 1.36

$$\frac{1}{17}bd^3x^{17} + \frac{1}{13}(3bcd^2 + ad^3)x^{13} + \frac{1}{3}(bc^2d + acd^2)x^9 + \frac{1}{5}(bc^3 + 3ac^2d)x^5 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)*(d*x^4 + c)^3,x, algorithm="maxima")`

[Out]  $\frac{1}{17}bd^3x^{17} + \frac{1}{13}(3b^2cd^2 + a^2d^3)x^{13} + \frac{1}{3}(b^2c^2d + a^2cd^2)x^9 + \frac{1}{5}(b^2c^3 + 3a^2c^2d)x^5 + a^2c^3x$

**Fricas [A]** time = 0.191661, size = 1, normalized size = 0.01

$$\frac{1}{17}x^{17}d^3b + \frac{3}{13}x^{13}d^2cb + \frac{1}{13}x^{13}d^3a + \frac{1}{3}x^9dc^2b + \frac{1}{3}x^9d^2ca + \frac{1}{5}x^5c^3b + \frac{3}{5}x^5dc^2a + xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)*(d*x^4 + c)^3,x, algorithm="fricas")`

[Out]  $\frac{1}{17}x^{17}d^3b + \frac{3}{13}x^{13}d^2c^2b + \frac{1}{13}x^{13}d^3a + \frac{1}{3}x^9d^2c^2b + \frac{1}{3}x^9d^2ca + \frac{1}{5}x^5c^3b + \frac{3}{5}x^5d^2c^2a + xc^3a$

**Sympy [A]** time = 0.122, size = 76, normalized size = 1.09

$$ac^3x + \frac{bd^3x^{17}}{17} + x^{13}\left(\frac{ad^3}{13} + \frac{3bcd^2}{13}\right) + x^9\left(\frac{acd^2}{3} + \frac{bc^2d}{3}\right) + x^5\left(\frac{3ac^2d}{5} + \frac{bc^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**4+a)*(d*x**4+c)**3,x)`

[Out]  $a^2c^3x + b^2d^3x^{17}/17 + x^{13}(ad^3/13 + 3b^2cd^2/13) + x^9(acd^2/3 + bc^2d/3) + x^5(3a^2c^2d/5 + b^2c^3/5)$

**GIAC/XCAS [A]** time = 0.211977, size = 100, normalized size = 1.43

$$\frac{1}{17}bd^3x^{17} + \frac{3}{13}bcd^2x^{13} + \frac{1}{13}ad^3x^{13} + \frac{1}{3}bc^2dx^9 + \frac{1}{3}acd^2x^9 + \frac{1}{5}bc^3x^5 + \frac{3}{5}ac^2dx^5 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)*(d*x^4 + c)^3,x, algorithm="giac")`

[Out]  $\frac{1}{17}bd^3x^{17} + \frac{3}{13}b^2cd^2x^{13} + \frac{1}{13}a^2d^3x^{13} + \frac{1}{3}b^2c^2d^2x^9 + \frac{1}{3}a^2cd^2x^9 + \frac{1}{5}b^2c^3x^5 + \frac{3}{5}a^2c^2d^2x^5 + a^2c^3x$

### 3.49 $\int (a + bx^4) (c + dx^4)^2 dx$

**Optimal.** Leaf size=50

$$\frac{1}{9}dx^9(ad + 2bc) + \frac{1}{5}cx^5(2ad + bc) + ac^2x + \frac{1}{13}bd^2x^{13}$$

[Out]  $a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^{13})/13$

**Rubi [A]** time = 0.0740207, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{1}{9}dx^9(ad + 2bc) + \frac{1}{5}cx^5(2ad + bc) + ac^2x + \frac{1}{13}bd^2x^{13}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)\*(c + d\*x^4)^2, x]

[Out]  $a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^{13})/13$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^2x^{13}}{13} + c^2 \int a dx + \frac{cx^5(2ad + bc)}{5} + \frac{dx^9(ad + 2bc)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)\*(d\*x\*\*4+c)\*\*2, x)

[Out]  $b*d^2*x^{13}/13 + c^2*Integral(a, x) + c*x^5*(2*a*d + b*c)/5 + d*x^9*(a*d + 2*b*c)/9$

**Mathematica [A]** time = 0.0200953, size = 50, normalized size = 1.

$$\frac{1}{9}dx^9(ad + 2bc) + \frac{1}{5}cx^5(2ad + bc) + ac^2x + \frac{1}{13}bd^2x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)\*(c + d\*x^4)^2, x]

[Out]  $a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^{13})/13$

**Maple [A]** time = 0.001, size = 49, normalized size = 1.

$$\frac{bd^2x^{13}}{13} + \frac{(ad^2 + 2bcd)x^9}{9} + \frac{(2acd + bc^2)x^5}{5} + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)\*(d\*x^4+c)^2, x)

[Out]  $1/13*b*d^2*x^{13}+1/9*(a*d^2+2*b*c*d)*x^9+1/5*(2*a*c*d+b*c^2)*x^5+a*c^2*x$

**Maxima [A]** time = 1.41285, size = 65, normalized size = 1.3

$$\frac{1}{13}bd^2x^{13} + \frac{1}{9}(2bcd + ad^2)x^9 + \frac{1}{5}(bc^2 + 2acd)x^5 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)*(d*x^4 + c)^2,x, algorithm="maxima")`

[Out]  $1/13*b*d^2*x^{13} + 1/9*(2*b*c*d + a*d^2)*x^9 + 1/5*(b*c^2 + 2*a*c*d)*x^5 + a*c^2*x$

**Fricas [A]** time = 0.189588, size = 1, normalized size = 0.02

$$\frac{1}{13}x^{13}d^2b + \frac{2}{9}x^9dcb + \frac{1}{9}x^9d^2a + \frac{1}{5}x^5c^2b + \frac{2}{5}x^5dca + xc^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)*(d*x^4 + c)^2,x, algorithm="fricas")`

[Out]  $1/13*x^{13}*d^2*b + 2/9*x^9*d*c*b + 1/9*x^9*d^2*a + 1/5*x^5*c^2*b + 2/5*x^5*d*c*a + x*c^2*a$

**Sympy [A]** time = 0.104548, size = 53, normalized size = 1.06

$$ac^2x + \frac{bd^2x^{13}}{13} + x^9\left(\frac{ad^2}{9} + \frac{2bcd}{9}\right) + x^5\left(\frac{2acd}{5} + \frac{bc^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**4+a)*(d*x**4+c)**2,x)`

[Out]  $a*c**2*x + b*d**2*x**13/13 + x**9*(a*d**2/9 + 2*b*c*d/9) + x**5*(2*a*c*d/5 + b*c**2/5)$

**GIAC/XCAS [A]** time = 0.211371, size = 68, normalized size = 1.36

$$\frac{1}{13}bd^2x^{13} + \frac{2}{9}bcdx^9 + \frac{1}{9}ad^2x^9 + \frac{1}{5}bc^2x^5 + \frac{2}{5}acdx^5 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)*(d*x^4 + c)^2,x, algorithm="giac")`

[Out]  $1/13*b*d^2*x^{13} + 2/9*b*c*d*x^9 + 1/9*a*d^2*x^9 + 1/5*b*c^2*x^5 + 2/5*a*c*d*x^5 + a*c^2*x$

### 3.50 $\int (a + bx^4)(c + dx^4) dx$

**Optimal.** Leaf size=28

$$\frac{1}{5}x^5(ad + bc) + acx + \frac{1}{9}bdx^9$$

[Out]  $a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9$

**Rubi [A]** time = 0.0381033, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{1}{5}x^5(ad + bc) + acx + \frac{1}{9}bdx^9$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)\*(c + d\*x^4), x]

[Out]  $a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bdx^9}{9} + c \int a dx + x^5 \left( \frac{ad}{5} + \frac{bc}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)\*(d\*x\*\*4+c), x)

[Out]  $b*d*x**9/9 + c*Integral(a, x) + x**5*(a*d/5 + b*c/5)$

**Mathematica [A]** time = 0.00912336, size = 28, normalized size = 1.

$$\frac{1}{5}x^5(ad + bc) + acx + \frac{1}{9}bdx^9$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)\*(c + d\*x^4), x]

[Out]  $a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9$

**Maple [A]** time = 0.001, size = 25, normalized size = 0.9

$$acx + \frac{(ad + bc)x^5}{5} + \frac{bdx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)\*(d\*x^4+c), x)

[Out]  $a*c*x+1/5*(a*d+b*c)*x^5+1/9*b*d*x^9$

---

**Maxima [A]** time = 1.36402, size = 32, normalized size = 1.14

$$\frac{1}{9} bdx^9 + \frac{1}{5} (bc + ad)x^5 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)\*(d\*x^4 + c),x, algorithm="maxima")

[Out] 1/9\*b\*d\*x^9 + 1/5\*(b\*c + a\*d)\*x^5 + a\*c\*x

---

**Fricas [A]** time = 0.188586, size = 1, normalized size = 0.04

$$\frac{1}{9}x^9db + \frac{1}{5}x^5cb + \frac{1}{5}x^5da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)\*(d\*x^4 + c),x, algorithm="fricas")

[Out] 1/9\*x^9\*d\*b + 1/5\*x^5\*c\*b + 1/5\*x^5\*d\*a + x\*c\*a

---

**Sympy [A]** time = 0.071019, size = 26, normalized size = 0.93

$$acx + \frac{bdx^9}{9} + x^5 \left( \frac{ad}{5} + \frac{bc}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*(d\*x\*\*4+c),x)

[Out] a\*c\*x + b\*d\*x\*\*9/9 + x\*\*5\*(a\*d/5 + b\*c/5)

---

**GIAC/XCAS [A]** time = 0.210788, size = 35, normalized size = 1.25

$$\frac{1}{9} bdx^9 + \frac{1}{5} bcx^5 + \frac{1}{5} adx^5 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)\*(d\*x^4 + c),x, algorithm="giac")

[Out] 1/9\*b\*d\*x^9 + 1/5\*b\*c\*x^5 + 1/5\*a\*d\*x^5 + a\*c\*x

### 3.51 $\int \frac{a+bx^4}{c+dx^4} dx$

**Optimal.** Leaf size=223

$$\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{4\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{4\sqrt{2}c^{3/4}d^{5/4}} \\ + \frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}d^{5/4}} + \frac{bx}{d}$$

[Out] (b\*x)/d + ((b\*c - a\*d)\*ArcTan[1 - (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)])/(2\*Sqrt[2]\*c^(3/4)\*d^(5/4)) - ((b\*c - a\*d)\*ArcTan[1 + (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)])/(2\*Sqrt[2]\*c^(3/4)\*d^(5/4)) + ((b\*c - a\*d)\*Log[Sqrt[c] - Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2])/(4\*Sqrt[2]\*c^(3/4)\*d^(5/4)) - ((b\*c - a\*d)\*Log[Sqrt[c] + Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2])/(4\*Sqrt[2]\*c^(3/4)\*d^(5/4))

**Rubi [A]** time = 0.328354, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{4\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{4\sqrt{2}c^{3/4}d^{5/4}} \\ + \frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}d^{5/4}} + \frac{bx}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)/(c + d\*x^4), x]

[Out] (b\*x)/d + ((b\*c - a\*d)\*ArcTan[1 - (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)])/(2\*Sqrt[2]\*c^(3/4)\*d^(5/4)) - ((b\*c - a\*d)\*ArcTan[1 + (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)])/(2\*Sqrt[2]\*c^(3/4)\*d^(5/4)) + ((b\*c - a\*d)\*Log[Sqrt[c] - Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2])/(4\*Sqrt[2]\*c^(3/4)\*d^(5/4)) - ((b\*c - a\*d)\*Log[Sqrt[c] + Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2])/(4\*Sqrt[2]\*c^(3/4)\*d^(5/4))

**Rubi in Sympy [A]** time = 57.1622, size = 204, normalized size = 0.91

$$\frac{bx}{d} - \frac{\sqrt{2}(ad-bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{8c^{3/4}d^{5/4}} + \frac{\sqrt{2}(ad-bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{8c^{3/4}d^{5/4}} \\ - \frac{\sqrt{2}(ad-bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{4c^{3/4}d^{5/4}} + \frac{\sqrt{2}(ad-bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{4c^{3/4}d^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)/(d\*x\*\*4+c), x)

[Out] b\*x/d - sqrt(2)\*(a\*d - b\*c)\*log(-sqrt(2)\*c\*\*(1/4)\*d\*\*(1/4)\*x + sqrt(c) + sqrt(d)\*x\*\*2)/(8\*c\*\*(3/4)\*d\*\*(5/4)) + sqrt(2)\*(a\*d - b\*c)\*log(sqrt(2)\*c\*\*(1/4)\*d\*\*(1/4)\*x + sqrt(c) + sqrt(d)\*x\*\*2)/(8\*c\*\*(3/4)\*d\*\*(5/4)) - sqrt(2)\*(a\*d - b\*c)\*atan(1 - sqrt(2)\*d\*\*(1/4)\*x/c\*\*(1/4))/(4\*c\*\*(3/4)\*d\*\*(5/4)) + sqrt(2)\*(a\*d - b\*c)\*atan(1 + sqrt(2)\*d\*\*(1/4)\*x/c\*\*(1/4))/(4\*c\*\*(3/4)\*d\*\*(5/4))

**Mathematica [A]** time = 0.227625, size = 196, normalized size = 0.88

$$\frac{\sqrt{2}(bc - ad) \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{c} + \sqrt{d}x^2\right) - \sqrt{2}(bc - ad) \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{c} + \sqrt{d}x^2\right) + 2\sqrt{2}(bc - ad) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{d}}{\sqrt[4]{c}}\right)}{8c^{3/4}d^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)/(c + d\*x^4), x]

[Out] (8\*b\*c^(3/4)\*d^(1/4)\*x + 2\*Sqrt[2]\*(b\*c - a\*d)\*ArcTan[1 - (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)] - 2\*Sqrt[2]\*(b\*c - a\*d)\*ArcTan[1 + (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)] + Sqrt[2]\*(b\*c - a\*d)\*Log[Sqrt[c] - Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2] - Sqrt[2]\*(b\*c - a\*d)\*Log[Sqrt[c] + Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2])/(8\*c^(3/4)\*d^(5/4))

**Maple [A]** time = 0.01, size = 266, normalized size = 1.2

$$\begin{aligned} & \frac{bx}{d} + \frac{\sqrt{2}a}{4c} \sqrt[4]{\frac{c}{d}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}} - 1\right) - \frac{\sqrt{2}b}{4d} \sqrt[4]{\frac{c}{d}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}} - 1\right) \\ & + \frac{\sqrt{2}a}{8c} \sqrt[4]{\frac{c}{d}} \ln\left(1\left(x^2 + \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)\left(x^2 - \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)^{-1}\right) \\ & - \frac{\sqrt{2}b}{8d} \sqrt[4]{\frac{c}{d}} \ln\left(1\left(x^2 + \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)\left(x^2 - \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)^{-1}\right) \\ & + \frac{\sqrt{2}a}{4c} \sqrt[4]{\frac{c}{d}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}} + 1\right) - \frac{\sqrt{2}b}{4d} \sqrt[4]{\frac{c}{d}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}} + 1\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)/(d\*x^4+c), x)

[Out] b\*x/d+1/4\*(c/d)^(1/4)/c\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x-1)\*a-1/4/d\*(c/d)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x-1)\*b+1/8\*(c/d)^(1/4)/c\*2^(1/2)\*ln((x^2+(c/d)^(1/4)\*x\*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)\*x\*2^(1/2)+(c/d)^(1/2)))\*a-1/8/d\*(c/d)^(1/4)\*2^(1/2)\*ln((x^2+(c/d)^(1/4)\*x\*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)\*x\*2^(1/2)+(c/d)^(1/2)))\*b+1/4\*(c/d)^(1/4)/c\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x+1)\*a-1/4/d\*(c/d)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x+1)\*b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)/(d\*x^4 + c), x, algorithm="maxima")

[Out] Exception raised: ValueError



**Fricas** [A] time = 0.239847, size = 743, normalized size = 3.33

$$4d \left( -\frac{b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4}{c^3d^5} \right)^{\frac{1}{4}} \arctan \left( \frac{cd \left( -\frac{b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4}{c^3d^5} \right)^{\frac{1}{4}}}{(bc-ad)x + (bc-ad) \sqrt{\frac{c^2d^2 \sqrt{-\frac{b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4}{c^3d^5}} + (b^2c^2 - 2abcd + a^2d^2)}}}{(bc-ad)x + (bc-ad) \sqrt{\frac{c^2d^2 \sqrt{-\frac{b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4}{c^3d^5}} + (b^2c^2 - 2abcd + a^2d^2)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)/(d\*x^4 + c), x, algorithm="fricas")

[Out] 
$$-1/4 * (4*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^{1/4} * \arctan(-c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^{1/4} / ((b*c - a*d)*x + (b*c - a*d)*\sqrt{(c^2*d^2*\sqrt{-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5)} + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2}) / (b^2*c^2 - 2*a*b*c*d + a^2*d^2))) - d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^{1/4} * \log(c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^{1/4} / (b*c - a*d)*x) + d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^{1/4} * \log(-c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^{1/4} / (b*c - a*d)*x) - 4*b*x)/d$$

**Sympy** [A] time = 2.23477, size = 87, normalized size = 0.39

$$\frac{bx}{d} + \text{RootSum} \left( 256t^4c^3d^5 + a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4, \left( t \mapsto t \log \left( \frac{4tcd}{ad - bc} + x \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)/(d\*x\*\*4+c), x)

[Out] 
$$b*x/d + \text{RootSum}(256*_t**4*c**3*d**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, \text{Lambda}(_t, _t*\log(4*_t*c*d/(a*d - b*c) + x)))$$

**GIAC/XCAS** [A] time = 0.220027, size = 331, normalized size = 1.48

$$\frac{bx}{d} - \frac{\sqrt{2} \left( (cd^3)^{\frac{1}{4}} bc - (cd^3)^{\frac{1}{4}} ad \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{c}{d} \right)^{\frac{1}{4}}} \right)}{4cd^2} - \frac{\sqrt{2} \left( (cd^3)^{\frac{1}{4}} bc - (cd^3)^{\frac{1}{4}} ad \right) \arctan \left( \frac{\sqrt{2} \left( 2x - \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{c}{d} \right)^{\frac{1}{4}}} \right)}{4cd^2} - \frac{\sqrt{2} \left( (cd^3)^{\frac{1}{4}} bc - (cd^3)^{\frac{1}{4}} ad \right) \ln \left( x^2 + \sqrt{2}x \left( \frac{c}{d} \right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}} \right)}{8cd^2} + \frac{\sqrt{2} \left( (cd^3)^{\frac{1}{4}} bc - (cd^3)^{\frac{1}{4}} ad \right) \ln \left( x^2 - \sqrt{2}x \left( \frac{c}{d} \right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}} \right)}{8cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)/(d\*x^4 + c), x, algorithm="giac")

```
[Out] b*x/d - 1/4*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c*d^2) - 1/4*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c*d^2) - 1/8*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*ln(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^2) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*ln(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^2)
```

$$3.52 \quad \int \frac{a+bx^4}{(c+dx^4)^2} dx$$

**Optimal.** Leaf size=245

$$\begin{aligned} & \frac{(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{16\sqrt{2}c^{7/4}d^{5/4}} + \frac{(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{16\sqrt{2}c^{7/4}d^{5/4}} \\ & - \frac{(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}} + \frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}d^{5/4}} - \frac{x(bc-ad)}{4cd(c+dx^4)} \end{aligned}$$

[Out]  $-\left((b*c - a*d)*x\right)/(4*c*d*(c + d*x^4)) - \left((b*c + 3*a*d)*\text{ArcTan}\left[1 - \left(\text{Sqrt}[2]*d^{(1/4)}*x\right)/c^{(1/4)}\right]\right)/(8*\text{Sqrt}[2]*c^{(7/4)}*d^{(5/4)}) + \left((b*c + 3*a*d)*\text{ArcTan}\left[1 + \left(\text{Sqrt}[2]*d^{(1/4)}*x\right)/c^{(1/4)}\right]\right)/(8*\text{Sqrt}[2]*c^{(7/4)}*d^{(5/4)}) - \left((b*c + 3*a*d)*\text{Log}\left[\text{Sqrt}[c] - \text{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \text{Sqrt}[d]*x^2\right]\right)/(16*\text{Sqrt}[2]*c^{(7/4)}*d^{(5/4)}) + \left((b*c + 3*a*d)*\text{Log}\left[\text{Sqrt}[c] + \text{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \text{Sqrt}[d]*x^2\right]\right)/(16*\text{Sqrt}[2]*c^{(7/4)}*d^{(5/4)})$

**Rubi [A]** time = 0.312072, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$\begin{aligned} & \frac{(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{16\sqrt{2}c^{7/4}d^{5/4}} + \frac{(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{16\sqrt{2}c^{7/4}d^{5/4}} \\ & - \frac{(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}} + \frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}d^{5/4}} - \frac{x(bc-ad)}{4cd(c+dx^4)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)/(c + d\*x^4)^2, x]

[Out]  $-\left((b*c - a*d)*x\right)/(4*c*d*(c + d*x^4)) - \left((b*c + 3*a*d)*\text{ArcTan}\left[1 - \left(\text{Sqrt}[2]*d^{(1/4)}*x\right)/c^{(1/4)}\right]\right)/(8*\text{Sqrt}[2]*c^{(7/4)}*d^{(5/4)}) + \left((b*c + 3*a*d)*\text{ArcTan}\left[1 + \left(\text{Sqrt}[2]*d^{(1/4)}*x\right)/c^{(1/4)}\right]\right)/(8*\text{Sqrt}[2]*c^{(7/4)}*d^{(5/4)}) - \left((b*c + 3*a*d)*\text{Log}\left[\text{Sqrt}[c] - \text{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \text{Sqrt}[d]*x^2\right]\right)/(16*\text{Sqrt}[2]*c^{(7/4)}*d^{(5/4)}) + \left((b*c + 3*a*d)*\text{Log}\left[\text{Sqrt}[c] + \text{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \text{Sqrt}[d]*x^2\right]\right)/(16*\text{Sqrt}[2]*c^{(7/4)}*d^{(5/4)})$

**Rubi in Sympy [A]** time = 59.1342, size = 226, normalized size = 0.92

$$\begin{aligned} & \frac{x(ad-bc)}{4cd(c+dx^4)} - \frac{\sqrt{2}(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{32c^{7/4}d^{5/4}} \\ & + \frac{\sqrt{2}(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{32c^{7/4}d^{5/4}} \\ & - \frac{\sqrt{2}(3ad+bc)\text{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{16c^{7/4}d^{5/4}} + \frac{\sqrt{2}(3ad+bc)\text{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{16c^{7/4}d^{5/4}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)/(d\*x\*\*4+c)\*\*2, x)

[Out]  $x*(a*d - b*c)/(4*c*d*(c + d*x**4)) - \text{sqrt}(2)*(3*a*d + b*c)*\log(-\text{sqrt}(2)*c**(1/4)*d**(1/4)*x + \text{sqrt}(c) + \text{sqrt}(d)*x**2)/(32*c**(7/4)*d**(5/4)) + \text{sqrt}(2)*(3*a*d + b*c)*\log(\text{sqrt}(2)*c**(1/4)*d**(1/4)*x + \text{sqrt}(c) + \text{sqrt}(d)*x**2)/(32*c**(7/4)*d**(5/4))$

$$x + \sqrt{c} + \sqrt{d}x^2 / (32c^{7/4}d^{5/4}) - \sqrt{2} \cdot (3a + b\sqrt{c}) \cdot \operatorname{atan}\left(1 - \frac{\sqrt{2}d^{1/4}x}{c^{1/4}}\right) / (16c^{7/4}d^{5/4}) + \sqrt{2} \cdot (3a + b\sqrt{c}) \cdot \operatorname{atan}\left(1 + \frac{\sqrt{2}d^{1/4}x}{c^{1/4}}\right) / (16c^{7/4}d^{5/4})$$

**Mathematica [A]** time = 0.290427, size = 212, normalized size = 0.87

$$\frac{-\frac{8c^{3/4}\sqrt[4]{dx(bc-ad)}}{c+dx^4} - \sqrt{2}(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right) + \sqrt{2}(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right) - 2\sqrt{2}(3ad + \sqrt{2}c^{7/4}d^{5/4})}{32c^{7/4}d^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)/(c + d\*x^4)^2, x]

[Out]  $\left(\frac{-8c^{3/4}d^{1/4}(b\sqrt{c} - a\sqrt{d})x}{(c + dx^4)} - 2\sqrt{2}(b\sqrt{c} + 3a\sqrt{d})\operatorname{ArcTan}\left[1 - \frac{\sqrt{2}d^{1/4}x}{c^{1/4}}\right] + 2\sqrt{2}(b\sqrt{c} + 3a\sqrt{d})\operatorname{ArcTan}\left[1 + \frac{\sqrt{2}d^{1/4}x}{c^{1/4}}\right] - \sqrt{2}(b\sqrt{c} + 3a\sqrt{d})\operatorname{Log}\left[\sqrt{c} - \sqrt{2}d^{1/4}x + \sqrt{dx^2}\right] + \sqrt{2}(b\sqrt{c} + 3a\sqrt{d})\operatorname{Log}\left[\sqrt{c} + \sqrt{2}d^{1/4}x + \sqrt{dx^2}\right]\right) / (32c^{7/4}d^{5/4})$

**Maple [A]** time = 0.013, size = 295, normalized size = 1.2

$$\begin{aligned} & \frac{(ad-bc)x}{4cd(dx^4+c)} + \frac{3\sqrt{2}a}{16c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right) + \frac{\sqrt{2}b}{16cd}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right) \\ & + \frac{3\sqrt{2}a}{32c^2}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2 + \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)\left(x^2 - \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)^{-1}\right) \\ & + \frac{\sqrt{2}b}{32cd}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2 + \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)\left(x^2 - \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)^{-1}\right) \\ & + \frac{3\sqrt{2}a}{16c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right) + \frac{\sqrt{2}b}{16cd}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)/(d\*x^4+c)^2, x)

[Out]  $\frac{1}{4} \cdot \frac{(a\sqrt{d} - b\sqrt{c})}{c} \cdot \frac{x}{d} \cdot \frac{1}{(d x^4 + c)} + \frac{3}{16} \cdot \frac{1}{c^2} \cdot \left(\frac{c}{d}\right)^{1/4} \cdot 2^{1/2} \cdot \arctan\left(\frac{2^{1/2}}{(c/d)^{1/4}} x - 1\right) \cdot \frac{a + 1/16}{c} \cdot \left(\frac{c}{d}\right)^{1/4} \cdot 2^{1/2} \cdot \arctan\left(\frac{2^{1/2}}{(c/d)^{1/4}} x - 1\right) \cdot \frac{b + 3/32}{c} \cdot \left(\frac{c}{d}\right)^{1/4} \cdot 2^{1/2} \cdot \ln\left(\frac{(x^2 + (c/d)^{1/4} x \sqrt{2} + \sqrt{c/d})}{(x^2 - (c/d)^{1/4} x \sqrt{2} + \sqrt{c/d})}\right) \cdot \frac{a + 1/32}{c} \cdot \left(\frac{c}{d}\right)^{1/4} \cdot 2^{1/2} \cdot \ln\left(\frac{(x^2 + (c/d)^{1/4} x \sqrt{2} + \sqrt{c/d})}{(x^2 - (c/d)^{1/4} x \sqrt{2} + \sqrt{c/d})}\right) \cdot \frac{b + 3/32}{c} \cdot \left(\frac{c}{d}\right)^{1/4} \cdot 2^{1/2} \cdot \arctan\left(\frac{2^{1/2}}{(c/d)^{1/4}} x + 1\right) \cdot \frac{a + 1/16}{c} \cdot \left(\frac{c}{d}\right)^{1/4} \cdot 2^{1/2} \cdot \arctan\left(\frac{2^{1/2}}{(c/d)^{1/4}} x + 1\right) \cdot b$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)/(d\*x^4 + c)^2, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.240921, size = 838, normalized size = 3.42

$$4 (cd^2x^4 + c^2d) \left( -\frac{b^4c^4 + 12ab^3c^3d + 54a^2b^2c^2d^2 + 108a^3bcd^3 + 81a^4d^4}{c^7d^5} \right)^{\frac{1}{4}} \arctan \left( \frac{c^2d \left( -\frac{b^4c^4 + 12ab^3c^3d + 54a^2b^2c^2d^2 + 108a^3bcd^3 + 81a^4d^4}{c^7d^5} \right)}{(bc+3ad)x + (bc+3ad)\sqrt{\frac{c^4d^2 \sqrt{-\frac{b^4c^4 + 12ab^3c^3d + 54a^2b^2c^2d^2 + 108a^3bcd^3 + 81a^4d^4}{c^7d^5}}}{b^2c^2 + 6abcd}}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)/(d\*x^4 + c)^2, x, algorithm="fricas")

[Out] 
$$-1/16 * (4 * (c*d^2*x^4 + c^2*d) * (- (b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4) / (c^7*d^5))^{1/4} * \arctan(c^2*d * (- (b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4) / (c^7*d^5))^{1/4} / ((b*c + 3*a*d)*x + (b*c + 3*a*d)*\sqrt{(c^4*d^2 * \sqrt{- (b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4) / (c^7*d^5)}) + (b^2*c^2 + 6*a*b*c*d + 9*a^2*d^2)*x^2} / (b^2*c^2 + 6*a*b*c*d + 9*a^2*d^2))) - (c*d^2*x^4 + c^2*d) * (- (b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4) / (c^7*d^5))^{1/4} * \log(c^2*d * (- (b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4) / (c^7*d^5))^{1/4} + (b*c + 3*a*d)*x) + (c*d^2*x^4 + c^2*d) * (- (b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4) / (c^7*d^5))^{1/4} * \log(-c^2*d * (- (b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4) / (c^7*d^5))^{1/4} + (b*c + 3*a*d)*x) + 4 * (b*c - a*d)*x) / (c*d^2*x^4 + c^2*d)$$

**Sympy** [A] time = 3.20672, size = 112, normalized size = 0.46

$$\frac{x(ad - bc)}{4c^2d + 4cd^2x^4} + \text{RootSum} \left( 65536t^4c^7d^5 + 81a^4d^4 + 108a^3bcd^3 + 54a^2b^2c^2d^2 + 12ab^3c^3d + b^4c^4, \left( t \mapsto t \log \left( \frac{16tc^2d}{3ad + bc} + x \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)/(d\*x\*\*4+c)\*\*2, x)

[Out] 
$$x*(a*d - b*c) / (4*c**2*d + 4*c*d**2*x**4) + \text{RootSum}(65536*_t**4*c**7*d**5 + 81*a**4*d**4 + 108*a**3*b*c*d**3 + 54*a**2*b**2*c**2*d**2 + 12*a*b**3*c**3*d + b**4*c**4, \text{Lambda}(_t, _t \log(16*_t*c**2*d / (3*a*d + b*c) + x)))$$

**GIAC/XCAS [A]** time = 0.219457, size = 359, normalized size = 1.47

$$\frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc + 3(cd^3)^{\frac{1}{4}}ad\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16c^2d^2} + \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc + 3(cd^3)^{\frac{1}{4}}ad\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16c^2d^2} + \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc + 3(cd^3)^{\frac{1}{4}}ad\right) \ln\left(x^2 + \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{32c^2d^2} - \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc + 3(cd^3)^{\frac{1}{4}}ad\right) \ln\left(x^2 - \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{32c^2d^2} - \frac{bcx - adx}{4(dx^4 + c)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)/(d\*x^4 + c)^2,x, algorithm="giac")

[Out] 1/16\*sqrt(2)\*((c\*d^3)^(1/4)\*b\*c + 3\*(c\*d^3)^(1/4)\*a\*d)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(c/d)^(1/4))/(c/d)^(1/4))/(c^2\*d^2) + 1/16\*sqrt(2)\*((c\*d^3)^(1/4)\*b\*c + 3\*(c\*d^3)^(1/4)\*a\*d)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)\*(c/d)^(1/4))/(c/d)^(1/4))/(c^2\*d^2) + 1/32\*sqrt(2)\*((c\*d^3)^(1/4)\*b\*c + 3\*(c\*d^3)^(1/4)\*a\*d)\*ln(x^2 + sqrt(2)\*x\*(c/d)^(1/4) + sqrt(c/d))/(c^2\*d^2) - 1/32\*sqrt(2)\*((c\*d^3)^(1/4)\*b\*c + 3\*(c\*d^3)^(1/4)\*a\*d)\*ln(x^2 - sqrt(2)\*x\*(c/d)^(1/4) + sqrt(c/d))/(c^2\*d^2) - 1/4\*(b\*c\*x - a\*d\*x)/((d\*x^4 + c)\*c\*d)

### 3.53 $\int \frac{a+bx^4}{(c+dx^4)^3} dx$

**Optimal.** Leaf size=273

$$\begin{aligned} & -\frac{3(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{128\sqrt{2}c^{11/4}d^{5/4}} \\ & +\frac{3(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{128\sqrt{2}c^{11/4}d^{5/4}}-\frac{3(7ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}} \\ & +\frac{3(7ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}+1\right)}{64\sqrt{2}c^{11/4}d^{5/4}}+\frac{x(7ad+bc)}{32c^2d(c+dx^4)}-\frac{x(bc-ad)}{8cd(c+dx^4)^2} \end{aligned}$$

[Out]  $-\left((b^*c - a^*d)^*x\right)/\left(8^*c^*d^*(c + d^*x^4)^2\right) + \left((b^*c + 7^*a^*d)^*x\right)/\left(32^*c^{\wedge}2^*d^*(c + d^*x^4)\right) - \left(3^*(b^*c + 7^*a^*d)^*ArcTan\left[1 - \left(\sqrt{2}\right)^*d^{\wedge}(1/4)^*x/c^{\wedge}(1/4)\right]\right)/\left(64^*\sqrt{2}^*c^{\wedge}(11/4)^*d^{\wedge}(5/4)\right) + \left(3^*(b^*c + 7^*a^*d)^*ArcTan\left[1 + \left(\sqrt{2}\right)^*d^{\wedge}(1/4)^*x/c^{\wedge}(1/4)\right]\right)/\left(64^*\sqrt{2}^*c^{\wedge}(11/4)^*d^{\wedge}(5/4)\right) - \left(3^*(b^*c + 7^*a^*d)^*Log\left[\sqrt{2}^*c^{\wedge}(1/4)^*d^{\wedge}(1/4)^*x + \sqrt{2}^*c^{\wedge}(1/4)^*d^{\wedge}(1/4)^*x + \sqrt{d}^*x^2\right]\right)/\left(128^*\sqrt{2}^*c^{\wedge}(11/4)^*d^{\wedge}(5/4)\right) + \left(3^*(b^*c + 7^*a^*d)^*Log\left[\sqrt{2}^*c^{\wedge}(1/4)^*d^{\wedge}(1/4)^*x + \sqrt{2}^*c^{\wedge}(1/4)^*d^{\wedge}(1/4)^*x + \sqrt{d}^*x^2\right]\right)/\left(128^*\sqrt{2}^*c^{\wedge}(11/4)^*d^{\wedge}(5/4)\right)$

**Rubi [A]** time = 0.356832, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$

$$\begin{aligned} & -\frac{3(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{128\sqrt{2}c^{11/4}d^{5/4}} \\ & +\frac{3(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{128\sqrt{2}c^{11/4}d^{5/4}}-\frac{3(7ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}} \\ & +\frac{3(7ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}+1\right)}{64\sqrt{2}c^{11/4}d^{5/4}}+\frac{x(7ad+bc)}{32c^2d(c+dx^4)}-\frac{x(bc-ad)}{8cd(c+dx^4)^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)/(c + d\*x^4)^3, x]

[Out]  $-\left((b^*c - a^*d)^*x\right)/\left(8^*c^*d^*(c + d^*x^4)^2\right) + \left((b^*c + 7^*a^*d)^*x\right)/\left(32^*c^{\wedge}2^*d^*(c + d^*x^4)\right) - \left(3^*(b^*c + 7^*a^*d)^*ArcTan\left[1 - \left(\sqrt{2}\right)^*d^{\wedge}(1/4)^*x/c^{\wedge}(1/4)\right]\right)/\left(64^*\sqrt{2}^*c^{\wedge}(11/4)^*d^{\wedge}(5/4)\right) + \left(3^*(b^*c + 7^*a^*d)^*ArcTan\left[1 + \left(\sqrt{2}\right)^*d^{\wedge}(1/4)^*x/c^{\wedge}(1/4)\right]\right)/\left(64^*\sqrt{2}^*c^{\wedge}(11/4)^*d^{\wedge}(5/4)\right) - \left(3^*(b^*c + 7^*a^*d)^*Log\left[\sqrt{2}^*c^{\wedge}(1/4)^*d^{\wedge}(1/4)^*x + \sqrt{2}^*c^{\wedge}(1/4)^*d^{\wedge}(1/4)^*x + \sqrt{d}^*x^2\right]\right)/\left(128^*\sqrt{2}^*c^{\wedge}(11/4)^*d^{\wedge}(5/4)\right) + \left(3^*(b^*c + 7^*a^*d)^*Log\left[\sqrt{2}^*c^{\wedge}(1/4)^*d^{\wedge}(1/4)^*x + \sqrt{2}^*c^{\wedge}(1/4)^*d^{\wedge}(1/4)^*x + \sqrt{d}^*x^2\right]\right)/\left(128^*\sqrt{2}^*c^{\wedge}(11/4)^*d^{\wedge}(5/4)\right)$

**Rubi in Sympy [A]** time = 65.6386, size = 258, normalized size = 0.95

$$\begin{aligned} & \frac{x(ad-bc)}{8cd(c+dx^4)^2} + \frac{x(7ad+bc)}{32c^2d(c+dx^4)} - \frac{3\sqrt{2}(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{256c^{\frac{11}{4}}d^{\frac{5}{4}}} \\ & + \frac{3\sqrt{2}(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{256c^{\frac{11}{4}}d^{\frac{5}{4}}} \\ & - \frac{3\sqrt{2}(7ad+bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{128c^{\frac{11}{4}}d^{\frac{5}{4}}} + \frac{3\sqrt{2}(7ad+bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{128c^{\frac{11}{4}}d^{\frac{5}{4}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**4+a)/(d*x**4+c)**3,x)`

[Out]  $x*(a*d - b*c)/(8*c*d*(c + d*x**4)**2) + x*(7*a*d + b*c)/(32*c**2*d*(c + d*x**4)) - 3*\sqrt{2}*(7*a*d + b*c)*\log(-\sqrt{2}*c**(1/4)*d**(1/4)*x + \sqrt{c} + \sqrt{d}*x**2)/(256*c**(11/4)*d**(5/4)) + 3*\sqrt{2}*(7*a*d + b*c)*\log(\sqrt{2}*c**(1/4)*d**(1/4)*x + \sqrt{c} + \sqrt{d}*x**2)/(256*c**(11/4)*d**(5/4)) - 3*\sqrt{2}*(7*a*d + b*c)*\operatorname{atan}(1 - \sqrt{2}*d**(1/4)*x/c**(1/4))/(128*c**(11/4)*d**(5/4)) + 3*\sqrt{2}*(7*a*d + b*c)*\operatorname{atan}(1 + \sqrt{2}*d**(1/4)*x/c**(1/4))/(128*c**(11/4)*d**(5/4))$

**Mathematica [A]** time = 0.371676, size = 243, normalized size = 0.89

$$\frac{-\frac{32c^{7/4}\sqrt[4]{dx(bc-ad)}}{(c+dx^4)^2} + \frac{8c^{3/4}\sqrt[4]{dx(7ad+bc)}}{c+dx^4} - 3\sqrt{2}(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right) + 3\sqrt{2}(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{256c^{11/4}d^{5/4}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^4)/(c + d*x^4)^3,x]`

[Out]  $((-32*c^{(7/4)}*d^{(1/4)}*(b*c - a*d)*x)/(c + d*x^4)^2 + (8*c^{(3/4)}*d^{(1/4)}*(b*c + 7*a*d)*x)/(c + d*x^4) - 6*\operatorname{Sqrt}[2]*(b*c + 7*a*d)*\operatorname{ArcTan}[1 - (\operatorname{Sqrt}[2]*d^{(1/4)}*x)/c^{(1/4)}] + 6*\operatorname{Sqrt}[2]*(b*c + 7*a*d)*\operatorname{ArcTan}[1 + (\operatorname{Sqrt}[2]*d^{(1/4)}*x)/c^{(1/4)}] - 3*\operatorname{Sqrt}[2]*(b*c + 7*a*d)*\operatorname{Log}[\operatorname{Sqrt}[c] - \operatorname{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \operatorname{Sqrt}[d]*x^2] + 3*\operatorname{Sqrt}[2]*(b*c + 7*a*d)*\operatorname{Log}[\operatorname{Sqrt}[c] + \operatorname{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \operatorname{Sqrt}[d]*x^2])/(256*c^{(11/4)}*d^{(5/4)})$

**Maple [A]** time = 0.016, size = 314, normalized size = 1.2

$$\begin{aligned} & \frac{1}{(dx^4 + c)^2} \left( \frac{(7ad + bc)x^5}{32c^2} + \frac{(11ad - 3bc)x}{32cd} \right) \\ & + \frac{21\sqrt{2}a}{128c^3} \sqrt[4]{\frac{c}{d}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}} + 1\right) + \frac{3\sqrt{2}b}{128c^2d} \sqrt[4]{\frac{c}{d}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}} + 1\right) \\ & + \frac{21\sqrt{2}a}{128c^3} \sqrt[4]{\frac{c}{d}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}} - 1\right) + \frac{3\sqrt{2}b}{128c^2d} \sqrt[4]{\frac{c}{d}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}} - 1\right) \\ & + \frac{21\sqrt{2}a}{256c^3} \sqrt[4]{\frac{c}{d}} \ln\left(1\left(x^2 + \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)\left(x^2 - \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)^{-1}\right) \\ & + \frac{3\sqrt{2}b}{256c^2d} \sqrt[4]{\frac{c}{d}} \ln\left(1\left(x^2 + \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)\left(x^2 - \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)^{-1}\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+a)/(d*x^4+c)^3,x)`

[Out]  $(1/32*(7*a*d+b*c)/c^2*x^5+1/32*(11*a*d-3*b*c)/c/d*x)/(d*x^4+c)^2+21/128/c^3*(c/d)^{(1/4)}*2^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/(c/d)^{(1/4)}*x+1)*a+3/128/c^3/d*(c/d)^{(1/4)}*2^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/(c/d)^{(1/4)}*x+1)*b+21/128/c^3*(c/d)^{(1/4)}*2^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/(c/d)^{(1/4)}*x-1)*a+3/128/c^3/d*(c/d)^{(1/4)}*2^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/(c/d)^{(1/4)}*x-1)*b+21/256/c^3*(c/d)^{(1/4)}*2^{(1/2)}*\ln((x^2+(c/d)^{(1/4)}*x*2^{(1/2)}+(c/d)^{(1/2)})/(x^2-(c/d)^{(1/4)}*x*2^{(1/2)}+(c/d)^{(1/2)}))*a+3/256/c^3/d$



$$\frac{(c/d)^{1/4} \cdot 2^{1/2} \cdot \ln((x^2 + (c/d)^{1/4} \cdot x \cdot 2^{1/2} + (c/d)^{1/2}))}{x^2 - (c/d)^{1/4} \cdot x \cdot 2^{1/2} + (c/d)^{1/2}} \cdot b$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)/(d\*x^4 + c)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.239178, size = 941, normalized size = 3.45

$$4(bcd + 7ad^2)x^5 - 12(c^2d^3x^8 + 2c^3d^2x^4 + c^4d) \left( -\frac{b^4c^4 + 28ab^3c^3d + 294a^2b^2c^2d^2 + 1372a^3bcd^3 + 2401a^4d^4}{c^{11}d^5} \right)^{1/4} \arctan \left( \frac{\dots}{(bc+7ad)x+(bc+7a)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)/(d\*x^4 + c)^3,x, algorithm="fricas")

[Out] 1/128\*(4\*(b\*c\*d + 7\*a\*d^2)\*x^5 - 12\*(c^2\*d^3\*x^8 + 2\*c^3\*d^2\*x^4 + c^4\*d) \* (- (b^4\*c^4 + 28\*a\*b^3\*c^3\*d + 294\*a^2\*b^2\*c^2\*d^2 + 1372\*a^3\*b\*c\*d^3 + 2401\*a^4\*d^4)/(c^11\*d^5))^(1/4) \* arctan(c^3\*d\*(-(b^4\*c^4 + 28\*a\*b^3\*c^3\*d + 294\*a^2\*b^2\*c^2\*d^2 + 1372\*a^3\*b\*c\*d^3 + 2401\*a^4\*d^4)/(c^11\*d^5))^(1/4) / ((b\*c + 7\*a\*d)\*x + (b\*c + 7\*a\*d) \* sqrt((c^6\*d^2\*sqrt(-(b^4\*c^4 + 28\*a\*b^3\*c^3\*d + 294\*a^2\*b^2\*c^2\*d^2 + 1372\*a^3\*b\*c\*d^3 + 2401\*a^4\*d^4)/(c^11\*d^5)) + (b^2\*c^2 + 14\*a\*b\*c\*d + 49\*a^2\*d^2)\*x^2) / (b^2\*c^2 + 14\*a\*b\*c\*d + 49\*a^2\*d^2))) + 3\*(c^2\*d^3\*x^8 + 2\*c^3\*d^2\*x^4 + c^4\*d) \* (- (b^4\*c^4 + 28\*a\*b^3\*c^3\*d + 294\*a^2\*b^2\*c^2\*d^2 + 1372\*a^3\*b\*c\*d^3 + 2401\*a^4\*d^4)/(c^11\*d^5))^(1/4) \* log(3\*c^3\*d\*(-(b^4\*c^4 + 28\*a\*b^3\*c^3\*d + 294\*a^2\*b^2\*c^2\*d^2 + 1372\*a^3\*b\*c\*d^3 + 2401\*a^4\*d^4)/(c^11\*d^5))^(1/4) + 3\*(b\*c + 7\*a\*d)\*x) - 3\*(c^2\*d^3\*x^8 + 2\*c^3\*d^2\*x^4 + c^4\*d) \* (- (b^4\*c^4 + 28\*a\*b^3\*c^3\*d + 294\*a^2\*b^2\*c^2\*d^2 + 1372\*a^3\*b\*c\*d^3 + 2401\*a^4\*d^4)/(c^11\*d^5))^(1/4) \* log(-3\*c^3\*d\*(-(b^4\*c^4 + 28\*a\*b^3\*c^3\*d + 294\*a^2\*b^2\*c^2\*d^2 + 1372\*a^3\*b\*c\*d^3 + 2401\*a^4\*d^4)/(c^11\*d^5))^(1/4) + 3\*(b\*c + 7\*a\*d)\*x) - 4\*(3\*b\*c^2 - 11\*a\*c\*d)\*x) / (c^2\*d^3\*x^8 + 2\*c^3\*d^2\*x^4 + c^4\*d)

**Sympy [A]** time = 6.69677, size = 151, normalized size = 0.55

$$\frac{x^5(7ad^2 + bcd) + x(11acd - 3bc^2)}{32c^4d + 64c^3d^2x^4 + 32c^2d^3x^8} + \text{RootSum} \left( 268435456t^4c^{11}d^5 + 194481a^4d^4 + 111132a^3bcd^3 + 23814a^2b^2c^2d^2 + 2268ab^3c^3d + 81b^4c^4, \left( t \mapsto t \log \left( \frac{128tc^3}{21ad + \dots} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)/(d\*x\*\*4+c)\*\*3,x)

[Out] (x\*\*5\*(7\*a\*d\*\*2 + b\*c\*d) + x\*(11\*a\*c\*d - 3\*b\*c\*\*2))/(32\*c\*\*4\*d + 64\*c\*\*3\*d\*\*2\*x\*\*4 + 32\*c\*\*2\*d\*\*3\*x\*\*8) + RootSum(268435456\*\_t\*\*4\*c\*\*11\*d\*\*5 + 194481\*a\*\*4\*d\*\*4 + 111132\*a\*\*3\*b\*c\*d\*\*3 + 23814\*a\*\*2\*b\*\*2\*c\*\*2\*d\*\*2 + 2268\*a\*b\*\*3\*c\*\*3\*d + 81\*b\*\*4\*c\*\*4, Lambda(\_t, \_

$t \cdot \log(128 \cdot t \cdot c^{**3} \cdot d / (21 \cdot a \cdot d + 3 \cdot b \cdot c) + x))$

**GIAC/XCAS [A]** time = 0.221907, size = 386, normalized size = 1.41

$$\frac{3\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc + 7(cd^3)^{\frac{1}{4}}ad\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128c^3d^2} + \frac{3\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc + 7(cd^3)^{\frac{1}{4}}ad\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128c^3d^2} + \frac{3\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc + 7(cd^3)^{\frac{1}{4}}ad\right) \ln\left(x^2 + \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{256c^3d^2} - \frac{3\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc + 7(cd^3)^{\frac{1}{4}}ad\right) \ln\left(x^2 - \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{256c^3d^2} + \frac{bcdx^5 + 7ad^2x^5 - 3bc^2x + 11acdx}{32(dx^4 + c)^2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)/(d\*x^4 + c)^3,x, algorithm="giac")

[Out] 3/128\*sqrt(2)\*((c\*d^3)^(1/4)\*b\*c + 7\*(c\*d^3)^(1/4)\*a\*d)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(c/d)^(1/4))/(c/d)^(1/4))/(c^3\*d^2) + 3/128\*sqrt(2)\*((c\*d^3)^(1/4)\*b\*c + 7\*(c\*d^3)^(1/4)\*a\*d)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)\*(c/d)^(1/4))/(c/d)^(1/4))/(c^3\*d^2) + 3/256\*sqrt(2)\*((c\*d^3)^(1/4)\*b\*c + 7\*(c\*d^3)^(1/4)\*a\*d)\*ln(x^2 + sqrt(2)\*x\*(c/d)^(1/4) + sqrt(c/d))/(c^3\*d^2) - 3/256\*sqrt(2)\*((c\*d^3)^(1/4)\*b\*c + 7\*(c\*d^3)^(1/4)\*a\*d)\*ln(x^2 - sqrt(2)\*x\*(c/d)^(1/4) + sqrt(c/d))/(c^3\*d^2) + 1/32\*(b\*c\*d\*x^5 + 7\*a\*d^2\*x^5 - 3\*b\*c^2\*x + 11\*a\*c\*d\*x)/((d\*x^4 + c)^2\*c^2\*d)

### 3.54 $\int (a + bx^4)^2 (c + dx^4)^4 dx$

**Optimal.** Leaf size=154

$$\frac{1}{17}d^2x^{17}(a^2d^2 + 8abcd + 6b^2c^2) + \frac{4}{13}cdx^{13}(a^2d^2 + 3abcd + b^2c^2) + \frac{1}{9}c^2x^9(6a^2d^2 + 8abcd + b^2c^2) + a^2c^4x + \frac{2}{5}ac^3x^5(2ad + bc) + \frac{2}{21}bd^3x^{21}(ad + 2bc) + \frac{1}{25}b^2d^4x^{25}$$

[Out]  $a^2c^4x + (2ac^3(b^2c + 2ad)x^5)/5 + (c^2(b^2c^2 + 8abcd + 6a^2d^2)x^9)/9 + (4cdx^{13}(a^2d^2 + 3abcd + b^2c^2))/13 + (d^2(6b^2c^2 + 8abcd + a^2d^2)x^{17})/17 + (2bd^3(2b^2c + ad)x^{21})/21 + (b^2d^4x^{25})/25$

**Rubi [A]** time = 0.235883, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{1}{17}d^2x^{17}(a^2d^2 + 8abcd + 6b^2c^2) + \frac{4}{13}cdx^{13}(a^2d^2 + 3abcd + b^2c^2) + \frac{1}{9}c^2x^9(6a^2d^2 + 8abcd + b^2c^2) + a^2c^4x + \frac{2}{5}ac^3x^5(2ad + bc) + \frac{2}{21}bd^3x^{21}(ad + 2bc) + \frac{1}{25}b^2d^4x^{25}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^2\*(c + d\*x^4)^4, x]

[Out]  $a^2c^4x + (2ac^3(b^2c + 2ad)x^5)/5 + (c^2(b^2c^2 + 8abcd + 6a^2d^2)x^9)/9 + (4cdx^{13}(a^2d^2 + 3abcd + b^2c^2))/13 + (d^2(6b^2c^2 + 8abcd + a^2d^2)x^{17})/17 + (2bd^3(2b^2c + ad)x^{21})/21 + (b^2d^4x^{25})/25$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2ac^3x^5(2ad + bc)}{5} + \frac{b^2d^4x^{25}}{25} + \frac{2bd^3x^{21}(ad + 2bc)}{21} + c^4 \int a^2 dx + \frac{c^2x^9(6a^2d^2 + 8abcd + b^2c^2)}{9} + \frac{4cdx^{13}(a^2d^2 + 3abcd + b^2c^2)}{13} + \frac{d^2x^{17}(a^2d^2 + 8abcd + 6b^2c^2)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)\*\*2\*(d\*x\*\*4+c)\*\*4, x)

[Out]  $2ac^3x^5(2ad + b^2c)/5 + b^2d^4x^{25}/25 + 2bd^3x^{21}(ad + 2bc)/21 + c^4 \text{Integral}(a^2, x) + c^2x^9(6a^2d^2 + 8abcd + b^2c^2)/9 + 4cdx^{13}(a^2d^2 + 3abcd + b^2c^2)/13 + d^2x^{17}(a^2d^2 + 8abcd + 6b^2c^2)/17$

**Mathematica [A]** time = 0.0624063, size = 154, normalized size = 1.

$$\frac{1}{17}d^2x^{17}(a^2d^2 + 8abcd + 6b^2c^2) + \frac{4}{13}cdx^{13}(a^2d^2 + 3abcd + b^2c^2) + \frac{1}{9}c^2x^9(6a^2d^2 + 8abcd + b^2c^2) + a^2c^4x + \frac{2}{5}ac^3x^5(2ad + bc) + \frac{2}{21}bd^3x^{21}(ad + 2bc) + \frac{1}{25}b^2d^4x^{25}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)^2\*(c + d\*x^4)^4, x]

[Out]  $a^2c^4x + (2ac^3(bc + 2ad)x^5)/5 + (c^2(b^2c^2 + 8abc^2d + 6a^2d^2)x^9)/9 + (4c^2d(b^2c^2 + 3abc^2d + a^2d^2)x^{13})/13 + (d^2(6b^2c^2 + 8abc^2d + a^2d^2)x^{17})/17 + (2bd^3(2bc + ad)x^{21})/21 + (b^2d^4x^{25})/25$

**Maple [A]** time = 0.001, size = 163, normalized size = 1.1

$$\frac{b^2d^4x^{25}}{25} + \frac{(2abd^4 + 4b^2cd^3)x^{21}}{21} + \frac{(a^2d^4 + 8abcd^3 + 6b^2c^2d^2)x^{17}}{17} + \frac{(4a^2cd^3 + 12abc^2d^2 + 4b^2c^3d)x^{13}}{13} + \frac{(6a^2c^2d^2 + 8abc^3d + b^2c^4)x^9}{9} + \frac{(4a^2c^3d + 2abc^4)x^5}{5} + a^2c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+a)^2*(d*x^4+c)^4,x)`

[Out]  $1/25*b^2*d^4*x^25 + 1/21*(2*a*b*d^4 + 4*b^2*c*d^3)*x^21 + 1/17*(a^2*d^4 + 8*a*b*c*d^3 + 6*b^2*c^2*d^2)*x^17 + 1/13*(4*a^2*c*d^3 + 12*a*b*c^2*d^2 + 4*b^2*c^3*d)*x^13 + 1/9*(6*a^2*c^2*d^2 + 8*a*b*c^3*d + b^2*c^4)*x^9 + 1/5*(4*a^2*c^3*d + 2*a*b*c^4)*x^5 + a^2*c^4*x$

**Maxima [A]** time = 1.37164, size = 213, normalized size = 1.38

$$\frac{1}{25}b^2d^4x^{25} + \frac{2}{21}(2b^2cd^3 + abd^4)x^{21} + \frac{1}{17}(6b^2c^2d^2 + 8abcd^3 + a^2d^4)x^{17} + \frac{4}{13}(b^2c^3d + 3abc^2d^2 + a^2cd^3)x^{13} + \frac{1}{9}(b^2c^4 + 8abc^3d + 6a^2c^2d^2)x^9 + a^2c^4x + \frac{2}{5}(abc^4 + 2a^2c^3d)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^2*(d*x^4 + c)^4,x, algorithm="maxima")`

[Out]  $1/25*b^2*d^4*x^25 + 2/21*(2*b^2*c*d^3 + a*b*d^4)*x^21 + 1/17*(6*b^2*c^2*d^2 + 8*a*b*c*d^3 + a^2*d^4)*x^17 + 4/13*(b^2*c^3*d + 3*a*b*c^2*d^2 + a^2*c*d^3)*x^13 + 1/9*(b^2*c^4 + 8*a*b*c^3*d + 6*a^2*c^2*d^2)*x^9 + a^2*c^4*x + 2/5*(a*b*c^4 + 2*a^2*c^3*d)*x^5$

**Fricas [A]** time = 0.188992, size = 1, normalized size = 0.01

$$\frac{1}{25}x^{25}d^4b^2 + \frac{4}{21}x^{21}d^3cb^2 + \frac{2}{21}x^{21}d^4ba + \frac{6}{17}x^{17}d^2c^2b^2 + \frac{8}{17}x^{17}d^3cba + \frac{1}{17}x^{17}d^4a^2 + \frac{4}{13}x^{13}dc^3b^2 + \frac{12}{13}x^{13}d^2c^2ba + \frac{4}{13}x^{13}d^3ca^2 + \frac{1}{9}x^9c^4b^2 + \frac{8}{9}x^9dc^3ba + \frac{2}{3}x^9d^2c^2a^2 + \frac{2}{5}x^5c^4ba + \frac{4}{5}x^5dc^3a^2 + xc^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^2*(d*x^4 + c)^4,x, algorithm="fricas")`

[Out]  $1/25*x^25*d^4*b^2 + 4/21*x^21*d^3*c*b^2 + 2/21*x^21*d^4*b*a + 6/17*x^17*d^2*c^2*b^2 + 8/17*x^17*d^3*c*b*a + 1/17*x^17*d^4*a^2 + 4/13*x^13*d^2*c^3*b^2 + 12/13*x^13*d^3*c^2*b*a + 4/13*x^13*d^3*c*a^2 + 1/9*x^9*c^4*b^2 + 8/9*x^9*d^3*c^3*b*a + 2/3*x^9*d^2*c^2*a^2 + 2/5*x^5*c^4*b*a + 4/5*x^5*d^3*c^3*a^2 + x*c^4*a^2$

**Sympy [A]** time = 0.182983, size = 185, normalized size = 1.2

$$a^2c^4x + \frac{b^2d^4x^{25}}{25} + x^{21} \left( \frac{2abd^4}{21} + \frac{4b^2cd^3}{21} \right) + x^{17} \left( \frac{a^2d^4}{17} + \frac{8abcd^3}{17} + \frac{6b^2c^2d^2}{17} \right) \\ + x^{13} \left( \frac{4a^2cd^3}{13} + \frac{12abc^2d^2}{13} + \frac{4b^2c^3d}{13} \right) + x^9 \left( \frac{2a^2c^2d^2}{3} + \frac{8abc^3d}{9} + \frac{b^2c^4}{9} \right) + x^5 \left( \frac{4a^2c^3d}{5} + \frac{2abc^4}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*2\*(d\*x\*\*4+c)\*\*4,x)

[Out] a\*\*2\*c\*\*4\*x + b\*\*2\*d\*\*4\*x\*\*25/25 + x\*\*21\*(2\*a\*b\*d\*\*4/21 + 4\*b\*\*2\*c\*d\*\*3/21) + x\*\*17\*(a\*\*2\*d\*\*4/17 + 8\*a\*b\*c\*d\*\*3/17 + 6\*b\*\*2\*c\*\*2\*d\*\*2/17) + x\*\*13\*(4\*a\*\*2\*c\*d\*\*3/13 + 12\*a\*b\*c\*\*2\*d\*\*2/13 + 4\*b\*\*2\*c\*\*3\*d/13) + x\*\*9\*(2\*a\*\*2\*c\*\*2\*d\*\*2/3 + 8\*a\*b\*c\*\*3\*d/9 + b\*\*2\*c\*\*4/9) + x\*\*5\*(4\*a\*\*2\*c\*\*3\*d/5 + 2\*a\*b\*c\*\*4/5)

**GIAC/XCAS [A]** time = 0.209819, size = 234, normalized size = 1.52

$$\frac{1}{25}b^2d^4x^{25} + \frac{4}{21}b^2cd^3x^{21} + \frac{2}{21}abd^4x^{21} + \frac{6}{17}b^2c^2d^2x^{17} + \frac{8}{17}abcd^3x^{17} + \frac{1}{17}a^2d^4x^{17} + \frac{4}{13}b^2c^3dx^{13} \\ + \frac{12}{13}abc^2d^2x^{13} + \frac{4}{13}a^2cd^3x^{13} + \frac{1}{9}b^2c^4x^9 + \frac{8}{9}abc^3dx^9 + \frac{2}{3}a^2c^2d^2x^9 + \frac{2}{5}abc^4x^5 + \frac{4}{5}a^2c^3dx^5 + a^2c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^2\*(d\*x^4 + c)^4,x, algorithm="giac")

[Out] 1/25\*b^2\*d^4\*x^25 + 4/21\*b^2\*c\*d^3\*x^21 + 2/21\*a\*b\*d^4\*x^21 + 6/17\*b^2\*c^2\*d^2\*x^17 + 8/17\*a\*b\*c\*d^3\*x^17 + 1/17\*a^2\*d^4\*x^17 + 4/13\*b^2\*c^3\*d\*x^13 + 12/13\*a\*b\*c^2\*d^2\*x^13 + 4/13\*a^2\*c\*d^3\*x^13 + 1/9\*b^2\*c^4\*x^9 + 8/9\*a\*b\*c^3\*d\*x^9 + 2/3\*a^2\*c^2\*d^2\*x^9 + 2/5\*a\*b\*c^4\*x^5 + 4/5\*a^2\*c^3\*d\*x^5 + a^2\*c^4\*x

### 3.55 $\int (a + bx^4)^2 (c + dx^4)^3 dx$

**Optimal.** Leaf size=122

$$\frac{1}{13}dx^{13} (a^2d^2 + 6abcd + 3b^2c^2) + \frac{1}{9}cx^9 (3a^2d^2 + 6abcd + b^2c^2) + a^2c^3x + \frac{1}{5}ac^2x^5(3ad + 2bc) + \frac{1}{17}bd^2x^{17}(2ad + 3bc) + \frac{1}{21}b^2d^3x^{21}$$

[Out]  $a^2c^3x + (a^2c^2(2bc + 3ad)x^5)/5 + (c(b^2c^2 + 6abc^2d + 3a^2d^2)x^9)/9 + (d(3b^2c^2 + 6abc^2d + a^2d^2)x^{13})/13 + (bd^2(3bc + 2ad)x^{17})/17 + (b^2d^3x^{21})/21$

**Rubi [A]** time = 0.172716, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{1}{13}dx^{13} (a^2d^2 + 6abcd + 3b^2c^2) + \frac{1}{9}cx^9 (3a^2d^2 + 6abcd + b^2c^2) + a^2c^3x + \frac{1}{5}ac^2x^5(3ad + 2bc) + \frac{1}{17}bd^2x^{17}(2ad + 3bc) + \frac{1}{21}b^2d^3x^{21}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^2\*(c + d\*x^4)^3,x]

[Out]  $a^2c^3x + (a^2c^2(2bc + 3ad)x^5)/5 + (c(b^2c^2 + 6abc^2d + 3a^2d^2)x^9)/9 + (d(3b^2c^2 + 6abc^2d + a^2d^2)x^{13})/13 + (bd^2(3bc + 2ad)x^{17})/17 + (b^2d^3x^{21})/21$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{ac^2x^5(3ad + 2bc)}{5} + \frac{b^2d^3x^{21}}{21} + \frac{bd^2x^{17}(2ad + 3bc)}{17} + c^3 \int a^2 dx + \frac{cx^9(3a^2d^2 + 6abcd + b^2c^2)}{9} + \frac{dx^{13}(a^2d^2 + 6abcd + 3b^2c^2)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)\*\*2\*(d\*x\*\*4+c)\*\*3,x)

[Out]  $a^2c^3x + (a^2c^2(2bc + 3ad)x^5)/5 + b^2d^3x^{21}/21 + bd^2x^{17}(2ad + 3bc)/17 + c^3 \text{Integral}(a^2, x) + cx^9(3a^2d^2 + 6abcd + b^2c^2)/9 + dx^{13}(a^2d^2 + 6abcd + 3b^2c^2)/13$

**Mathematica [A]** time = 0.0428412, size = 122, normalized size = 1.

$$\frac{1}{13}dx^{13} (a^2d^2 + 6abcd + 3b^2c^2) + \frac{1}{9}cx^9 (3a^2d^2 + 6abcd + b^2c^2) + a^2c^3x + \frac{1}{5}ac^2x^5(3ad + 2bc) + \frac{1}{17}bd^2x^{17}(2ad + 3bc) + \frac{1}{21}b^2d^3x^{21}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)^2\*(c + d\*x^4)^3,x]

[Out]  $a^2c^3x + (a^2c^2(2bc + 3ad)x^5)/5 + (c(b^2c^2 + 6abc^2d + 3a^2d^2)x^9)/9 + (d(3b^2c^2 + 6abc^2d + a^2d^2)x^{13})/13 + (bd^2(3bc + 2ad)x^{17})/17 + (b^2d^3x^{21})/21$

$$3)/13 + (b^2 d^2 (3 b^2 c + 2 a^2 d) x^{17})/17 + (b^2 d^3 x^{21})/21$$

**Maple [A]** time = 0.002, size = 125, normalized size = 1.

$$\frac{b^2 d^3 x^{21}}{21} + \frac{(2 a b d^3 + 3 b^2 c d^2) x^{17}}{17} + \frac{(a^2 d^3 + 6 a b c d^2 + 3 b^2 c^2 d) x^{13}}{13} + \frac{(3 a^2 c d^2 + 6 a b c^2 d + b^2 c^3) x^9}{9} + \frac{(3 a^2 c^2 d + 2 a b c^3) x^5}{5} + a^2 c^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^2\*(d\*x^4+c)^3,x)

[Out] 1/21\*b^2\*d^3\*x^21+1/17\*(2\*a\*b\*d^3+3\*b^2\*c\*d^2)\*x^17+1/13\*(a^2\*d^3+6\*a\*b\*c\*d^2+3\*b^2\*c^2\*d)\*x^13+1/9\*(3\*a^2\*c\*d^2+6\*a\*b\*c^2\*d+b^2\*c^3)\*x^9+1/5\*(3\*a^2\*c^2\*d+2\*a\*b\*c^3)\*x^5+a^2\*c^3\*x

**Maxima [A]** time = 1.35818, size = 167, normalized size = 1.37

$$\frac{1}{21} b^2 d^3 x^{21} + \frac{1}{17} (3 b^2 c d^2 + 2 a b d^3) x^{17} + \frac{1}{13} (3 b^2 c^2 d + 6 a b c d^2 + a^2 d^3) x^{13} + \frac{1}{9} (b^2 c^3 + 6 a b c^2 d + 3 a^2 c d^2) x^9 + a^2 c^3 x + \frac{1}{5} (2 a b c^3 + 3 a^2 c^2 d) x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^2\*(d\*x^4 + c)^3,x, algorithm="maxima")

[Out] 1/21\*b^2\*d^3\*x^21 + 1/17\*(3\*b^2\*c\*d^2 + 2\*a\*b\*d^3)\*x^17 + 1/13\*(3\*b^2\*c^2\*d + 6\*a\*b\*c\*d^2 + a^2\*d^3)\*x^13 + 1/9\*(b^2\*c^3 + 6\*a\*b\*c^2\*d + 3\*a^2\*c\*d^2)\*x^9 + a^2\*c^3\*x + 1/5\*(2\*a\*b\*c^3 + 3\*a^2\*c^2\*d)\*x^5

**Fricas [A]** time = 0.191059, size = 1, normalized size = 0.01

$$\frac{1}{21} x^{21} d^3 b^2 + \frac{3}{17} x^{17} d^2 c b^2 + \frac{2}{17} x^{17} d^3 b a + \frac{3}{13} x^{13} d c^2 b^2 + \frac{6}{13} x^{13} d^2 c b a + \frac{1}{13} x^{13} d^3 a^2 + \frac{1}{9} x^9 c^3 b^2 + \frac{2}{3} x^9 d c^2 b a + \frac{1}{3} x^9 d^2 c a^2 + \frac{2}{5} x^5 c^3 b a + \frac{3}{5} x^5 d c^2 a^2 + x c^3 a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^2\*(d\*x^4 + c)^3,x, algorithm="fricas")

[Out] 1/21\*x^21\*d^3\*b^2 + 3/17\*x^17\*d^2\*c\*b^2 + 2/17\*x^17\*d^3\*b\*a + 3/13\*x^13\*d\*c^2\*b^2 + 6/13\*x^13\*d^2\*c\*b\*a + 1/13\*x^13\*d^3\*a^2 + 1/9\*x^9\*c^3\*b^2 + 2/3\*x^9\*d\*c^2\*b\*a + 1/3\*x^9\*d^2\*c\*a^2 + 2/5\*x^5\*c^3\*b\*a + 3/5\*x^5\*d\*c^2\*a^2 + x\*c^3\*a^2

**Sympy [A]** time = 0.156153, size = 139, normalized size = 1.14

$$a^2 c^3 x + \frac{b^2 d^3 x^{21}}{21} + x^{17} \left( \frac{2 a b d^3}{17} + \frac{3 b^2 c d^2}{17} \right) + x^{13} \left( \frac{a^2 d^3}{13} + \frac{6 a b c d^2}{13} + \frac{3 b^2 c^2 d}{13} \right) + x^9 \left( \frac{a^2 c d^2}{3} + \frac{2 a b c^2 d}{3} + \frac{b^2 c^3}{9} \right) + x^5 \left( \frac{3 a^2 c^2 d}{5} + \frac{2 a b c^3}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*2\*(d\*x\*\*4+c)\*\*3,x)

[Out] a\*\*2\*c\*\*3\*x + b\*\*2\*d\*\*3\*x\*\*21/21 + x\*\*17\*(2\*a\*b\*d\*\*3/17 + 3\*b\*\*2\*c\*d\*\*2/17) + x\*\*13\*(a\*\*2\*d\*\*3/13 + 6\*a\*b\*c\*d\*\*2/13 + 3\*b\*\*2\*c\*\*2\*d/13) + x\*\*9\*(a\*\*2\*c\*d\*\*2/3 + 2\*a\*b\*c\*\*2\*d/3 + b\*\*2\*c\*\*3/9) + x\*\*5\*(3\*a\*\*2\*c\*\*2\*d/5 + 2\*a\*b\*c\*\*3/5)

---

**GIAC/XCAS [A]** time = 0.212041, size = 178, normalized size = 1.46

$$\frac{1}{21} b^2 d^3 x^{21} + \frac{3}{17} b^2 c d^2 x^{17} + \frac{2}{17} a b d^3 x^{17} + \frac{3}{13} b^2 c^2 d x^{13} + \frac{6}{13} a b c d^2 x^{13} + \frac{1}{13} a^2 d^3 x^{13} + \frac{1}{9} b^2 c^3 x^9 + \frac{2}{3} a b c^2 d x^9 + \frac{1}{3} a^2 c d^2 x^9 + \frac{2}{5} a b c^3 x^5 + \frac{3}{5} a^2 c^2 d x^5 + a^2 c^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^2\*(d\*x^4 + c)^3,x, algorithm="giac")

[Out] 1/21\*b^2\*d^3\*x^21 + 3/17\*b^2\*c\*d^2\*x^17 + 2/17\*a\*b\*d^3\*x^17 + 3/13\*b^2\*c^2\*d\*x^13 + 6/13\*a\*b\*c\*d^2\*x^13 + 1/13\*a^2\*d^3\*x^13 + 1/9\*b^2\*c^3\*x^9 + 2/3\*a\*b\*c^2\*d\*x^9 + 1/3\*a^2\*c\*d^2\*x^9 + 2/5\*a\*b\*c^3\*x^5 + 3/5\*a^2\*c^2\*d\*x^5 + a^2\*c^3\*x



### 3.56 $\int (a + bx^4)^2 (c + dx^4)^2 dx$

**Optimal.** Leaf size=82

$$\frac{1}{9}x^9 (a^2d^2 + 4abcd + b^2c^2) + a^2c^2x + \frac{2}{13}bdx^{13}(ad + bc) + \frac{2}{5}acx^5(ad + bc) + \frac{1}{17}b^2d^2x^{17}$$

[Out]  $a^2c^2x + (2ac(b^2c + a^2d)x^5)/5 + ((b^2c^2 + 4ab^2cd + a^2d^2)x^9)/9 + (2bd^2(b^2c + a^2d)x^{13})/13 + (b^2d^2x^{17})/17$

**Rubi [A]** time = 0.118699, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{1}{9}x^9 (a^2d^2 + 4abcd + b^2c^2) + a^2c^2x + \frac{2}{13}bdx^{13}(ad + bc) + \frac{2}{5}acx^5(ad + bc) + \frac{1}{17}b^2d^2x^{17}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^2\*(c + d\*x^4)^2, x]

[Out]  $a^2c^2x + (2ac(b^2c + a^2d)x^5)/5 + ((b^2c^2 + 4ab^2cd + a^2d^2)x^9)/9 + (2bd^2(b^2c + a^2d)x^{13})/13 + (b^2d^2x^{17})/17$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2acx^5(ad + bc)}{5} + \frac{b^2d^2x^{17}}{17} + \frac{2bdx^{13}(ad + bc)}{13} + c^2 \int a^2 dx + x^9 \left( \frac{a^2d^2}{9} + \frac{4abcd}{9} + \frac{b^2c^2}{9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)\*\*2\*(d\*x\*\*4+c)\*\*2, x)

[Out]  $2acx^5(a^2d + b^2c)/5 + b^2d^2x^{17}/17 + 2bd^2x^{13}(a^2d + b^2c)/13 + c^2 \int a^2 dx + x^9(a^2d^2/9 + 4ab^2cd/9 + b^2c^2/9)$

**Mathematica [A]** time = 0.0316722, size = 82, normalized size = 1.

$$\frac{1}{9}x^9 (a^2d^2 + 4abcd + b^2c^2) + a^2c^2x + \frac{2}{13}bdx^{13}(ad + bc) + \frac{2}{5}acx^5(ad + bc) + \frac{1}{17}b^2d^2x^{17}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)^2\*(c + d\*x^4)^2, x]

[Out]  $a^2c^2x + (2ac(b^2c + a^2d)x^5)/5 + ((b^2c^2 + 4ab^2cd + a^2d^2)x^9)/9 + (2bd^2(b^2c + a^2d)x^{13})/13 + (b^2d^2x^{17})/17$

**Maple [A]** time = 0.001, size = 87, normalized size = 1.1

$$\frac{b^2d^2x^{17}}{17} + \frac{(2abd^2 + 2b^2cd)x^{13}}{13} + \frac{(a^2d^2 + 4cabd + b^2c^2)x^9}{9} + \frac{(2a^2cd + 2abc^2)x^5}{5} + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+a)^2*(d*x^4+c)^2,x)`

[Out]  $\frac{1}{17}b^2d^2x^{17} + \frac{1}{13}(2ab^2d + 2b^2cd)x^{13} + \frac{1}{9}(a^2d^2 + 4a^2b^2cd + 4a^2b^2c^2d)x^9 + \frac{1}{5}(2a^2c^2d + 2a^2b^2c^2d)x^5 + a^2c^2x$

**Maxima [A]** time = 1.37829, size = 111, normalized size = 1.35

$$\frac{1}{17}b^2d^2x^{17} + \frac{2}{13}(b^2cd + abd^2)x^{13} + \frac{1}{9}(b^2c^2 + 4abcd + a^2d^2)x^9 + \frac{2}{5}(abc^2 + a^2cd)x^5 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^2*(d*x^4 + c)^2,x, algorithm="maxima")`

[Out]  $\frac{1}{17}b^2d^2x^{17} + \frac{2}{13}(b^2cd + a^2bd^2)x^{13} + \frac{1}{9}(b^2c^2 + 4a^2b^2cd + a^2d^2)x^9 + \frac{2}{5}(a^2b^2c^2 + a^2c^2d)x^5 + a^2c^2x$

**Fricas [A]** time = 0.190083, size = 1, normalized size = 0.01

$$\frac{1}{17}x^{17}d^2b^2 + \frac{2}{13}x^{13}dcb^2 + \frac{2}{13}x^{13}d^2ba + \frac{1}{9}x^9c^2b^2 + \frac{4}{9}x^9dcba + \frac{1}{9}x^9d^2a^2 + \frac{2}{5}x^5c^2ba + \frac{2}{5}x^5dca^2 + xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^2*(d*x^4 + c)^2,x, algorithm="fricas")`

[Out]  $\frac{1}{17}x^{17}d^2b^2 + \frac{2}{13}x^{13}d^2c^2b^2 + \frac{2}{13}x^{13}d^2b^2a + \frac{1}{9}x^9c^2b^2 + \frac{4}{9}x^9d^2c^2ba + \frac{1}{9}x^9d^2a^2 + \frac{2}{5}x^5c^2ba + \frac{2}{5}x^5d^2ca^2 + xc^2a^2$

**Sympy [A]** time = 0.138062, size = 97, normalized size = 1.18

$$a^2c^2x + \frac{b^2d^2x^{17}}{17} + x^{13}\left(\frac{2abd^2}{13} + \frac{2b^2cd}{13}\right) + x^9\left(\frac{a^2d^2}{9} + \frac{4abcd}{9} + \frac{b^2c^2}{9}\right) + x^5\left(\frac{2a^2cd}{5} + \frac{2abc^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**4+a)**2*(d*x**4+c)**2,x)`

[Out]  $a^2c^2x + b^2d^2x^{17}/17 + x^{13}(2ab^2d^2/13 + 2b^2cd^2/13) + x^9(a^2d^2/9 + 4a^2b^2cd/9 + b^2c^2d/9) + x^5(2a^2cd/5 + 2a^2b^2c^2/5)$

**GIAC/XCAS [A]** time = 0.209447, size = 123, normalized size = 1.5

$$\frac{1}{17}b^2d^2x^{17} + \frac{2}{13}b^2cdx^{13} + \frac{2}{13}abd^2x^{13} + \frac{1}{9}b^2c^2x^9 + \frac{4}{9}abcdx^9 + \frac{1}{9}a^2d^2x^9 + \frac{2}{5}abc^2x^5 + \frac{2}{5}a^2cdx^5 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^2*(d*x^4 + c)^2,x, algorithm="giac")`

[Out]  $\frac{1}{17}b^2d^2x^{17} + \frac{2}{13}b^2cd^2x^{13} + \frac{2}{13}a^2bd^2x^{13} + \frac{1}{9}b^2c^2x^9 + \frac{4}{9}a^2b^2cdx^9 + \frac{1}{9}a^2d^2x^9 + \frac{2}{5}a^2b^2c^2x^5 + \frac{2}{5}a^2cdx^5 + a^2c^2x$

$$3.57 \quad \int (a + bx^4)^2 (c + dx^4) dx$$

**Optimal.** Leaf size=50

$$a^2cx + \frac{1}{9}bx^9(2ad + bc) + \frac{1}{5}ax^5(ad + 2bc) + \frac{1}{13}b^2dx^{13}$$

[Out]  $a^2c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^{13})/13$

**Rubi [A]** time = 0.0734854, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$a^2cx + \frac{1}{9}bx^9(2ad + bc) + \frac{1}{5}ax^5(ad + 2bc) + \frac{1}{13}b^2dx^{13}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^2\*(c + d\*x^4), x]

[Out]  $a^2c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^{13})/13$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$a^2 \int c dx + \frac{ax^5(ad + 2bc)}{5} + \frac{b^2dx^{13}}{13} + \frac{bx^9(2ad + bc)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)\*\*2\*(d\*x\*\*4+c), x)

[Out]  $a**2*Integral(c, x) + a*x**5*(a*d + 2*b*c)/5 + b**2*d*x**13/13 + b*x**9*(2*a*d + b*c)/9$

**Mathematica [A]** time = 0.0136399, size = 50, normalized size = 1.

$$a^2cx + \frac{1}{9}bx^9(2ad + bc) + \frac{1}{5}ax^5(ad + 2bc) + \frac{1}{13}b^2dx^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)^2\*(c + d\*x^4), x]

[Out]  $a^2c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^{13})/13$

**Maple [A]** time = 0.002, size = 49, normalized size = 1.

$$\frac{b^2dx^{13}}{13} + \frac{(2abd + b^2c)x^9}{9} + \frac{(a^2d + 2abc)x^5}{5} + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^2\*(d\*x^4+c), x)

[Out]  $\frac{1}{13}b^2d^2x^{13} + \frac{1}{9}(2ab^2d + b^2c)x^9 + \frac{1}{5}(a^2d + 2abc)x^5 + a^2cx$

**Maxima [A]** time = 1.39505, size = 65, normalized size = 1.3

$$\frac{1}{13}b^2dx^{13} + \frac{1}{9}(b^2c + 2abd)x^9 + \frac{1}{5}(2abc + a^2d)x^5 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^2*(d*x^4 + c),x, algorithm="maxima")`

[Out]  $\frac{1}{13}b^2d^2x^{13} + \frac{1}{9}(b^2c + 2ab^2d)x^9 + \frac{1}{5}(2abc + a^2d)x^5 + a^2cx$

**Fricas [A]** time = 0.192196, size = 1, normalized size = 0.02

$$\frac{1}{13}x^{13}db^2 + \frac{1}{9}x^9cb^2 + \frac{2}{9}x^9dba + \frac{2}{5}x^5cba + \frac{1}{5}x^5da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^2*(d*x^4 + c),x, algorithm="fricas")`

[Out]  $\frac{1}{13}x^{13}d^2b^2 + \frac{1}{9}x^9c^2b^2 + \frac{2}{9}x^9d^2b^2a + \frac{2}{5}x^5c^2b^2a + \frac{1}{5}x^5d^2a^2 + x^2c^2a^2$

**Sympy [A]** time = 0.107578, size = 53, normalized size = 1.06

$$a^2cx + \frac{b^2dx^{13}}{13} + x^9\left(\frac{2abd}{9} + \frac{b^2c}{9}\right) + x^5\left(\frac{a^2d}{5} + \frac{2abc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**4+a)**2*(d*x**4+c),x)`

[Out]  $a^2cx + b^2d^2x^{13}/13 + x^9(2ab^2d/9 + b^2c/9) + x^5(a^2d/5 + 2abc/5)$

**GIAC/XCAS [A]** time = 0.214651, size = 68, normalized size = 1.36

$$\frac{1}{13}b^2dx^{13} + \frac{1}{9}b^2cx^9 + \frac{2}{9}abdx^9 + \frac{2}{5}abcx^5 + \frac{1}{5}a^2dx^5 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^2*(d*x^4 + c),x, algorithm="giac")`

[Out]  $\frac{1}{13}b^2d^2x^{13} + \frac{1}{9}b^2c^2x^9 + \frac{2}{9}a^2b^2d^2x^9 + \frac{2}{5}a^2b^2c^2x^5 + \frac{1}{5}a^2d^2x^5 + a^2c^2x$

$$3.58 \quad \int \frac{(a+bx^4)^2}{c+dx^4} dx$$

**Optimal.** Leaf size=253

$$\begin{aligned} & -\frac{(bc-ad)^2 \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{4\sqrt{2}c^{3/4}d^{9/4}} + \frac{(bc-ad)^2 \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{4\sqrt{2}c^{3/4}d^{9/4}} \\ & -\frac{(bc-ad)^2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}} + \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}} + 1\right)}{2\sqrt{2}c^{3/4}d^{9/4}} - \frac{bx(bc-2ad)}{d^2} + \frac{b^2x^5}{5d} \end{aligned}$$

[Out]  $-\left(\frac{b(b^2c - 2ad)x}{d^2} + \frac{b^2x^5}{5d}\right) - \left(\frac{(b^2c - a^2d)^2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}d^{1/4}x}{c^{1/4}}\right]}{2\sqrt{2}c^{3/4}d^{9/4}} + \frac{(b^2c - a^2d)^2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}d^{1/4}x}{c^{1/4}}\right]}{2\sqrt{2}c^{3/4}d^{9/4}} - \frac{(b^2c - a^2d)^2 \operatorname{Log}\left[\sqrt{c} - \sqrt{2}d^{1/4}x + \sqrt{d}x^2\right]}{4\sqrt{2}c^{3/4}d^{9/4}} + \frac{(b^2c - a^2d)^2 \operatorname{Log}\left[\sqrt{c} + \sqrt{2}d^{1/4}x + \sqrt{d}x^2\right]}{4\sqrt{2}c^{3/4}d^{9/4}}\right)$

**Rubi [A]** time = 0.414989, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$\begin{aligned} & -\frac{(bc-ad)^2 \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{4\sqrt{2}c^{3/4}d^{9/4}} + \frac{(bc-ad)^2 \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{4\sqrt{2}c^{3/4}d^{9/4}} \\ & -\frac{(bc-ad)^2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}} + \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}} + 1\right)}{2\sqrt{2}c^{3/4}d^{9/4}} - \frac{bx(bc-2ad)}{d^2} + \frac{b^2x^5}{5d} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{(a + b^2x^4)^2}{(c + dx^4)}, x\right]$

[Out]  $-\left(\frac{b(b^2c - 2ad)x}{d^2} + \frac{b^2x^5}{5d}\right) - \left(\frac{(b^2c - a^2d)^2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}d^{1/4}x}{c^{1/4}}\right]}{2\sqrt{2}c^{3/4}d^{9/4}} + \frac{(b^2c - a^2d)^2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}d^{1/4}x}{c^{1/4}}\right]}{2\sqrt{2}c^{3/4}d^{9/4}} - \frac{(b^2c - a^2d)^2 \operatorname{Log}\left[\sqrt{c} - \sqrt{2}d^{1/4}x + \sqrt{d}x^2\right]}{4\sqrt{2}c^{3/4}d^{9/4}} + \frac{(b^2c - a^2d)^2 \operatorname{Log}\left[\sqrt{c} + \sqrt{2}d^{1/4}x + \sqrt{d}x^2\right]}{4\sqrt{2}c^{3/4}d^{9/4}}\right)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & \frac{b^2x^5}{5d} + \frac{(2ad - bc) \int b dx}{d^2} - \frac{\sqrt{2}(ad - bc)^2 \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{8c^{3/4}d^{9/4}} \\ & + \frac{\sqrt{2}(ad - bc)^2 \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{8c^{3/4}d^{9/4}} \\ & - \frac{\sqrt{2}(ad - bc)^2 \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{4c^{3/4}d^{9/4}} + \frac{\sqrt{2}(ad - bc)^2 \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{4c^{3/4}d^{9/4}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}\left(\frac{(b^2x^4+a)^2}{(d^2x^4+c)}, x\right)$

[Out]  $b^2x^5/(5d) + (2ad - b^2c) \operatorname{Integral}(b, x)/d^2 - \sqrt{2} \left( \frac{(ad - b^2c)^2 \log(-\sqrt{2}c^{1/4}d^{1/4}x + \sqrt{c}) + \sqrt{2}d^{9/4}x^2}{8c^{3/4}d^{9/4}} + \sqrt{2} \left( \frac{(ad - b^2c)^2 \log(\sqrt{2}c^{1/4}d^{1/4}x + \sqrt{c})}{8c^{3/4}d^{9/4}} + \frac{(ad - b^2c)^2 \log(\sqrt{2}c^{1/4}d^{1/4}x + \sqrt{c})}{8c^{3/4}d^{9/4}} \right) \right)$

$$c^{** (1/4)} * d^{** (1/4)} * x + \text{sqrt}(c) + \text{sqrt}(d) * x^{** 2}) / (8 * c^{** (3/4)} * d^{** (9/4)}) - \text{sqrt}(2) * (a * d - b * c)^{** 2} * \text{atan}(1 - \text{sqrt}(2) * d^{** (1/4)} * x / c^{** (1/4)}) / (4 * c^{** (3/4)} * d^{** (9/4)}) + \text{sqrt}(2) * (a * d - b * c)^{** 2} * \text{atan}(1 + \text{sqrt}(2) * d^{** (1/4)} * x / c^{** (1/4)}) / (4 * c^{** (3/4)} * d^{** (9/4)})$$

**Mathematica [A]** time = 0.192653, size = 231, normalized size = 0.91

$$-40bc^{3/4}\sqrt[4]{dx}(bc - 2ad) - 5\sqrt{2}(bc - ad)^2 \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right) + 5\sqrt{2}(bc - ad)^2 \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right) -$$

$$40c^{3/4}d^{9/4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)^2/(c + d\*x^4), x]

[Out] (-40\*b\*c^(3/4)\*d^(1/4)\*(b\*c - 2\*a\*d)\*x + 8\*b^2\*c^(3/4)\*d^(5/4)\*x^5 - 10\*Sqrt[2]\*(b\*c - a\*d)^2\*ArcTan[1 - (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)] + 10\*Sqrt[2]\*(b\*c - a\*d)^2\*ArcTan[1 + (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)] - 5\*Sqrt[2]\*(b\*c - a\*d)^2\*Log[Sqrt[c] - Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2] + 5\*Sqrt[2]\*(b\*c - a\*d)^2\*Log[Sqrt[c] + Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2])/(40\*c^(3/4)\*d^(9/4))

**Maple [B]** time = 0.002, size = 436, normalized size = 1.7

$$\begin{aligned} & \frac{b^2x^5}{5d} + 2\frac{abx}{d} - \frac{b^2xc}{d^2} + \frac{\sqrt{2}a^2}{4c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}} + 1\right) - \frac{\sqrt{2}ab}{2d}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}} + 1\right) \\ & + \frac{c\sqrt{2}b^2}{4d^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}} + 1\right) + \frac{\sqrt{2}a^2}{4c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}} - 1\right) \\ & - \frac{\sqrt{2}ab}{2d}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}} - 1\right) + \frac{c\sqrt{2}b^2}{4d^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}} - 1\right) \\ & + \frac{\sqrt{2}a^2}{8c}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2 + \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)\left(x^2 - \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)^{-1}\right) \\ & - \frac{\sqrt{2}ab}{4d}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2 + \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)\left(x^2 - \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)^{-1}\right) \\ & + \frac{c\sqrt{2}b^2}{8d^2}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2 + \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)\left(x^2 - \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)^{-1}\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^2/(d\*x^4+c), x)

[Out] 1/5\*b^2\*x^5/d+2\*b/d\*a\*x-b^2/d^2\*x\*c+1/4\*(c/d)^(1/4)/c^2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x+1)\*a^2-1/2/d\*(c/d)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x+1)\*a\*b+1/4/d^2\*(c/d)^(1/4)\*c^2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x+1)\*b^2+1/4\*(c/d)^(1/4)/c^2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x-1)\*a^2-1/2/d\*(c/d)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x-1)\*a\*b+1/4/d^2\*(c/d)^(1/4)\*c^2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x-1)\*b^2+1/8\*(c/d)^(1/4)/c^2^(1/2)\*ln((x^2+(c/d)^(1/4)\*x\*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)\*x\*2^(1/2)+(c/d)^(1/2))) \* a^2-1/4/d\*(c/d)^(1/4)\*2^(1/2)\*ln((x^2+(c/d)^(1/4)\*x\*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)\*x\*2^(1/2)+(c/d)^(1/2))) \* a\*b+1/8/d^2\*(c/d)^(1/4)\*c^2^(1/2)\*ln((x^2+(c/d)^(1/4)\*x\*2^(1/2)+(c/d)^(1/2))

$) / (x^2 - (c/d)^{1/4} * x * 2^{1/2} + (c/d)^{1/2})) * b^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^2/(d\*x^4 + c), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.240962, size = 1457, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^2/(d\*x^4 + c), x, algorithm="fricas")

[Out] 
$$\frac{1}{20} (4b^2d^5x^5 - 20d^2(-b^8c^8 - 8ab^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7b^1c^1d^7 + a^8d^8) / (c^3d^9))^{1/4} \arctan\left(\frac{c^2d^2(-b^8c^8 - 8ab^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7b^1c^1d^7 + a^8d^8) / (c^3d^9))^{1/4}}{(b^2c^2 - 2abc^1d + a^2d^2)x + (b^2c^2 - 2abc^1d + a^2d^2)}\right) \sqrt{(c^2d^4 \sqrt{-b^8c^8 - 8ab^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7b^1c^1d^7 + a^8d^8} / (c^3d^9)) + (b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3b^1c^1d^3 + a^4d^4)x^2} / (b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3b^1c^1d^3 + a^4d^4)) + 5d^2(-b^8c^8 - 8ab^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7b^1c^1d^7 + a^8d^8) / (c^3d^9))^{1/4} \log\left(\frac{c^2d^2(-b^8c^8 - 8ab^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7b^1c^1d^7 + a^8d^8) / (c^3d^9))^{1/4}}{-c^2d^2(-b^8c^8 - 8ab^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7b^1c^1d^7 + a^8d^8) / (c^3d^9))^{1/4}}\right) + (b^2c^2 - 2abc^1d + a^2d^2)x - 5d^2(-b^8c^8 - 8ab^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7b^1c^1d^7 + a^8d^8) / (c^3d^9))^{1/4} \log(-c^2d^2(-b^8c^8 - 8ab^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7b^1c^1d^7 + a^8d^8) / (c^3d^9))^{1/4}} + (b^2c^2 - 2abc^1d + a^2d^2)x - 20(b^2c - 2ab^1d)x / d^2$$

**Sympy [A]** time = 3.58949, size = 187, normalized size = 0.74

$$\frac{b^2x^5}{5d} + \text{RootSum}\left(\frac{256t^4c^3d^9 + a^8d^8 - 8a^7bcd^7 + 28a^6b^2c^2d^6 - 56a^5b^3c^3d^5 + 70a^4b^4c^4d^4 - 56a^3b^5c^5d^3 + 28a^2b^6c^6d^2 - 8ab^7c^7d + b^8d^8}{d^2}\right) + \frac{x(2abd - b^2c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*2/(d\*x\*\*4+c), x)

[Out]  $b^2 x^5 / (5d) + \text{RootSum}(256 t^4 c^3 d^9 + a^8 d^8 - 8 a^7 b c d^7 + 28 a^6 b^2 c^2 d^6 - 56 a^5 b^3 c^3 d^5 + 70 a^4 b^4 c^4 d^4 - 56 a^3 b^5 c^5 d^3 + 28 a^2 b^6 c^6 d^2 - 8 a b^7 c^7 d + b^8 c^8, \text{Lambda}(t, t \log(4 t c d^2 / (a^2 d^2 - 2 a b c d + b^2 c^2) + x))) + x^2 (2 a b d - b^2 c) / d^2$

**GIAC/XCAS [A]** time = 0.220611, size = 477, normalized size = 1.89

$$\frac{\sqrt{2} \left( (cd^3)^{\frac{1}{4}} b^2 c^2 - 2 (cd^3)^{\frac{1}{4}} abcd + (cd^3)^{\frac{1}{4}} a^2 d^2 \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{c}{d} \right)^{\frac{1}{4}}} \right)}{4 cd^3} + \frac{\sqrt{2} \left( (cd^3)^{\frac{1}{4}} b^2 c^2 - 2 (cd^3)^{\frac{1}{4}} abcd + (cd^3)^{\frac{1}{4}} a^2 d^2 \right) \arctan \left( \frac{\sqrt{2} \left( 2x - \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{c}{d} \right)^{\frac{1}{4}}} \right)}{4 cd^3} + \frac{\sqrt{2} \left( (cd^3)^{\frac{1}{4}} b^2 c^2 - 2 (cd^3)^{\frac{1}{4}} abcd + (cd^3)^{\frac{1}{4}} a^2 d^2 \right) \ln \left( x^2 + \sqrt{2} x \left( \frac{c}{d} \right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}} \right)}{8 cd^3} - \frac{\sqrt{2} \left( (cd^3)^{\frac{1}{4}} b^2 c^2 - 2 (cd^3)^{\frac{1}{4}} abcd + (cd^3)^{\frac{1}{4}} a^2 d^2 \right) \ln \left( x^2 - \sqrt{2} x \left( \frac{c}{d} \right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}} \right)}{8 cd^3} + \frac{b^2 d^4 x^5 - 5 b^2 c d^3 x + 10 a b d^4 x}{5 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^2/(d*x^4 + c),x, algorithm="giac")`

[Out]  $\frac{1}{4} \sqrt{2} \left( (c^3 d)^{\frac{1}{4}} b^2 c^2 - 2 (c^3 d)^{\frac{1}{4}} a b c d + (c^3 d)^{\frac{1}{4}} a^2 d^2 \right) \arctan \left( \frac{1}{2} \sqrt{2} \left( 2x + \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right) \right) / (c/d)^{\frac{1}{4}} / (c^3 d) + \frac{1}{4} \sqrt{2} \left( (c^3 d)^{\frac{1}{4}} b^2 c^2 - 2 (c^3 d)^{\frac{1}{4}} a b c d + (c^3 d)^{\frac{1}{4}} a^2 d^2 \right) \arctan \left( \frac{1}{2} \sqrt{2} \left( 2x - \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right) \right) / (c/d)^{\frac{1}{4}} / (c^3 d) + \frac{1}{8} \sqrt{2} \left( (c^3 d)^{\frac{1}{4}} b^2 c^2 - 2 (c^3 d)^{\frac{1}{4}} a b c d + (c^3 d)^{\frac{1}{4}} a^2 d^2 \right) \ln \left( x^2 + \sqrt{2} x \left( \frac{c}{d} \right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}} \right) / (c^3 d) - \frac{1}{8} \sqrt{2} \left( (c^3 d)^{\frac{1}{4}} b^2 c^2 - 2 (c^3 d)^{\frac{1}{4}} a b c d + (c^3 d)^{\frac{1}{4}} a^2 d^2 \right) \ln \left( x^2 - \sqrt{2} x \left( \frac{c}{d} \right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}} \right) / (c^3 d) + \frac{1}{5} (b^2 d^4 x^5 - 5 b^2 c d^3 x + 10 a b d^4 x) / d^5$



$$3.59 \quad \int \frac{(a+bx^4)^2}{(c+dx^4)^2} dx$$

**Optimal.** Leaf size=291

$$\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{16\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{16\sqrt{2}c^{7/4}d^{9/4}} + \frac{(bc-ad)(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}d^{9/4}} + \frac{x(bc-ad)^2}{4cd^2(c+dx^4)} + \frac{b^2x}{d^2}$$

[Out]  $(b^2x)/d^2 + ((b^2c - a^2d)^2x)/(4c^2d^2(c + dx^4)) + ((b^2c - a^2d)(5b^2c + 3a^2d) \operatorname{ArcTan}[1 - (\operatorname{Sqrt}[2]d^{1/4}x)/c^{1/4}])/(8\operatorname{Sqrt}[2]c^{7/4}d^{9/4}) - ((b^2c - a^2d)(5b^2c + 3a^2d) \operatorname{ArcTan}[1 + (\operatorname{Sqrt}[2]d^{1/4}x)/c^{1/4}])/(8\operatorname{Sqrt}[2]c^{7/4}d^{9/4}) + ((b^2c - a^2d)(5b^2c + 3a^2d) \operatorname{Log}[\operatorname{Sqrt}[c] - \operatorname{Sqrt}[2]c^{1/4}d^{1/4}x + \operatorname{Sqrt}[d]x^2])/(16\operatorname{Sqrt}[2]c^{7/4}d^{9/4}) - ((b^2c - a^2d)(5b^2c + 3a^2d) \operatorname{Log}[\operatorname{Sqrt}[c] + \operatorname{Sqrt}[2]c^{1/4}d^{1/4}x + \operatorname{Sqrt}[d]x^2])/(16\operatorname{Sqrt}[2]c^{7/4}d^{9/4})$

**Rubi [A]** time = 0.722594, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{16\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{16\sqrt{2}c^{7/4}d^{9/4}} + \frac{(bc-ad)(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}d^{9/4}} + \frac{x(bc-ad)^2}{4cd^2(c+dx^4)} + \frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^2/(c + d\*x^4)^2, x]

[Out]  $(b^2x)/d^2 + ((b^2c - a^2d)^2x)/(4c^2d^2(c + dx^4)) + ((b^2c - a^2d)(5b^2c + 3a^2d) \operatorname{ArcTan}[1 - (\operatorname{Sqrt}[2]d^{1/4}x)/c^{1/4}])/(8\operatorname{Sqrt}[2]c^{7/4}d^{9/4}) - ((b^2c - a^2d)(5b^2c + 3a^2d) \operatorname{ArcTan}[1 + (\operatorname{Sqrt}[2]d^{1/4}x)/c^{1/4}])/(8\operatorname{Sqrt}[2]c^{7/4}d^{9/4}) + ((b^2c - a^2d)(5b^2c + 3a^2d) \operatorname{Log}[\operatorname{Sqrt}[c] - \operatorname{Sqrt}[2]c^{1/4}d^{1/4}x + \operatorname{Sqrt}[d]x^2])/(16\operatorname{Sqrt}[2]c^{7/4}d^{9/4}) - ((b^2c - a^2d)(5b^2c + 3a^2d) \operatorname{Log}[\operatorname{Sqrt}[c] + \operatorname{Sqrt}[2]c^{1/4}d^{1/4}x + \operatorname{Sqrt}[d]x^2])/(16\operatorname{Sqrt}[2]c^{7/4}d^{9/4})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int b^2 dx}{d^2} + \frac{x(ad-bc)^2}{4cd^2(c+dx^4)} - \frac{\sqrt{2}(ad-bc)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{32c^{7/4}d^{9/4}} + \frac{\sqrt{2}(ad-bc)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{32c^{7/4}d^{9/4}} - \frac{\sqrt{2}(ad-bc)(3ad+5bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{16c^{7/4}d^{9/4}} + \frac{\sqrt{2}(ad-bc)(3ad+5bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{16c^{7/4}d^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**4+a)**2/(d*x**4+c)**2,x)`

[Out]  $\text{Integral}(b^{**2}, x)/d^{**2} + x*(a*d - b*c)^{**2}/(4*c*d^{**2}*(c + d*x^{**4})) - \sqrt{2}*(a*d - b*c)*(3*a*d + 5*b*c)*\log(-\sqrt{2}*c^{**1/4}*d^{**1/4}*(1/4)*x + \sqrt{c} + \sqrt{d}*x^{**2})/(32*c^{**7/4}*d^{**9/4}) + \sqrt{2}*(a*d - b*c)*(3*a*d + 5*b*c)*\log(\sqrt{2}*c^{**1/4}*d^{**1/4}*x + \sqrt{c} + \sqrt{d}*x^{**2})/(32*c^{**7/4}*d^{**9/4}) - \sqrt{2}*(a*d - b*c)*(3*a*d + 5*b*c)*\text{atan}(1 - \sqrt{2}*d^{**1/4}*x/c^{**1/4})/(16*c^{**7/4}*d^{**9/4}) + \sqrt{2}*(a*d - b*c)*(3*a*d + 5*b*c)*\text{atan}(1 + \sqrt{2}*d^{**1/4}*x/c^{**1/4})/(16*c^{**7/4}*d^{**9/4})$

**Mathematica [A]** time = 0.282857, size = 298, normalized size = 1.02

$$\frac{\sqrt{2}(-3a^2d^2-2abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{dx^2}\right)}{c^{7/4}} - \frac{\sqrt{2}(-3a^2d^2-2abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{dx^2}\right)}{c^{7/4}} + \frac{2\sqrt{2}(-3a^2d^2-2abcd+5b^2c^2)\tan^{-1}\left(\frac{x\sqrt{2}\sqrt[4]{c}+\sqrt{d}}{\sqrt{c}}\right)}{32d^{9/4}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^4)^2/(c + d*x^4)^2,x]`

[Out]  $(32*b^2*d^{1/4}*x + (8*d^{1/4}*(b*c - a*d)^2*x)/(c*(c + d*x^4)) + (2*\text{Sqrt}[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*d^{1/4}*x)/c^{1/4}])/c^{7/4} - (2*\text{Sqrt}[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*d^{1/4}*x)/c^{1/4}])/c^{7/4} + (\text{Sqrt}[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*\text{Log}[\text{Sqrt}[c] - \text{Sqrt}[2]*c^{1/4}*d^{1/4}*x + \text{Sqrt}[d]*x^2])/c^{7/4} - (\text{Sqrt}[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*\text{Log}[\text{Sqrt}[c] + \text{Sqrt}[2]*c^{1/4}*d^{1/4}*x + \text{Sqrt}[d]*x^2])/c^{7/4})/(32*d^{9/4})$

**Maple [B]** time = 0.002, size = 475, normalized size = 1.6

$$\begin{aligned} & \frac{b^2x}{d^2} + \frac{xa^2}{4c(dx^4+c)} - \frac{axb}{2d(dx^4+c)} + \frac{cxb^2}{4d^2(dx^4+c)} + \frac{3\sqrt{2}a^2\sqrt[4]{c}}{16c^2\sqrt[4]{d}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{c}} - 1\right) \\ & + \frac{\sqrt{2}ab\sqrt[4]{c}}{8cd\sqrt[4]{d}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{c}} - 1\right) - \frac{5\sqrt{2}b^2\sqrt[4]{c}}{16d^2\sqrt[4]{d}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{c}} - 1\right) \\ & + \frac{3\sqrt{2}a^2\sqrt[4]{c}}{32c^2\sqrt[4]{d}} \ln\left(1\left(x^2 + \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)\left(x^2 - \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)^{-1}\right) \\ & + \frac{\sqrt{2}ab\sqrt[4]{c}}{16cd\sqrt[4]{d}} \ln\left(1\left(x^2 + \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)\left(x^2 - \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)^{-1}\right) \\ & - \frac{5\sqrt{2}b^2\sqrt[4]{c}}{32d^2\sqrt[4]{d}} \ln\left(1\left(x^2 + \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)\left(x^2 - \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)^{-1}\right) \\ & + \frac{3\sqrt{2}a^2\sqrt[4]{c}}{16c^2\sqrt[4]{d}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{c}} + 1\right) + \frac{\sqrt{2}ab\sqrt[4]{c}}{8cd\sqrt[4]{d}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{c}} + 1\right) \\ & - \frac{5\sqrt{2}b^2\sqrt[4]{c}}{16d^2\sqrt[4]{d}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{c}} + 1\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+a)^2/(d*x^4+c)^2,x)`

[Out] 
$$\begin{aligned} & b^2 x/d^2 + 1/4 c x/(d x^4 + c) a^2 - 1/2 d x/(d x^4 + c) a b + 1/4 d^2 c x \\ & / (d x^4 + c) b^2 + 3/16 c^2 (c/d)^{1/4} 2^{1/2} \arctan(2^{1/2}/(c/d)^{1/4} \\ & (1/4) x - 1) a^2 + 1/8 d/c (c/d)^{1/4} 2^{1/2} \arctan(2^{1/2}/(c/d)^{1/4} \\ & (1/4) x - 1) a b - 5/16 d^2 (c/d)^{1/4} 2^{1/2} \arctan(2^{1/2}/(c/d)^{1/4} \\ & (1/4) x - 1) b^2 + 3/32 c^2 (c/d)^{1/4} 2^{1/2} \ln((x^2 + (c/d)^{1/4} x^* \\ & 2^{1/2} + (c/d)^{1/2})/(x^2 - (c/d)^{1/4} x^* 2^{1/2} + (c/d)^{1/2})) a^2 \\ & + 1/16 d/c (c/d)^{1/4} 2^{1/2} \ln((x^2 + (c/d)^{1/4} x^* 2^{1/2} + (c/d)^{1/2}) \\ & / (x^2 - (c/d)^{1/4} x^* 2^{1/2} + (c/d)^{1/2})) a b - 5/32 d^2 (c/d)^{1/4} \\ & 2^{1/2} \ln((x^2 + (c/d)^{1/4} x^* 2^{1/2} + (c/d)^{1/2})/(x^2 - (c/d)^{1/4} x^* \\ & 2^{1/2} + (c/d)^{1/2})) b^2 + 3/16 c^2 (c/d)^{1/4} 2^{1/2} \arctan(2^{1/2}/(c/d)^{1/4} \\ & x + 1) a^2 + 1/8 d/c (c/d)^{1/4} 2^{1/2} \arctan(2^{1/2}/(c/d)^{1/4} x + 1) a b \\ & - 5/16 d^2 (c/d)^{1/4} 2^{1/2} \arctan(2^{1/2}/(c/d)^{1/4} x + 1) b^2 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^2/(d*x^4 + c)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.256329, size = 1592, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^2/(d*x^4 + c)^2,x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & 1/16 (16 b^2 c d x^5 - 4 (c d^3 x^4 + c^2 d^2) (-625 b^8 c^8 - 1000 a b^7 c^7 d - 900 a^2 b^6 c^6 d^2 + 1640 a^3 b^5 c^5 d^3 + 646 a^4 b^4 c^4 d^4 - 984 a^5 b^3 c^3 d^5 - 324 a^6 b^2 c^2 d^6 + 216 a^7 b c d^7 + 81 a^8 d^8) / (c^7 d^9))^{1/4} \arctan(-c^2 d^2 (-625 b^8 c^8 - 1000 a b^7 c^7 d - 900 a^2 b^6 c^6 d^2 + 1640 a^3 b^5 c^5 d^3 + 646 a^4 b^4 c^4 d^4 - 984 a^5 b^3 c^3 d^5 - 324 a^6 b^2 c^2 d^6 + 216 a^7 b c d^7 + 81 a^8 d^8) / (c^7 d^9))^{1/4} / ((5 b^2 c^2 - 2 a b c d - 3 a^2 d^2) x + (5 b^2 c^2 - 2 a b c d - 3 a^2 d^2) \sqrt{(c^4 d^4 \sqrt{-625 b^8 c^8 - 1000 a b^7 c^7 d - 900 a^2 b^6 c^6 d^2 + 1640 a^3 b^5 c^5 d^3 + 646 a^4 b^4 c^4 d^4 - 984 a^5 b^3 c^3 d^5 - 324 a^6 b^2 c^2 d^6 + 216 a^7 b c d^7 + 81 a^8 d^8} / (c^7 d^9)) + (25 b^4 c^4 - 20 a b^3 c^3 d - 26 a^2 b^2 c^2 d^2 + 12 a^3 b c d^3 + 9 a^4 d^4) x^2) / (25 b^4 c^4 - 20 a b^3 c^3 d - 26 a^2 b^2 c^2 d^2 + 12 a^3 b c d^3 + 9 a^4 d^4)) + (c d^3 x^4 + c^2 d^2) (-625 b^8 c^8 - 1000 a b^7 c^7 d - 900 a^2 b^6 c^6 d^2 + 1640 a^3 b^5 c^5 d^3 + 646 a^4 b^4 c^4 d^4 - 984 a^5 b^3 c^3 d^5 - 324 a^6 b^2 c^2 d^6 + 216 a^7 b c d^7 + 81 a^8 d^8) / (c^7 d^9)^{1/4} \log(c^2 d^2 (-625 b^8 c^8 - 1000 a b^7 c^7 d - 900 a^2 b^6 c^6 d^2 + 1640 a^3 b^5 c^5 d^3 + 646 a^4 b^4 c^4 d^4 - 984 a^5 b^3 c^3 d^5 - 324 a^6 b^2 c^2 d^6 + 216 a^7 b c d^7 + 81 a^8 d^8) / (c^7 d^9))^{1/4} - (5 b^2 c^2 - 2 a b c d - 3 a^2 d^2) x - (c d^3 x^4 + c^2 d^2) (-625 b^8 c^8 - 1000 a b^7 c^7 d - 900 a^2 b^6 c^6 d^2 + 1640 a^3 b^5 c^5 d^3 + 646 a^4 b^4 c^4 d^4 - 984 a^5 b^3 c^3 d^5 - 324 a^6 b^2 c^2 d^6 + 216 a^7 b c d^7 + 81 a^8 d^8) / (c^7 d^9))^{1/4} \log(-c^2 d^2 (-625 b^8 c^8 - 1000 a b^7 c^7 d - 900 a^2 b^6 c^6 d^2 + 1640 a^3 b^5 c^5 d^3 + 646 a^4 b^4 c^4 d^4 - 984 a^5 b^3 c^3 d^5 - 324 a^6 b^2 c^2 d^6 + 216 a^7 b c d^7 + 81 a^8 d^8) / (c^7 d^9))^{1/4} - (5 b^2 c^2 - 2 a b c d - 3 a^2 d^2) x + 4 (5 b^2 c^2 - 2 a b c d + a^2 d^2) x / (c d^3 x^4 + c^2 d^2) \end{aligned}$$

**Sympy [A]** time = 6.90248, size = 219, normalized size = 0.75

$$\frac{b^2x}{d^2} + \frac{x(a^2d^2 - 2abcd + b^2c^2)}{4c^2d^2 + 4cd^3x^4} + \text{RootSum}\left(65536t^4c^7d^9 + 81a^8d^8 + 216a^7bcd^7 - 324a^6b^2c^2d^6 - 984a^5b^3c^3d^5 + 646a^4b^4c^4d^4 + 1640a^3b^5c^5d^3 - 900a^2b^6c^6d^2 - 1000ab^7c^7d + 625b^8c^8, \text{Lambda}(\_t, \_t \log(16\_t c^2 d^2 / (3a^2 d^2 + 2ab^2 c^2) + x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*2/(d\*x\*\*4+c)\*\*2,x)

[Out] b\*\*2\*x/d\*\*2 + x\*(a\*\*2\*d\*\*2 - 2\*a\*b\*c\*d + b\*\*2\*c\*\*2)/(4\*c\*\*2\*d\*\*2 + 4\*c\*d\*\*3\*x\*\*4) + RootSum(65536\*\_t\*\*4\*c\*\*7\*d\*\*9 + 81\*a\*\*8\*d\*\*8 + 216\*a\*\*7\*b\*c\*d\*\*7 - 324\*a\*\*6\*b\*\*2\*c\*\*2\*d\*\*6 - 984\*a\*\*5\*b\*\*3\*c\*\*3\*d\*\*5 + 646\*a\*\*4\*b\*\*4\*c\*\*4\*d\*\*4 + 1640\*a\*\*3\*b\*\*5\*c\*\*5\*d\*\*3 - 900\*a\*\*2\*b\*\*6\*c\*\*6\*d\*\*2 - 1000\*a\*b\*\*7\*c\*\*7\*d + 625\*b\*\*8\*c\*\*8, Lambda(\_t, \_t\*log(16\*\_t\*c\*\*2\*d\*\*2/(3\*a\*\*2\*d\*\*2 + 2\*a\*b\*\*2\*c\*\*2) + x)))

**GIAC/XCAS [A]** time = 0.222355, size = 508, normalized size = 1.75

$$\frac{b^2x}{d^2} - \frac{\sqrt{2}\left(5(cd^3)^{\frac{1}{4}}b^2c^2 - 2(cd^3)^{\frac{1}{4}}abcd - 3(cd^3)^{\frac{1}{4}}a^2d^2\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16c^2d^3} - \frac{\sqrt{2}\left(5(cd^3)^{\frac{1}{4}}b^2c^2 - 2(cd^3)^{\frac{1}{4}}abcd - 3(cd^3)^{\frac{1}{4}}a^2d^2\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16c^2d^3} - \frac{\sqrt{2}\left(5(cd^3)^{\frac{1}{4}}b^2c^2 - 2(cd^3)^{\frac{1}{4}}abcd - 3(cd^3)^{\frac{1}{4}}a^2d^2\right) \ln\left(x^2 + \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{32c^2d^3} + \frac{\sqrt{2}\left(5(cd^3)^{\frac{1}{4}}b^2c^2 - 2(cd^3)^{\frac{1}{4}}abcd - 3(cd^3)^{\frac{1}{4}}a^2d^2\right) \ln\left(x^2 - \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{32c^2d^3} + \frac{b^2c^2x - 2abcdx + a^2d^2x}{4(dx^4 + c)cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^2/(d\*x^4 + c)^2,x, algorithm="giac")

[Out] b^2\*x/d^2 - 1/16\*sqrt(2)\*(5\*(c\*d^3)^(1/4)\*b^2\*c^2 - 2\*(c\*d^3)^(1/4)\*a\*b\*c\*d - 3\*(c\*d^3)^(1/4)\*a^2\*d^2)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(c/d)^(1/4))/(c/d)^(1/4))/(c^2\*d^3) - 1/16\*sqrt(2)\*(5\*(c\*d^3)^(1/4)\*b^2\*c^2 - 2\*(c\*d^3)^(1/4)\*a\*b\*c\*d - 3\*(c\*d^3)^(1/4)\*a^2\*d^2)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)\*(c/d)^(1/4))/(c/d)^(1/4))/(c^2\*d^3) - 1/32\*sqrt(2)\*(5\*(c\*d^3)^(1/4)\*b^2\*c^2 - 2\*(c\*d^3)^(1/4)\*a\*b\*c\*d - 3\*(c\*d^3)^(1/4)\*a^2\*d^2)\*ln(x^2 + sqrt(2)\*x\*(c/d)^(1/4) + sqrt(c/d))/(c^2\*d^3) + 1/32\*sqrt(2)\*(5\*(c\*d^3)^(1/4)\*b^2\*c^2 - 2\*(c\*d^3)^(1/4)\*a\*b\*c\*d - 3\*(c\*d^3)^(1/4)\*a^2\*d^2)\*ln(x^2 - sqrt(2)\*x\*(c/d)^(1/4) + sqrt(c/d))/(c^2\*d^3) + 1/4\*(b^2\*c^2\*x - 2\*a\*b\*c\*d\*x + a^2\*d^2\*x)/((d\*x^4 + c)\*c\*d^2)

$$3.60 \quad \int \frac{(a+bx^4)^2}{(c+dx^4)^3} dx$$

**Optimal.** Leaf size=349

$$\begin{aligned} & - \frac{(21a^2d^2 + 6abcd + 5b^2c^2) \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{128\sqrt{2}c^{11/4}d^{9/4}} \\ & + \frac{(21a^2d^2 + 6abcd + 5b^2c^2) \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{128\sqrt{2}c^{11/4}d^{9/4}} \\ & - \frac{(21a^2d^2 + 6abcd + 5b^2c^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}} + \frac{(21a^2d^2 + 6abcd + 5b^2c^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}} + 1\right)}{64\sqrt{2}c^{11/4}d^{9/4}} \\ & - \frac{x(bc - ad)(7ad + 5bc)}{32c^2d^2(c + dx^4)} - \frac{x(a + bx^4)(bc - ad)}{8cd(c + dx^4)^2} \end{aligned}$$

[Out]  $-\left((b^*c - a^*d) * x^*(a + b^*x^4)\right) / \left(8^*c^*d^*(c + d^*x^4)^2\right) - \left((b^*c - a^*d) * (5^*b^*c + 7^*a^*d) * x\right) / \left(32^*c^2 * d^2 * (c + d^*x^4)\right) - \left((5^*b^2 * c^2 + 6^*a^*b^*c * d + 21^*a^2 * d^2) * \text{ArcTan}\left[1 - \left(\text{Sqrt}[2] * d^{(1/4)} * x\right) / c^{(1/4)}\right]\right) / \left(64^* \text{Sqrt}[2] * c^{(11/4)} * d^{(9/4)}\right) + \left((5^*b^2 * c^2 + 6^*a^*b^*c * d + 21^*a^2 * d^2) * \text{ArcTan}\left[1 + \left(\text{Sqrt}[2] * d^{(1/4)} * x\right) / c^{(1/4)}\right]\right) / \left(64^* \text{Sqrt}[2] * c^{(11/4)} * d^{(9/4)}\right) - \left((5^*b^2 * c^2 + 6^*a^*b^*c * d + 21^*a^2 * d^2) * \text{Log}\left[\text{Sqrt}[c] - \text{Sqrt}[2] * c^{(1/4)} * d^{(1/4)} * x + \text{Sqrt}[d] * x^2\right]\right) / \left(128^* \text{Sqrt}[2] * c^{(11/4)} * d^{(9/4)}\right) + \left((5^*b^2 * c^2 + 6^*a^*b^*c * d + 21^*a^2 * d^2) * \text{Log}\left[\text{Sqrt}[c] + \text{Sqrt}[2] * c^{(1/4)} * d^{(1/4)} * x + \text{Sqrt}[d] * x^2\right]\right) / \left(128^* \text{Sqrt}[2] * c^{(11/4)} * d^{(9/4)}\right)$

**Rubi [A]** time = 0.549538, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\begin{aligned} & - \frac{(21a^2d^2 + 6abcd + 5b^2c^2) \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{128\sqrt{2}c^{11/4}d^{9/4}} \\ & + \frac{(21a^2d^2 + 6abcd + 5b^2c^2) \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{128\sqrt{2}c^{11/4}d^{9/4}} \\ & - \frac{(21a^2d^2 + 6abcd + 5b^2c^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}} + \frac{(21a^2d^2 + 6abcd + 5b^2c^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}} + 1\right)}{64\sqrt{2}c^{11/4}d^{9/4}} \\ & - \frac{x(bc - ad)(7ad + 5bc)}{32c^2d^2(c + dx^4)} - \frac{x(a + bx^4)(bc - ad)}{8cd(c + dx^4)^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^2/(c + d\*x^4)^3, x]

[Out]  $-\left((b^*c - a^*d) * x^*(a + b^*x^4)\right) / \left(8^*c^*d^*(c + d^*x^4)^2\right) - \left((b^*c - a^*d) * (5^*b^*c + 7^*a^*d) * x\right) / \left(32^*c^2 * d^2 * (c + d^*x^4)\right) - \left((5^*b^2 * c^2 + 6^*a^*b^*c * d + 21^*a^2 * d^2) * \text{ArcTan}\left[1 - \left(\text{Sqrt}[2] * d^{(1/4)} * x\right) / c^{(1/4)}\right]\right) / \left(64^* \text{Sqrt}[2] * c^{(11/4)} * d^{(9/4)}\right) + \left((5^*b^2 * c^2 + 6^*a^*b^*c * d + 21^*a^2 * d^2) * \text{ArcTan}\left[1 + \left(\text{Sqrt}[2] * d^{(1/4)} * x\right) / c^{(1/4)}\right]\right) / \left(64^* \text{Sqrt}[2] * c^{(11/4)} * d^{(9/4)}\right) - \left((5^*b^2 * c^2 + 6^*a^*b^*c * d + 21^*a^2 * d^2) * \text{Log}\left[\text{Sqrt}[c] - \text{Sqrt}[2] * c^{(1/4)} * d^{(1/4)} * x + \text{Sqrt}[d] * x^2\right]\right) / \left(128^* \text{Sqrt}[2] * c^{(11/4)} * d^{(9/4)}\right) + \left((5^*b^2 * c^2 + 6^*a^*b^*c * d + 21^*a^2 * d^2) * \text{Log}\left[\text{Sqrt}[c] + \text{Sqrt}[2] * c^{(1/4)} * d^{(1/4)} * x + \text{Sqrt}[d] * x^2\right]\right) / \left(128^* \text{Sqrt}[2] * c^{(11/4)} * d^{(9/4)}\right)$

**Rubi in Sympy [A]** time = 82.4633, size = 337, normalized size = 0.97

$$\frac{x(a+bx^4)(ad-bc)}{8cd(c+dx^4)^2} + \frac{x(ad-bc)(7ad+5bc)}{32c^2d^2(c+dx^4)}$$

$$\frac{\sqrt{2}(21a^2d^2+6abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{256c^{\frac{11}{4}}d^{\frac{9}{4}}}$$

$$+ \frac{\sqrt{2}(21a^2d^2+6abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)}{256c^{\frac{11}{4}}d^{\frac{9}{4}}}$$

$$- \frac{\sqrt{2}(21a^2d^2+6abcd+5b^2c^2)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{128c^{\frac{11}{4}}d^{\frac{9}{4}}}$$

$$+ \frac{\sqrt{2}(21a^2d^2+6abcd+5b^2c^2)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{128c^{\frac{11}{4}}d^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**4+a)**2/(d*x**4+c)**3,x)`

[Out]  $x(a+b*x^4)(a*d-b*c)/(8*c*d*(c+d*x^4)^2) + x(a*d-b*c)$   
 $(7*a*d+5*b*c)/(32*c^2*d^2*(c+d*x^4)) - \operatorname{sqrt}(2)*(21*a^2*d^2$   
 $+6*a*b*c*d+5*b^2*c^2)*\log(-\operatorname{sqrt}(2)*c^{(1/4)}*d^{(1/4)}*x$   
 $+ \operatorname{sqrt}(c) + \operatorname{sqrt}(d)*x^2)/(256*c^{(11/4)}*d^{(9/4)}) + \operatorname{sqrt}(2)*(21*a^2*d^2$   
 $+6*a*b*c*d+5*b^2*c^2)*\log(\operatorname{sqrt}(2)*c^{(1/4)}*d^{(1/4)}$   
 $*x + \operatorname{sqrt}(c) + \operatorname{sqrt}(d)*x^2)/(256*c^{(11/4)}*d^{(9/4)}) - \operatorname{sqrt}(2)*$   
 $(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*\operatorname{atan}(1-\operatorname{sqrt}(2)*d^{(1/4)}$   
 $*x/c^{(1/4)})/(128*c^{(11/4)}*d^{(9/4)}) + \operatorname{sqrt}(2)*(21*a^2*d^2+6$   
 $*a*b*c*d+5*b^2*c^2)*\operatorname{atan}(1+\operatorname{sqrt}(2)*d^{(1/4)}*x/c^{(1/4)})/(1$   
 $28*c^{(11/4)}*d^{(9/4)})$

**Mathematica [A]** time = 0.320891, size = 319, normalized size = 0.91

$$-\sqrt{2}(21a^2d^2+6abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right) + \sqrt{2}(21a^2d^2+6abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx}+\sqrt{c}+\sqrt{dx^2}\right)$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^4)^2/(c + d*x^4)^3,x]`

[Out]  $((32*c^{(7/4)}*d^{(1/4)}*(b*c-a*d)^2*x)/(c+d*x^4)^2 - (8*c^{(3/4)}*d^{(1/4)}*(9*b^2*c^2 - 2*a*b*c*d - 7*a^2*d^2)*x)/(c+d*x^4) - 2*\operatorname{Sqrt}[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*\operatorname{ArcTan}[1 - (\operatorname{Sqrt}[2]*d^{(1/4)}*x)/c^{(1/4)}] + 2*\operatorname{Sqrt}[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*\operatorname{ArcTan}[1 + (\operatorname{Sqrt}[2]*d^{(1/4)}*x)/c^{(1/4)}] - \operatorname{Sqrt}[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*\operatorname{Log}[\operatorname{Sqrt}[c] - \operatorname{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \operatorname{Sqrt}[d]*x^2] + \operatorname{Sqrt}[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*\operatorname{Log}[\operatorname{Sqrt}[c] + \operatorname{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \operatorname{Sqrt}[d]*x^2)]/(256*c^{(11/4)}*d^{(9/4)})$

**Maple [A]** time = 0.002, size = 499, normalized size = 1.4

$$\begin{aligned} & \frac{1}{(dx^4 + c)^2} \left( \frac{(7a^2d^2 + 2cabd - 9b^2c^2)x^5}{32c^2d} + \frac{(11a^2d^2 - 6cabd - 5b^2c^2)x}{32d^2c} \right) \\ & + \frac{21\sqrt{2}a^2}{128c^3} \sqrt[4]{\frac{c}{d}} \arctan \left( x\sqrt{2} \frac{1}{\sqrt[4]{\frac{c}{d}}} - 1 \right) \\ & + \frac{3\sqrt{2}ab}{64c^2d} \sqrt[4]{\frac{c}{d}} \arctan \left( x\sqrt{2} \frac{1}{\sqrt[4]{\frac{c}{d}}} - 1 \right) + \frac{5\sqrt{2}b^2}{128d^2c} \sqrt[4]{\frac{c}{d}} \arctan \left( x\sqrt{2} \frac{1}{\sqrt[4]{\frac{c}{d}}} - 1 \right) \\ & + \frac{21\sqrt{2}a^2}{256c^3} \sqrt[4]{\frac{c}{d}} \ln \left( 1 \left( x^2 + \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}} \right) \left( x^2 - \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}} \right)^{-1} \right) \\ & + \frac{3\sqrt{2}ab}{128c^2d} \sqrt[4]{\frac{c}{d}} \ln \left( 1 \left( x^2 + \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}} \right) \left( x^2 - \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}} \right)^{-1} \right) \\ & + \frac{5\sqrt{2}b^2}{256d^2c} \sqrt[4]{\frac{c}{d}} \ln \left( 1 \left( x^2 + \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}} \right) \left( x^2 - \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}} \right)^{-1} \right) \\ & + \frac{21\sqrt{2}a^2}{128c^3} \sqrt[4]{\frac{c}{d}} \arctan \left( x\sqrt{2} \frac{1}{\sqrt[4]{\frac{c}{d}}} + 1 \right) \\ & + \frac{3\sqrt{2}ab}{64c^2d} \sqrt[4]{\frac{c}{d}} \arctan \left( x\sqrt{2} \frac{1}{\sqrt[4]{\frac{c}{d}}} + 1 \right) + \frac{5\sqrt{2}b^2}{128d^2c} \sqrt[4]{\frac{c}{d}} \arctan \left( x\sqrt{2} \frac{1}{\sqrt[4]{\frac{c}{d}}} + 1 \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^2/(d\*x^4+c)^3,x)

[Out] (1/32\*(7\*a^2\*d^2+2\*a\*b\*c\*d-9\*b^2\*c^2)/c^2/d\*x^5+1/32\*(11\*a^2\*d^2-6\*a\*b\*c\*d-5\*b^2\*c^2)/d^2/c\*x)/(d\*x^4+c)^2+21/128/c^3\*(c/d)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x-1)\*a^2+3/64/c^2/d\*(c/d)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x-1)\*a\*b+5/128/c/d^2\*(c/d)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x-1)\*b^2+21/256/c^3\*(c/d)^(1/4)\*2^(1/2)\*ln((x^2+(c/d)^(1/4)\*x\*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)\*x\*2^(1/2)+(c/d)^(1/2)))\*a^2+3/128/c^2/d\*(c/d)^(1/4)\*2^(1/2)\*ln((x^2+(c/d)^(1/4)\*x\*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)\*x\*2^(1/2)+(c/d)^(1/2)))\*a\*b+5/256/c/d^2\*(c/d)^(1/4)\*2^(1/2)\*ln((x^2+(c/d)^(1/4)\*x\*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)\*x\*2^(1/2)+(c/d)^(1/2)))\*b^2+21/128/c^3\*(c/d)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x+1)\*a^2+3/64/c^2/d\*(c/d)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x+1)\*a\*b+5/128/c/d^2\*(c/d)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x+1)\*b^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^2/(d\*x^4 + c)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.258428, size = 1692, normalized size = 4.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^2/(d\*x^4 + c)^3,x, algorithm="fricas")

[Out] 
$$-1/128*(4*(9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^5 + 4*(c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^{11}*d^9))^{1/4}*\arctan(c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^{11}*d^9))^{1/4}/((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*x + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*\sqrt{(c^6*d^4*\sqrt{-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^{11}*d^9))} + (25*b^4*c^4 + 60*a*b^3*c^3*d + 246*a^2*b^2*c^2*d^2 + 252*a^3*b*c*d^3 + 441*a^4*d^4)*x^2)/(25*b^4*c^4 + 60*a*b^3*c^3*d + 246*a^2*b^2*c^2*d^2 + 252*a^3*b*c*d^3 + 441*a^4*d^4))) - (c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^{11}*d^9))^{1/4}*\log(c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^{11}*d^9))^{1/4} + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + (c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^{11}*d^9))^{1/4}*\log(-c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^{11}*d^9))^{1/4} + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + 4*(5*b^2*c^3 + 6*a*b*c^2*d - 11*a^2*c*d^2)*x)/(c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)$$

**Sympy [A]** time = 20.5634, size = 264, normalized size = 0.76

$$\frac{x^5(7a^2d^3 + 2abcd^2 - 9b^2c^2d) + x(11a^2cd^2 - 6abc^2d - 5b^2c^3)}{32c^4d^2 + 64c^3d^3x^4 + 32c^2d^4x^8} + \text{RootSum}\left(268435456t^4c^{11}d^9 + 194481a^8d^8 + 222264a^7bcd^7 + 280476a^6b^2c^2d^6 + 176904a^5b^3c^3d^5 + 112806a^4b^4c^4d^4 + 42120a^3b^5c^5d^3 + 15900a^2b^6c^6d^2 + 3000a^2b^7c^7d + 625b^8c^8, \text{Lambda}(t, t \log(128t^3d^2/(21a^2d^2 + 6ab^2cd + 5b^2c^2) + x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*2/(d\*x\*\*4+c)\*\*3,x)

[Out] 
$$(x^{5*(7*a^{2*d^{**3}} + 2*a*b*c*d^{**2} - 9*b^{**2}*c^{**2}*d) + x*(11*a^{**2}*c^{**2}*d^{**2} - 6*a*b*c^{**2}*d - 5*b^{**2}*c^{**3})}/(32*c^{**4}*d^{**2} + 64*c^{**3}*d^{**3}*x^{**4} + 32*c^{**2}*d^{**4}*x^{**8}) + \text{RootSum}(268435456*_t^{**4}*c^{**11}*d^{**9} + 194481*a^{**8}*d^{**8} + 222264*a^{**7}*b*c*d^{**7} + 280476*a^{**6}*b^{**2}*c^{**2}*d^{**6} + 176904*a^{**5}*b^{**3}*c^{**3}*d^{**5} + 112806*a^{**4}*b^{**4}*c^{**4}*d^{**4} + 42120*a^{**3}*b^{**5}*c^{**5}*d^{**3} + 15900*a^{**2}*b^{**6}*c^{**6}*d^{**2} + 3000*a*b^{**7}*c^{**7}*d + 625*b^{**8}*c^{**8}, \text{Lambda}(_t, _t \log(128*_t*c^{**3}*d^{**2}/(21*a^{**2}*d^{**2} + 6*a*b^2*c*d + 5*b^{**2}*c^{**2}) + x)))$$



**GIAC/XCAS [A]** time = 0.223269, size = 549, normalized size = 1.57

$$\frac{\sqrt{2}\left(5 (cd^3)^{\frac{1}{4}} b^2 c^2 + 6 (cd^3)^{\frac{1}{4}} abcd + 21 (cd^3)^{\frac{1}{4}} a^2 d^2\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128 c^3 d^3}$$

$$+ \frac{\sqrt{2}\left(5 (cd^3)^{\frac{1}{4}} b^2 c^2 + 6 (cd^3)^{\frac{1}{4}} abcd + 21 (cd^3)^{\frac{1}{4}} a^2 d^2\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128 c^3 d^3}$$

$$+ \frac{\sqrt{2}\left(5 (cd^3)^{\frac{1}{4}} b^2 c^2 + 6 (cd^3)^{\frac{1}{4}} abcd + 21 (cd^3)^{\frac{1}{4}} a^2 d^2\right) \ln\left(x^2 + \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{256 c^3 d^3}$$

$$- \frac{\sqrt{2}\left(5 (cd^3)^{\frac{1}{4}} b^2 c^2 + 6 (cd^3)^{\frac{1}{4}} abcd + 21 (cd^3)^{\frac{1}{4}} a^2 d^2\right) \ln\left(x^2 - \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{256 c^3 d^3}$$

$$- \frac{9 b^2 c^2 d x^5 - 2 abcd^2 x^5 - 7 a^2 d^3 x^5 + 5 b^2 c^3 x + 6 abc^2 d x - 11 a^2 c d^2 x}{32 (dx^4 + c)^2 c^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^2/(d\*x^4 + c)^3,x, algorithm="giac")

[Out] 1/128\*sqrt(2)\*(5\*(c\*d^3)^(1/4)\*b^2\*c^2 + 6\*(c\*d^3)^(1/4)\*a\*b\*c\*d + 21\*(c\*d^3)^(1/4)\*a^2\*d^2)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(c/d)^(1/4))/(c/d)^(1/4))/(c^3\*d^3) + 1/128\*sqrt(2)\*(5\*(c\*d^3)^(1/4)\*b^2\*c^2 + 6\*(c\*d^3)^(1/4)\*a\*b\*c\*d + 21\*(c\*d^3)^(1/4)\*a^2\*d^2)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)\*(c/d)^(1/4))/(c/d)^(1/4))/(c^3\*d^3) + 1/256\*sqrt(2)\*(5\*(c\*d^3)^(1/4)\*b^2\*c^2 + 6\*(c\*d^3)^(1/4)\*a\*b\*c\*d + 21\*(c\*d^3)^(1/4)\*a^2\*d^2)\*ln(x^2 + sqrt(2)\*x\*(c/d)^(1/4) + sqrt(c/d))/(c^3\*d^3) - 1/256\*sqrt(2)\*(5\*(c\*d^3)^(1/4)\*b^2\*c^2 + 6\*(c\*d^3)^(1/4)\*a\*b\*c\*d + 21\*(c\*d^3)^(1/4)\*a^2\*d^2)\*ln(x^2 - sqrt(2)\*x\*(c/d)^(1/4) + sqrt(c/d))/(c^3\*d^3) - 1/32\*(9\*b^2\*c^2\*d\*x^5 - 2\*a\*b\*c\*d^2\*x^5 - 7\*a^2\*d^3\*x^5 + 5\*b^2\*c^3\*x + 6\*a\*b\*c^2\*d\*x - 11\*a^2\*c\*d^2\*x)/((d\*x^4 + c)^2\*c^2\*d^2)

$$3.61 \quad \int \frac{(c+dx^4)^4}{a+bx^4} dx$$

**Optimal.** Leaf size=332

$$\begin{aligned} & - \frac{(bc-ad)^4 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}b^{17/4}} + \frac{(bc-ad)^4 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}b^{17/4}} \\ & - \frac{(bc-ad)^4 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{17/4}} + \frac{(bc-ad)^4 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}b^{17/4}} \\ & + \frac{dx(2bc-ad)(a^2d^2 - 2abcd + 2b^2c^2)}{b^4} + \frac{d^2x^5(a^2d^2 - 4abcd + 6b^2c^2)}{5b^3} + \frac{d^3x^9(4bc-ad)}{9b^2} + \frac{d^4x^{13}}{13b} \end{aligned}$$

[Out]  $(d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^3*(4*b*c - a*d)*x^9)/(9*b^2) + (d^4*x^13)/(13*b) - ((b*c - a*d)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c - a*d)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(17/4)) - ((b*c - a*d)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c - a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(17/4))$

**Rubi [A]** time = 0.581279, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$\begin{aligned} & - \frac{(bc-ad)^4 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}b^{17/4}} + \frac{(bc-ad)^4 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}b^{17/4}} \\ & - \frac{(bc-ad)^4 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{17/4}} + \frac{(bc-ad)^4 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}b^{17/4}} \\ & + \frac{dx(2bc-ad)(a^2d^2 - 2abcd + 2b^2c^2)}{b^4} + \frac{d^2x^5(a^2d^2 - 4abcd + 6b^2c^2)}{5b^3} + \frac{d^3x^9(4bc-ad)}{9b^2} + \frac{d^4x^{13}}{13b} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^4/(a + b\*x^4), x]

[Out]  $(d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^3*(4*b*c - a*d)*x^9)/(9*b^2) + (d^4*x^13)/(13*b) - ((b*c - a*d)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c - a*d)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(17/4)) - ((b*c - a*d)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c - a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(17/4))$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & \frac{d^4x^{13}}{13b} - \frac{d^3x^9(ad-4bc)}{9b^2} + \frac{d^2x^5(a^2d^2 - 4abcd + 6b^2c^2)}{5b^3} - \frac{(ad-2bc)(a^2d^2 - 2abcd + 2b^2c^2)}{b^4} \int d dx \\ & - \frac{\sqrt{2}(ad-bc)^4 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{8a^{\frac{3}{4}}b^{\frac{17}{4}}} + \frac{\sqrt{2}(ad-bc)^4 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{8a^{\frac{3}{4}}b^{\frac{17}{4}}} \\ & - \frac{\sqrt{2}(ad-bc)^4 \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{17}{4}}} + \frac{\sqrt{2}(ad-bc)^4 \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{17}{4}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((d*x**4+c)**4/(b*x**4+a),x)`

[Out]  $d^4 x^{13}/(13b) - d^3 x^9 (a d - 4 b c)/(9 b^2) + d^2 x^5 (a^2 d^2 - 4 a b c d + 6 b^2 c^2)/(5 b^3) - (a d - 2 b c) (a^2 d^2 - 2 a b c d + 2 b^2 c^2) \text{Integral}(d, x)/b^4 - \sqrt{2} (a d - b c)^4 \log(-\sqrt{2} a^{1/4} b^{1/4} x + \sqrt{a} + \sqrt{b} x^2)/(8 a^{3/4} b^{17/4}) + \sqrt{2} (a d - b c)^4 \log(\sqrt{2} a^{1/4} b^{1/4} x + \sqrt{a} + \sqrt{b} x^2)/(8 a^{3/4} b^{17/4}) - \sqrt{2} (a d - b c)^4 \text{atan}(1 - \sqrt{2} b^{1/4} x/a^{1/4})/(4 a^{3/4} b^{17/4}) + \sqrt{2} (a d - b c)^4 \text{atan}(1 + \sqrt{2} b^{1/4} x/a^{1/4})/(4 a^{3/4} b^{17/4})$

**Mathematica [A]** time = 0.329157, size = 322, normalized size = 0.97

$$\frac{585\sqrt{2}(bc-ad)^4 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx+\sqrt{a}+\sqrt{bx^2}}\right)}{a^{3/4}} + \frac{585\sqrt{2}(bc-ad)^4 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx+\sqrt{a}+\sqrt{bx^2}}\right)}{a^{3/4}} - \frac{1170\sqrt{2}(bc-ad)^4 \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{1170\sqrt{2}(bc-ad)^4 \tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{a^{3/4}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c + d*x^4)^4/(a + b*x^4),x]`

[Out]  $(4680 b^{1/4} d^4 (4 b^3 c^3 - 6 a b^2 c^2 d + 4 a^2 b c d^2 - a^3 d^3) x + 936 b^{5/4} d^4 (6 b^2 c^2 - 4 a b c d + a^2 d^2) x^5 + 520 b^{9/4} d^4 (4 b c - a d) x^9 + 360 b^{13/4} d^4 x^{13} - (1170 \sqrt{2} (b c - a d)^4 \text{ArcTan}[1 - (\sqrt{2} b^{1/4} x)/a^{1/4}])/a^{3/4} + (1170 \sqrt{2} (b c - a d)^4 \text{ArcTan}[1 + (\sqrt{2} b^{1/4} x)/a^{1/4}])/a^{3/4} - (585 \sqrt{2} (b c - a d)^4 \text{Log}[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} x + \sqrt{b} x^2])/a^{3/4} + (585 \sqrt{2} (b c - a d)^4 \text{Log}[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} x + \sqrt{b} x^2])/a^{3/4})/(4680 b^{17/4})$

**Maple [B]** time = 0.008, size = 837, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^4+c)^4/(b*x^4+a),x)`

[Out]  $1/13 d^4 x^{13}/b + 1/4 (a/b)^{1/4} / a^2 (1/2) \arctan(2^{1/2}/(a/b)^{1/4} x + 1) c^4 - 4/5 d^3/b^2 x^5 a^2 c + 1/4 (a/b)^{1/4} / a^2 (1/2) \arctan(2^{1/2}/(a/b)^{1/4} x - 1) c^4 - 1/9 d^4/b^2 x^9 a^4/9 d^3/b^2 x^9 c + 1/5 d^4/b^3 x^5 a^2 + 6/5 d^2/b^2 x^5 c^2 + 4 d^3/b^3 a^2 c^2 x + 3/2/b^2 (a/b)^{1/4} a^2 (1/2) \arctan(2^{1/2}/(a/b)^{1/4} x - 1) c^2 d^2 - 1/2/b^3 (a/b)^{1/4} a^2 (1/2) \ln((x^2 + (a/b)^{1/4} x^2 (1/2) + (a/b)^{1/2})/(x^2 - (a/b)^{1/4} x^2 (1/2) + (a/b)^{1/2})) c^2 d^3 - 6 d^2/b^2 a^2 c^2 x + 1/8 (a/b)^{1/4} / a^2 (1/2) \ln((x^2 + (a/b)^{1/4} x^2 (1/2) + (a/b)^{1/2})/(x^2 - (a/b)^{1/4} x^2 (1/2) + (a/b)^{1/2})) c^4 - d^4/b^4 a^3 x + 4 d/b^2 c^3 x + 3/4/b^2 (a/b)^{1/4} a^2 (1/2) \ln((x^2 + (a/b)^{1/4} x^2 (1/2) + (a/b)^{1/2})/(x^2 - (a/b)^{1/4} x^2 (1/2) + (a/b)^{1/2})) c^2 d^2 - 1/b^3 (a/b)^{1/4} a^2 (1/2) \arctan(2^{1/2}/(a/b)^{1/4} x + 1) c^2 d^3 + 3/2/b^2 (a/b)^{1/4} a^2 (1/2) \arctan(2^{1/2}/(a/b)^{1/4} x + 1) c^2 d^2 - 1/b^3 (a/b)^{1/4} a^2 (1/2) \arctan(2^{1/2}/(a/b)^{1/4} x - 1) c^2 d^3 + 1/8/b^4 (a/b)^{1/4} a^3 (1/2) \ln((x^2 + (a/b)^{1/4} x^2 (1/2) + (a/b)^{1/2})/(x^2 - (a/b)^{1/4} x^2 (1/2) + (a/b)^{1/2})) d^4 - 1/2/b^2 (a/b)^{1/4} a^2 (1/2) \ln((x^2 + (a/b)^{1/4} x^2 (1/2) + (a/b)^{1/2})/(x^2 - (a/b)^{1/4} x^2 (1/2) + (a/b)^{1/2})) c^3 d - 1/b^2 (a/b)^{1/4} a^2 (1/2) \arctan(2^{1/2}/(a/b)^{1/4} x + 1) c^3 d + 1/4/b^4 (a/b)^{1/4} a^3 (1/2) \arctan(2^{1/2}/(a/b)^{1/4} x + 1) d^4 - 1/b^2 (a/b)^{1/4} a^2 (1/2) \arctan(2^{1/2}/(a/b)^{1/4} x - 1) c^3 d + 1/4/b^4 (a/b)^{1/4} a^3 (1/2) \arctan(2^{1/2}/(a/b)^{1/4} x - 1) d^4$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^4/(b\*x^4 + a),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.258891, size = 2939, normalized size = 8.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^4/(b\*x^4 + a),x, algorithm="fricas")

[Out] 
$$\frac{1}{2340} \cdot (180 \cdot b^3 \cdot d^4 \cdot x^{13} + 260 \cdot (4 \cdot b^3 \cdot c \cdot d^3 - a \cdot b^2 \cdot d^4) \cdot x^9 + 468 \cdot (6 \cdot b^3 \cdot c^2 \cdot d^2 - 4 \cdot a \cdot b^2 \cdot c \cdot d^3 + a^2 \cdot b \cdot d^4) \cdot x^5 - 2340 \cdot b^4 \cdot (- (b^{16} \cdot c^{16} - 16 \cdot a \cdot b^{15} \cdot c^{15} \cdot d + 120 \cdot a^2 \cdot b^{14} \cdot c^{14} \cdot d^2 - 560 \cdot a^3 \cdot b^{13} \cdot c^{13} \cdot d^3 + 1820 \cdot a^4 \cdot b^{12} \cdot c^{12} \cdot d^4 - 4368 \cdot a^5 \cdot b^{11} \cdot c^{11} \cdot d^5 + 8008 \cdot a^6 \cdot b^{10} \cdot c^{10} \cdot d^6 - 11440 \cdot a^7 \cdot b^9 \cdot c^9 \cdot d^7 + 12870 \cdot a^8 \cdot b^8 \cdot c^8 \cdot d^8 - 11440 \cdot a^9 \cdot b^7 \cdot c^7 \cdot d^9 + 8008 \cdot a^{10} \cdot b^6 \cdot c^6 \cdot d^{10} - 4368 \cdot a^{11} \cdot b^5 \cdot c^5 \cdot d^{11} + 1820 \cdot a^{12} \cdot b^4 \cdot c^4 \cdot d^{12} - 560 \cdot a^{13} \cdot b^3 \cdot c^3 \cdot d^{13} + 120 \cdot a^{14} \cdot b^2 \cdot c^2 \cdot d^{14} - 16 \cdot a^{15} \cdot b \cdot c \cdot d^{15} + a^{16} \cdot d^{16}) / (a^3 \cdot b^{17}))^{1/4} \cdot \arctan(a \cdot b^4 \cdot (- (b^{16} \cdot c^{16} - 16 \cdot a \cdot b^{15} \cdot c^{15} \cdot d + 120 \cdot a^2 \cdot b^{14} \cdot c^{14} \cdot d^2 - 560 \cdot a^3 \cdot b^{13} \cdot c^{13} \cdot d^3 + 1820 \cdot a^4 \cdot b^{12} \cdot c^{12} \cdot d^4 - 4368 \cdot a^5 \cdot b^{11} \cdot c^{11} \cdot d^5 + 8008 \cdot a^6 \cdot b^{10} \cdot c^{10} \cdot d^6 - 11440 \cdot a^7 \cdot b^9 \cdot c^9 \cdot d^7 + 12870 \cdot a^8 \cdot b^8 \cdot c^8 \cdot d^8 - 11440 \cdot a^9 \cdot b^7 \cdot c^7 \cdot d^9 + 8008 \cdot a^{10} \cdot b^6 \cdot c^6 \cdot d^{10} - 4368 \cdot a^{11} \cdot b^5 \cdot c^5 \cdot d^{11} + 1820 \cdot a^{12} \cdot b^4 \cdot c^4 \cdot d^{12} - 560 \cdot a^{13} \cdot b^3 \cdot c^3 \cdot d^{13} + 120 \cdot a^{14} \cdot b^2 \cdot c^2 \cdot d^{14} - 16 \cdot a^{15} \cdot b \cdot c \cdot d^{15} + a^{16} \cdot d^{16}) / (a^3 \cdot b^{17}))^{1/4} / ((b^4 \cdot c^4 - 4 \cdot a \cdot b^3 \cdot c^3 \cdot d + 6 \cdot a^2 \cdot b^2 \cdot c^2 \cdot d^2 - 4 \cdot a^3 \cdot b \cdot c \cdot d^3 + a^4 \cdot d^4) \cdot x + (b^4 \cdot c^4 - 4 \cdot a \cdot b^3 \cdot c^3 \cdot d + 6 \cdot a^2 \cdot b^2 \cdot c^2 \cdot d^2 - 4 \cdot a^3 \cdot b \cdot c \cdot d^3 + a^4 \cdot d^4) \cdot \sqrt{(a^2 \cdot b^8 \cdot \sqrt{(- (b^{16} \cdot c^{16} - 16 \cdot a \cdot b^{15} \cdot c^{15} \cdot d + 120 \cdot a^2 \cdot b^{14} \cdot c^{14} \cdot d^2 - 560 \cdot a^3 \cdot b^{13} \cdot c^{13} \cdot d^3 + 1820 \cdot a^4 \cdot b^{12} \cdot c^{12} \cdot d^4 - 4368 \cdot a^5 \cdot b^{11} \cdot c^{11} \cdot d^5 + 8008 \cdot a^6 \cdot b^{10} \cdot c^{10} \cdot d^6 - 11440 \cdot a^7 \cdot b^9 \cdot c^9 \cdot d^7 + 12870 \cdot a^8 \cdot b^8 \cdot c^8 \cdot d^8 - 11440 \cdot a^9 \cdot b^7 \cdot c^7 \cdot d^9 + 8008 \cdot a^{10} \cdot b^6 \cdot c^6 \cdot d^{10} - 4368 \cdot a^{11} \cdot b^5 \cdot c^5 \cdot d^{11} + 1820 \cdot a^{12} \cdot b^4 \cdot c^4 \cdot d^{12} - 560 \cdot a^{13} \cdot b^3 \cdot c^3 \cdot d^{13} + 120 \cdot a^{14} \cdot b^2 \cdot c^2 \cdot d^{14} - 16 \cdot a^{15} \cdot b \cdot c \cdot d^{15} + a^{16} \cdot d^{16}) / (a^3 \cdot b^{17}))^{1/4}} + (b^8 \cdot c^8 - 8 \cdot a \cdot b^7 \cdot c^7 \cdot d + 28 \cdot a^2 \cdot b^6 \cdot c^6 \cdot d^2 - 56 \cdot a^3 \cdot b^5 \cdot c^5 \cdot d^3 + 70 \cdot a^4 \cdot b^4 \cdot c^4 \cdot d^4 - 56 \cdot a^5 \cdot b^3 \cdot c^3 \cdot d^5 + 28 \cdot a^6 \cdot b^2 \cdot c^2 \cdot d^6 - 8 \cdot a^7 \cdot b \cdot c \cdot d^7 + a^8 \cdot d^8) \cdot x^2) / (b^8 \cdot c^8 - 8 \cdot a \cdot b^7 \cdot c^7 \cdot d + 28 \cdot a^2 \cdot b^6 \cdot c^6 \cdot d^2 - 56 \cdot a^3 \cdot b^5 \cdot c^5 \cdot d^3 + 70 \cdot a^4 \cdot b^4 \cdot c^4 \cdot d^4 - 56 \cdot a^5 \cdot b^3 \cdot c^3 \cdot d^5 + 28 \cdot a^6 \cdot b^2 \cdot c^2 \cdot d^6 - 8 \cdot a^7 \cdot b \cdot c \cdot d^7 + a^8 \cdot d^8) \cdot x^2) + 585 \cdot b^4 \cdot (- (b^{16} \cdot c^{16} - 16 \cdot a \cdot b^{15} \cdot c^{15} \cdot d + 120 \cdot a^2 \cdot b^{14} \cdot c^{14} \cdot d^2 - 560 \cdot a^3 \cdot b^{13} \cdot c^{13} \cdot d^3 + 1820 \cdot a^4 \cdot b^{12} \cdot c^{12} \cdot d^4 - 4368 \cdot a^5 \cdot b^{11} \cdot c^{11} \cdot d^5 + 8008 \cdot a^6 \cdot b^{10} \cdot c^{10} \cdot d^6 - 11440 \cdot a^7 \cdot b^9 \cdot c^9 \cdot d^7 + 12870 \cdot a^8 \cdot b^8 \cdot c^8 \cdot d^8 - 11440 \cdot a^9 \cdot b^7 \cdot c^7 \cdot d^9 + 8008 \cdot a^{10} \cdot b^6 \cdot c^6 \cdot d^{10} - 4368 \cdot a^{11} \cdot b^5 \cdot c^5 \cdot d^{11} + 1820 \cdot a^{12} \cdot b^4 \cdot c^4 \cdot d^{12} - 560 \cdot a^{13} \cdot b^3 \cdot c^3 \cdot d^{13} + 120 \cdot a^{14} \cdot b^2 \cdot c^2 \cdot d^{14} - 16 \cdot a^{15} \cdot b \cdot c \cdot d^{15} + a^{16} \cdot d^{16}) / (a^3 \cdot b^{17}))^{1/4} \cdot \log(a \cdot b^4 \cdot (- (b^{16} \cdot c^{16} - 16 \cdot a \cdot b^{15} \cdot c^{15} \cdot d + 120 \cdot a^2 \cdot b^{14} \cdot c^{14} \cdot d^2 - 560 \cdot a^3 \cdot b^{13} \cdot c^{13} \cdot d^3 + 1820 \cdot a^4 \cdot b^{12} \cdot c^{12} \cdot d^4 - 4368 \cdot a^5 \cdot b^{11} \cdot c^{11} \cdot d^5 + 8008 \cdot a^6 \cdot b^{10} \cdot c^{10} \cdot d^6 - 11440 \cdot a^7 \cdot b^9 \cdot c^9 \cdot d^7 + 12870 \cdot a^8 \cdot b^8 \cdot c^8 \cdot d^8 - 11440 \cdot a^9 \cdot b^7 \cdot c^7 \cdot d^9 + 8008 \cdot a^{10} \cdot b^6 \cdot c^6 \cdot d^{10} - 4368 \cdot a^{11} \cdot b^5 \cdot c^5 \cdot d^{11} + 1820 \cdot a^{12} \cdot b^4 \cdot c^4 \cdot d^{12} - 560 \cdot a^{13} \cdot b^3 \cdot c^3 \cdot d^{13} + 120 \cdot a^{14} \cdot b^2 \cdot c^2 \cdot d^{14} - 16 \cdot a^{15} \cdot b \cdot c \cdot d^{15} + a^{16} \cdot d^{16}) / (a^3 \cdot b^{17}))^{1/4} + (b^4 \cdot c^4 - 4 \cdot a \cdot b^3 \cdot c^3 \cdot d + 6 \cdot a^2 \cdot b^2 \cdot c^2 \cdot d^2 - 4 \cdot a^3 \cdot b \cdot c \cdot d^3 + a^4 \cdot d^4) \cdot x - 585 \cdot b^4 \cdot (- (b^{16} \cdot c^{16} - 16 \cdot a \cdot b^{15} \cdot c^{15} \cdot d + 120 \cdot a^2 \cdot b^{14} \cdot c^{14} \cdot d^2 - 560 \cdot a^3 \cdot b^{13} \cdot c^{13} \cdot d^3 + 1820 \cdot a^4 \cdot b^{12} \cdot c^{12} \cdot d^4 - 4368 \cdot a^5 \cdot b^{11} \cdot c^{11} \cdot d^5 + 8008 \cdot a^6 \cdot b^{10} \cdot c^{10} \cdot d^6 - 11440 \cdot a^7 \cdot b^9 \cdot c^9 \cdot d^7 + 12870 \cdot a^8 \cdot b^8 \cdot c^8 \cdot d^8 - 11440 \cdot a^9 \cdot b^7 \cdot c^7 \cdot d^9 + 8008 \cdot a^{10} \cdot b^6 \cdot c^6 \cdot d^{10} - 4368 \cdot a^{11} \cdot b^5 \cdot c^5 \cdot d^{11} + 1820 \cdot a^{12} \cdot b^4 \cdot c^4 \cdot d^{12} - 560 \cdot a^{13} \cdot b^3 \cdot c^3 \cdot d^{13} + 120 \cdot a^{14} \cdot b^2 \cdot c^2 \cdot d^{14} - 16 \cdot a^{15} \cdot b \cdot c \cdot d^{15} + a^{16} \cdot d^{16}) / (a^3 \cdot b^{17}))^{1/4}$$

$$17))^{1/4} \log(-a^4 b^4 (-b^{16} c^{16} - 16 a^4 b^{15} c^{15} d + 120 a^4 b^{14} c^{14} d^2 - 560 a^4 b^{13} c^{13} d^3 + 1820 a^4 b^{12} c^{12} d^4 - 4368 a^4 b^{11} c^{11} d^5 + 8008 a^4 b^{10} c^{10} d^6 - 11440 a^4 b^9 c^9 d^7 + 12870 a^4 b^8 c^8 d^8 - 11440 a^4 b^7 c^7 d^9 + 8008 a^4 b^6 c^6 d^{10} - 4368 a^4 b^5 c^5 d^{11} + 1820 a^4 b^4 c^4 d^{12} - 560 a^4 b^3 c^3 d^{13} + 120 a^4 b^2 c^2 d^{14} - 16 a^4 b c d^{15} + a^4 d^{16}) / (a^3 b^{17})^{1/4} + (b^4 c^4 - 4 a^3 b^3 c^3 d + 6 a^2 b^2 c^2 d^2 - 4 a^3 b^3 c^3 d + a^4 d^4) x + 2340 (4 b^3 c^3 d - 6 a^2 b^2 c^2 d^2 + 4 a^2 b^3 c^3 d - a^3 d^4) x) / b^4$$

**Sympy [A]** time = 9.62074, size = 430, normalized size = 1.3

$$\text{RootSum}\left(256t^4 a^3 b^{17} + a^{16} d^{16} - 16a^{15} b c d^{15} + 120a^{14} b^2 c^2 d^{14} - 560a^{13} b^3 c^3 d^{13} + 1820a^{12} b^4 c^4 d^{12} - 4368a^{11} b^5 c^5 d^{11} + 8008a^{10} b^6 c^6 d^{10} - 11440a^9 b^7 c^7 d^9 + 12870a^8 b^8 c^8 d^8 - 11440a^7 b^9 c^9 d^7 + 8008a^6 b^{10} c^{10} d^6 - 4368a^5 b^{11} c^{11} d^5 + 1820a^4 b^{12} c^{12} d^4 - 560a^3 b^{13} c^{13} d^3 + 120a^2 b^{14} c^{14} d^2 - 16a b^{15} c^{15} d + a^{16} d^{16}\right) / (a^3 b^{17})^{1/4} + (b^4 c^4 - 4 a^3 b^3 c^3 d + 6 a^2 b^2 c^2 d^2 - 4 a^3 b^3 c^3 d + a^4 d^4) x + 2340 (4 b^3 c^3 d - 6 a^2 b^2 c^2 d^2 + 4 a^2 b^3 c^3 d - a^3 d^4) x) / b^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*4+c)\*\*4/(b\*x\*\*4+a),x)

[Out] RootSum(256\*\_t\*\*4\*a\*\*3\*b\*\*17 + a\*\*16\*d\*\*16 - 16\*a\*\*15\*b\*c\*d\*\*15 + 120\*a\*\*14\*b\*\*2\*c\*\*2\*d\*\*14 - 560\*a\*\*13\*b\*\*3\*c\*\*3\*d\*\*13 + 1820\*a\*\*12\*b\*\*4\*c\*\*4\*d\*\*12 - 4368\*a\*\*11\*b\*\*5\*c\*\*5\*d\*\*11 + 8008\*a\*\*10\*b\*\*6\*c\*\*6\*d\*\*10 - 11440\*a\*\*9\*b\*\*7\*c\*\*7\*d\*\*9 + 12870\*a\*\*8\*b\*\*8\*c\*\*8\*d\*\*8 - 11440\*a\*\*7\*b\*\*9\*c\*\*9\*d\*\*7 + 8008\*a\*\*6\*b\*\*10\*c\*\*10\*d\*\*6 - 4368\*a\*\*5\*b\*\*11\*c\*\*11\*d\*\*5 + 1820\*a\*\*4\*b\*\*12\*c\*\*12\*d\*\*4 - 560\*a\*\*3\*b\*\*13\*c\*\*13\*d\*\*3 + 120\*a\*\*2\*b\*\*14\*c\*\*14\*d\*\*2 - 16\*a\*b\*\*15\*c\*\*15\*d + b\*\*16\*c\*\*16, Lambda(\_t, \_t\*log(4\*\_t\*a\*b\*\*4/(a\*\*4\*d\*\*4 - 4\*a\*\*3\*b\*c\*d\*\*3 + 6\*a\*\*2\*b\*\*2\*c\*\*2\*d\*\*2 - 4\*a\*b\*\*3\*c\*\*3\*d + b\*\*4\*c\*\*4) + x))) + d\*\*4\*x\*\*13/(13\*b) - x\*\*9\*(a\*d\*\*4 - 4\*b\*c\*d\*\*3)/(9\*b\*\*2) + x\*\*5\*(a\*\*2\*d\*\*4 - 4\*a\*b\*c\*d\*\*3 + 6\*b\*\*2\*c\*\*2\*d\*\*2)/(5\*b\*\*3) - x\*(a\*\*3\*d\*\*4 - 4\*a\*\*2\*b\*c\*d\*\*3 + 6\*a\*b\*\*2\*c\*\*2\*d\*\*2 - 4\*b\*\*3\*c\*\*3\*d)/b\*\*4

**GIAC/XCAS [A]** time = 0.219731, size = 833, normalized size = 2.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^4/(b\*x^4 + a),x, algorithm="giac")

[Out] 1/4\*sqrt(2)\*((a\*b^3)^(1/4)\*b^4\*c^4 - 4\*(a\*b^3)^(1/4)\*a\*b^3\*c^3\*d + 6\*(a\*b^3)^(1/4)\*a^2\*b^2\*c^2\*d^2 - 4\*(a\*b^3)^(1/4)\*a^3\*b\*c\*d^3 + (a\*b^3)^(1/4)\*a^4\*d^4)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(a/b)^(1/4))/(a/b)^(1/4))/(a\*b^5) + 1/4\*sqrt(2)\*((a\*b^3)^(1/4)\*b^4\*c^4 - 4\*(a\*b^3)^(1/4)\*a\*b^3\*c^3\*d + 6\*(a\*b^3)^(1/4)\*a^2\*b^2\*c^2\*d^2 - 4\*(a\*b^3)^(1/4)\*a^3\*b\*c\*d^3 + (a\*b^3)^(1/4)\*a^4\*d^4)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)\*(a/b)^(1/4))/(a/b)^(1/4))/(a\*b^5) + 1/8\*sqrt(2)\*((a\*b^3)^(1/4)\*b^4\*c^4 - 4\*(a\*b^3)^(1/4)\*a\*b^3\*c^3\*d + 6\*(a\*b^3)^(1/4)\*a^2\*b^2\*c^2\*d^2 - 4\*(a\*b^3)^(1/4)\*a^3\*b\*c\*d^3 + (a\*b^3)^(1/4)\*a^4\*d^4)\*ln(x^2 + sqrt(2)\*x\*(a/b)^(1/4) + sqrt(a/b))/(a\*b^5) - 1/8\*sqrt(2)\*((a\*b^3)^(1/4)\*b^4\*c^4 - 4\*(a\*b^3)^(1/4)\*a\*b^3\*c^3\*d + 6\*(a\*b^3)^(1/4)\*a^2\*b^2\*c^2\*d^2 - 4\*(a\*b^3)^(1/4)\*a^3\*b\*c\*d^3 + (a\*b^3)^(1/4)\*a^4\*d^4)\*ln(x^2 - sqrt(2)\*x\*(a/b)^(1/4) + sqrt(a/b))/(a\*b^5) + 1/585\*(45\*b^12\*d^4\*x^13 + 260\*b^12\*c\*d^3\*x^9 - 65\*a\*b^11\*d^4\*x^9 + 702\*b^12\*c^2\*d^2\*x^5 - 468\*a\*b^11\*c\*d^3\*x^5 + 117\*a^2\*b^10\*d^4\*x^5 + 2340\*b^12\*c^3\*d\*x - 3510\*a\*b^11\*c^2\*d^2\*x + 2340\*a^2\*b^10\*c\*d^3\*x - 585\*a^3\*b^9\*d^4\*x)/b^13

$$3.62 \quad \int \frac{(c+dx^4)^3}{a+bx^4} dx$$

**Optimal.** Leaf size=288

$$\begin{aligned} & -\frac{(bc-ad)^3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}b^{13/4}} + \frac{(bc-ad)^3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}b^{13/4}} \\ & -\frac{(bc-ad)^3 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{13/4}} + \frac{(bc-ad)^3 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}b^{13/4}} \\ & + \frac{dx(a^2d^2 - 3abcd + 3b^2c^2)}{b^3} + \frac{d^2x^5(3bc - ad)}{5b^2} + \frac{d^3x^9}{9b} \end{aligned}$$

[Out] (d\*(3\*b^2\*c^2 - 3\*a\*b\*c\*d + a^2\*d^2)\*x)/b^3 + (d^2\*(3\*b\*c - a\*d)\*x^5)/(5\*b^2) + (d^3\*x^9)/(9\*b) - ((b\*c - a\*d)^3\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/ (2\*Sqrt[2]\*a^(3/4)\*b^(13/4)) + ((b\*c - a\*d)^3\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/ (2\*Sqrt[2]\*a^(3/4)\*b^(13/4)) - ((b\*c - a\*d)^3\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/ (4\*Sqrt[2]\*a^(3/4)\*b^(13/4)) + ((b\*c - a\*d)^3\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/ (4\*Sqrt[2]\*a^(3/4)\*b^(13/4))

**Rubi [A]** time = 0.450354, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$\begin{aligned} & -\frac{(bc-ad)^3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}b^{13/4}} + \frac{(bc-ad)^3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}b^{13/4}} \\ & -\frac{(bc-ad)^3 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{13/4}} + \frac{(bc-ad)^3 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}b^{13/4}} \\ & + \frac{dx(a^2d^2 - 3abcd + 3b^2c^2)}{b^3} + \frac{d^2x^5(3bc - ad)}{5b^2} + \frac{d^3x^9}{9b} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^3/(a + b\*x^4), x]

[Out] (d\*(3\*b^2\*c^2 - 3\*a\*b\*c\*d + a^2\*d^2)\*x)/b^3 + (d^2\*(3\*b\*c - a\*d)\*x^5)/(5\*b^2) + (d^3\*x^9)/(9\*b) - ((b\*c - a\*d)^3\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/ (2\*Sqrt[2]\*a^(3/4)\*b^(13/4)) + ((b\*c - a\*d)^3\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/ (2\*Sqrt[2]\*a^(3/4)\*b^(13/4)) - ((b\*c - a\*d)^3\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/ (4\*Sqrt[2]\*a^(3/4)\*b^(13/4)) + ((b\*c - a\*d)^3\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/ (4\*Sqrt[2]\*a^(3/4)\*b^(13/4))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & \frac{d^3x^9}{9b} - \frac{d^2x^5(ad - 3bc)}{5b^2} + \frac{(a^2d^2 - 3abcd + 3b^2c^2) \int dx}{b^3} \\ & + \frac{\sqrt{2}(ad - bc)^3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{8a^{\frac{3}{4}}b^{\frac{13}{4}}} - \frac{\sqrt{2}(ad - bc)^3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{8a^{\frac{3}{4}}b^{\frac{13}{4}}} \\ & + \frac{\sqrt{2}(ad - bc)^3 \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{13}{4}}} - \frac{\sqrt{2}(ad - bc)^3 \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{13}{4}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((d*x**4+c)**3/(b*x**4+a),x)`

[Out]  $d^{*3}x^{*9}/(9*b) - d^{*2}x^{*5}(a*d - 3*b*c)/(5*b^{*2}) + (a^{*2}d^{*2} - 3*a*b*c*d + 3*b^{*2}c^{*2})\text{Integral}(d, x)/b^{*3} + \sqrt{2}*(a*d - b*c)^{*3}\log(-\sqrt{2}*a^{*(1/4)}*b^{*(1/4)}*x + \sqrt{a} + \sqrt{b})*x^{*2}/(8*a^{*(3/4)}*b^{*(13/4)}) - \sqrt{2}*(a*d - b*c)^{*3}\log(\sqrt{2}*a^{*(1/4)}*b^{*(1/4)}*x + \sqrt{a} + \sqrt{b})*x^{*2}/(8*a^{*(3/4)}*b^{*(13/4)}) + \sqrt{2}*(a*d - b*c)^{*3}\text{atan}(1 - \sqrt{2}*b^{*(1/4)}*x/a^{*(1/4)})/(4*a^{*(3/4)}*b^{*(13/4)}) - \sqrt{2}*(a*d - b*c)^{*3}\text{atan}(1 + \sqrt{2}*b^{*(1/4)}*x/a^{*(1/4)})/(4*a^{*(3/4)}*b^{*(13/4)})$

**Mathematica [A]** time = 0.234732, size = 271, normalized size = 0.94

$$-72a^{3/4}b^{5/4}d^2x^5(ad - 3bc) + 40a^{3/4}b^{9/4}d^3x^9 + 360a^{3/4}\sqrt[4]{bdx}(a^2d^2 - 3abcd + 3b^2c^2) - 45\sqrt{2}(bc - ad)^3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \dots\right)$$

Antiderivative was successfully verified.

[In] `Integrate[(c + d*x^4)^3/(a + b*x^4),x]`

[Out]  $(360*a^{(3/4)}*b^{(1/4)}*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x - 72*a^{(3/4)}*b^{(5/4)}*d^2*(-3*b*c + a*d)*x^5 + 40*a^{(3/4)}*b^{(9/4)}*d^3*x^9 - 90*\text{Sqrt}[2]*(b*c - a*d)^3*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*x)/a^{(1/4)}] + 90*\text{Sqrt}[2]*(b*c - a*d)^3*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*x)/a^{(1/4)}] - 45*\text{Sqrt}[2]*(b*c - a*d)^3*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2] + 45*\text{Sqrt}[2]*(b*c - a*d)^3*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(360*a^{(3/4)}*b^{(13/4)})$

**Maple [B]** time = 0.002, size = 627, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^4+c)^3/(b*x^4+a),x)`

[Out]  $1/9*d^3*x^9/b - 1/5*d^3/b^2*x^5*a + 3/5*d^2/b*x^5*c + d^3/b^3*a^2*x - 3*d^2/b^2*a*c*x + 3*d/b*c^2*x - 1/4/b^3*(a/b)^{(1/4)}*a^2*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)*d^3 + 3/4/b^2*(a/b)^{(1/4)}*a^2*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)*c*d^2 - 3/4/b*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)*c^2*d + 1/4*(a/b)^{(1/4)}/a^2*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)*c^3 - 1/4/b^3*(a/b)^{(1/4)}*a^2*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)*d^3 + 3/4/b^2*(a/b)^{(1/4)}*a^2*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)*c*d^2 - 3/4/b*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)*c^2*d + 1/4*(a/b)^{(1/4)}/a^2*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)*c^3 - 1/8/b^3*(a/b)^{(1/4)}*a^2*2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*x*2^{(1/2)}+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*x*2^{(1/2)}+(a/b)^{(1/2)})))*d^3 + 3/8/b^2*(a/b)^{(1/4)}*a^2*2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*x*2^{(1/2)}+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*x*2^{(1/2)}+(a/b)^{(1/2)})))*c*d^2 - 3/8/b*(a/b)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*x*2^{(1/2)}+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*x*2^{(1/2)}+(a/b)^{(1/2)})))*c^2*d + 1/8*(a/b)^{(1/4)}/a^2*2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*x*2^{(1/2)}+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*x*2^{(1/2)}+(a/b)^{(1/2)})))*c^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^3/(b\*x^4 + a),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.250542, size = 2198, normalized size = 7.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^3/(b\*x^4 + a),x, algorithm="fricas")

[Out] 
$$\frac{1}{180} (20b^2d^3x^9 + 36(3b^2c^2d^2 - ab^2d^3)x^5 + 180b^3(- (b^{12}c^{12} - 12a^2b^{11}c^{11}d + 66a^2b^{10}c^{10}d^2 - 220a^3b^9c^9d^3 + 495a^4b^8c^8d^4 - 792a^5b^7c^7d^5 + 924a^6b^6c^6d^6 - 792a^7b^5c^5d^7 + 495a^8b^4c^4d^8 - 220a^9b^3c^3d^9 + 66a^{10}b^2c^2d^{10} - 12a^{11}b^1c^1d^{11} + a^{12}d^{12}) / (a^3b^{13}))^{1/4} \arctan(-ab^3(- (b^{12}c^{12} - 12a^2b^{11}c^{11}d + 66a^2b^{10}c^{10}d^2 - 220a^3b^9c^9d^3 + 495a^4b^8c^8d^4 - 792a^5b^7c^7d^5 + 924a^6b^6c^6d^6 - 792a^7b^5c^5d^7 + 495a^8b^4c^4d^8 - 220a^9b^3c^3d^9 + 66a^{10}b^2c^2d^{10} - 12a^{11}b^1c^1d^{11} + a^{12}d^{12}) / (a^3b^{13}))^{1/4} / ((b^3c^3 - 3a^2b^2c^2d + 3a^2b^1c^1d^2 - a^3d^3)x + (b^3c^3 - 3a^2b^2c^2d + 3a^2b^1c^1d^2 - a^3d^3)) \sqrt{(a^2b^6 \sqrt{- (b^{12}c^{12} - 12a^2b^{11}c^{11}d + 66a^2b^{10}c^{10}d^2 - 220a^3b^9c^9d^3 + 495a^4b^8c^8d^4 - 792a^5b^7c^7d^5 + 924a^6b^6c^6d^6 - 792a^7b^5c^5d^7 + 495a^8b^4c^4d^8 - 220a^9b^3c^3d^9 + 66a^{10}b^2c^2d^{10} - 12a^{11}b^1c^1d^{11} + a^{12}d^{12}) / (a^3b^{13}))} + (b^6c^6 - 6a^5b^5c^5d + 15a^2b^4c^4d^2 - 20a^3b^3c^3d^3 + 15a^4b^2c^2d^4 - 6a^5b^1c^1d^5 + a^6d^6))x^2) / (b^6c^6 - 6a^5b^5c^5d + 15a^2b^4c^4d^2 - 20a^3b^3c^3d^3 + 15a^4b^2c^2d^4 - 6a^5b^1c^1d^5 + a^6d^6))) - 45b^3(- (b^{12}c^{12} - 12a^2b^{11}c^{11}d + 66a^2b^{10}c^{10}d^2 - 220a^3b^9c^9d^3 + 495a^4b^8c^8d^4 - 792a^5b^7c^7d^5 + 924a^6b^6c^6d^6 - 792a^7b^5c^5d^7 + 495a^8b^4c^4d^8 - 220a^9b^3c^3d^9 + 66a^{10}b^2c^2d^{10} - 12a^{11}b^1c^1d^{11} + a^{12}d^{12}) / (a^3b^{13}))^{1/4} \log(ab^3(- (b^{12}c^{12} - 12a^2b^{11}c^{11}d + 66a^2b^{10}c^{10}d^2 - 220a^3b^9c^9d^3 + 495a^4b^8c^8d^4 - 792a^5b^7c^7d^5 + 924a^6b^6c^6d^6 - 792a^7b^5c^5d^7 + 495a^8b^4c^4d^8 - 220a^9b^3c^3d^9 + 66a^{10}b^2c^2d^{10} - 12a^{11}b^1c^1d^{11} + a^{12}d^{12}) / (a^3b^{13}))^{1/4} - (b^3c^3 - 3a^2b^2c^2d + 3a^2b^1c^1d^2 - a^3d^3)x) + 45b^3(- (b^{12}c^{12} - 12a^2b^{11}c^{11}d + 66a^2b^{10}c^{10}d^2 - 220a^3b^9c^9d^3 + 495a^4b^8c^8d^4 - 792a^5b^7c^7d^5 + 924a^6b^6c^6d^6 - 792a^7b^5c^5d^7 + 495a^8b^4c^4d^8 - 220a^9b^3c^3d^9 + 66a^{10}b^2c^2d^{10} - 12a^{11}b^1c^1d^{11} + a^{12}d^{12}) / (a^3b^{13}))^{1/4} \log(-ab^3(- (b^{12}c^{12} - 12a^2b^{11}c^{11}d + 66a^2b^{10}c^{10}d^2 - 220a^3b^9c^9d^3 + 495a^4b^8c^8d^4 - 792a^5b^7c^7d^5 + 924a^6b^6c^6d^6 - 792a^7b^5c^5d^7 + 495a^8b^4c^4d^8 - 220a^9b^3c^3d^9 + 66a^{10}b^2c^2d^{10} - 12a^{11}b^1c^1d^{11} + a^{12}d^{12}) / (a^3b^{13}))^{1/4} - (b^3c^3 - 3a^2b^2c^2d + 3a^2b^1c^1d^2 - a^3d^3)x) + 180(3b^2c^2d - 3a^2b^1c^1d^2 + a^2d^3)x) / b^3$$

**Sympy [A]** time = 6.1807, size = 301, normalized size = 1.05

$$\text{RootSum}\left(256t^4a^3b^{13} + a^{12}d^{12} - 12a^{11}bcd^{11} + 66a^{10}b^2c^2d^{10} - 220a^9b^3c^3d^9 + 495a^8b^4c^4d^8 - 792a^7b^5c^5d^7 + 924a^6b^6c^6d^6 - \dots\right) + \frac{d^3x^9}{9b} - \frac{x^5(ad^3 - 3bcd^2)}{5b^2} + \frac{x(a^2d^3 - 3abcd^2 + 3b^2c^2d)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*4+c)\*\*3/(b\*x\*\*4+a),x)



```
[Out] RootSum(256*_t**4*a**3*b**13 + a**12*d**12 - 12*a**11*b*c*d**11 +
66*a**10*b**2*c**2*d**10 - 220*a**9*b**3*c**3*d**9 + 495*a**8*b*
*4*c**4*d**8 - 792*a**7*b**5*c**5*d**7 + 924*a**6*b**6*c**6*d**6
- 792*a**5*b**7*c**7*d**5 + 495*a**4*b**8*c**8*d**4 - 220*a**3*b*
*9*c**9*d**3 + 66*a**2*b**10*c**10*d**2 - 12*a*b**11*c**11*d + b*
*12*c**12, Lambda(_t, _t*log(-4*_t*a*b**3/(a**3*d**3 - 3*a**2*b*c
*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x))) + d**3*x**9/(9*b) - x
**5*(a*d**3 - 3*b*c*d**2)/(5*b**2) + x*(a**2*d**3 - 3*a*b*c*d**2
+ 3*b**2*c**2*d)/b**3
```

**GIAC/XCAS [A]** time = 0.218108, size = 649, normalized size = 2.25

$$\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3(ab^3)^{\frac{1}{4}}ab^2c^2d + 3(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^4}$$

$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3(ab^3)^{\frac{1}{4}}ab^2c^2d + 3(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^4}$$

$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3(ab^3)^{\frac{1}{4}}ab^2c^2d + 3(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right) \ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8ab^4}$$

$$- \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3(ab^3)^{\frac{1}{4}}ab^2c^2d + 3(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right) \ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8ab^4}$$

$$+ \frac{5b^8d^3x^9 + 27b^8cd^2x^5 - 9ab^7d^3x^5 + 135b^8c^2dx - 135ab^7cd^2x + 45a^2b^6d^3x}{45b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4 + c)^3/(b*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d
+ 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/4*s
qrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(
a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/8*sqrt(2
)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3
)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*ln(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(a*b^4) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*
c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 -
(a*b^3)^(1/4)*a^3*d^3)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b
))/(a*b^4) + 1/45*(5*b^8*d^3*x^9 + 27*b^8*c*d^2*x^5 - 9*a*b^7*d^3
*x^5 + 135*b^8*c^2*d*x - 135*a*b^7*c*d^2*x + 45*a^2*b^6*d^3*x)/b^
9
```

$$3.63 \quad \int \frac{(c+dx^4)^2}{a+bx^4} dx$$

**Optimal.** Leaf size=253

$$\begin{aligned} & -\frac{(bc-ad)^2 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}b^{9/4}} + \frac{(bc-ad)^2 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}b^{9/4}} \\ & -\frac{(bc-ad)^2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}} + \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}b^{9/4}} + \frac{dx(2bc-ad)}{b^2} + \frac{d^2x^5}{5b} \end{aligned}$$

[Out] (d\*(2\*b\*c - a\*d)\*x)/b^2 + (d^2\*x^5)/(5\*b) - ((b\*c - a\*d)^2\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(2\*Sqrt[2]\*a^(3/4)\*b^(9/4)) + ((b\*c - a\*d)^2\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(2\*Sqrt[2]\*a^(3/4)\*b^(9/4)) - ((b\*c - a\*d)^2\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*b^(9/4)) + ((b\*c - a\*d)^2\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*b^(9/4))

**Rubi [A]** time = 0.400891, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$\begin{aligned} & -\frac{(bc-ad)^2 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}b^{9/4}} + \frac{(bc-ad)^2 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}b^{9/4}} \\ & -\frac{(bc-ad)^2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}} + \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}b^{9/4}} + \frac{dx(2bc-ad)}{b^2} + \frac{d^2x^5}{5b} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^2/(a + b\*x^4), x]

[Out] (d\*(2\*b\*c - a\*d)\*x)/b^2 + (d^2\*x^5)/(5\*b) - ((b\*c - a\*d)^2\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(2\*Sqrt[2]\*a^(3/4)\*b^(9/4)) + ((b\*c - a\*d)^2\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(2\*Sqrt[2]\*a^(3/4)\*b^(9/4)) - ((b\*c - a\*d)^2\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*b^(9/4)) + ((b\*c - a\*d)^2\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*b^(9/4))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & \frac{d^2x^5}{5b} - \frac{(ad-2bc) \int d dx}{b^2} - \frac{\sqrt{2}(ad-bc)^2 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{8a^{\frac{3}{4}}b^{\frac{9}{4}}} \\ & + \frac{\sqrt{2}(ad-bc)^2 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{8a^{\frac{3}{4}}b^{\frac{9}{4}}} \\ & - \frac{\sqrt{2}(ad-bc)^2 \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{9}{4}}} + \frac{\sqrt{2}(ad-bc)^2 \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{9}{4}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((d\*x\*\*4+c)\*\*2/(b\*x\*\*4+a), x)

[Out] d\*\*2\*x\*\*5/(5\*b) - (a\*d - 2\*b\*c)\*Integral(d, x)/b\*\*2 - sqrt(2)\*(a\*d - b\*c)\*\*2\*log(-sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*x + sqrt(a) + sqrt(b)\*x\*\*2)/(8\*a\*\*(3/4)\*b\*\*(9/4)) + sqrt(2)\*(a\*d - b\*c)\*\*2\*log(sqrt(2)\*

$$a^{1/4} b^{1/4} x + \sqrt{a} + \sqrt{b} x^2) / (8 a^{3/4} b^{9/4}) - \sqrt{2} (a d - b c)^2 \operatorname{atan}(1 - \sqrt{2} b^{1/4} x / a^{1/4}) / (4 a^{3/4} b^{9/4}) + \sqrt{2} (a d - b c)^2 \operatorname{atan}(1 + \sqrt{2} b^{1/4} x / a^{1/4}) / (4 a^{3/4} b^{9/4})$$

**Mathematica [A]** time = 0.172543, size = 231, normalized size = 0.91

$$8a^{3/4}b^{5/4}d^2x^5 - 40a^{3/4}\sqrt[4]{b}dx(ad - 2bc) - 5\sqrt{2}(bc - ad)^2 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{bx^2}\right) + 5\sqrt{2}(bc - ad)^2 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} - \sqrt{bx^2}\right) / 40a^{3/4}b^{9/4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^4)^2/(a + b\*x^4), x]

[Out] (-40\*a^(3/4)\*b^(1/4)\*d\*(-2\*b\*c + a\*d)\*x + 8\*a^(3/4)\*b^(5/4)\*d^2\*x^5 - 10\*Sqrt[2]\*(b\*c - a\*d)^2\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)] + 10\*Sqrt[2]\*(b\*c - a\*d)^2\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)] - 5\*Sqrt[2]\*(b\*c - a\*d)^2\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2] + 5\*Sqrt[2]\*(b\*c - a\*d)^2\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(40\*a^(3/4)\*b^(9/4))

**Maple [B]** time = 0.003, size = 436, normalized size = 1.7

$$\begin{aligned} & \frac{d^2x^5}{5b} - \frac{ad^2x}{b^2} + 2\frac{dxc}{b} + \frac{a\sqrt{2}d^2}{4b^2}\sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) \\ & - \frac{\sqrt{2}cd}{2b}\sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) + \frac{\sqrt{2}c^2}{4a}\sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) \\ & + \frac{a\sqrt{2}d^2}{8b^2}\sqrt[4]{\frac{a}{b}} \ln\left(1\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \\ & - \frac{\sqrt{2}cd}{4b}\sqrt[4]{\frac{a}{b}} \ln\left(1\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \\ & + \frac{\sqrt{2}c^2}{8a}\sqrt[4]{\frac{a}{b}} \ln\left(1\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \\ & + \frac{a\sqrt{2}d^2}{4b^2}\sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) - \frac{\sqrt{2}cd}{2b}\sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) \\ & + \frac{\sqrt{2}c^2}{4a}\sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^4+c)^2/(b\*x^4+a), x)

[Out] 1/5\*d^2\*x^5/b - d^2/b^2\*a\*x + 2\*d/b\*x\*c + 1/4/b^2\*(a/b)^(1/4)\*a^2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x - 1)\*d^2 - 1/2/b\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x - 1)\*c\*d + 1/4\*(a/b)^(1/4)/a^2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x - 1)\*c^2 + 1/8/b^2\*(a/b)^(1/4)\*a^2^(1/2)\*ln((x^2 + (a/b)^(1/4)\*x\*2^(1/2) + (a/b)^(1/2))/(x^2 - (a/b)^(1/4)\*x\*2^(1/2) + (a/b)^(1/2))) \* d^2 - 1/4/b\*(a/b)^(1/4)\*2^(1/2)\*ln((x^2 + (a/b)^(1/4)\*x\*2^(1/2) + (a/b)^(1/2))/(x^2 - (a/b)^(1/4)\*x\*2^(1/2) + (a/b)^(1/2))) \* c

$$\begin{aligned} & *d+1/8*(a/b)^{(1/4)}/a*2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*x*2^{(1/2)}+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*x*2^{(1/2)}+(a/b)^{(1/2)})) *c^2+1/4/b^2*(a/b)^{(1/4)}*a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)*d^2-1/2/b*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)*c*d+1/4*(a/b)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)*c^2 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^2/(b\*x^4 + a),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.243164, size = 1458, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^2/(b\*x^4 + a),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/20*(4*b*d^2*x^5 - 20*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^{(1/4)}*\arctan(a*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^{(1/4)})/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\sqrt{(a^2*b^4*\sqrt{-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9)) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x^2}/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4))) + 5*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^{(1/4)}*\log(a*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^{(1/4)}) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) - 5*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^{(1/4)}*\log(-a*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^{(1/4)}) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) + 20*(2*b*c*d - a*d^2)*x/b^2 \end{aligned}$$

**Sympy [A]** time = 3.88518, size = 187, normalized size = 0.74

$$\begin{aligned} & \text{RootSum}\left(256t^4a^3b^9 + a^8d^8 - 8a^7bcd^7 + 28a^6b^2c^2d^6 - 56a^5b^3c^3d^5 + 70a^4b^4c^4d^4 - 56a^3b^5c^5d^3 + 28a^2b^6c^6d^2 - 8ab^7c^7d + b^8\right) \\ & + \frac{d^2x^5}{5b} - \frac{x(ad^2 - 2bcd)}{b^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*4+c)\*\*2/(b\*x\*\*4+a),x)

[Out] RootSum(256\*\_t\*\*4\*a\*\*3\*b\*\*9 + a\*\*8\*d\*\*8 - 8\*a\*\*7\*b\*c\*d\*\*7 + 28\*a\*\*6\*b\*\*2\*c\*\*2\*d\*\*6 - 56\*a\*\*5\*b\*\*3\*c\*\*3\*d\*\*5 + 70\*a\*\*4\*b\*\*4\*c\*\*4\*d\*\*4 - 56\*a\*\*3\*b\*\*5\*c\*\*5\*d\*\*3 + 28\*a\*\*2\*b\*\*6\*c\*\*6\*d\*\*2 - 8\*a\*b\*\*7\*c\*\*7\*d + b\*\*8\*c\*\*8, Lambda(\_t, \_t\*log(4\*\_t\*a\*b\*\*2/(a\*\*2\*d\*\*2 - 2\*a\*b\*c\*d + b\*\*2\*c\*\*2) + x))) + d\*\*2\*x\*\*5/(5\*b) - x\*(a\*d\*\*2 - 2\*b\*c\*d)/b\*\*2

**GIAC/XCAS [A]** time = 0.218389, size = 477, normalized size = 1.89

$$\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c^2 - 2(ab^3)^{\frac{1}{4}}abcd + (ab^3)^{\frac{1}{4}}a^2d^2\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}$$

$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c^2 - 2(ab^3)^{\frac{1}{4}}abcd + (ab^3)^{\frac{1}{4}}a^2d^2\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}$$

$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c^2 - 2(ab^3)^{\frac{1}{4}}abcd + (ab^3)^{\frac{1}{4}}a^2d^2\right) \ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8ab^3}$$

$$- \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c^2 - 2(ab^3)^{\frac{1}{4}}abcd + (ab^3)^{\frac{1}{4}}a^2d^2\right) \ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8ab^3}$$

$$+ \frac{b^4d^2x^5 + 10b^4cdx - 5ab^3d^2x}{5b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^2/(b\*x^4 + a),x, algorithm="giac")

[Out] 1/4\*sqrt(2)\*((a\*b^3)^(1/4)\*b^2\*c^2 - 2\*(a\*b^3)^(1/4)\*a\*b\*c\*d + (a\*b^3)^(1/4)\*a^2\*d^2)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(a/b)^(1/4)))/(a/b)^(1/4))/(a\*b^3) + 1/4\*sqrt(2)\*((a\*b^3)^(1/4)\*b^2\*c^2 - 2\*(a\*b^3)^(1/4)\*a\*b\*c\*d + (a\*b^3)^(1/4)\*a^2\*d^2)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)\*(a/b)^(1/4)))/(a/b)^(1/4))/(a\*b^3) + 1/8\*sqrt(2)\*((a\*b^3)^(1/4)\*b^2\*c^2 - 2\*(a\*b^3)^(1/4)\*a\*b\*c\*d + (a\*b^3)^(1/4)\*a^2\*d^2)\*ln(x^2 + sqrt(2)\*x\*(a/b)^(1/4) + sqrt(a/b))/(a\*b^3) - 1/8\*sqrt(2)\*((a\*b^3)^(1/4)\*b^2\*c^2 - 2\*(a\*b^3)^(1/4)\*a\*b\*c\*d + (a\*b^3)^(1/4)\*a^2\*d^2)\*ln(x^2 - sqrt(2)\*x\*(a/b)^(1/4) + sqrt(a/b))/(a\*b^3) + 1/5\*(b^4\*d^2\*x^5 + 10\*b^4\*c\*d\*x - 5\*a\*b^3\*d^2\*x)/b^5

### 3.64 $\int \frac{c+dx^4}{a+bx^4} dx$

**Optimal.** Leaf size=223

$$\begin{aligned} & -\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}b^{5/4}} \\ & -\frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{5/4}} + \frac{dx}{b} \end{aligned}$$

[Out] (d\*x)/b - ((b\*c - a\*d)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(2\*Sqrt[2]\*a^(3/4)\*b^(5/4)) + ((b\*c - a\*d)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(2\*Sqrt[2]\*a^(3/4)\*b^(5/4)) - ((b\*c - a\*d)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*b^(5/4)) + ((b\*c - a\*d)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*b^(5/4))

**Rubi [A]** time = 0.298679, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$\begin{aligned} & -\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}b^{5/4}} \\ & -\frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{5/4}} + \frac{dx}{b} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)/(a + b\*x^4), x]

[Out] (d\*x)/b - ((b\*c - a\*d)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(2\*Sqrt[2]\*a^(3/4)\*b^(5/4)) + ((b\*c - a\*d)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(2\*Sqrt[2]\*a^(3/4)\*b^(5/4)) - ((b\*c - a\*d)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*b^(5/4)) + ((b\*c - a\*d)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*b^(5/4))

**Rubi in Sympy [A]** time = 61.2056, size = 204, normalized size = 0.91

$$\begin{aligned} & \frac{dx}{b} + \frac{\sqrt{2}(ad-bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{8a^{\frac{3}{4}}b^{\frac{5}{4}}} - \frac{\sqrt{2}(ad-bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{8a^{\frac{3}{4}}b^{\frac{5}{4}}} \\ & + \frac{\sqrt{2}(ad-bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{5}{4}}} - \frac{\sqrt{2}(ad-bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{5}{4}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((d\*x\*\*4+c)/(b\*x\*\*4+a), x)

[Out] d\*x/b + sqrt(2)\*(a\*d - b\*c)\*log(-sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*x + sqrt(a) + sqrt(b)\*x\*\*2)/(8\*a\*\*(3/4)\*b\*\*(5/4)) - sqrt(2)\*(a\*d - b\*c)\*log(sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*x + sqrt(a) + sqrt(b)\*x\*\*2)/(8\*a\*\*(3/4)\*b\*\*(5/4)) + sqrt(2)\*(a\*d - b\*c)\*atan(1 - sqrt(2)\*b\*\*(1/4)\*x/a\*\*(1/4))/(4\*a\*\*(3/4)\*b\*\*(5/4)) - sqrt(2)\*(a\*d - b\*c)\*atan(1 + sqrt(2)\*b\*\*(1/4)\*x/a\*\*(1/4))/(4\*a\*\*(3/4)\*b\*\*(5/4))

**Mathematica [A]** time = 0.214753, size = 196, normalized size = 0.88

$$\frac{8a^{3/4}\sqrt[4]{b}dx - \sqrt{2}(bc - ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2\right) + \sqrt{2}(bc - ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2\right) - 2\sqrt{2}(bc - ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x - \sqrt{a} - \sqrt{b}x^2}\right)}{8a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^4)/(a + b\*x^4), x]

[Out]  $(8*a^{3/4}*b^{1/4}*d*x - 2*\text{Sqrt}[2]*(b*c - a*d)*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{1/4}*x)/a^{1/4}]] + 2*\text{Sqrt}[2]*(b*c - a*d)*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{1/4}*x)/a^{1/4}] - \text{Sqrt}[2]*(b*c - a*d)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2] + \text{Sqrt}[2]*(b*c - a*d)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2])/(8*a^{3/4}*b^{5/4})$

**Maple [A]** time = 0.005, size = 266, normalized size = 1.2

$$\begin{aligned} & \frac{dx}{b} - \frac{\sqrt{2}d}{4b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right) + \frac{\sqrt{2}c}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right) \\ & - \frac{\sqrt{2}d}{8b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \\ & + \frac{\sqrt{2}c}{8a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \\ & - \frac{\sqrt{2}d}{4b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right) + \frac{\sqrt{2}c}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^4+c)/(b\*x^4+a), x)

[Out]  $d*x/b - 1/4/b*(a/b)^{1/4}*2^{1/2}*arctan(2^{1/2}/(a/b)^{1/4}*x - 1)*d + 1/4*(a/b)^{1/4}/a*2^{1/2}*arctan(2^{1/2}/(a/b)^{1/4}*x - 1)*c - 1/8/b*(a/b)^{1/4}*2^{1/2}*ln((x^2+(a/b)^{1/4}*x*2^{1/2}+(a/b)^{1/2}))/((x^2-(a/b)^{1/4}*x*2^{1/2}+(a/b)^{1/2})))*d + 1/8*(a/b)^{1/4}/a*2^{1/2}*ln((x^2+(a/b)^{1/4}*x*2^{1/2}+(a/b)^{1/2}))/((x^2-(a/b)^{1/4}*x*2^{1/2}+(a/b)^{1/2})))*c - 1/4/b*(a/b)^{1/4}*2^{1/2}*arctan(2^{1/2}/(a/b)^{1/4}*x + 1)*d + 1/4*(a/b)^{1/4}/a*2^{1/2}*arctan(2^{1/2}/(a/b)^{1/4}*x + 1)*c$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)/(b\*x^4 + a), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.238155, size = 743, normalized size = 3.33

$$4b \left( -\frac{b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4}{a^3b^5} \right)^{\frac{1}{4}} \arctan \left( \frac{ab \left( -\frac{b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4}{a^3b^5} \right)^{\frac{1}{4}}}{(bc-ad)x + (bc-ad) \sqrt{\frac{a^2b^2 \sqrt{-\frac{b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4}{a^3b^5}} + (b^2c^2 - 2abcd + a^2d^2)}}{b^2c^2 - 2abcd + a^2d^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)/(b\*x^4 + a), x, algorithm="fricas")

[Out]  $\frac{1}{4} (4*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*d^4)/(a^3*b^5))^{(1/4)} * \arctan(-a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*d^4)/(a^3*b^5))^{(1/4)}) / ((b*c - a*d)*x + (b*c - a*d)*\sqrt{(a^2*b^2*\sqrt{(a^2*b^2*\sqrt{(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*d^4)/(a^3*b^5))} + (b^2*c^2 - 2*a*b^3*c*d^3 + a^4*d^4)/(a^3*b^5))} - 2*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*d^4) / (a^3*b^5)) + (b^2*c^2 - 2*a*b^3*c*d^3 + a^4*d^4)/(a^3*b^5))^{(1/4)} * \log(a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*d^4)/(a^3*b^5))^{(1/4)} - (b*c - a*d)*x) + b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*d^4)/(a^3*b^5))^{(1/4)} * \log(a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*d^4)/(a^3*b^5))^{(1/4)} - (b*c - a*d)*x) + b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*d^4)/(a^3*b^5))^{(1/4)} * \log(-a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*d^4)/(a^3*b^5))^{(1/4)} - (b*c - a*d)*x) + 4*d*x) / b$

**Sympy [A]** time = 2.37949, size = 87, normalized size = 0.39

$$\text{RootSum} \left( 256t^4a^3b^5 + a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4, \left( t \mapsto t \log \left( -\frac{4tab}{ad-bc} + x \right) \right) \right) + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*4+c)/(b\*x\*\*4+a), x)

[Out]  $\text{RootSum}(256*_t**4*a**3*b**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, \text{Lambda}(_t, _t*\log(-4*_t*a*b/(a*d - b*c) + x))) + d*x/b$

**GIAC/XCAS [A]** time = 0.217809, size = 331, normalized size = 1.48

$$\frac{dx}{b} + \frac{\sqrt{2} \left( (ab^3)^{\frac{1}{4}} bc - (ab^3)^{\frac{1}{4}} ad \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{a}{b} \right)^{\frac{1}{4}}} \right)}{4ab^2} + \frac{\sqrt{2} \left( (ab^3)^{\frac{1}{4}} bc - (ab^3)^{\frac{1}{4}} ad \right) \arctan \left( \frac{\sqrt{2} \left( 2x - \sqrt{2} \left( \frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{a}{b} \right)^{\frac{1}{4}}} \right)}{4ab^2} + \frac{\sqrt{2} \left( (ab^3)^{\frac{1}{4}} bc - (ab^3)^{\frac{1}{4}} ad \right) \ln \left( x^2 + \sqrt{2}x \left( \frac{a}{b} \right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}} \right)}{8ab^2} - \frac{\sqrt{2} \left( (ab^3)^{\frac{1}{4}} bc - (ab^3)^{\frac{1}{4}} ad \right) \ln \left( x^2 - \sqrt{2}x \left( \frac{a}{b} \right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}} \right)}{8ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)/(b\*x^4 + a), x, algorithm="giac")



```
[Out] d*x/b + 1/4*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^2) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^2)
```

$$3.65 \quad \int \frac{1}{(a+bx^4)(c+dx^4)} dx$$

**Optimal.** Leaf size=449

$$\begin{aligned} & \frac{b^{3/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}(bc-ad)} \\ & - \frac{b^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}(bc-ad)} + \frac{d^{3/4} \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{4\sqrt{2}c^{3/4}(bc-ad)} \\ & - \frac{d^{3/4} \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{4\sqrt{2}c^{3/4}(bc-ad)} + \frac{d^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)} - \frac{d^{3/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}} + 1\right)}{2\sqrt{2}c^{3/4}(bc-ad)} \end{aligned}$$

[Out]  $-(b^{3/4} \text{ArcTan}[1 - (\text{Sqrt}[2] * b^{1/4} * x)/a^{1/4}]) / (2 * \text{Sqrt}[2] * a^{3/4} * (b * c - a * d)) + (b^{3/4} \text{ArcTan}[1 + (\text{Sqrt}[2] * b^{1/4} * x)/a^{1/4}]) / (2 * \text{Sqrt}[2] * a^{3/4} * (b * c - a * d)) + (d^{3/4} \text{ArcTan}[1 - (\text{Sqrt}[2] * d^{1/4} * x)/c^{1/4}]) / (2 * \text{Sqrt}[2] * c^{3/4} * (b * c - a * d)) - (d^{3/4} \text{ArcTan}[1 + (\text{Sqrt}[2] * d^{1/4} * x)/c^{1/4}]) / (2 * \text{Sqrt}[2] * c^{3/4} * (b * c - a * d)) - (b^{3/4} \text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2] * a^{1/4} * b^{1/4} * x + \text{Sqrt}[b] * x^2]) / (4 * \text{Sqrt}[2] * a^{3/4} * (b * c - a * d)) + (b^{3/4} \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] * a^{1/4} * b^{1/4} * x + \text{Sqrt}[b] * x^2]) / (4 * \text{Sqrt}[2] * a^{3/4} * (b * c - a * d)) + (d^{3/4} \text{Log}[\text{Sqrt}[c] - \text{Sqrt}[2] * c^{1/4} * d^{1/4} * x + \text{Sqrt}[d] * x^2]) / (4 * \text{Sqrt}[2] * c^{3/4} * (b * c - a * d)) - (d^{3/4} \text{Log}[\text{Sqrt}[c] + \text{Sqrt}[2] * c^{1/4} * d^{1/4} * x + \text{Sqrt}[d] * x^2]) / (4 * \text{Sqrt}[2] * c^{3/4} * (b * c - a * d))$

**Rubi [A]** time = 0.566816, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$\begin{aligned} & \frac{b^{3/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}(bc-ad)} \\ & - \frac{b^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}(bc-ad)} + \frac{d^{3/4} \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{4\sqrt{2}c^{3/4}(bc-ad)} \\ & - \frac{d^{3/4} \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{4\sqrt{2}c^{3/4}(bc-ad)} + \frac{d^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)} - \frac{d^{3/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}} + 1\right)}{2\sqrt{2}c^{3/4}(bc-ad)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)\*(c + d\*x^4)), x]

[Out]  $-(b^{3/4} \text{ArcTan}[1 - (\text{Sqrt}[2] * b^{1/4} * x)/a^{1/4}]) / (2 * \text{Sqrt}[2] * a^{3/4} * (b * c - a * d)) + (b^{3/4} \text{ArcTan}[1 + (\text{Sqrt}[2] * b^{1/4} * x)/a^{1/4}]) / (2 * \text{Sqrt}[2] * a^{3/4} * (b * c - a * d)) + (d^{3/4} \text{ArcTan}[1 - (\text{Sqrt}[2] * d^{1/4} * x)/c^{1/4}]) / (2 * \text{Sqrt}[2] * c^{3/4} * (b * c - a * d)) - (d^{3/4} \text{ArcTan}[1 + (\text{Sqrt}[2] * d^{1/4} * x)/c^{1/4}]) / (2 * \text{Sqrt}[2] * c^{3/4} * (b * c - a * d)) - (b^{3/4} \text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2] * a^{1/4} * b^{1/4} * x + \text{Sqrt}[b] * x^2]) / (4 * \text{Sqrt}[2] * a^{3/4} * (b * c - a * d)) + (b^{3/4} \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] * a^{1/4} * b^{1/4} * x + \text{Sqrt}[b] * x^2]) / (4 * \text{Sqrt}[2] * a^{3/4} * (b * c - a * d)) + (d^{3/4} \text{Log}[\text{Sqrt}[c] - \text{Sqrt}[2] * c^{1/4} * d^{1/4} * x + \text{Sqrt}[d] * x^2]) / (4 * \text{Sqrt}[2] * c^{3/4} * (b * c - a * d)) - (d^{3/4} \text{Log}[\text{Sqrt}[c] + \text{Sqrt}[2] * c^{1/4} * d^{1/4} * x + \text{Sqrt}[d] * x^2]) / (4 * \text{Sqrt}[2] * c^{3/4} * (b * c - a * d))$

**Rubi in Sympy [A]** time = 119.517, size = 400, normalized size = 0.89

$$\frac{\sqrt{2}d^{\frac{3}{4}} \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{8c^{\frac{3}{4}}(ad-bc)} + \frac{\sqrt{2}d^{\frac{3}{4}} \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{8c^{\frac{3}{4}}(ad-bc)}$$

$$- \frac{\sqrt{2}d^{\frac{3}{4}} \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{4c^{\frac{3}{4}}(ad-bc)} + \frac{\sqrt{2}d^{\frac{3}{4}} \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{4c^{\frac{3}{4}}(ad-bc)} + \frac{\sqrt{2}b^{\frac{3}{4}} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{8a^{\frac{3}{4}}(ad-bc)}$$

$$- \frac{\sqrt{2}b^{\frac{3}{4}} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{8a^{\frac{3}{4}}(ad-bc)} + \frac{\sqrt{2}b^{\frac{3}{4}} \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}(ad-bc)} - \frac{\sqrt{2}b^{\frac{3}{4}} \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(b*x**4+a)/(d*x**4+c),x)`

[Out] `-sqrt(2)*d**(3/4)*log(-sqrt(2)*c**(1/4)*d**(1/4)*x + sqrt(c) + sqrt(d)*x**2)/(8*c**(3/4)*(a*d - b*c)) + sqrt(2)*d**(3/4)*log(sqrt(2)*c**(1/4)*d**(1/4)*x + sqrt(c) + sqrt(d)*x**2)/(8*c**(3/4)*(a*d - b*c)) - sqrt(2)*d**(3/4)*atan(1 - sqrt(2)*d**(1/4)*x/c**(1/4))/(4*c**(3/4)*(a*d - b*c)) + sqrt(2)*d**(3/4)*atan(1 + sqrt(2)*d**(1/4)*x/c**(1/4))/(4*c**(3/4)*(a*d - b*c)) + sqrt(2)*b**(3/4)*log(-sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(8*a**(3/4)*(a*d - b*c)) - sqrt(2)*b**(3/4)*log(sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(8*a**(3/4)*(a*d - b*c)) + sqrt(2)*b**(3/4)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*(a*d - b*c)) - sqrt(2)*b**(3/4)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*(a*d - b*c))`

**Mathematica [A]** time = 0.281602, size = 340, normalized size = 0.76

$$a^{3/4}d^{3/4} \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right) - a^{3/4}d^{3/4} \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right) + 2a^{3/4}d^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right) - 2a^{3/4}d^{3/4} \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)$$

Antiderivative was successfully verified.

[In] `Integrate[1/((a + b*x^4)*(c + d*x^4)),x]`

[Out] `(-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(3/4)*c^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(3/4)*d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] - a^(3/4)*d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))`

**Maple [A]** time = 0.002, size = 320, normalized size = 0.7

$$\begin{aligned} & \frac{d\sqrt{2}}{(8ad - 8bc)c} \sqrt[4]{\frac{c}{d}} \ln \left( 1 \left( x^2 + \sqrt[4]{\frac{c}{d}} x \sqrt{2} + \sqrt{\frac{c}{d}} \right) \left( x^2 - \sqrt[4]{\frac{c}{d}} x \sqrt{2} + \sqrt{\frac{c}{d}} \right)^{-1} \right) \\ & + \frac{d\sqrt{2}}{(4ad - 4bc)c} \sqrt[4]{\frac{c}{d}} \arctan \left( x \sqrt{2} \frac{1}{\sqrt[4]{\frac{c}{d}}} + 1 \right) + \frac{d\sqrt{2}}{(4ad - 4bc)c} \sqrt[4]{\frac{c}{d}} \arctan \left( x \sqrt{2} \frac{1}{\sqrt[4]{\frac{c}{d}}} - 1 \right) \\ & - \frac{b\sqrt{2}}{(8ad - 8bc)a} \sqrt[4]{\frac{a}{b}} \ln \left( 1 \left( x^2 + \sqrt[4]{\frac{a}{b}} x \sqrt{2} + \sqrt{\frac{a}{b}} \right) \left( x^2 - \sqrt[4]{\frac{a}{b}} x \sqrt{2} + \sqrt{\frac{a}{b}} \right)^{-1} \right) \\ & - \frac{b\sqrt{2}}{(4ad - 4bc)a} \sqrt[4]{\frac{a}{b}} \arctan \left( x \sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} + 1 \right) - \frac{b\sqrt{2}}{(4ad - 4bc)a} \sqrt[4]{\frac{a}{b}} \arctan \left( x \sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} - 1 \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)/(d\*x^4+c), x)

[Out] 1/8\*d/(a\*d-b\*c)\*(c/d)^(1/4)/c\*2^(1/2)\*ln((x^2+(c/d)^(1/4)\*x\*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)\*x\*2^(1/2)+(c/d)^(1/2)))+1/4\*d/(a\*d-b\*c)\*(c/d)^(1/4)/c\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x+1)+1/4\*d/(a\*d-b\*c)\*(c/d)^(1/4)/c\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x-1)-1/8\*b/(a\*d-b\*c)\*(a/b)^(1/4)/a\*2^(1/2)\*ln((x^2+(a/b)^(1/4)\*x\*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*x\*2^(1/2)+(a/b)^(1/2)))-1/4\*b/(a\*d-b\*c)\*(a/b)^(1/4)/a\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)-1/4\*b/(a\*d-b\*c)\*(a/b)^(1/4)/a\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)\*(d\*x^4 + c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.395315, size = 1472, normalized size = 3.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)\*(d\*x^4 + c)),x, algorithm="fricas")

[Out] (-b^3/(a^3\*b^4\*c^4 - 4\*a^4\*b^3\*c^3\*d + 6\*a^5\*b^2\*c^2\*d^2 - 4\*a^6\*b\*c\*d^3 + a^7\*d^4))^(1/4)\*arctan(-(a\*b\*c - a^2\*d)\*(-b^3/(a^3\*b^4\*c^4 - 4\*a^4\*b^3\*c^3\*d + 6\*a^5\*b^2\*c^2\*d^2 - 4\*a^6\*b\*c\*d^3 + a^7\*d^4))^(1/4)/(b\*x + b\*sqrt((b^2\*x^2 + (a^2\*b^2\*c^2 - 2\*a^3\*b\*c\*d + a^4\*d^2)\*sqrt(-b^3/(a^3\*b^4\*c^4 - 4\*a^4\*b^3\*c^3\*d + 6\*a^5\*b^2\*c^2\*d^2 - 4\*a^6\*b\*c\*d^3 + a^7\*d^4))/b^2)))) - (-d^3/(b^4\*c^7 - 4\*a\*b^3\*c^6\*d + 6\*a^2\*b^2\*c^5\*d^2 - 4\*a^3\*b\*c^4\*d^3 + a^4\*c^3\*d^4))^(1/4)\*arctan(-(b\*c^2 - a\*c\*d)\*(-d^3/(b^4\*c^7 - 4\*a\*b^3\*c^6\*d + 6\*a^2\*b^2\*c^5\*d^2 - 4\*a^3\*b\*c^4\*d^3 + a^4\*c^3\*d^4))^(1/4)/(d\*x + d\*sqrt((d^2\*x^2 + (b^2\*c^4 - 2\*a\*b\*c^3\*d + a^2\*c^2\*d^2)\*sqrt(-d^3/(b^4\*c^7 - 4\*a\*b^3\*c^6\*d + 6\*a^2\*b^2\*c^5\*d^2 - 4\*a^3\*b\*c^4\*d^3 + a^4\*c^3\*d^4))/d^2)))) + 1/4\*(-b^3/(a^3\*b^4\*c^4 - 4\*a^4\*b^3\*c^3\*d + 6\*a^5\*b^2\*c^2\*d^2 - 4\*a^6\*b\*c\*d^3 + a^7\*d^4))^(1/4)\*sqrt((b^2\*x^2 + (a^2\*b^2\*c^2 - 2\*a^3\*b\*c\*d + a^4\*d^2)\*sqrt(-b^3/(a^3\*b^4\*c^4 - 4\*a^4\*b^3\*c^3\*d + 6\*a^5\*b^2\*c^2\*d^2 - 4\*a^6\*b\*c\*d^3 + a^7\*d^4))/b^2)))/(b\*x + b\*sqrt((b^2\*x^2 + (a^2\*b^2\*c^2 - 2\*a^3\*b\*c\*d + a^4\*d^2)\*sqrt(-b^3/(a^3\*b^4\*c^4 - 4\*a^4\*b^3\*c^3\*d + 6\*a^5\*b^2\*c^2\*d^2 - 4\*a^6\*b\*c\*d^3 + a^7\*d^4))/b^2)))) + 1/4\*(-d^3/(b^4\*c^7 - 4\*a\*b^3\*c^6\*d + 6\*a^2\*b^2\*c^5\*d^2 - 4\*a^3\*b\*c^4\*d^3 + a^4\*c^3\*d^4))^(1/4)\*sqrt((d^2\*x^2 + (b^2\*c^4 - 2\*a\*b\*c^3\*d + a^2\*c^2\*d^2)\*sqrt(-d^3/(b^4\*c^7 - 4\*a\*b^3\*c^6\*d + 6\*a^2\*b^2\*c^5\*d^2 - 4\*a^3\*b\*c^4\*d^3 + a^4\*c^3\*d^4))/d^2))))/(d\*x + d\*sqrt((d^2\*x^2 + (b^2\*c^4 - 2\*a\*b\*c^3\*d + a^2\*c^2\*d^2)\*sqrt(-d^3/(b^4\*c^7 - 4\*a\*b^3\*c^6\*d + 6\*a^2\*b^2\*c^5\*d^2 - 4\*a^3\*b\*c^4\*d^3 + a^4\*c^3\*d^4))/d^2))))

$$\begin{aligned} & (a^5 b^2 c^2 d^2 - 4 a^6 b^3 c d^3 + a^7 d^4)^{1/4} \log(bx + (abc - a^2 d) \cdot (-b^3 / (a^3 b^4 c^4 - 4 a^4 b^3 c^3 d + 6 a^5 b^2 c^2 d^2 - 4 a^6 b^3 c d^3 + a^7 d^4))^{1/4}) \\ & - 1/4 \cdot (-b^3 / (a^3 b^4 c^4 - 4 a^4 b^3 c^3 d + 6 a^5 b^2 c^2 d^2 - 4 a^6 b^3 c d^3 + a^7 d^4))^{1/4} \log(bx - (abc - a^2 d) \cdot (-b^3 / (a^3 b^4 c^4 - 4 a^4 b^3 c^3 d + 6 a^5 b^2 c^2 d^2 - 4 a^6 b^3 c d^3 + a^7 d^4))^{1/4}) \\ & - 1/4 \cdot (-d^3 / (b^4 c^7 - 4 a b^3 c^6 d + 6 a^2 b^2 c^5 d^2 - 4 a^3 b^3 c^4 d^3 + a^4 c^3 d^4))^{1/4} \log(dx + (b^2 c^2 - a^2 c d) \cdot (-d^3 / (b^4 c^7 - 4 a b^3 c^6 d + 6 a^2 b^2 c^5 d^2 - 4 a^3 b^3 c^4 d^3 + a^4 c^3 d^4))^{1/4}) \\ & + 1/4 \cdot (-d^3 / (b^4 c^7 - 4 a b^3 c^6 d + 6 a^2 b^2 c^5 d^2 - 4 a^3 b^3 c^4 d^3 + a^4 c^3 d^4))^{1/4} \log(dx - (b^2 c^2 - a^2 c d) \cdot (-d^3 / (b^4 c^7 - 4 a b^3 c^6 d + 6 a^2 b^2 c^5 d^2 - 4 a^3 b^3 c^4 d^3 + a^4 c^3 d^4))^{1/4}) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)/(d\*x\*\*4+c), x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)\*(d\*x^4 + c)), x, algorithm="giac")

[Out] integrate(1/((b\*x^4 + a)\*(d\*x^4 + c)), x)

$$3.66 \quad \int \frac{1}{(a+bx^4)(c+dx^4)^2} dx$$

**Optimal.** Leaf size=513

$$\begin{aligned} & \frac{b^{7/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{b^{7/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2} \\ & - \frac{b^{7/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{b^{7/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} \\ & + \frac{d^{3/4}(7bc-3ad) \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2} \\ & - \frac{d^{3/4}(7bc-3ad) \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2} + \frac{d^{3/4}(7bc-3ad) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2} \\ & - \frac{d^{3/4}(7bc-3ad) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}} + 1\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2} - \frac{dx}{4c(c+dx^4)(bc-ad)} \end{aligned}$$

[Out]  $-(d*x)/(4*c*(b*c - a*d)*(c + d*x^4)) - (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (b^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*d)^2)$

**Rubi [A]** time = 0.884336, antiderivative size = 513, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\begin{aligned} & \frac{b^{7/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{b^{7/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2} \\ & - \frac{b^{7/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{b^{7/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} \\ & + \frac{d^{3/4}(7bc-3ad) \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2} \\ & - \frac{d^{3/4}(7bc-3ad) \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2} + \frac{d^{3/4}(7bc-3ad) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2} \\ & - \frac{d^{3/4}(7bc-3ad) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}} + 1\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2} - \frac{dx}{4c(c+dx^4)(bc-ad)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)\*(c + d\*x^4)^2), x]

[Out]  $-(d*x)/(4*c*(b*c - a*d)*(c + d*x^4)) - (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (b^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*d)^2)$

$$\begin{aligned} & /4) * x) / c^{(1/4)}] / (8 * \text{Sqrt}[2] * c^{(7/4)} * (b * c - a * d)^2) - (d^{(3/4)} * (7 * \\ & b * c - 3 * a * d) * \text{ArcTan}[1 + (\text{Sqrt}[2] * d^{(1/4)} * x) / c^{(1/4)}] / (8 * \text{Sqrt}[2] * \\ & c^{(7/4)} * (b * c - a * d)^2) - (b^{(7/4)} * \text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2] * a^{(1/4)} * b \\ & ^{(1/4)} * x + \text{Sqrt}[b] * x^2]) / (4 * \text{Sqrt}[2] * a^{(3/4)} * (b * c - a * d)^2) + (b^{(7/4)} * \\ & \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] * a^{(1/4)} * b^{(1/4)} * x + \text{Sqrt}[b] * x^2]) / (4 * \text{S} \\ & \text{qrt}[2] * a^{(3/4)} * (b * c - a * d)^2) + (d^{(3/4)} * (7 * b * c - 3 * a * d) * \text{Log}[\text{Sqrt} \\ & [c] - \text{Sqrt}[2] * c^{(1/4)} * d^{(1/4)} * x + \text{Sqrt}[d] * x^2]) / (16 * \text{Sqrt}[2] * c^{(7/4)} \\ & * (b * c - a * d)^2) - (d^{(3/4)} * (7 * b * c - 3 * a * d) * \text{Log}[\text{Sqrt}[c] + \text{Sqrt}[2] \\ & ] * c^{(1/4)} * d^{(1/4)} * x + \text{Sqrt}[d] * x^2]) / (16 * \text{Sqrt}[2] * c^{(7/4)} * (b * c - a * \\ & d)^2) \end{aligned}$$

**Rubi in Sympy [A]** time = 174.262, size = 474, normalized size = 0.92

$$\begin{aligned} & \frac{dx}{4c(c + dx^4)(ad - bc)} - \frac{\sqrt{2}d^{3/4}(3ad - 7bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{32c^{7/4}(ad - bc)^2} \\ & + \frac{\sqrt{2}d^{3/4}(3ad - 7bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{32c^{7/4}(ad - bc)^2} - \frac{\sqrt{2}d^{3/4}(3ad - 7bc)\operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{16c^{7/4}(ad - bc)^2} \\ & + \frac{\sqrt{2}d^{3/4}(3ad - 7bc)\operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{16c^{7/4}(ad - bc)^2} - \frac{\sqrt{2}b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{8a^{3/4}(ad - bc)^2} \\ & + \frac{\sqrt{2}b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{8a^{3/4}(ad - bc)^2} - \frac{\sqrt{2}b^{7/4}\operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{4a^{3/4}(ad - bc)^2} + \frac{\sqrt{2}b^{7/4}\operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{4a^{3/4}(ad - bc)^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(b*x**4+a)/(d*x**4+c)**2,x)`

[Out]  $d * x / (4 * c * (c + d * x^4) * (a * d - b * c)) - \text{sqrt}(2) * d^{(3/4)} * (3 * a * d - 7 * b * c) * \log(-\text{sqrt}(2) * c^{(1/4)} * d^{(1/4)} * x + \text{sqrt}(c) + \text{sqrt}(d) * x^2) / (32 * c^{(7/4)} * (a * d - b * c)^2) + \text{sqrt}(2) * d^{(3/4)} * (3 * a * d - 7 * b * c) * \log(\text{sqrt}(2) * c^{(1/4)} * d^{(1/4)} * x + \text{sqrt}(c) + \text{sqrt}(d) * x^2) / (32 * c^{(7/4)} * (a * d - b * c)^2) - \text{sqrt}(2) * d^{(3/4)} * (3 * a * d - 7 * b * c) * \operatorname{atan}(1 - \text{sqrt}(2) * d^{(1/4)} * x / c^{(1/4)}) / (16 * c^{(7/4)} * (a * d - b * c)^2) + \text{sqrt}(2) * d^{(3/4)} * (3 * a * d - 7 * b * c) * \operatorname{atan}(1 + \text{sqrt}(2) * d^{(1/4)} * x / c^{(1/4)}) / (16 * c^{(7/4)} * (a * d - b * c)^2) - \text{sqrt}(2) * b^{(7/4)} * \log(-\text{sqrt}(2) * a^{(1/4)} * b^{(1/4)} * x + \text{sqrt}(a) + \text{sqrt}(b) * x^2) / (8 * a^{(3/4)} * (a * d - b * c)^2) + \text{sqrt}(2) * b^{(7/4)} * \log(\text{sqrt}(2) * a^{(1/4)} * b^{(1/4)} * x + \text{sqrt}(a) + \text{sqrt}(b) * x^2) / (8 * a^{(3/4)} * (a * d - b * c)^2) - \text{sqrt}(2) * b^{(7/4)} * \operatorname{atan}(1 - \text{sqrt}(2) * b^{(1/4)} * x / a^{(1/4)}) / (4 * a^{(3/4)} * (a * d - b * c)^2) + \text{sqrt}(2) * b^{(7/4)} * \operatorname{atan}(1 + \text{sqrt}(2) * b^{(1/4)} * x / a^{(1/4)}) / (4 * a^{(3/4)} * (a * d - b * c)^2)$

**Mathematica [A]** time = 0.561561, size = 498, normalized size = 0.97

$$8a^{3/4}c^{3/4}dx(ad - bc) - 2\sqrt{2}a^{3/4}d^{3/4}(c + dx^4)(3ad - 7bc)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right) + 2\sqrt{2}a^{3/4}d^{3/4}(c + dx^4)(3ad - 7bc)\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)$$

Antiderivative was successfully verified.

[In] `Integrate[1/((a + b*x^4)*(c + d*x^4)^2),x]`

[Out]  $(8 * a^{(3/4)} * c^{(3/4)} * d * (- (b * c) + a * d) * x - 8 * \text{Sqrt}[2] * b^{(7/4)} * c^{(7/4)} * (c + d * x^4) * \text{ArcTan}[1 - (\text{Sqrt}[2] * b^{(1/4)} * x) / a^{(1/4)}] + 8 * \text{Sqrt}[2] * b^{(7/4)} * c^{(7/4)} * (c + d * x^4) * \text{ArcTan}[1 + (\text{Sqrt}[2] * b^{(1/4)} * x) / a^{(1/4)}]) - 2 * \text{Sqrt}[2] * a^{(3/4)} * d^{(3/4)} * (-7 * b * c + 3 * a * d) * (c + d * x^4) * \text{ArcTan}[1 - (\text{Sqrt}[2] * d^{(1/4)} * x) / c^{(1/4)}] + 2 * \text{Sqrt}[2] * a^{(3/4)} * d^{(3/4)} * (-7 * b * c + 3 * a * d) * (c + d * x^4) * \text{ArcTan}[1 + (\text{Sqrt}[2] * d^{(1/4)} * x) / c^{(1/4)}] - 4 * \text{Sqrt}[2] * b^{(7/4)} * c^{(7/4)} * (c + d * x^4) * \text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2] * a$

$$\begin{aligned} & \sqrt[4]{b} \sqrt[4]{x} + \sqrt{b} x^2 + 4 \sqrt{2} b^{7/4} c^{7/4} (c + d x^4) \operatorname{Log}[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} x + \sqrt{b} x^2] + \sqrt{2} a^{3/4} d^{3/4} (7 b c - 3 a d) (c + d x^4) \operatorname{Log}[\sqrt{c} - \sqrt{2} c^{1/4} d^{1/4} x + \sqrt{d} x^2] + \sqrt{2} a^{3/4} d^{3/4} (-7 b c + 3 a d) (c + d x^4) \operatorname{Log}[\sqrt{c} + \sqrt{2} c^{1/4} d^{1/4} x + \sqrt{d} x^2] / (32 a^{3/4} c^{7/4} (b c - a d)^2 (c + d x^4)) \end{aligned}$$

**Maple [A]** time = 0.003, size = 550, normalized size = 1.1

$$\begin{aligned} & \frac{d^2 x a}{4 (a d - b c)^2 c (d x^4 + c)} - \frac{d x b}{4 (a d - b c)^2 (d x^4 + c)} \\ & + \frac{3 d^2 \sqrt{2} a}{16 (a d - b c)^2 c^2 \sqrt[4]{d}} \arctan\left(x \sqrt{2} \frac{1}{\sqrt[4]{\frac{c}{d}}} - 1\right) - \frac{7 d \sqrt{2} b}{16 (a d - b c)^2 c \sqrt[4]{d}} \arctan\left(x \sqrt{2} \frac{1}{\sqrt[4]{\frac{c}{d}}} - 1\right) \\ & + \frac{3 d^2 \sqrt{2} a}{32 (a d - b c)^2 c^2 \sqrt[4]{d}} \ln\left(1 \left(x^2 + \sqrt[4]{\frac{c}{d}} x \sqrt{2} + \sqrt{\frac{c}{d}}\right) \left(x^2 - \sqrt[4]{\frac{c}{d}} x \sqrt{2} + \sqrt{\frac{c}{d}}\right)^{-1}\right) \\ & - \frac{7 d \sqrt{2} b}{32 (a d - b c)^2 c \sqrt[4]{d}} \ln\left(1 \left(x^2 + \sqrt[4]{\frac{c}{d}} x \sqrt{2} + \sqrt{\frac{c}{d}}\right) \left(x^2 - \sqrt[4]{\frac{c}{d}} x \sqrt{2} + \sqrt{\frac{c}{d}}\right)^{-1}\right) \\ & + \frac{3 d^2 \sqrt{2} a}{16 (a d - b c)^2 c^2 \sqrt[4]{d}} \arctan\left(x \sqrt{2} \frac{1}{\sqrt[4]{\frac{c}{d}}} + 1\right) - \frac{7 d \sqrt{2} b}{16 (a d - b c)^2 c \sqrt[4]{d}} \arctan\left(x \sqrt{2} \frac{1}{\sqrt[4]{\frac{c}{d}}} + 1\right) \\ & + \frac{b^2 \sqrt{2}}{8 (a d - b c)^2 a \sqrt[4]{b}} \ln\left(1 \left(x^2 + \sqrt[4]{\frac{a}{b}} x \sqrt{2} + \sqrt{\frac{a}{b}}\right) \left(x^2 - \sqrt[4]{\frac{a}{b}} x \sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \\ & + \frac{b^2 \sqrt{2}}{4 (a d - b c)^2 a \sqrt[4]{b}} \arctan\left(x \sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) + \frac{b^2 \sqrt{2}}{4 (a d - b c)^2 a \sqrt[4]{b}} \arctan\left(x \sqrt{2} \frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)/(d\*x^4+c)^2,x)

[Out]  $\frac{1}{4} d^2 / (a d - b c)^2 / c x / (d x^4 + c) + a^{-1/4} d / (a d - b c)^2 x / (d x^4 + c) + b + 3/16 d^2 / (a d - b c)^2 / c^2 (c/d)^{1/4} 2^{1/2} \arctan(2^{1/2} / (c/d)^{1/4} x - 1) + a - 7/16 d / (a d - b c)^2 / c (c/d)^{1/4} 2^{1/2} \arctan(2^{1/2} / (c/d)^{1/4} x - 1) + b + 3/32 d^2 / (a d - b c)^2 / c^2 (c/d)^{1/4} 2^{1/2} \ln((x^2 + (c/d)^{1/4} x 2^{1/2} + (c/d)^{1/2}) / (x^2 - (c/d)^{1/4} x 2^{1/2} + (c/d)^{1/2})) + a - 7/32 d / (a d - b c)^2 / c (c/d)^{1/4} 2^{1/2} \ln((x^2 + (c/d)^{1/4} x 2^{1/2} + (c/d)^{1/2}) / (x^2 - (c/d)^{1/4} x 2^{1/2} + (c/d)^{1/2})) + b + 3/16 d^2 / (a d - b c)^2 / c^2 (c/d)^{1/4} 2^{1/2} \arctan(2^{1/2} / (c/d)^{1/4} x + 1) + a - 7/16 d / (a d - b c)^2 / c (c/d)^{1/4} 2^{1/2} \arctan(2^{1/2} / (c/d)^{1/4} x + 1) + b + 1/8 b^2 / (a d - b c)^2 (a/b)^{1/4} / a 2^{1/2} \ln((x^2 + (a/b)^{1/4} x 2^{1/2} + (a/b)^{1/2}) / (x^2 - (a/b)^{1/4} x 2^{1/2} + (a/b)^{1/2})) + 1/4 b^2 / (a d - b c)^2 (a/b)^{1/4} / a 2^{1/2} \arctan(2^{1/2} / (a/b)^{1/4} x + 1) + 1/4 b^2 / (a d - b c)^2 (a/b)^{1/4} / a 2^{1/2} \arctan(2^{1/2} / (a/b)^{1/4} x - 1)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)\*(d\*x^4 + c)^2),x, algorithm="maxima")



[Out] Exception raised: ValueError

**Fricas** [A] time = 21.891, size = 3665, normalized size = 7.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)\*(d\*x^4 + c)^2),x, algorithm="fricas")

[Out] 
$$-1/16*(16*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^{10}*b*c*d^7 + a^{11}*d^8))^{1/4}*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*\arctan((-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^{10}*b*c*d^7 + a^{11}*d^8))^{1/4}*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2))/(b^2*x + b^2*\sqrt{(b^4*x^2 + (a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*\sqrt{-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^{10}*b*c*d^7 + a^{11}*d^8)}}/b^4))) + 4*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^{15} - 8*a*b^7*c^{14}*d + 28*a^2*b^6*c^{13}*d^2 - 56*a^3*b^5*c^{12}*d^3 + 70*a^4*b^4*c^{11}*d^4 - 56*a^5*b^3*c^{10}*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^{1/4}*\arctan(-(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^{15} - 8*a*b^7*c^{14}*d + 28*a^2*b^6*c^{13}*d^2 - 56*a^3*b^5*c^{12}*d^3 + 70*a^4*b^4*c^{11}*d^4 - 56*a^5*b^3*c^{10}*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8)))^{1/4}/((7*b*c*d - 3*a*d^2)*x + (7*b*c*d - 3*a*d^2)*\sqrt{((49*b^2*c^2*d^2 - 42*a*b*c*d^3 + 9*a^2*d^4)*x^2 + (b^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4)*\sqrt{-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^{15} - 8*a*b^7*c^{14}*d + 28*a^2*b^6*c^{13}*d^2 - 56*a^3*b^5*c^{12}*d^3 + 70*a^4*b^4*c^{11}*d^4 - 56*a^5*b^3*c^{10}*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))}) - 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^{10}*b*c*d^7 + a^{11}*d^8))^{1/4}*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*\log(b^2*x + (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^{10}*b*c*d^7 + a^{11}*d^8))^{1/4}*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)) + 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^{10}*b*c*d^7 + a^{11}*d^8))^{1/4}*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*\log(b^2*x - (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^{10}*b*c*d^7 + a^{11}*d^8))^{1/4}*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)) - ((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^{15} - 8*a*b^7*c^{14}*d + 28*a^2*b^6*c^{13}*d^2 - 56*a^3*b^5*c^{12}*d^3 + 70*a^4*b^4*c^{11}*d^4 - 56*a^5*b^3*c^{10}*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^{1/4}*\log(-(7*b*c*d - 3*a*d^2)*x + (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^{15} - 8*a*b^7*c^{14}*d + 28*a^2*b^6*c^{13}*d^2 - 56*a^3*b^5*c^{12}*d^3 + 70*a^4*b^4*c^{11}*d^4 - 56*a^5*b^3*c^{10}*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^{1/4})) + ((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^{15} - 8*a*b^7*c^{14}*d + 28*a^2*b^6*c^{13}*d^2 - 56*a^3*b^5*c^{12}*d^3 + 70*a^4*b^4*c^{11}*d^4 - 56*a^5*b^3*c^{10}*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^{1/4}*\log(-(7*b*c*d - 3*a*d^2)*x - (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^{15} - 8*a*b^7*c^{14}*d + 28*a^2*b^6*c^{13}*d^2 - 56*a^3*b^5*c^{12}*d^3 + 70*a^4*b^4*c^{11}*d^4 - 56*a^5*b^3*c^{10}*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^{1/4}))$$

$$\frac{3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^{(1/4)} + 4*d*x)/((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)/(d\*x\*\*4+c)\*\*2,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.228134, size = 900, normalized size = 1.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)\*(d\*x^4 + c)^2),x, algorithm="giac")

[Out]  $\frac{1}{2}*(a*b^3)^{1/4}*b*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{1/4}))/((a/b)^{1/4})*\sqrt{2}*(a*b^2*c^2 - 2*\sqrt{2}*a^2*b*c*d + \sqrt{2}*a^3*d^2) + 1/2*(a*b^3)^{1/4}*b*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{1/4}))/((a/b)^{1/4})*\sqrt{2}*(a*b^2*c^2 - 2*\sqrt{2}*a^2*b*c*d + \sqrt{2}*a^3*d^2) + 1/4*(a*b^3)^{1/4}*b*\ln(x^2 + \sqrt{2}*x*(a/b)^{1/4} + \sqrt{a/b}))/(\sqrt{2}*(a*b^2*c^2 - 2*\sqrt{2}*a^2*b*c*d + \sqrt{2}*a^3*d^2) - 1/4*(a*b^3)^{1/4}*b*\ln(x^2 - \sqrt{2}*x*(a/b)^{1/4} + \sqrt{a/b}))/(\sqrt{2}*(a*b^2*c^2 - 2*\sqrt{2}*a^2*b*c*d + \sqrt{2}*a^3*d^2) - 1/8*(7*(c*d^3)^{1/4}*b*c - 3*(c*d^3)^{1/4}*a*d)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(c/d)^{1/4}))/((c/d)^{1/4}))/(\sqrt{2}*(b^2*c^4 - 2*\sqrt{2}*a*b*c^3*d + \sqrt{2}*a^2*c^2*d^2) - 1/8*(7*(c*d^3)^{1/4}*b*c - 3*(c*d^3)^{1/4}*a*d)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(c/d)^{1/4}))/((c/d)^{1/4}))/(\sqrt{2}*(b^2*c^4 - 2*\sqrt{2}*a*b*c^3*d + \sqrt{2}*a^2*c^2*d^2) - 1/16*(7*(c*d^3)^{1/4}*b*c - 3*(c*d^3)^{1/4}*a*d)*\ln(x^2 + \sqrt{2}*x*(c/d)^{1/4} + \sqrt{c/d}))/(\sqrt{2}*(b^2*c^4 - 2*\sqrt{2}*a*b*c^3*d + \sqrt{2}*a^2*c^2*d^2) + 1/16*(7*(c*d^3)^{1/4}*b*c - 3*(c*d^3)^{1/4}*a*d)*\ln(x^2 - \sqrt{2}*x*(c/d)^{1/4} + \sqrt{c/d}))/(\sqrt{2}*(b^2*c^4 - 2*\sqrt{2}*a*b*c^3*d + \sqrt{2}*a^2*c^2*d^2) - 1/4*d*x)/((d*x^4 + c)*(b*c^2 - a*c*d))$

$$3.67 \quad \int \frac{(c+dx^4)^5}{(a+bx^4)^2} dx$$

**Optimal.** Leaf size=407

$$\begin{aligned} & - \frac{(bc-ad)^4(17ad+3bc) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{21/4}} \\ & + \frac{(bc-ad)^4(17ad+3bc) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{21/4}} \\ & - \frac{(bc-ad)^4(17ad+3bc) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{21/4}} \\ & + \frac{(bc-ad)^4(17ad+3bc) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2}a^{7/4}b^{21/4}} + \frac{d^3x^5(3a^2d^2 - 10abcd + 10b^2c^2)}{5b^4} \\ & + \frac{d^2x(-4a^3d^3 + 15a^2bcd^2 - 20ab^2c^2d + 10b^3c^3)}{b^5} + \frac{x(bc-ad)^5}{4ab^5(a+bx^4)} + \frac{d^4x^9(5bc-2ad)}{9b^3} + \frac{d^5x^{13}}{13b^2} \end{aligned}$$

[Out]  $(d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5)/(5*b^4) + (d^4*(5*b*c - 2*a*d)*x^9)/(9*b^3) + (d^5*x^13)/(13*b^2) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^4)) - ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*b^(21/4)) - ((b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(21/4))$

**Rubi [A]** time = 0.785902, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\begin{aligned} & - \frac{(bc-ad)^4(17ad+3bc) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{21/4}} \\ & + \frac{(bc-ad)^4(17ad+3bc) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{21/4}} \\ & - \frac{(bc-ad)^4(17ad+3bc) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{21/4}} \\ & + \frac{(bc-ad)^4(17ad+3bc) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2}a^{7/4}b^{21/4}} + \frac{d^3x^5(3a^2d^2 - 10abcd + 10b^2c^2)}{5b^4} \\ & + \frac{d^2x(-4a^3d^3 + 15a^2bcd^2 - 20ab^2c^2d + 10b^3c^3)}{b^5} + \frac{x(bc-ad)^5}{4ab^5(a+bx^4)} + \frac{d^4x^9(5bc-2ad)}{9b^3} + \frac{d^5x^{13}}{13b^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^5/(a + b\*x^4)^2, x]

[Out]  $(d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5)/(5*b^4) + (d^4*(5*b*c - 2*a*d)*x^9)/(9*b^3) + (d^5*x^13)/(13*b^2) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^4)) - ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*b^(21/4)) - ((b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(21/4))$

\* a\*d)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2)]/(16  
\*Sqrt[2]\*a^(7/4)\*b^(21/4))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned}
 & -d^2(4a^3d^3 - 15a^2bcd^2 + 20ab^2c^2d - 10b^3c^3) \int \frac{1}{b^5} dx + \frac{d^5x^{13}}{13b^2} \\
 & - \frac{d^4x^9(2ad - 5bc)}{9b^3} + \frac{d^3x^5(3a^2d^2 - 10abcd + 10b^2c^2)}{5b^4} - \frac{x(ad - bc)^5}{4ab^5(a + bx^4)} \\
 & - \frac{\sqrt{2}(ad - bc)^4(17ad + 3bc) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{32a^{\frac{7}{4}}b^{\frac{21}{4}}} \\
 & + \frac{\sqrt{2}(ad - bc)^4(17ad + 3bc) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{32a^{\frac{7}{4}}b^{\frac{21}{4}}} \\
 & - \frac{\sqrt{2}(ad - bc)^4(17ad + 3bc) \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{21}{4}}} + \frac{\sqrt{2}(ad - bc)^4(17ad + 3bc) \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{21}{4}}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((d\*x\*\*4+c)\*\*5/(b\*x\*\*4+a)\*\*2,x)

[Out] -d\*\*2\*(4\*a\*\*3\*d\*\*3 - 15\*a\*\*2\*b\*c\*d\*\*2 + 20\*a\*b\*\*2\*c\*\*2\*d - 10\*b\*\*3\*c\*\*3)\*Integral(b\*\*(-5), x) + d\*\*5\*x\*\*13/(13\*b\*\*2) - d\*\*4\*x\*\*9\*(2\*a\*d - 5\*b\*c)/(9\*b\*\*3) + d\*\*3\*x\*\*5\*(3\*a\*\*2\*d\*\*2 - 10\*a\*b\*c\*d + 10\*b\*\*2\*c\*\*2)/(5\*b\*\*4) - x\*(a\*d - b\*c)\*\*5/(4\*a\*b\*\*5\*(a + b\*x\*\*4)) - sqrt(2)\*(a\*d - b\*c)\*\*4\*(17\*a\*d + 3\*b\*c)\*log(-sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*x + sqrt(a) + sqrt(b)\*x\*\*2)/(32\*a\*\*(7/4)\*b\*\*(21/4)) + sqrt(2)\*(a\*d - b\*c)\*\*4\*(17\*a\*d + 3\*b\*c)\*log(sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*x + sqrt(a) + sqrt(b)\*x\*\*2)/(32\*a\*\*(7/4)\*b\*\*(21/4)) - sqrt(2)\*(a\*d - b\*c)\*\*4\*(17\*a\*d + 3\*b\*c)\*atan(1 - sqrt(2)\*b\*\*(1/4)\*x/a\*\*(1/4))/(16\*a\*\*(7/4)\*b\*\*(21/4)) + sqrt(2)\*(a\*d - b\*c)\*\*4\*(17\*a\*d + 3\*b\*c)\*atan(1 + sqrt(2)\*b\*\*(1/4)\*x/a\*\*(1/4))/(16\*a\*\*(7/4)\*b\*\*(21/4))

**Mathematica [A]** time = 0.673226, size = 391, normalized size = 0.96

$$\frac{585\sqrt{2}(bc-ad)^4(17ad+3bc) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{a^{7/4}} + \frac{585\sqrt{2}(bc-ad)^4(17ad+3bc) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{a^{7/4}} - \frac{1170\sqrt{2}(bc-ad)^4(17ad+3bc) \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{a^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^4)^5/(a + b\*x^4)^2,x]

[Out] (18720\*b^(1/4)\*d^2\*(10\*b^3\*c^3 - 20\*a\*b^2\*c^2\*d + 15\*a^2\*b\*c\*d^2 - 4\*a^3\*d^3)\*x + 3744\*b^(5/4)\*d^3\*(10\*b^2\*c^2 - 10\*a\*b\*c\*d + 3\*a^2\*d^2)\*x^5 + 2080\*b^(9/4)\*d^4\*(5\*b\*c - 2\*a\*d)\*x^9 + 1440\*b^(13/4)\*d^5\*x^13 + (4680\*b^(1/4)\*(b\*c - a\*d)^5\*x)/(a\*(a + b\*x^4)) - (1170\*Sqrt[2]\*(b\*c - a\*d)^4\*(3\*b\*c + 17\*a\*d)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/a^(7/4) + (1170\*Sqrt[2]\*(b\*c - a\*d)^4\*(3\*b\*c + 17\*a\*d)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/a^(7/4) - (585\*Sqrt[2]\*(b\*c - a\*d)^4\*(3\*b\*c + 17\*a\*d)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/a^(7/4) + (585\*Sqrt[2]\*(b\*c - a\*d)^4\*(3\*b\*c + 17\*a\*d)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/a^(7/4))/(18720\*b^(21/4))

**Maple [B]** time = 0.018, size = 1118, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^4+c)^5/(b*x^4+a)^2,x)`

[Out] 
$$\frac{1}{13}d^5x^{13}/b^2 - 5/4/b^3x/(b^4x^4+a)^2 + 20d^3/b^3a^2x + 15d^4/b^4a^2c^2x + 10d^2/b^2c^3x - 4d^5/b^5a^3x - 2d^4/b^3x^5a^2c + 5/9d^4/b^2x^9c + 45/8/b^3a^2(a/b)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/b)^{1/4}x-1) + c^2d^3 + 5/16/b^4a^2(a/b)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/b)^{1/4}x-1) + c^4d - 65/32/b^4a^2(a/b)^{1/4}2^{1/2}\ln((x^2+(a/b)^{1/4}x^2)^{1/2}+(a/b)^{1/4})/(x^2-(a/b)^{1/4}x^2)^{1/2}+(a/b)^{1/4}) + c^2d^4 + 45/16/b^3a^2(a/b)^{1/4}2^{1/2}\ln((x^2+(a/b)^{1/4}x^2)^{1/2}+(a/b)^{1/4})/(x^2-(a/b)^{1/4}x^2)^{1/2}+(a/b)^{1/4}) + c^2d^3 + 5/32/b^4a^2(a/b)^{1/4}2^{1/2}\ln((x^2+(a/b)^{1/4}x^2)^{1/2}+(a/b)^{1/4})/(x^2-(a/b)^{1/4}x^2)^{1/2}+(a/b)^{1/4}) + c^4d + 5/4/b^4x^5a^3/(b^4x^4+a)^2 + c^2d^4 - 5/2/b^3x^2a^2/(b^4x^4+a)^2 + 5/2/b^2x^2a/(b^4x^4+a)^2 + c^3d^2 + 17/16/b^5a^3(a/b)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/b)^{1/4}x+1) + d^5 - 25/8/b^2(a/b)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/b)^{1/4}x+1) + c^3d^2 + 17/16/b^5a^3(a/b)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/b)^{1/4}x-1) + d^5 - 25/8/b^2(a/b)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/b)^{1/4}x-1) + c^3d^2 + 17/32/b^5a^3(a/b)^{1/4}2^{1/2}\ln((x^2+(a/b)^{1/4}x^2)^{1/2}+(a/b)^{1/4})/(x^2-(a/b)^{1/4}x^2)^{1/2}+(a/b)^{1/4}) + d^5 - 25/16/b^2(a/b)^{1/4}2^{1/2}\ln((x^2+(a/b)^{1/4}x^2)^{1/2}+(a/b)^{1/4})/(x^2-(a/b)^{1/4}x^2)^{1/2}+(a/b)^{1/4}) + c^3d^2 + 3/16/a^2(a/b)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/b)^{1/4}x+1) + c^5 + 3/16/a^2(a/b)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/b)^{1/4}x-1) + c^5 + 3/32/a^2(a/b)^{1/4}2^{1/2}\ln((x^2+(a/b)^{1/4}x^2)^{1/2}+(a/b)^{1/4})/(x^2-(a/b)^{1/4}x^2)^{1/2}+(a/b)^{1/4}) + c^5 - 1/4/b^5x^2a^4/(b^4x^4+a)^2 + d^5 - 65/16/b^4a^2(a/b)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/b)^{1/4}x+1) + c^2d^4 + 45/8/b^3a^2(a/b)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/b)^{1/4}x+1) + c^2d^3 + 5/16/b^4a^2(a/b)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/b)^{1/4}x+1) + c^4d - 65/16/b^4a^2(a/b)^{1/4}2^{1/2}\arctan(2^{1/2}/(a/b)^{1/4}x-1) + c^2d^4 - 2/9d^5/b^3x^9a + 3/5d^5/b^4x^5a^2 + 2d^3/b^2x^5c^2 + 1/4x/a/(b^4x^4+a)^2 + c^5$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^4 + c)^5/(b*x^4 + a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.268173, size = 3853, normalized size = 9.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^4 + c)^5/(b*x^4 + a)^2,x, algorithm="fricas")`

[Out] 
$$\frac{1}{9360}(720a^4b^4d^5x^{17} + 80(65a^4b^4c^2d^4 - 17a^2b^3d^5)x^{13} + 208(90a^4b^4c^2d^3 - 65a^2b^3c^2d^4 + 17a^3b^2d^5)x^9 + 1872(50a^4b^4c^3d^2 - 90a^2b^3c^2d^3 + 65a^3b^2c^2d^4 - 17a^4b^2d^5)x^5 - 2340(a^6b^4x^4 + a^2b^5)^2 - (81b^20c^20 + 540a^4b^19c^19d - 4050a^2b^18c^18d^2 - 15780a^3b^17c^17d^3 + 132205a^4b^16c^16d^4 - 13264a^5b^15c^15d^5 - 1960920a^6b^14c^14d^6 + 6137200a^7b^13c^13d^7 - 500110a^8b^12c^12d^8 - 48530040a^9b^11c^11d^9 + 174873556a^10b^10c^10d^10 - 360900280a^11b^9c^9d^11 + 517559250a^12b^8c^8d^12 - 548231440a^13b^7c^7d^13 + 438700840a^14b^6c^6d^14 - 266040144a^15b^5c^5d^15 + 120836285a^16b^4c^4d^16 -$$

$$\begin{aligned}
& 39944900*a^{17}*b^3*c^3*d^{17} + 9094830*a^{18}*b^2*c^2*d^{18} - 1277380 \\
& *a^{19}*b*c*d^{19} + 83521*a^{20}*d^{20})/(a^7*b^{21})^{(1/4)}*\arctan(a^2*b^b \\
& 5*(-(81*b^{20}*c^{20} + 540*a*b^{19}*c^{19}*d - 4050*a^2*b^{18}*c^{18}*d^2 - \\
& 15780*a^3*b^{17}*c^{17}*d^3 + 132205*a^4*b^{16}*c^{16}*d^4 - 13264*a^5*b^{15}*c^{15}*d^5 - \\
& 1960920*a^6*b^{14}*c^{14}*d^6 + 6137200*a^7*b^{13}*c^{13}*d^7 - 500110*a^8*b^{12}*c^{12}*d^8 - \\
& 48530040*a^9*b^{11}*c^{11}*d^9 + 174873556*a^{10}*b^{10}*c^{10}*d^{10} - 360900280*a^{11}*b^9*c^9*d^{11} + \\
& 517559250*a^{12}*b^8*c^8*d^{12} - 548231440*a^{13}*b^7*c^7*d^{13} + 438700840*a^{14}*b^6*c^6*d^{14} - \\
& 266040144*a^{15}*b^5*c^5*d^{15} + 120836285*a^{16}*b^4*c^4*d^{16} - 39944900*a^{17}*b^3*c^3*d^{17} + \\
& 9094830*a^{18}*b^2*c^2*d^{18} - 1277380*a^{19}*b*c*d^{19} + 83521*a^{20}*d^{20})/(a^7*b^{21})^{(1/4)})/( \\
& (3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*x + (3*b^5*c^5 + 5*a*b^4*c^4*d \\
& - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*\sqrt{(a^4*b^{10}*\sqrt{-(81*b^{20}*c^{20} + 540*a*b^{19}*c^{19}*d - \\
& 4050*a^2*b^{18}*c^{18}*d^2 - 15780*a^3*b^{17}*c^{17}*d^3 + 132205*a^4*b^{16}*c^{16}*d^4 - 13264*a^5*b^{15}*c^{15}*d^5 - \\
& 1960920*a^6*b^{14}*c^{14}*d^6 + 6137200*a^7*b^{13}*c^{13}*d^7 - 500110*a^8*b^{12}*c^{12}*d^8 - 48530040*a^9*b^{11}*c^{11}*d^9 + \\
& 174873556*a^{10}*b^{10}*c^{10}*d^{10} - 360900280*a^{11}*b^9*c^9*d^{11} + 517559250*a^{12}*b^8*c^8*d^{12} - 548231440*a^{13}*b^7*c^7*d^{13} + \\
& 438700840*a^{14}*b^6*c^6*d^{14} - 266040144*a^{15}*b^5*c^5*d^{15} + 120836285*a^{16}*b^4*c^4*d^{16} - 39944900*a^{17}*b^3*c^3*d^{17} \\
& + 9094830*a^{18}*b^2*c^2*d^{18} - 1277380*a^{19}*b*c*d^{19} + 83521*a^{20}*d^{20})/(a^7*b^{21})) + (9*b^{10}*c^{10} + 30*a*b^9*c^9*d - 275*a^2*b^8* \\
& c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285* \\
& a^8*b^2*c^2*d^8 - 2210*a^9*b*c*d^9 + 289*a^{10}*d^{10})*x^2)/(9*b^{10}* \\
& c^{10} + 30*a*b^9*c^9*d - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 \\
& + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4 \\
& *d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 - 2210*a^9*b* \\
& c*d^9 + 289*a^{10}*d^{10}))) + 585*(a*b^6*x^4 + a^2*b^5)*(-(81*b^{20}* \\
& c^{20} + 540*a*b^{19}*c^{19}*d - 4050*a^2*b^{18}*c^{18}*d^2 - 15780*a^3*b^{17}*c^{17}*d^3 + 132205*a^4*b^{16}*c^{16}*d^4 - 13264*a^5*b^{15}*c^{15}*d^5 - \\
& 1960920*a^6*b^{14}*c^{14}*d^6 + 6137200*a^7*b^{13}*c^{13}*d^7 - 500110*a^8*b^{12}*c^{12}*d^8 - 48530040*a^9*b^{11}*c^{11}*d^9 + 174873556*a^{10}*b^{10}*c^{10}*d^{10} - \\
& 360900280*a^{11}*b^9*c^9*d^{11} + 517559250*a^{12}*b^8*c^8*d^{12} - 548231440*a^{13}*b^7*c^7*d^{13} + 438700840*a^{14}*b^6*c^6*d^{14} - 266040144*a^{15}*b^5*c^5*d^{15} + \\
& 120836285*a^{16}*b^4*c^4*d^{16} - 39944900*a^{17}*b^3*c^3*d^{17} + 9094830*a^{18}*b^2*c^2*d^{18} - 1277380*a^{19}*b*c*d^{19} + 83521*a^{20}*d^{20})/(a^7*b^{21})^{(1/4)}* \\
& \log(a^2*b^b5*(-(81*b^{20}*c^{20} + 540*a*b^{19}*c^{19}*d - 4050*a^2*b^{18}*c^{18}*d^2 - 15780*a^3*b^{17}*c^{17}*d^3 + 132205*a^4*b^{16}*c^{16}*d^4 - 13264*a^5*b^{15}*c^{15}*d^5 - \\
& 1960920*a^6*b^{14}*c^{14}*d^6 + 6137200*a^7*b^{13}*c^{13}*d^7 - 500110*a^8*b^{12}*c^{12}*d^8 - 48530040*a^9*b^{11}*c^{11}*d^9 + 174873556*a^{10}*b^{10}*c^{10}*d^{10} - \\
& 360900280*a^{11}*b^9*c^9*d^{11} + 517559250*a^{12}*b^8*c^8*d^{12} - 548231440*a^{13}*b^7*c^7*d^{13} + 438700840*a^{14}*b^6*c^6*d^{14} - 266040144*a^{15}*b^5*c^5*d^{15} + \\
& 120836285*a^{16}*b^4*c^4*d^{16} - 39944900*a^{17}*b^3*c^3*d^{17} + 9094830*a^{18}*b^2*c^2*d^{18} - 1277380*a^{19}*b*c*d^{19} + 83521*a^{20}*d^{20})/(a^7*b^{21})^{(1/4)} + (3* \\
& b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*x) - 585*(a*b^6*x^4 + a^2*b^5)*(-(81*b^{20}*c^{20} + 540*a*b^{19}*c^{19}*d - 4050*a^2*b^{18}*c^{18}*d^2 - 15780*a^3*b^{17}*c^{17}*d^3 + 132205*a^4*b^{16}*c^{16}*d^4 - 13264*a^5*b^{15}*c^{15}*d^5 - \\
& 1960920*a^6*b^{14}*c^{14}*d^6 + 6137200*a^7*b^{13}*c^{13}*d^7 - 500110*a^8*b^{12}*c^{12}*d^8 - 48530040*a^9*b^{11}*c^{11}*d^9 + 174873556*a^{10}*b^{10}*c^{10}*d^{10} - \\
& 360900280*a^{11}*b^9*c^9*d^{11} + 517559250*a^{12}*b^8*c^8*d^{12} - 548231440*a^{13}*b^7*c^7*d^{13} + 438700840*a^{14}*b^6*c^6*d^{14} - 266040144*a^{15}*b^5*c^5*d^{15} + \\
& 120836285*a^{16}*b^4*c^4*d^{16} - 39944900*a^{17}*b^3*c^3*d^{17} + 9094830*a^{18}*b^2*c^2*d^{18} - 1277380*a^{19}*b*c*d^{19} + 83521*a^{20}*d^{20})/(a^7*b^{21})^{(1/4)}* \\
& \log(- \\
& a^2*b^b5*(-(81*b^{20}*c^{20} + 540*a*b^{19}*c^{19}*d - 4050*a^2*b^{18}*c^{18}*d^2 - 15780*a^3*b^{17}*c^{17}*d^3 + 132205*a^4*b^{16}*c^{16}*d^4 - 13264*a^5*b^{15}*c^{15}*d^5 - \\
& 1960920*a^6*b^{14}*c^{14}*d^6 + 6137200*a^7*b^{13}*c^{13}*d^7 - 500110*a^8*b^{12}*c^{12}*d^8 - 48530040*a^9*b^{11}*c^{11}*d^9 + 174873556*a^{10}*b^{10}*c^{10}*d^{10} - \\
& 360900280*a^{11}*b^9*c^9*d^{11} + 517559250*a^{12}*b^8*c^8*d^{12} - 548231440*a^{13}*b^7*c^7*d^{13} + 438700840*a^{14}*b^6*c^6*d^{14} - 266040144*a^{15}*b^5*c^5*d^{15} + \\
& 120836285*a^{16}*b^4*c^4*d^{16} - 39944900*a^{17}*b^3*c^3*d^{17} + 9094830*a^{18}*b^2*c^2*d^{18} - 1277380*a^{19}*b*c*d^{19} + 83521*a^{20}*d^{20})/(a^7*b^{21})^{(1/4)} + (3* \\
& b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*x) + 2340*(b^5*c^5 - 5* \\
& a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 90*a^3*b^2*c^2*d^3 + 65*a^4*b* \\
& c*d^4 - 17*a^5*d^5)*x)/(a*b^6*x^4 + a^2*b^5)
\end{aligned}$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*4+c)\*\*5/(b\*x\*\*4+a)\*\*2,x)

[Out] Timed out

---

**GIAC/XCAS [A]** time = 0.221219, size = 1077, normalized size = 2.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^5/(b\*x^4 + a)^2,x, algorithm="giac")

[Out] 
$$\frac{1}{16} \sqrt{2} (3 (a b^3)^{1/4} b^5 c^5 + 5 (a b^3)^{1/4} a b^4 c^4 d - 50 (a b^3)^{1/4} a^2 b^3 c^3 d^2 + 90 (a b^3)^{1/4} a^3 b^2 c^2 d^3 - 65 (a b^3)^{1/4} a^4 b c d^4 + 17 (a b^3)^{1/4} a^5 d^5) \arctan\left(\frac{1/2 \sqrt{2} (2x + \sqrt{2} (a/b)^{1/4})}{(a/b)^{1/4}}\right) / (a^2 b^6) + \frac{1}{16} \sqrt{2} (3 (a b^3)^{1/4} b^5 c^5 + 5 (a b^3)^{1/4} a b^4 c^4 d - 50 (a b^3)^{1/4} a^2 b^3 c^3 d^2 + 90 (a b^3)^{1/4} a^3 b^2 c^2 d^3 - 65 (a b^3)^{1/4} a^4 b c d^4 + 17 (a b^3)^{1/4} a^5 d^5) \arctan\left(\frac{1/2 \sqrt{2} (2x - \sqrt{2} (a/b)^{1/4})}{(a/b)^{1/4}}\right) / (a^2 b^6) + \frac{1}{32} \sqrt{2} (3 (a b^3)^{1/4} b^5 c^5 + 5 (a b^3)^{1/4} a b^4 c^4 d - 50 (a b^3)^{1/4} a^2 b^3 c^3 d^2 + 90 (a b^3)^{1/4} a^3 b^2 c^2 d^3 - 65 (a b^3)^{1/4} a^4 b c d^4 + 17 (a b^3)^{1/4} a^5 d^5) \ln(x^2 + \sqrt{2} x (a/b)^{1/4} + \sqrt{a/b}) / (a^2 b^6) - \frac{1}{32} \sqrt{2} (3 (a b^3)^{1/4} b^5 c^5 + 5 (a b^3)^{1/4} a b^4 c^4 d - 50 (a b^3)^{1/4} a^2 b^3 c^3 d^2 + 90 (a b^3)^{1/4} a^3 b^2 c^2 d^3 - 65 (a b^3)^{1/4} a^4 b c d^4 + 17 (a b^3)^{1/4} a^5 d^5) \ln(x^2 - \sqrt{2} x (a/b)^{1/4} + \sqrt{a/b}) / (a^2 b^6) + \frac{1}{4} (b^5 c^5 x - 5 a b^4 c^4 d x + 10 a^2 b^3 c^3 d^2 x - 10 a^3 b^2 c^2 d^3 x + 5 a^4 b c d^4 x - a^5 d^5 x) / ((b x^4 + a) a b^5) + \frac{1}{585} (45 b^{24} d^5 x^{13} + 325 b^{24} c d^4 x^9 - 130 a b^{23} d^5 x^9 + 1170 b^{24} c^2 d^3 x^5 - 1170 a b^{23} c d^4 x^5 + 351 a^2 b^{22} d^5 x^5 + 5850 b^{24} c^3 d^2 x - 11700 a b^{23} c^2 d^3 x + 8775 a^2 b^{22} c d^4 x - 2340 a^3 b^{21} d^5 x) / b^{26}$$

$$3.68 \quad \int \frac{(c+dx^4)^4}{(a+bx^4)^2} dx$$

**Optimal.** Leaf size=357

$$\begin{aligned} & \frac{(bc-ad)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{17/4}} \\ & + \frac{(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{17/4}} \\ & - \frac{(bc-ad)^3(13ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{17/4}} + \frac{(bc-ad)^3(13ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{17/4}} \\ & + \frac{d^2x(3a^2d^2-8abcd+6b^2c^2)}{b^4} + \frac{x(bc-ad)^4}{4ab^4(a+bx^4)} + \frac{2d^3x^5(2bc-ad)}{5b^3} + \frac{d^4x^9}{9b^2} \end{aligned}$$

[Out]  $(d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^5)/(5*b^3) + (d^4*x^9)/(9*b^2) + ((b*c - a*d)^4*x)/(4*a*b^4*(a + b*x^4)) - ((b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(17/4)) - ((b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(17/4))$

**Rubi [A]** time = 0.718939, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\begin{aligned} & \frac{(bc-ad)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{17/4}} \\ & + \frac{(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{17/4}} \\ & - \frac{(bc-ad)^3(13ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{17/4}} + \frac{(bc-ad)^3(13ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{17/4}} \\ & + \frac{d^2x(3a^2d^2-8abcd+6b^2c^2)}{b^4} + \frac{x(bc-ad)^4}{4ab^4(a+bx^4)} + \frac{2d^3x^5(2bc-ad)}{5b^3} + \frac{d^4x^9}{9b^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^4/(a + b\*x^4)^2, x]

[Out]  $(d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^5)/(5*b^3) + (d^4*x^9)/(9*b^2) + ((b*c - a*d)^4*x)/(4*a*b^4*(a + b*x^4)) - ((b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(17/4)) - ((b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(17/4))$



**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$d^2 (3a^2 d^2 - 8abcd + 6b^2 c^2) \int \frac{1}{b^4} dx + \frac{d^4 x^9}{9b^2} - \frac{2d^3 x^5 (ad - 2bc)}{5b^3} \\ + \frac{x(ad - bc)^4}{4ab^4(a + bx^4)} + \frac{\sqrt{2}(ad - bc)^3(13ad + 3bc) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{32a^{7/4}b^{17/4}} \\ - \frac{\sqrt{2}(ad - bc)^3(13ad + 3bc) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{32a^{7/4}b^{17/4}} \\ + \frac{\sqrt{2}(ad - bc)^3(13ad + 3bc) \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{16a^{7/4}b^{17/4}} - \frac{\sqrt{2}(ad - bc)^3(13ad + 3bc) \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{16a^{7/4}b^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((d*x**4+c)**4/(b*x**4+a)**2,x)`

[Out] `d**2*(3*a**2*d**2 - 8*a*b*c*d + 6*b**2*c**2)*Integral(b**(-4), x) + d**4*x**9/(9*b**2) - 2*d**3*x**5*(a*d - 2*b*c)/(5*b**3) + x*(a*d - b*c)**4/(4*a*b**4*(a + b*x**4)) + sqrt(2)*(a*d - b*c)**3*(13*a*d + 3*b*c)*log(-sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(32*a**(7/4)*b**(17/4)) - sqrt(2)*(a*d - b*c)**3*(13*a*d + 3*b*c)*log(sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(32*a**(7/4)*b**(17/4)) + sqrt(2)*(a*d - b*c)**3*(13*a*d + 3*b*c)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(16*a**(7/4)*b**(17/4)) - sqrt(2)*(a*d - b*c)**3*(13*a*d + 3*b*c)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(16*a**(7/4)*b**(17/4))`

**Mathematica [A]** time = 0.501044, size = 341, normalized size = 0.96

$$\frac{45\sqrt{2}(ad-bc)^3(13ad+3bc) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{a^{7/4}} + \frac{45\sqrt{2}(bc-ad)^3(13ad+3bc) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{a^{7/4}} + \frac{90\sqrt{2}(ad-bc)^3(13ad+3bc) \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{a^{7/4}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c + d*x^4)^4/(a + b*x^4)^2,x]`

[Out] `(1440*b^(1/4)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 576*b^(5/4)*d^3*(2*b*c - a*d)*x^5 + 160*b^(9/4)*d^4*x^9 + (360*b^(1/4)*(b*c - a*d)^4*x)/(a*(a + b*x^4)) + (90*Sqrt[2]*(-(b*c) + a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (90*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (45*Sqrt[2]*(-(b*c) + a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (45*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(1440*b^(1/4)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 576*b^(5/4)*d^3*(2*b*c - a*d)*x^5 + 160*b^(9/4)*d^4*x^9 + (360*b^(1/4)*(b*c - a*d)^4*x)/(a*(a + b*x^4)) + (90*Sqrt[2]*(-(b*c) + a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (90*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (45*Sqrt[2]*(-(b*c) + a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (45*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))`

**Maple [B]** time = 0.017, size = 885, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^4+c)^4/(b*x^4+a)^2,x)`

[Out] `1/9*d^4*x^9/b^2-8*d^3/b^3*a*c*x+3*d^4/b^4*a^2*x+6*d^2/b^2*c^2*x-1/b^3*x*a^2/(b*x^4+a)*c*d^3+3/2/b^2*x*a/(b*x^4+a)*c^2*d^2-13/16/b^4*a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d^4-15/`

$$\begin{aligned}
& 8/b^2 * (a/b)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)}/(a/b)^{(1/4)} * x+1) * c^2 * d^2 \\
& - 13/16/b^4 * a^2 * (a/b)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)}/(a/b)^{(1/4)} * x-1) \\
& * d^4 - 15/8/b^2 * (a/b)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)}/(a/b)^{(1/4)} * x-1) \\
& * c^2 * d^2 - 13/32/b^4 * a^2 * (a/b)^{(1/4)} * 2^{(1/2)} * \ln((x^2+(a/b)^{(1/4)} * x \\
& * 2^{(1/2)}+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)} * x * 2^{(1/2)}+(a/b)^{(1/2)})) * d^4 \\
& - 15/16/b^2 * (a/b)^{(1/4)} * 2^{(1/2)} * \ln((x^2+(a/b)^{(1/4)} * x * 2^{(1/2)}+(a/b)^{(1/2)}) \\
& / (x^2-(a/b)^{(1/4)} * x * 2^{(1/2)}+(a/b)^{(1/2)})) * c^2 * d^2 - 1/b * x / \\
& (b * x^4+a) * c^3 * d+3/16/a^2 * (a/b)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)}/(a/b)^{(1/4)} * x+1) \\
& * c^4+3/16/a^2 * (a/b)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)}/(a/b)^{(1/4)} * x-1) * c^4+3/32/a^2 * \\
& (a/b)^{(1/4)} * 2^{(1/2)} * \ln((x^2+(a/b)^{(1/4)} * x * 2^{(1/2)}+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)} * x * 2^{(1/2)} \\
& +(a/b)^{(1/2)})) * c^4+1/4/b^4 * x * a^3/(b * x^4+a) * d^4+9/4/b^3 * a * (a/b)^{(1/4)} * 2^{(1/2)} * \arctan \\
& (2^{(1/2)}/(a/b)^{(1/4)} * x+1) * c^3 * d+9/4/b^3 * a * (a/b)^{(1/4)} * 2^{(1/2)} * \arctan \\
& (2^{(1/2)}/(a/b)^{(1/4)} * x-1) * c^3 * d-2/5 * d^4/b^3 * x^5 * a+4/5 * d^3/b^2 * x^5 * c+1/4 * x/a / \\
& (b * x^4+a) * c^4+1/4/b * a * (a/b)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)}/(a/b)^{(1/4)} * x-1) * c^3 * d+9/8/b^3 * a * \\
& (a/b)^{(1/4)} * 2^{(1/2)} * \ln((x^2+(a/b)^{(1/4)} * x * 2^{(1/2)}+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)} * x * 2^{(1/2)} \\
& +(a/b)^{(1/2)})) * c^3 * d+1/8/b * a * (a/b)^{(1/4)} * 2^{(1/2)} * \ln((x^2+(a/b)^{(1/4)} * x * 2^{(1/2)}+(a/b)^{(1/2)}) \\
& / (x^2-(a/b)^{(1/4)} * x * 2^{(1/2)}+(a/b)^{(1/2)})) * c^3 * d
\end{aligned}$$


---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^4/(b\*x^4 + a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.2572, size = 3083, normalized size = 8.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^4/(b\*x^4 + a)^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned}
& 1/720 * (80 * a * b^3 * d^4 * x^{13} + 16 * (36 * a * b^3 * c * d^3 - 13 * a^2 * b^2 * d^4) * x \\
& ^9 + 144 * (30 * a * b^3 * c^2 * d^2 - 36 * a^2 * b^2 * c * d^3 + 13 * a^3 * b * d^4) * x^5 \\
& + 180 * (a * b^5 * x^4 + a^2 * b^4) * (- (81 * b^{16} * c^{16} + 432 * a * b^{15} * c^{15} * d \\
& - 2376 * a^2 * b^{14} * c^{14} * d^2 - 8304 * a^3 * b^{13} * c^{13} * d^3 + 45724 * a^4 * b^{12} * c^{12} * d^4 \\
& + 20400 * a^5 * b^{11} * c^{11} * d^5 - 434808 * a^6 * b^{10} * c^{10} * d^6 + 772112 * a^7 * b^9 * c^9 * d^7 \\
& + 617958 * a^8 * b^8 * c^8 * d^8 - 4810608 * a^9 * b^7 * c^7 * d^9 + 9723912 * a^{10} * b^6 * c^6 * d^{10} \\
& - 11486160 * a^{11} * b^5 * c^5 * d^{11} + 8923164 * a^{12} * b^4 * c^4 * d^{12} - 4651504 * a^{13} * b^3 * c^3 * d^{13} \\
& + 1577784 * a^{14} * b^2 * c^2 * d^{14} - 316368 * a^{15} * b * c * d^{15} + 28561 * a^{16} * d^{16}) / (a \\
& ^7 * b^{17})^{(1/4)} * \arctan(-a^2 * b^4 * (- (81 * b^{16} * c^{16} + 432 * a * b^{15} * c^{15} * d \\
& - 2376 * a^2 * b^{14} * c^{14} * d^2 - 8304 * a^3 * b^{13} * c^{13} * d^3 + 45724 * a^4 * b^{12} * c^{12} * d^4 \\
& + 20400 * a^5 * b^{11} * c^{11} * d^5 - 434808 * a^6 * b^{10} * c^{10} * d^6 + 772112 * a^7 * b^9 * c^9 * d^7 \\
& + 617958 * a^8 * b^8 * c^8 * d^8 - 4810608 * a^9 * b^7 * c^7 * d^9 + 9723912 * a^{10} * b^6 * c^6 * d^{10} \\
& - 11486160 * a^{11} * b^5 * c^5 * d^{11} + 8923164 * a^{12} * b^4 * c^4 * d^{12} - 4651504 * a^{13} * b^3 * c^3 * d^{13} \\
& + 1577784 * a^{14} * b^2 * c^2 * d^{14} - 316368 * a^{15} * b * c * d^{15} + 28561 * a^{16} * d^{16}) / (a^7 * b^{17})^{(1/4)} / ((3 * b^4 * c^4 + 4 * a * b^3 * c^3 * d - 30 * a^2 * b^2 * c^2 * d^2 + 36 * a^3 * b * c * d^3 - 13 * a^4 * d^4) * x + (3 * b^4 * c^4 + 4 * a * b^3 * c^3 * d - 30 * a^2 * b^2 * c^2 * d^2 + 36 * a^3 * b * c * d^3 - 13 * a^4 * d^4) * \sqrt{(a^4 * b^8 * \sqrt{- (81 * b^{16} * c^{16} + 432 * a * b^{15} * c^{15} * d - 2376 * a^2 * b^{14} * c^{14} * d^2 - 8304 * a^3 * b^{13} * c^{13} * d^3 + 45724 * a^4 * b^{12} * c^{12} * d^4 + 20400 * a^5 * b^{11} * c^{11} * d^5 - 434808 * a^6 * b^{10} * c^{10} * d^6 + 772112 * a^7 * b^9 * c^9 * d^7 + 617958 * a^8 * b^8 * c^8 * d^8 - 4810608 * a^9 * b^7 * c^7 * d^9 + 9723912 * a^{10} * b^6 * c^6 * d^{10} - 11486160 * a^{11} * b^5 * c^5 * d^{11} + 8923164 * a^{12} * b^4 * c^4 * d^{12} - 4651504 * a^{13} * b^3 * c^3 * d^{13} + 1577784 * a^{14} * b^2 * c^2 * d^{14} - 3
\end{aligned}$$

$$\begin{aligned}
& 16368*a^{15}*b*c*d^{15} + 28561*a^{16}*d^{16})/(a^7*b^{17})) + (9*b^8*c^8 + \\
& 24*a*b^7*c^7*d - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110 \\
& *a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 - \\
& 936*a^7*b*c*d^7 + 169*a^8*d^8)*x^2)/(9*b^8*c^8 + 24*a*b^7*c^7*d - \\
& 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 \\
& - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 - 936*a^7*b*c*d^7 + \\
& 169*a^8*d^8)))) - 45*(a*b^5*x^4 + a^2*b^4)*(-(81*b^16*c^16 + 432 \\
& *a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 \\
& + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6* \\
& b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - \\
& 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a \\
& ^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c \\
& ^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 2856 \\
& 1*a^16*d^16)/(a^7*b^{17}))^{(1/4)} * \log(a^2*b^4*(-(81*b^16*c^16 + 432* \\
& a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + \\
& 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b \\
& ^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - \\
& 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a \\
& ^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^ \\
& ^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561 \\
& *a^16*d^16)/(a^7*b^{17}))^{(1/4)} - (3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a \\
& ^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*x) + 45*(a*b^5*x^4 \\
& + a^2*b^4)*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^ \\
& ^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400 \\
& *a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^ \\
& ^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 972391 \\
& 2*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b \\
& ^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^ \\
& ^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^{17}))^{(1/4)} * \log \\
& (-a^2*b^4*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^ \\
& ^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400 \\
& *a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^ \\
& ^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 972391 \\
& 2*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b \\
& ^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^ \\
& ^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^{17}))^{(1/4)} - \\
& (3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 \\
& - 13*a^4*d^4)*x) + 180*(b^4*c^4 - 4*a*b^3*c^3*d + 30*a^2*b^2*c^2* \\
& d^2 - 36*a^3*b*c*d^3 + 13*a^4*d^4)*x)/(a*b^5*x^4 + a^2*b^4)
\end{aligned}$$

**Sympy [A]** time = 89.2013, size = 466, normalized size = 1.31

$$\frac{x(a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4)}{4a^2b^4 + 4ab^5x^4}$$

$$\begin{aligned}
& + \text{RootSum}\left(65536t^4a^7b^{17} + 28561a^{16}d^{16} - 316368a^{15}bcd^{15} + 1577784a^{14}b^2c^2d^{14} - 4651504a^{13}b^3c^3d^{13} + 8923164a^{12}b^4c^4d^{12}\right. \\
& \left. + \frac{d^4x^9}{9b^2} - \frac{x^5(2ad^4 - 4bcd^3)}{5b^3} + \frac{x(3a^2d^4 - 8abcd^3 + 6b^2c^2d^2)}{b^4}\right)
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*4+c)\*\*4/(b\*x\*\*4+a)\*\*2,x)

[Out] x\*(a\*\*4\*d\*\*4 - 4\*a\*\*3\*b\*c\*d\*\*3 + 6\*a\*\*2\*b\*\*2\*c\*\*2\*d\*\*2 - 4\*a\*b\*\*3\*c\*\*3\*d + b\*\*4\*c\*\*4)/(4\*a\*\*2\*b\*\*4 + 4\*a\*b\*\*5\*x\*\*4) + RootSum(65536\*\_t\*\*4\*a\*\*7\*b\*\*17 + 28561\*a\*\*16\*d\*\*16 - 316368\*a\*\*15\*b\*c\*d\*\*15 + 1577784\*a\*\*14\*b\*\*2\*c\*\*2\*d\*\*14 - 4651504\*a\*\*13\*b\*\*3\*c\*\*3\*d\*\*13 + 8923164\*a\*\*12\*b\*\*4\*c\*\*4\*d\*\*12 - 11486160\*a\*\*11\*b\*\*5\*c\*\*5\*d\*\*11 + 9723912\*a\*\*10\*b\*\*6\*c\*\*6\*d\*\*10 - 4810608\*a\*\*9\*b\*\*7\*c\*\*7\*d\*\*9 + 617958\*a\*\*8\*b\*\*8\*c\*\*8\*d\*\*8 + 772112\*a\*\*7\*b\*\*9\*c\*\*9\*d\*\*7 - 434808\*a\*\*6\*b\*\*10\*c\*\*10\*d\*\*6 + 20400\*a\*\*5\*b\*\*11\*c\*\*11\*d\*\*5 + 45724\*a\*\*4\*b\*\*12\*c\*\*12\*d\*\*4 - 8304\*a\*\*3\*b\*\*13\*c\*\*13\*d\*\*3 - 2376\*a\*\*2\*b\*\*14\*c\*\*14\*d\*\*2 + 432\*a\*b\*\*15\*c\*\*15\*d + 81\*b\*\*16\*c\*\*16, Lambda(\_t, \_t\*log(-16\*\_t\*a\*\*2\*b\*\*4/(13\*a\*\*4\*d\*\*4 - 36\*a\*\*3\*b\*c\*d\*\*3 + 30\*a\*\*2\*b\*\*2\*c\*\*2\*d\*\*2 - 4\*a\*b\*\*3\*c\*\*3\*d - 3\*b\*\*4\*c\*\*4) + x))) + d\*\*4\*x\*\*9/(9\*b\*\*2) - x\*\*5\*(2\*a\*d\*\*4 - 4\*b\*c\*d\*\*3)/(5\*b\*\*3) + x\*(3\*a\*\*2\*d\*\*4 - 8\*a\*b\*c\*d\*\*3 + 6\*b\*\*2\*c\*\*2\*d\*\*2)/b\*\*4

**GIAC/XCAS [A]** time = 0.220568, size = 867, normalized size = 2.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^4/(b\*x^4 + a)^2,x, algorithm="giac")

[Out] 
$$\begin{aligned} & 1/16*\sqrt{2}*(3*(a*b^3)^{(1/4)}*b^4*c^4 + 4*(a*b^3)^{(1/4)}*a*b^3*c^3 \\ & *d - 30*(a*b^3)^{(1/4)}*a^2*b^2*c^2*d^2 + 36*(a*b^3)^{(1/4)}*a^3*b*c \\ & *d^3 - 13*(a*b^3)^{(1/4)}*a^4*d^4)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}) \\ & *(a/b)^{(1/4)})/(a/b)^{(1/4)}/(a^2*b^5) + 1/16*\sqrt{2}*(3*(a*b^3)^{(1/4)} \\ & *b^4*c^4 + 4*(a*b^3)^{(1/4)}*a*b^3*c^3*d - 30*(a*b^3)^{(1/4)}*a^2 \\ & *b^2*c^2*d^2 + 36*(a*b^3)^{(1/4)}*a^3*b*c*d^3 - 13*(a*b^3)^{(1/4)}*a^4 \\ & *d^4)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)}) \\ & /((a^2*b^5) + 1/32*\sqrt{2}*(3*(a*b^3)^{(1/4)}*b^4*c^4 + 4*(a*b^3)^{(1/4)} \\ & *a*b^3*c^3*d - 30*(a*b^3)^{(1/4)}*a^2*b^2*c^2*d^2 + 36*(a*b^3)^{(1/4)} \\ & *a^3*b*c*d^3 - 13*(a*b^3)^{(1/4)}*a^4*d^4)*\ln(x^2 + \sqrt{2}*x*( \\ & a/b)^{(1/4)} + \sqrt{a/b})/(a^2*b^5) - 1/32*\sqrt{2}*(3*(a*b^3)^{(1/4)} \\ & *b^4*c^4 + 4*(a*b^3)^{(1/4)}*a*b^3*c^3*d - 30*(a*b^3)^{(1/4)}*a^2*b^2 \\ & *c^2*d^2 + 36*(a*b^3)^{(1/4)}*a^3*b*c*d^3 - 13*(a*b^3)^{(1/4)}*a^4*d^4 \\ & *\ln(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(a^2*b^5) + 1/4*(b \\ & ^4*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3* \\ & x + a^4*d^4*x)/((b*x^4 + a)*a*b^4) + 1/45*(5*b^16*d^4*x^9 + 36*b^16 \\ & *c*d^3*x^5 - 18*a*b^15*d^4*x^5 + 270*b^16*c^2*d^2*x - 360*a*b^15 \\ & *c*d^3*x + 135*a^2*b^14*d^4*x)/b^18 \end{aligned}$$

$$3.69 \quad \int \frac{(c+dx^4)^3}{(a+bx^4)^2} dx$$

**Optimal.** Leaf size=317

$$\begin{aligned} & \frac{3(bc-ad)^2(3ad+bc) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{13/4}} \\ & + \frac{3(bc-ad)^2(3ad+bc) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{13/4}} - \frac{3(bc-ad)^2(3ad+bc) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}} \\ & + \frac{3(bc-ad)^2(3ad+bc) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2}a^{7/4}b^{13/4}} + \frac{d^2x(3bc-2ad)}{b^3} + \frac{x(bc-ad)^3}{4ab^3(a+bx^4)} + \frac{d^3x^5}{5b^2} \end{aligned}$$

[Out]  $(d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^5)/(5*b^2) + ((b*c - a*d)^3*x)/(4*a*b^3*(a + b*x^4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{1/4}*x)/a^{1/4}])/(8*\text{Sqrt}[2]*a^{7/4}*b^{13/4}) + (3*(b*c - a*d)^2*(b*c + 3*a*d)*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{1/4}*x)/a^{1/4}])/(8*\text{Sqrt}[2]*a^{7/4}*b^{13/4}) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{7/4}*b^{13/4}) + (3*(b*c - a*d)^2*(b*c + 3*a*d)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{7/4}*b^{13/4})$

**Rubi [A]** time = 0.611067, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\begin{aligned} & \frac{3(bc-ad)^2(3ad+bc) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{13/4}} \\ & + \frac{3(bc-ad)^2(3ad+bc) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{13/4}} - \frac{3(bc-ad)^2(3ad+bc) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}} \\ & + \frac{3(bc-ad)^2(3ad+bc) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2}a^{7/4}b^{13/4}} + \frac{d^2x(3bc-2ad)}{b^3} + \frac{x(bc-ad)^3}{4ab^3(a+bx^4)} + \frac{d^3x^5}{5b^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^3/(a + b\*x^4)^2, x]

[Out]  $(d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^5)/(5*b^2) + ((b*c - a*d)^3*x)/(4*a*b^3*(a + b*x^4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{1/4}*x)/a^{1/4}])/(8*\text{Sqrt}[2]*a^{7/4}*b^{13/4}) + (3*(b*c - a*d)^2*(b*c + 3*a*d)*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{1/4}*x)/a^{1/4}])/(8*\text{Sqrt}[2]*a^{7/4}*b^{13/4}) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{7/4}*b^{13/4}) + (3*(b*c - a*d)^2*(b*c + 3*a*d)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{7/4}*b^{13/4})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned}
 & -d^2(2ad - 3bc) \int \frac{1}{b^3} dx + \frac{d^3x^5}{5b^2} - \frac{x(ad - bc)^3}{4ab^3(a + bx^4)} \\
 & - \frac{3\sqrt{2}(ad - bc)^2(3ad + bc) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{32a^{\frac{7}{4}}b^{\frac{13}{4}}} \\
 & + \frac{3\sqrt{2}(ad - bc)^2(3ad + bc) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{32a^{\frac{7}{4}}b^{\frac{13}{4}}} \\
 & - \frac{3\sqrt{2}(ad - bc)^2(3ad + bc) \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{13}{4}}} + \frac{3\sqrt{2}(ad - bc)^2(3ad + bc) \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{13}{4}}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((d*x**4+c)**3/(b*x**4+a)**2,x)`

[Out] `-d**2*(2*a*d - 3*b*c)*Integral(b**(-3), x) + d**3*x**5/(5*b**2) - x*(a*d - b*c)**3/(4*a*b**3*(a + b*x**4)) - 3*sqrt(2)*(a*d - b*c)**2*(3*a*d + b*c)*log(-sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(32*a**(7/4)*b**(13/4)) + 3*sqrt(2)*(a*d - b*c)**2*(3*a*d + b*c)*log(sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(32*a**(7/4)*b**(13/4)) - 3*sqrt(2)*(a*d - b*c)**2*(3*a*d + b*c)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(16*a**(7/4)*b**(13/4)) + 3*sqrt(2)*(a*d - b*c)**2*(3*a*d + b*c)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(16*a**(7/4)*b**(13/4))`

**Mathematica [A]** time = 0.399996, size = 301, normalized size = 0.95

$$\frac{15\sqrt{2}(bc-ad)^2(3ad+bc) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{a^{7/4}} + \frac{15\sqrt{2}(bc-ad)^2(3ad+bc) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{a^{7/4}} - \frac{30\sqrt{2}(bc-ad)^2(3ad+bc) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{160b^{13/4}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c + d*x^4)^3/(a + b*x^4)^2,x]`

[Out] `(160*b^(1/4)*d^2*(3*b*c - 2*a*d)*x + 32*b^(5/4)*d^3*x^5 + (40*b^(1/4)*(b*c - a*d)^3*x)/(a*(a + b*x^4)) - (30*sqrt(2)*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 - (sqrt(2)*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (30*sqrt(2)*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 + (sqrt(2)*b^(1/4)*x)/a^(1/4)])/a^(7/4) - (15*sqrt(2)*(b*c - a*d)^2*(b*c + 3*a*d)*Log[sqrt(a) - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(b)*x^2])/a^(7/4) + (15*sqrt(2)*(b*c - a*d)^2*(b*c + 3*a*d)*Log[sqrt(a) + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(b)*x^2])/a^(7/4))/(160*b^(13/4))`

**Maple [B]** time = 0.001, size = 669, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^4+c)^3/(b*x^4+a)^2,x)`

[Out] `1/5*d^3*x^5/b^2 - 2*d^3/b^3*a*x + 3*d^2/b^2*x*c - 1/4/b^3*x*a^2/(b*x^4+a) + d^3+3/4/b^2*x*a/(b*x^4+a) + c*d^2-3/4/b*x/(b*x^4+a) + c^2*d+1/4*x/a/(b*x^4+a) + c^3+9/16/b^3*a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*d^3-15/16/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c*d^2+3/16/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/`

$$\begin{aligned} & (a/b)^{(1/4)} * x - 1) * c^2 * d + 3/16/a^2 * (a/b)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} \\ & ) / (a/b)^{(1/4)} * x - 1) * c^3 + 9/32/b^3 * a * (a/b)^{(1/4)} * 2^{(1/2)} * \ln((x^2 + (a/b)^{(1/4)} * x \\ & )^2 + (a/b)^{(1/2)}) / (x^2 - (a/b)^{(1/4)} * x^2 + (a/b)^{(1/2)}) * d^3 - 15/32/b^2 * (a/b)^{(1/4)} * 2^{(1/2)} * \ln((x^2 + (a/b)^{(1/4)} * x^2 \\ & )^2 + (a/b)^{(1/2)}) / (x^2 - (a/b)^{(1/4)} * x^2 + (a/b)^{(1/2)}) * c * d^2 + 3/32/b/a * (a/b)^{(1/4)} * 2^{(1/2)} * \ln((x^2 + (a/b)^{(1/4)} * x^2 + (a/b)^{(1/2)}) / (x \\ & )^2 - (a/b)^{(1/4)} * x^2 + (a/b)^{(1/2)}) * c^2 * d + 3/32/a^2 * (a/b)^{(1/4)} * 2^{(1/2)} * \ln((x^2 + (a/b)^{(1/4)} * x^2 + (a/b)^{(1/2)}) / (x \\ & )^2 - (a/b)^{(1/4)} * x^2 + (a/b)^{(1/2)}) * c^3 + 9/16/b^3 * a * (a/b)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/b)^{(1/4)} * x + 1) * d^3 - 15/16/b^2 * (a/b)^{(1/4)} \\ & ) * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/b)^{(1/4)} * x + 1) * c * d^2 + 3/16/b/a * (a/b)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/b)^{(1/4)} * x + 1) * c^2 * d + 3/16/a^2 * (a/b)^{(1/4)} \\ & ) * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/b)^{(1/4)} * x + 1) * c^3 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^3/(b\*x^4 + a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.248349, size = 2306, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^3/(b\*x^4 + a)^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/80 * (16 * a^2 * b^2 * d^3 * x^9 + 48 * (5 * a^2 * b^2 * c * d^2 - 3 * a^2 * b^2 * d^3) * x^5 - 6 \\ & 0 * (a^2 * b^4 * x^4 + a^2 * b^3) * (- (b^{12} * c^{12} + 4 * a^2 * b^{11} * c^{11} * d - 14 * a^2 * b^{10} * c^{10} * d^2 - 44 * a^3 * b^9 * c^9 * d^3 + 127 * a^4 * b^8 * c^8 * d^4 + 136 * a^5 * b^7 * c^7 * d^5 - 644 * a^6 * b^6 * c^6 * d^6 + 328 * a^7 * b^5 * c^5 * d^7 + 1039 * a^8 * b^4 * c^4 * d^8 - 1932 * a^9 * b^3 * c^3 * d^9 + 1458 * a^{10} * b^2 * c^2 * d^{10} - 540 * a^{11} * b * c * d^{11} + 81 * a^{12} * d^{12}) / (a^7 * b^{13})^{(1/4)} * \arctan(a^2 * b^3 * (- (b^{12} * c^{12} + 4 * a^2 * b^{11} * c^{11} * d - 14 * a^2 * b^{10} * c^{10} * d^2 - 44 * a^3 * b^9 * c^9 * d^3 + 127 * a^4 * b^8 * c^8 * d^4 + 136 * a^5 * b^7 * c^7 * d^5 - 644 * a^6 * b^6 * c^6 * d^6 + 328 * a^7 * b^5 * c^5 * d^7 + 1039 * a^8 * b^4 * c^4 * d^8 - 1932 * a^9 * b^3 * c^3 * d^9 + 1458 * a^{10} * b^2 * c^2 * d^{10} - 540 * a^{11} * b * c * d^{11} + 81 * a^{12} * d^{12}) / (a^7 * b^{13})^{(1/4)} / ((b^3 * c^3 + a * b^2 * c^2 * d - 5 * a^2 * b * c * d^2 + 3 * a^3 * d^3) * x + (b^3 * c^3 + a * b^2 * c^2 * d - 5 * a^2 * b * c * d^2 + 3 * a^3 * d^3) * \sqrt{(a^4 * b^6 * \sqrt{- (b^{12} * c^{12} + 4 * a^2 * b^{11} * c^{11} * d - 14 * a^2 * b^{10} * c^{10} * d^2 - 44 * a^3 * b^9 * c^9 * d^3 + 127 * a^4 * b^8 * c^8 * d^4 + 136 * a^5 * b^7 * c^7 * d^5 - 644 * a^6 * b^6 * c^6 * d^6 + 328 * a^7 * b^5 * c^5 * d^7 + 1039 * a^8 * b^4 * c^4 * d^8 - 1932 * a^9 * b^3 * c^3 * d^9 + 1458 * a^{10} * b^2 * c^2 * d^{10} - 540 * a^{11} * b * c * d^{11} + 81 * a^{12} * d^{12}) / (a^7 * b^{13})}) + (b^6 * c^6 + 2 * a * b^5 * c^5 * d - 9 * a^2 * b^4 * c^4 * d^2 - 4 * a^3 * b^3 * c^3 * d^3 + 31 * a^4 * b^2 * c^2 * d^4 - 30 * a^5 * b * c * d^5 + 9 * a^6 * d^6) * x^2) / (b^6 * c^6 + 2 * a * b^5 * c^5 * d - 9 * a^2 * b^4 * c^4 * d^2 - 4 * a^3 * b^3 * c^3 * d^3 + 31 * a^4 * b^2 * c^2 * d^4 - 30 * a^5 * b * c * d^5 + 9 * a^6 * d^6))) + 15 * (a^2 * b^4 * x^4 + a^2 * b^3) * (- (b^{12} * c^{12} + 4 * a^2 * b^{11} * c^{11} * d - 14 * a^2 * b^{10} * c^{10} * d^2 - 44 * a^3 * b^9 * c^9 * d^3 + 127 * a^4 * b^8 * c^8 * d^4 + 136 * a^5 * b^7 * c^7 * d^5 - 644 * a^6 * b^6 * c^6 * d^6 + 328 * a^7 * b^5 * c^5 * d^7 + 1039 * a^8 * b^4 * c^4 * d^8 - 1932 * a^9 * b^3 * c^3 * d^9 + 1458 * a^{10} * b^2 * c^2 * d^{10} - 540 * a^{11} * b * c * d^{11} + 81 * a^{12} * d^{12}) / (a^7 * b^{13})^{(1/4)} * \log(3 * a^2 * b^3 * (- (b^{12} * c^{12} + 4 * a^2 * b^{11} * c^{11} * d - 14 * a^2 * b^{10} * c^{10} * d^2 - 44 * a^3 * b^9 * c^9 * d^3 + 127 * a^4 * b^8 * c^8 * d^4 + 136 * a^5 * b^7 * c^7 * d^5 - 644 * a^6 * b^6 * c^6 * d^6 + 328 * a^7 * b^5 * c^5 * d^7 + 1039 * a^8 * b^4 * c^4 * d^8 - 1932 * a^9 * b^3 * c^3 * d^9 + 1458 * a^{10} * b^2 * c^2 * d^{10} - 540 * a^{11} * b * c * d^{11} + 81 * a^{12} * d^{12}) / (a^7 * b^{13})^{(1/4)} + 3 * (b^3 * c^3 + a * b^2 * c^2 * d - 5 * a^2 * b * c * d^2 + 3 * a^3 * d^3) * x) - 15 * (a^2 * b^4 * x^4 + a^2 * b^3) * (- (b^{12} * c^{12} + 4 * a^2 * b^{11} * c^{11} * d - 14 * a^2 * b^{10} * c^{10} * d^2 - 44 * a^3 * b^9 * c^9 * d^3 + 127 * a^4 * b^8 * c^8 * d^4 + 136 * a^5 * b^7 * c^7 * d^5 - 644 * a^6 * b^6 * c^6 * d^6 + 328 * a^7 * b^5 * c^5 * d^7 + 1039 * a^8 * b^4 * c^4 * d^8 - 1932 * a^9 * b^3 * c^3 * d^9 + 1458 * a^{10} * b^2 * c^2 * d^{10} - 540 * a^{11} * b * c * d^{11} + 81 * a^{12} * d^{12}) / (a^7 * b^{13})^{(1/4)} + 3 * (b^3 * c^3 + a * b^2 * c^2 * d - 5 * a^2 * b * c * d^2 + 3 * a^3 * d^3) * x) \end{aligned}$$

$$\frac{c^7 d^5 - 644 a^6 b^6 c^6 d^6 + 328 a^7 b^5 c^5 d^7 + 1039 a^8 b^4 c^4 d^8 - 1932 a^9 b^3 c^3 d^9 + 1458 a^{10} b^2 c^2 d^{10} - 540 a^{11} b c d^{11} + 81 a^{12} d^{12}}{(a^7 b^{13})^{1/4}} \log(-3 a^2 b^3 (-b^{12} c^{12} + 4 a b^{11} c^{11} d - 14 a^2 b^{10} c^{10} d^2 - 44 a^3 b^9 c^9 d^3 + 127 a^4 b^8 c^8 d^4 + 136 a^5 b^7 c^7 d^5 - 644 a^6 b^6 c^6 d^6 + 328 a^7 b^5 c^5 d^7 + 1039 a^8 b^4 c^4 d^8 - 1932 a^9 b^3 c^3 d^9 + 1458 a^{10} b^2 c^2 d^{10} - 540 a^{11} b c d^{11} + 81 a^{12} d^{12}) / (a^7 b^{13})^{1/4} + 3 (b^3 c^3 + a b^2 c^2 d - 5 a^2 b c d^2 + 3 a^3 d^3) x + 20 (b^3 c^3 - 3 a b^2 c^2 d + 15 a^2 b c d^2 - 9 a^3 d^3) x) / (a b^4 x^4 + a^2 b^3)$$

**Sympy [A]** time = 20.5834, size = 335, normalized size = 1.06

$$\frac{x(a^3 d^3 - 3 a^2 b c d^2 + 3 a b^2 c^2 d - b^3 c^3)}{4 a^2 b^3 + 4 a b^4 x^4} + \text{RootSum}\left(65536 t^4 a^7 b^{13} + 6561 a^{12} d^{12} - 43740 a^{11} b c d^{11} + 118098 a^{10} b^2 c^2 d^{10} - 156492 a^9 b^3 c^3 d^9 + 84159 a^8 b^4 c^4 d^8 + 26568 a^7 b^5 c^5 d^7 - 52164 a^6 b^6 c^6 d^6 + 11016 a^5 b^7 c^7 d^5 + 10287 a^4 b^8 c^8 d^4 - 3564 a^3 b^9 c^9 d^3 - 1134 a^2 b^{10} c^{10} d^2 + 324 a b^{11} c^{11} d + 81 b^{12} c^{12}, \text{Lambda}(\_t, \_t \log(16 \_t a^2 b^3 / (9 a^3 d^3 - 15 a^2 b c d^2 + 3 a b^2 c^2 d + 3 b^3 c^3) + x))\right) + d^3 x^5 / 5 b^2 - \frac{x(2 a d^3 - 3 b c d^2)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*4+c)\*\*3/(b\*x\*\*4+a)\*\*2,x)

[Out] -x\*(a\*\*3\*d\*\*3 - 3\*a\*\*2\*b\*c\*d\*\*2 + 3\*a\*b\*\*2\*c\*\*2\*d - b\*\*3\*c\*\*3)/(4\*a\*\*2\*b\*\*3 + 4\*a\*b\*\*4\*x\*\*4) + RootSum(65536\*\_t\*\*4\*a\*\*7\*b\*\*13 + 6561\*a\*\*12\*d\*\*12 - 43740\*a\*\*11\*b\*c\*d\*\*11 + 118098\*a\*\*10\*b\*\*2\*c\*\*2\*d\*\*10 - 156492\*a\*\*9\*b\*\*3\*c\*\*3\*d\*\*9 + 84159\*a\*\*8\*b\*\*4\*c\*\*4\*d\*\*8 + 26568\*a\*\*7\*b\*\*5\*c\*\*5\*d\*\*7 - 52164\*a\*\*6\*b\*\*6\*c\*\*6\*d\*\*6 + 11016\*a\*\*5\*b\*\*7\*c\*\*7\*d\*\*5 + 10287\*a\*\*4\*b\*\*8\*c\*\*8\*d\*\*4 - 3564\*a\*\*3\*b\*\*9\*c\*\*9\*d\*\*3 - 1134\*a\*\*2\*b\*\*10\*c\*\*10\*d\*\*2 + 324\*a\*b\*\*11\*c\*\*11\*d + 81\*b\*\*12\*c\*\*12, Lambda(\_t, \_t\*log(16\*\_t\*a\*\*2\*b\*\*3/(9\*a\*\*3\*d\*\*3 - 15\*a\*\*2\*b\*c\*d\*\*2 + 3\*a\*b\*\*2\*c\*\*2\*d + 3\*b\*\*3\*c\*\*3) + x))) + d\*\*3\*x\*\*5/(5\*b\*\*2) - x\*(2\*a\*d\*\*3 - 3\*b\*c\*d\*\*2)/b\*\*3

**GIAC/XCAS [A]** time = 0.221806, size = 670, normalized size = 2.11

$$\frac{3 \sqrt{2} \left( (ab^3)^{\frac{1}{4}} b^3 c^3 + (ab^3)^{\frac{1}{4}} ab^2 c^2 d - 5 (ab^3)^{\frac{1}{4}} a^2 b c d^2 + 3 (ab^3)^{\frac{1}{4}} a^3 d^3 \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{a}{b} \right)^{\frac{1}{4}}} \right)}{16 a^2 b^4} + \frac{3 \sqrt{2} \left( (ab^3)^{\frac{1}{4}} b^3 c^3 + (ab^3)^{\frac{1}{4}} ab^2 c^2 d - 5 (ab^3)^{\frac{1}{4}} a^2 b c d^2 + 3 (ab^3)^{\frac{1}{4}} a^3 d^3 \right) \arctan \left( \frac{\sqrt{2} \left( 2x - \sqrt{2} \left( \frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{a}{b} \right)^{\frac{1}{4}}} \right)}{16 a^2 b^4} + \frac{3 \sqrt{2} \left( (ab^3)^{\frac{1}{4}} b^3 c^3 + (ab^3)^{\frac{1}{4}} ab^2 c^2 d - 5 (ab^3)^{\frac{1}{4}} a^2 b c d^2 + 3 (ab^3)^{\frac{1}{4}} a^3 d^3 \right) \ln \left( x^2 + \sqrt{2} x \left( \frac{a}{b} \right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}} \right)}{32 a^2 b^4} + \frac{3 \sqrt{2} \left( (ab^3)^{\frac{1}{4}} b^3 c^3 + (ab^3)^{\frac{1}{4}} ab^2 c^2 d - 5 (ab^3)^{\frac{1}{4}} a^2 b c d^2 + 3 (ab^3)^{\frac{1}{4}} a^3 d^3 \right) \ln \left( x^2 - \sqrt{2} x \left( \frac{a}{b} \right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}} \right)}{32 a^2 b^4} + \frac{b^3 c^3 x - 3 a b^2 c^2 d x + 3 a^2 b c d^2 x - a^3 d^3 x}{4 (b x^4 + a) a b^3} + \frac{b^8 d^3 x^5 + 15 b^8 c d^2 x - 10 a b^7 d^3 x}{5 b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^3/(b\*x^4 + a)^2,x, algorithm="giac")

[Out] 3/16\*sqrt(2)\*((a\*b^3)^(1/4)\*b^3\*c^3 + (a\*b^3)^(1/4)\*a\*b^2\*c^2\*d - 5\*(a\*b^3)^(1/4)\*a^2\*b\*c\*d^2 + 3\*(a\*b^3)^(1/4)\*a^3\*d^3)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(a/b)^(1/4))/(a/b)^(1/4))/(a^2\*b^4) + 3/



$$\begin{aligned}
& 16 \sqrt{2} \left( (a^3 b)^{1/4} b^3 c^3 + (a^3 b)^{1/4} a^2 b^2 c^2 d - 5 \right. \\
& \left. (a^3 b)^{1/4} a^2 b^2 c^2 d^2 + 3 (a^3 b)^{1/4} a^3 d^3 \right) \arctan \left( \frac{1}{2} \sqrt{2} \right. \\
& \left. (2x - \sqrt{2} (a/b)^{1/4}) / (a/b)^{1/4} \right) / (a^2 b^4) + 3/32 \\
& \sqrt{2} \left( (a^3 b)^{1/4} b^3 c^3 + (a^3 b)^{1/4} a^2 b^2 c^2 d - 5 \left( \right. \right. \\
& \left. \left. (a^3 b)^{1/4} a^2 b^2 c^2 d^2 + 3 (a^3 b)^{1/4} a^3 d^3 \right) \ln(x^2 + \sqrt{2} \right. \\
& \left. x (a/b)^{1/4} + \sqrt{a/b}) \right) / (a^2 b^4) - 3/32 \sqrt{2} \left( (a^3 b)^{1/4} \right. \\
& \left. b^3 c^3 + (a^3 b)^{1/4} a^2 b^2 c^2 d - 5 (a^3 b)^{1/4} a^2 b^2 \right. \\
& \left. c^2 d^2 + 3 (a^3 b)^{1/4} a^3 d^3 \right) \ln(x^2 - \sqrt{2} x (a/b)^{1/4} \\
& + \sqrt{a/b}) / (a^2 b^4) + 1/4 (b^3 c^3 x - 3 a^2 b^2 c^2 d x + 3 a^2 \\
& b^2 c^2 d^2 x - a^3 d^3 x) / (b x^4 + a) a^3 b^3 + 1/5 (b^8 d^3 x^5 + \\
& 15 b^8 c^2 d^2 x - 10 a^2 b^7 d^3 x) / b^{10}
\end{aligned}$$

$$3.70 \quad \int \frac{(c+dx^4)^2}{(a+bx^4)^2} dx$$

**Optimal.** Leaf size=291

$$\begin{aligned} & - \frac{(bc-ad)(5ad+3bc) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{9/4}} \\ & + \frac{(bc-ad)(5ad+3bc) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{9/4}} - \frac{(bc-ad)(5ad+3bc) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{9/4}} \\ & + \frac{(bc-ad)(5ad+3bc) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2}a^{7/4}b^{9/4}} + \frac{x(bc-ad)^2}{4ab^2(a+bx^4)} + \frac{d^2x}{b^2} \end{aligned}$$

[Out]  $(d^2x)/b^2 + ((b^*c - a^*d)^2*x)/(4*a^*b^2*(a + b*x^4)) - ((b^*c - a^*d)*(3*b^*c + 5*a^*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(9/4)) + ((b^*c - a^*d)*(3*b^*c + 5*a^*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b^*c - a^*d)*(3*b^*c + 5*a^*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4)) + ((b^*c - a^*d)*(3*b^*c + 5*a^*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4))$

**Rubi [A]** time = 0.724132, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\begin{aligned} & - \frac{(bc-ad)(5ad+3bc) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{9/4}} \\ & + \frac{(bc-ad)(5ad+3bc) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{9/4}} - \frac{(bc-ad)(5ad+3bc) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{9/4}} \\ & + \frac{(bc-ad)(5ad+3bc) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2}a^{7/4}b^{9/4}} + \frac{x(bc-ad)^2}{4ab^2(a+bx^4)} + \frac{d^2x}{b^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^2/(a + b\*x^4)^2, x]

[Out]  $(d^2x)/b^2 + ((b^*c - a^*d)^2*x)/(4*a^*b^2*(a + b*x^4)) - ((b^*c - a^*d)*(3*b^*c + 5*a^*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(9/4)) + ((b^*c - a^*d)*(3*b^*c + 5*a^*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b^*c - a^*d)*(3*b^*c + 5*a^*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4)) + ((b^*c - a^*d)*(3*b^*c + 5*a^*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4))$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & d^2 \int \frac{1}{b^2} dx + \frac{x(ad-bc)^2}{4ab^2(a+bx^4)} + \frac{\sqrt{2}(ad-bc)(5ad+3bc) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{32a^{7/4}b^{9/4}} \\ & - \frac{\sqrt{2}(ad-bc)(5ad+3bc) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{32a^{7/4}b^{9/4}} \\ & + \frac{\sqrt{2}(ad-bc)(5ad+3bc) \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{16a^{7/4}b^{9/4}} - \frac{\sqrt{2}(ad-bc)(5ad+3bc) \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{16a^{7/4}b^{9/4}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((d*x**4+c)**2/(b*x**4+a)**2,x)`

[Out]  $d^{*2} \text{Integral}(b^{*(-2)}, x) + x^{*}(a^{*}d - b^{*}c)^{*2} / (4^{*}a^{*}b^{*2} (a + b^{*}x^{*4})) + \sqrt{2}^{*}(a^{*}d - b^{*}c)^{*}(5^{*}a^{*}d + 3^{*}b^{*}c)^{*} \log(-\sqrt{2}^{*}a^{*}(1/4)^{*}b^{*}(1/4)^{*}x + \sqrt{2}^{*}(a + \sqrt{b}^{*}x^{*2}) / (32^{*}a^{*}(7/4)^{*}b^{*}(9/4))) - \sqrt{2}^{*}(a^{*}d - b^{*}c)^{*}(5^{*}a^{*}d + 3^{*}b^{*}c)^{*} \log(\sqrt{2}^{*}a^{*}(1/4)^{*}b^{*}(1/4)^{*}x + \sqrt{2}^{*}(a + \sqrt{b}^{*}x^{*2}) / (32^{*}a^{*}(7/4)^{*}b^{*}(9/4))) + \sqrt{2}^{*}(a^{*}d - b^{*}c)^{*}(5^{*}a^{*}d + 3^{*}b^{*}c)^{*} \text{atan}(1 - \sqrt{2}^{*}b^{*}(1/4)^{*}x / a^{*}(1/4)) / (16^{*}a^{*}(7/4)^{*}b^{*}(9/4)) - \sqrt{2}^{*}(a^{*}d - b^{*}c)^{*}(5^{*}a^{*}d + 3^{*}b^{*}c)^{*} \text{atan}(1 + \sqrt{2}^{*}b^{*}(1/4)^{*}x / a^{*}(1/4)) / (16^{*}a^{*}(7/4)^{*}b^{*}(9/4))$

**Mathematica [A]** time = 0.299828, size = 297, normalized size = 1.02

$$\frac{\sqrt{2}(5a^2d^2 - 2abcd - 3b^2c^2) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{bx^2}\right)}{a^{7/4}} + \frac{\sqrt{2}(-5a^2d^2 + 2abcd + 3b^2c^2) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{bx^2}\right)}{a^{7/4}} + \frac{2\sqrt{2}(5a^2d^2 - 2abcd - 3b^2c^2) \tan^{-1}\left(\frac{x\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}} + 1\right)}{32b^{9/4}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c + d*x^4)^2/(a + b*x^4)^2,x]`

[Out]  $(32^{*}b^{*(1/4)}d^{*2}x + (8^{*}b^{*(1/4)}(b^{*}c - a^{*}d)^{*2}x) / (a^{*}(a + b^{*}x^{*4})) + (2^{*}\text{Sqrt}[2]^{*}(-3^{*}b^{*2}c^{*2} - 2^{*}a^{*}b^{*}c^{*}d + 5^{*}a^{*2}d^{*2})^{*}\text{ArcTan}[1 - (\text{Sqrt}[2]^{*}b^{*(1/4)}x) / a^{*(1/4)}]) / a^{*(7/4)} + (2^{*}\text{Sqrt}[2]^{*}(3^{*}b^{*2}c^{*2} + 2^{*}a^{*}b^{*}c^{*}d - 5^{*}a^{*2}d^{*2})^{*}\text{ArcTan}[1 + (\text{Sqrt}[2]^{*}b^{*(1/4)}x) / a^{*(1/4)}]) / a^{*(7/4)} + (\text{Sqrt}[2]^{*}(-3^{*}b^{*2}c^{*2} - 2^{*}a^{*}b^{*}c^{*}d + 5^{*}a^{*2}d^{*2})^{*}\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]^{*}a^{*(1/4)}b^{*(1/4)}x + \text{Sqrt}[b]^{*}x^{*2}]) / a^{*(7/4)} + (\text{Sqrt}[2]^{*}(3^{*}b^{*2}c^{*2} + 2^{*}a^{*}b^{*}c^{*}d - 5^{*}a^{*2}d^{*2})^{*}\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]^{*}a^{*(1/4)}b^{*(1/4)}x + \text{Sqrt}[b]^{*}x^{*2}]) / a^{*(7/4)}) / (32^{*}b^{*(9/4)})$

**Maple [B]** time = 0.014, size = 475, normalized size = 1.6

$$\begin{aligned} & \frac{d^2x}{b^2} + \frac{axd^2}{4b^2(bx^4+a)} - \frac{cxd}{2b(bx^4+a)} + \frac{xc^2}{4a(bx^4+a)} \\ & - \frac{5\sqrt{2}d^2}{16b^2} \sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) + \frac{\sqrt{2}cd}{8ab} \sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) \\ & + \frac{3\sqrt{2}c^2}{16a^2} \sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) - \frac{5\sqrt{2}d^2}{16b^2} \sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) \\ & + \frac{\sqrt{2}cd}{8ab} \sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) + \frac{3\sqrt{2}c^2}{16a^2} \sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) \\ & - \frac{5\sqrt{2}d^2}{32b^2} \sqrt[4]{\frac{a}{b}} \ln\left(1\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \\ & + \frac{\sqrt{2}cd}{16ab} \sqrt[4]{\frac{a}{b}} \ln\left(1\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \\ & + \frac{3\sqrt{2}c^2}{32a^2} \sqrt[4]{\frac{a}{b}} \ln\left(1\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^4+c)^2/(b*x^4+a)^2,x)`

[Out] 
$$\begin{aligned} & d^2x/b^2+1/4/b^2*x*a/(b*x^4+a)*d^2-1/2/b*x/(b*x^4+a)*c*d+1/4*x/a \\ & / (b*x^4+a)*c^2-5/16/b^2*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)*d^2+1/8/b/a*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)*c*d+3/16/a^2*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)*c^2-5/16/b^2*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)*d^2+1/8/b/a*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)*c*d+3/16/a^2*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)*c^2-5/32/b^2*(a/b)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*x^2 \\ & ^{(1/2)+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*x*2^{(1/2)+(a/b)^{(1/2)})))*d^2+ \\ & 1/16/b/a*(a/b)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*x*2^{(1/2)+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*x*2^{(1/2)+(a/b)^{(1/2)})))*c*d+3/32/a^2*(a/b)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*x*2^{(1/2)+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*x*2^{(1/2)+(a/b)^{(1/2)})))*c^2 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^4 + c)^2/(b*x^4 + a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.245341, size = 1592, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^4 + c)^2/(b*x^4 + a)^2,x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & 1/16*(16*a*b*d^2*x^5 + 4*(a*b^3*x^4 + a^2*b^2)*(-81*b^8*c^8 + 21 \\ & 6*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a \\ & ^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 100 \\ & 0*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^{(1/4)}*\arctan(-a^2*b^2*(-( \\ & 81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5* \\ & c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2* \\ & c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^{(1/4)})/(3* \\ & b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*x + (3*b^2*c^2 + 2*a*b*c*d - 5*a \\ & ^2*d^2)*\sqrt{(a^4*b^4*\sqrt{-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a \\ & ^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640 \\ & *a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a \\ & ^8*d^8)/(a^7*b^9))} + (9*b^4*c^4 + 12*a*b^3*c^3*d - 26*a^2*b^2*c^2 \\ & *d^2 - 20*a^3*b*c*d^3 + 25*a^4*d^4)*x^2)/(9*b^4*c^4 + 12*a*b^3*c^ \\ & 3*d - 26*a^2*b^2*c^2*d^2 - 20*a^3*b*c*d^3 + 25*a^4*d^4))) - (a*b \\ & ^3*x^4 + a^2*b^2)*(-81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^ \\ & ^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3* \\ & c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/ \\ & (a^7*b^9))^{(1/4)}*\log(a^2*b^2*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 32 \\ & 4*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1 \\ & 640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 62 \\ & 5*a^8*d^8)/(a^7*b^9))^{(1/4)} - (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2) \\ & *x) + (a*b^3*x^4 + a^2*b^2)*(-81*b^8*c^8 + 216*a*b^7*c^7*d - 324 \\ & *a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 16 \\ & 40*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625 \\ & *a^8*d^8)/(a^7*b^9))^{(1/4)}*\log(-a^2*b^2*(-(81*b^8*c^8 + 216*a*b^7* \\ & c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4* \\ & c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b* \\ & c*d^7 + 625*a^8*d^8)/(a^7*b^9))^{(1/4)} - (3*b^2*c^2 + 2*a*b*c*d - \\ & 5*a^2*d^2)*x) + 4*(b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x)/(a*b^3*x^ \\ & 4 + a^2*b^2) \end{aligned}$$

**Sympy [A]** time = 7.23865, size = 219, normalized size = 0.75

$$\frac{x(a^2d^2 - 2abcd + b^2c^2)}{4a^2b^2 + 4ab^3x^4} + \text{RootSum}\left(65536t^4a^7b^9 + 625a^8d^8 - 1000a^7bcd^7 - 900a^6b^2c^2d^6 + 1640a^5b^3c^3d^5 + 646a^4b^4c^4d^4 - 984a^3b^5c^5d^3 - 324a^2b^6c^6d^2 + 216ab^7c^7d + 81b^8c^8, \text{Lambda}(t, t \log(-16t^2a^2b^2/(5a^2d^2 - 2ab^3cd - 3b^2c^2) + x))\right) + \frac{d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*4+c)\*\*2/(b\*x\*\*4+a)\*\*2,x)

[Out] x\*(a\*\*2\*d\*\*2 - 2\*a\*b\*c\*d + b\*\*2\*c\*\*2)/(4\*a\*\*2\*b\*\*2 + 4\*a\*b\*\*3\*x\*\*4) + RootSum(65536\*\_t\*\*4\*a\*\*7\*b\*\*9 + 625\*a\*\*8\*d\*\*8 - 1000\*a\*\*7\*b\*c\*d\*\*7 - 900\*a\*\*6\*b\*\*2\*c\*\*2\*d\*\*6 + 1640\*a\*\*5\*b\*\*3\*c\*\*3\*d\*\*5 + 646\*a\*\*4\*b\*\*4\*c\*\*4\*d\*\*4 - 984\*a\*\*3\*b\*\*5\*c\*\*5\*d\*\*3 - 324\*a\*\*2\*b\*\*6\*c\*\*6\*d\*\*2 + 216\*a\*b\*\*7\*c\*\*7\*d + 81\*b\*\*8\*c\*\*8, Lambda(\_t, \_t\*log(-16\*\_t\*a\*\*2\*b\*\*2/(5\*a\*\*2\*d\*\*2 - 2\*a\*b\*c\*d - 3\*b\*\*2\*c\*\*2) + x))) + d\*\*2\*x/b\*\*2

**GIAC/XCAS [A]** time = 0.222079, size = 508, normalized size = 1.75

$$\frac{\sqrt{2}\left(3(ab^3)^{\frac{1}{4}}b^2c^2 + 2(ab^3)^{\frac{1}{4}}abcd - 5(ab^3)^{\frac{1}{4}}a^2d^2\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{b^2} + \frac{\sqrt{2}\left(3(ab^3)^{\frac{1}{4}}b^2c^2 + 2(ab^3)^{\frac{1}{4}}abcd - 5(ab^3)^{\frac{1}{4}}a^2d^2\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3} + \frac{\sqrt{2}\left(3(ab^3)^{\frac{1}{4}}b^2c^2 + 2(ab^3)^{\frac{1}{4}}abcd - 5(ab^3)^{\frac{1}{4}}a^2d^2\right) \ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32a^2b^3} - \frac{\sqrt{2}\left(3(ab^3)^{\frac{1}{4}}b^2c^2 + 2(ab^3)^{\frac{1}{4}}abcd - 5(ab^3)^{\frac{1}{4}}a^2d^2\right) \ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32a^2b^3} + \frac{b^2c^2x - 2abcdx + a^2d^2x}{4(bx^4 + a)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^2/(b\*x^4 + a)^2,x, algorithm="giac")

[Out] d^2\*x/b^2 + 1/16\*sqrt(2)\*(3\*(a\*b^3)^(1/4)\*b^2\*c^2 + 2\*(a\*b^3)^(1/4)\*a\*b\*c\*d - 5\*(a\*b^3)^(1/4)\*a^2\*d^2)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(a/b)^(1/4))/(a/b)^(1/4))/(a^2\*b^3) + 1/16\*sqrt(2)\*(3\*(a\*b^3)^(1/4)\*b^2\*c^2 + 2\*(a\*b^3)^(1/4)\*a\*b\*c\*d - 5\*(a\*b^3)^(1/4)\*a^2\*d^2)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)\*(a/b)^(1/4))/(a/b)^(1/4))/(a^2\*b^3) + 1/32\*sqrt(2)\*(3\*(a\*b^3)^(1/4)\*b^2\*c^2 + 2\*(a\*b^3)^(1/4)\*a\*b\*c\*d - 5\*(a\*b^3)^(1/4)\*a^2\*d^2)\*ln(x^2 + sqrt(2)\*x\*(a/b)^(1/4) + sqrt(a/b))/(a^2\*b^3) - 1/32\*sqrt(2)\*(3\*(a\*b^3)^(1/4)\*b^2\*c^2 + 2\*(a\*b^3)^(1/4)\*a\*b\*c\*d - 5\*(a\*b^3)^(1/4)\*a^2\*d^2)\*ln(x^2 - sqrt(2)\*x\*(a/b)^(1/4) + sqrt(a/b))/(a^2\*b^3) + 1/4\*(b^2\*c^2\*x - 2\*a\*b\*c\*d\*x + a^2\*d^2\*x)/((b\*x^4 + a)\*a\*b^2)

$$3.71 \quad \int \frac{c+dx^4}{(a+bx^4)^2} dx$$

**Optimal.** Leaf size=245

$$\begin{aligned} & -\frac{(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{5/4}} \\ & -\frac{(ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{x(bc-ad)}{4ab(a+bx^4)} \end{aligned}$$

[Out] ((b\*c - a\*d)\*x)/(4\*a\*b\*(a + b\*x^4)) - ((3\*b\*c + a\*d)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(8\*Sqrt[2]\*a^(7/4)\*b^(5/4)) + ((3\*b\*c + a\*d)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(8\*Sqrt[2]\*a^(7/4)\*b^(5/4)) - ((3\*b\*c + a\*d)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(16\*Sqrt[2]\*a^(7/4)\*b^(5/4)) + ((3\*b\*c + a\*d)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(16\*Sqrt[2]\*a^(7/4)\*b^(5/4))

**Rubi [A]** time = 0.316828, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$\begin{aligned} & -\frac{(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}b^{5/4}} \\ & -\frac{(ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{x(bc-ad)}{4ab(a+bx^4)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)/(a + b\*x^4)^2, x]

[Out] ((b\*c - a\*d)\*x)/(4\*a\*b\*(a + b\*x^4)) - ((3\*b\*c + a\*d)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(8\*Sqrt[2]\*a^(7/4)\*b^(5/4)) + ((3\*b\*c + a\*d)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(8\*Sqrt[2]\*a^(7/4)\*b^(5/4)) - ((3\*b\*c + a\*d)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(16\*Sqrt[2]\*a^(7/4)\*b^(5/4)) + ((3\*b\*c + a\*d)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(16\*Sqrt[2]\*a^(7/4)\*b^(5/4))

**Rubi in Sympy [A]** time = 62.0599, size = 226, normalized size = 0.92

$$\begin{aligned} & \frac{x(ad-bc)}{4ab(a+bx^4)} - \frac{\sqrt{2}(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{32a^{7/4}b^{5/4}} \\ & + \frac{\sqrt{2}(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}+\sqrt{a}+\sqrt{bx^2}\right)}{32a^{7/4}b^{5/4}} \\ & - \frac{\sqrt{2}(ad+3bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{16a^{7/4}b^{5/4}} + \frac{\sqrt{2}(ad+3bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{16a^{7/4}b^{5/4}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((d\*x\*\*4+c)/(b\*x\*\*4+a)\*\*2, x)

[Out] -x\*(a\*d - b\*c)/(4\*a\*b\*(a + b\*x\*\*4)) - sqrt(2)\*(a\*d + 3\*b\*c)\*log(-sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*x + sqrt(a) + sqrt(b)\*x\*\*2)/(32\*a\*\*(7/4)\*b\*\*(5/4)) + sqrt(2)\*(a\*d + 3\*b\*c)\*log(sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)

$$\frac{\sqrt{x + \sqrt{a} + \sqrt{b}x^2}}{(32a^{7/4}b^{5/4})} - \sqrt{2} \frac{(ad + 3b^2c) \operatorname{atan}\left(1 - \sqrt{2} \frac{b^{1/4}x}{a^{1/4}}\right)}{(16a^{7/4}b^{5/4})} + \sqrt{2} \frac{(ad + 3b^2c) \operatorname{atan}\left(1 + \sqrt{2} \frac{b^{1/4}x}{a^{1/4}}\right)}{(16a^{7/4}b^{5/4})}$$

**Mathematica [A]** time = 0.30402, size = 212, normalized size = 0.87

$$\frac{-\frac{8a^{3/4}\sqrt[4]{bx(ad-bc)}}{a+bx^4} - \sqrt{2}(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right) + \sqrt{2}(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right) - 2\sqrt{2}(ad+3b^2c)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}}{\sqrt{a} + \sqrt{bx^2}}\right)}{32a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^4)/(a + b\*x^4)^2, x]

[Out]  $\frac{(-8a^{3/4}b^{1/4}(-b^2c + a^2d)x)/(a + b^2x^4) - 2\sqrt{2} \left( (3b^2c + a^2d) \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}b^{1/4}x}{a^{1/4}}\right] + (3b^2c + a^2d) \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}b^{1/4}x}{a^{1/4}}\right] - \sqrt{2} \left( (3b^2c + a^2d) \operatorname{Log}\left[\sqrt{a} - \sqrt{2} \frac{b^{1/4}x}{a^{1/4}}\right] + \sqrt{2} \frac{b^{1/4}x}{a^{1/4}} \right) \right)}{(32a^{7/4}b^{5/4})}$

**Maple [A]** time = 0.011, size = 295, normalized size = 1.2

$$\begin{aligned} & -\frac{(ad-bc)x}{4ab(bx^4+a)} + \frac{\sqrt{2}d}{16ab}\sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) + \frac{3\sqrt{2}c}{16a^2}\sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) \\ & + \frac{\sqrt{2}d}{16ab}\sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) + \frac{3\sqrt{2}c}{16a^2}\sqrt[4]{\frac{a}{b}} \arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) \\ & + \frac{\sqrt{2}d}{32ab}\sqrt[4]{\frac{a}{b}} \ln\left(1\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \\ & + \frac{3\sqrt{2}c}{32a^2}\sqrt[4]{\frac{a}{b}} \ln\left(1\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^4+c)/(b\*x^4+a)^2, x)

[Out]  $\frac{-1/4(a^2d-b^2c)/a/b^2x/(b^2x^4+a) + 1/16/a/b^2(a/b)^{1/4}2^{1/2} \operatorname{arctan}\left(\frac{2^{1/2}}{(a/b)^{1/4}x+1} \frac{d+3/16/a^2(a/b)^{1/4}2^{1/2} \operatorname{arctan}\left(\frac{2^{1/2}}{(a/b)^{1/4}x+1} \frac{c+1/16/a/b^2(a/b)^{1/4}2^{1/2} \operatorname{arctan}\left(\frac{2^{1/2}}{(a/b)^{1/4}x-1} \frac{d+3/16/a^2(a/b)^{1/4}2^{1/2} \operatorname{arctan}\left(\frac{2^{1/2}}{(a/b)^{1/4}x-1} \frac{c+1/32/a/b^2(a/b)^{1/4}2^{1/2} \ln\left(\frac{x^2+(a/b)^{1/4}x\sqrt{2}+(a/b)^{1/2}}{x^2-(a/b)^{1/4}x\sqrt{2}+(a/b)^{1/2}}\right)}{d+3/32/a^2(a/b)^{1/4}2^{1/2} \ln\left(\frac{x^2+(a/b)^{1/4}x\sqrt{2}+(a/b)^{1/2}}{x^2-(a/b)^{1/4}x\sqrt{2}+(a/b)^{1/2}}\right)}\right)}{c}\right)}{(b^2x^4+a)^2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)/(b\*x^4 + a)^2, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.229541, size = 838, normalized size = 3.42

$$4(ab^2x^4 + a^2b) \left( -\frac{81b^4c^4 + 108ab^3c^3d + 54a^2b^2c^2d^2 + 12a^3bcd^3 + a^4d^4}{a^7b^5} \right)^{\frac{1}{4}} \arctan \left( \frac{a^2b \left( -\frac{81b^4c^4 + 108ab^3c^3d + 54a^2b^2c^2d^2 + 12a^3bcd^3 + a^4d^4}{a^7b^5} \right)^{\frac{1}{4}}}{(3bc+ad)x + (3bc+ad) \sqrt{\frac{a^4b^2 \sqrt{-\frac{81b^4c^4 + 108ab^3c^3d + 54a^2b^2c^2d^2 + 12a^3bcd^3 + a^4d^4}{a^7b^5}}}{9b^2c^2 + 6abc}}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)/(b\*x^4 + a)^2, x, algorithm="fricas")

[Out] 
$$-1/16 * (4 * (a^2 * b^2 * x^4 + a^2 * b) * (-81 * b^4 * c^4 + 108 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 12 * a^3 * b * c * d^3 + a^4 * d^4) / (a^7 * b^5))^{1/4} * \arctan(a^2 * b * (-81 * b^4 * c^4 + 108 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 12 * a^3 * b * c * d^3 + a^4 * d^4) / (a^7 * b^5))^{1/4} / ((3 * b * c + a * d) * x + (3 * b * c + a * d) * \sqrt{(a^4 * b^2 * \sqrt{(-81 * b^4 * c^4 + 108 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 12 * a^3 * b * c * d^3 + a^4 * d^4) / (a^7 * b^5)}) + (9 * b^2 * c^2 + 6 * a * b * c * d + a^2 * d^2) * x^2} / (9 * b^2 * c^2 + 6 * a * b * c * d + a^2 * d^2)) - (a^2 * b^2 * x^4 + a^2 * b) * (-81 * b^4 * c^4 + 108 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 12 * a^3 * b * c * d^3 + a^4 * d^4) / (a^7 * b^5))^{1/4} * \log(a^2 * b * (-81 * b^4 * c^4 + 108 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 12 * a^3 * b * c * d^3 + a^4 * d^4) / (a^7 * b^5))^{1/4} + (3 * b * c + a * d) * x) + (a^2 * b^2 * x^4 + a^2 * b) * (-81 * b^4 * c^4 + 108 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 12 * a^3 * b * c * d^3 + a^4 * d^4) / (a^7 * b^5))^{1/4} * \log(-a^2 * b * (-81 * b^4 * c^4 + 108 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 12 * a^3 * b * c * d^3 + a^4 * d^4) / (a^7 * b^5))^{1/4} + (3 * b * c + a * d) * x) - 4 * (b * c - a * d) * x) / (a^2 * b^2 * x^4 + a^2 * b)$$

**Sympy** [A] time = 3.40875, size = 112, normalized size = 0.46

$$\frac{x(ad - bc)}{4a^2b + 4ab^2x^4} + \text{RootSum} \left( 65536t^4a^7b^5 + a^4d^4 + 12a^3bcd^3 + 54a^2b^2c^2d^2 + 108ab^3c^3d + 81b^4c^4, \left( t \mapsto t \log \left( \frac{16ta^2b}{ad + 3bc} + x \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*4+c)/(b\*x\*\*4+a)\*\*2, x)

[Out] 
$$-x * (a * d - b * c) / (4 * a^2 * b + 4 * a * b^2 * x^4) + \text{RootSum}(65536 * _t^4 * a^7 * b^5 + a^4 * d^4 + 12 * a^3 * b * c * d^3 + 54 * a^2 * b^2 * c^2 * d^2 + 108 * a * b^3 * c^3 * d + 81 * b^4 * c^4, \text{Lambda}(_t, _t * \log(16 * _t * a^2 * b / (a * d + 3 * b * c) + x)))$$



**GIAC/XCAS [A]** time = 0.220218, size = 359, normalized size = 1.47

$$\frac{\sqrt{2}\left(3(ab^3)^{\frac{1}{4}}bc + (ab^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^2} + \frac{\sqrt{2}\left(3(ab^3)^{\frac{1}{4}}bc + (ab^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^2} + \frac{\sqrt{2}\left(3(ab^3)^{\frac{1}{4}}bc + (ab^3)^{\frac{1}{4}}ad\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32a^2b^2} - \frac{\sqrt{2}\left(3(ab^3)^{\frac{1}{4}}bc + (ab^3)^{\frac{1}{4}}ad\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32a^2b^2} + \frac{bcx - adx}{4(bx^4 + a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)/(b\*x^4 + a)^2,x, algorithm="giac")

[Out] 1/16\*sqrt(2)\*(3\*(a\*b^3)^(1/4)\*b\*c + (a\*b^3)^(1/4)\*a\*d)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(a/b)^(1/4))/(a/b)^(1/4))/(a^2\*b^2) + 1/16\*sqrt(2)\*(3\*(a\*b^3)^(1/4)\*b\*c + (a\*b^3)^(1/4)\*a\*d)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)\*(a/b)^(1/4))/(a/b)^(1/4))/(a^2\*b^2) + 1/32\*sqrt(2)\*(3\*(a\*b^3)^(1/4)\*b\*c + (a\*b^3)^(1/4)\*a\*d)\*ln(x^2 + sqrt(2)\*x\*(a/b)^(1/4) + sqrt(a/b))/(a^2\*b^2) - 1/32\*sqrt(2)\*(3\*(a\*b^3)^(1/4)\*b\*c + (a\*b^3)^(1/4)\*a\*d)\*ln(x^2 - sqrt(2)\*x\*(a/b)^(1/4) + sqrt(a/b))/(a^2\*b^2) + 1/4\*(b\*c\*x - a\*d\*x)/((b\*x^4 + a)\*a\*b)

$$3.72 \quad \int \frac{1}{(a+bx^4)^2(c+dx^4)} dx$$

**Optimal.** Leaf size=513

$$\begin{aligned} & \frac{b^{3/4}(3bc - 7ad) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}(bc - ad)^2} \\ & + \frac{b^{3/4}(3bc - 7ad) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}(bc - ad)^2} - \frac{b^{3/4}(3bc - 7ad) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc - ad)^2} \\ & + \frac{b^{3/4}(3bc - 7ad) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2}a^{7/4}(bc - ad)^2} - \frac{d^{7/4} \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{4\sqrt{2}c^{3/4}(bc - ad)^2} \\ & + \frac{d^{7/4} \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{4\sqrt{2}c^{3/4}(bc - ad)^2} - \frac{d^{7/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc - ad)^2} \\ & + \frac{d^{7/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}} + 1\right)}{2\sqrt{2}c^{3/4}(bc - ad)^2} + \frac{bx}{4a(a + bx^4)(bc - ad)} \end{aligned}$$

[Out] (b\*x)/(4\*a\*(b\*c - a\*d)\*(a + b\*x^4)) - (b^(3/4)\*(3\*b\*c - 7\*a\*d)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)]/(8\*Sqrt[2]\*a^(7/4)\*(b\*c - a\*d)^2) + (b^(3/4)\*(3\*b\*c - 7\*a\*d)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)]/(8\*Sqrt[2]\*a^(7/4)\*(b\*c - a\*d)^2) - (d^(7/4)\*ArcTan[1 - (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)]/(2\*Sqrt[2]\*c^(3/4)\*(b\*c - a\*d)^2) + (d^(7/4)\*ArcTan[1 + (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)]/(2\*Sqrt[2]\*c^(3/4)\*(b\*c - a\*d)^2) - (b^(3/4)\*(3\*b\*c - 7\*a\*d)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(16\*Sqrt[2]\*a^(7/4)\*(b\*c - a\*d)^2) + (b^(3/4)\*(3\*b\*c - 7\*a\*d)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(16\*Sqrt[2]\*a^(7/4)\*(b\*c - a\*d)^2) - (d^(7/4)\*Log[Sqrt[c] - Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2])/(4\*Sqrt[2]\*c^(3/4)\*(b\*c - a\*d)^2) + (d^(7/4)\*Log[Sqrt[c] + Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2])/(4\*Sqrt[2]\*c^(3/4)\*(b\*c - a\*d)^2)

**Rubi [A]** time = 0.902967, antiderivative size = 513, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\begin{aligned} & \frac{b^{3/4}(3bc - 7ad) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}(bc - ad)^2} \\ & + \frac{b^{3/4}(3bc - 7ad) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}(bc - ad)^2} - \frac{b^{3/4}(3bc - 7ad) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc - ad)^2} \\ & + \frac{b^{3/4}(3bc - 7ad) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2}a^{7/4}(bc - ad)^2} - \frac{d^{7/4} \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{4\sqrt{2}c^{3/4}(bc - ad)^2} \\ & + \frac{d^{7/4} \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{4\sqrt{2}c^{3/4}(bc - ad)^2} - \frac{d^{7/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc - ad)^2} \\ & + \frac{d^{7/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}} + 1\right)}{2\sqrt{2}c^{3/4}(bc - ad)^2} + \frac{bx}{4a(a + bx^4)(bc - ad)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^2\*(c + d\*x^4)), x]

[Out] (b\*x)/(4\*a\*(b\*c - a\*d)\*(a + b\*x^4)) - (b^(3/4)\*(3\*b\*c - 7\*a\*d)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)]/(8\*Sqrt[2]\*a^(7/4)\*(b\*c -

$$\begin{aligned}
& a^*d)^2) + (b^{(3/4)} * (3*b*c - 7*a*d) * \text{ArcTan}[1 + (\text{Sqrt}[2] * b^{(1/4)} * x) \\
& / a^{(1/4)}]) / (8 * \text{Sqrt}[2] * a^{(7/4)} * (b*c - a*d)^2) - (d^{(7/4)} * \text{ArcTan}[1 \\
& - (\text{Sqrt}[2] * d^{(1/4)} * x) / c^{(1/4)}]) / (2 * \text{Sqrt}[2] * c^{(3/4)} * (b*c - a*d)^2) \\
& + (d^{(7/4)} * \text{ArcTan}[1 + (\text{Sqrt}[2] * d^{(1/4)} * x) / c^{(1/4)}]) / (2 * \text{Sqrt}[2] * c \\
& ^{(3/4)} * (b*c - a*d)^2) - (b^{(3/4)} * (3*b*c - 7*a*d) * \text{Log}[\text{Sqrt}[a] - \text{Sqr} \\
& \text{rt}[2] * a^{(1/4)} * b^{(1/4)} * x + \text{Sqrt}[b] * x^2]) / (16 * \text{Sqrt}[2] * a^{(7/4)} * (b*c \\
& - a*d)^2) + (b^{(3/4)} * (3*b*c - 7*a*d) * \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] * a^{(1/4)} \\
& ) * b^{(1/4)} * x + \text{Sqrt}[b] * x^2]) / (16 * \text{Sqrt}[2] * a^{(7/4)} * (b*c - a*d)^2) - \\
& (d^{(7/4)} * \text{Log}[\text{Sqrt}[c] - \text{Sqrt}[2] * c^{(1/4)} * d^{(1/4)} * x + \text{Sqrt}[d] * x^2]) / \\
& (4 * \text{Sqrt}[2] * c^{(3/4)} * (b*c - a*d)^2) + (d^{(7/4)} * \text{Log}[\text{Sqrt}[c] + \text{Sqrt}[2] \\
& ] * c^{(1/4)} * d^{(1/4)} * x + \text{Sqrt}[d] * x^2]) / (4 * \text{Sqrt}[2] * c^{(3/4)} * (b*c - a*d \\
& )^2)
\end{aligned}$$

**Rubi in Sympy [A]** time = 168.513, size = 474, normalized size = 0.92

$$\begin{aligned}
& -\frac{\sqrt{2}d^{7/4} \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{8c^{3/4}(ad-bc)^2} + \frac{\sqrt{2}d^{7/4} \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{8c^{3/4}(ad-bc)^2} \\
& -\frac{\sqrt{2}d^{7/4} \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{4c^{3/4}(ad-bc)^2} + \frac{\sqrt{2}d^{7/4} \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{4c^{3/4}(ad-bc)^2} \\
& -\frac{bx}{4a(a+bx^4)(ad-bc)} + \frac{\sqrt{2}b^{3/4}(7ad-3bc) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{32a^{7/4}(ad-bc)^2} \\
& -\frac{\sqrt{2}b^{3/4}(7ad-3bc) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{32a^{7/4}(ad-bc)^2} \\
& +\frac{\sqrt{2}b^{3/4}(7ad-3bc) \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{16a^{7/4}(ad-bc)^2} - \frac{\sqrt{2}b^{3/4}(7ad-3bc) \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{16a^{7/4}(ad-bc)^2}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(b*x**4+a)**2/(d*x**4+c), x)`

[Out] `-sqrt(2)*d**(7/4)*log(-sqrt(2)*c**(1/4)*d**(1/4)*x + sqrt(c) + sqrt(d)*x**2)/(8*c**(3/4)*(a*d - b*c)**2) + sqrt(2)*d**(7/4)*log(sqrt(2)*c**(1/4)*d**(1/4)*x + sqrt(c) + sqrt(d)*x**2)/(8*c**(3/4)*(a*d - b*c)**2) - sqrt(2)*d**(7/4)*atan(1 - sqrt(2)*d**(1/4)*x/c**(1/4))/(4*c**(3/4)*(a*d - b*c)**2) + sqrt(2)*d**(7/4)*atan(1 + sqrt(2)*d**(1/4)*x/c**(1/4))/(4*c**(3/4)*(a*d - b*c)**2) - b*x/(4*a*(a + b*x**4)*(a*d - b*c)) + sqrt(2)*b**(3/4)*(7*a*d - 3*b*c)*log(-sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(32*a**(7/4)*(a*d - b*c)**2) - sqrt(2)*b**(3/4)*(7*a*d - 3*b*c)*log(sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(32*a**(7/4)*(a*d - b*c)**2) + sqrt(2)*b**(3/4)*(7*a*d - 3*b*c)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(16*a**(7/4)*(a*d - b*c)**2) - sqrt(2)*b**(3/4)*(7*a*d - 3*b*c)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(16*a**(7/4)*(a*d - b*c)**2)`

**Mathematica [A]** time = 0.544791, size = 499, normalized size = 0.97

$$8a^{3/4}bc^{3/4}x(bc-ad) - 8\sqrt{2}a^{7/4}d^{7/4}(a+bx^4) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right) + 8\sqrt{2}a^{7/4}d^{7/4}(a+bx^4) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}} + 1\right) - 4\sqrt{2}a^{7/4}d^{7/4}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((a + b*x^4)^2*(c + d*x^4)), x]`

[Out] `(8*a^(3/4)*b*c^(3/4)*(b*c - a*d)*x - 2*Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(a + b*x^4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]`

$$\begin{aligned}
& + 2*\text{Sqrt}[2]*b^{(3/4)}*c^{(3/4)}*(3*b*c - 7*a*d)*(a + b*x^4)*\text{ArcTan}\left[1 + \frac{\text{Sqrt}[2]*b^{(1/4)}*x}{a^{(1/4)}}\right] - 8*\text{Sqrt}[2]*a^{(7/4)}*d^{(7/4)}*(a + b*x^4)*\text{ArcTan}\left[1 - \frac{\text{Sqrt}[2]*d^{(1/4)}*x}{c^{(1/4)}}\right] + 8*\text{Sqrt}[2]*a^{(7/4)}*d^{(7/4)}*(a + b*x^4)*\text{ArcTan}\left[1 + \frac{\text{Sqrt}[2]*d^{(1/4)}*x}{c^{(1/4)}}\right] - \text{Sqrt}[2]*b^{(3/4)}*c^{(3/4)}*(3*b*c - 7*a*d)*(a + b*x^4)*\text{Log}\left[\frac{\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2}{\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2}\right] - 4*\text{Sqrt}[2]*a^{(7/4)}*d^{(7/4)}*(a + b*x^4)*\text{Log}\left[\frac{\text{Sqrt}[c] - \text{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \text{Sqrt}[d]*x^2}{\text{Sqrt}[c] + \text{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \text{Sqrt}[d]*x^2}\right]\bigg/(32*a^{(7/4)}*c^{(3/4)}*(b*c - a*d)^2*(a + b*x^4))
\end{aligned}$$

**Maple [A]** time = 0.018, size = 550, normalized size = 1.1

$$\begin{aligned}
& \frac{d^2\sqrt{2}}{8(ad-bc)^2c}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2 + \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)\left(x^2 - \sqrt[4]{\frac{c}{d}}x\sqrt{2} + \sqrt{\frac{c}{d}}\right)^{-1}\right) \\
& + \frac{d^2\sqrt{2}}{4(ad-bc)^2c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}} + 1\right) + \frac{d^2\sqrt{2}}{4(ad-bc)^2c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}} - 1\right) \\
& - \frac{bxd}{4(ad-bc)^2(bx^4+a)} + \frac{b^2xc}{4(ad-bc)^2a(bx^4+a)} \\
& - \frac{7b\sqrt{2}d}{16(ad-bc)^2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) + \frac{3b^2\sqrt{2}c}{16(ad-bc)^2a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) \\
& - \frac{7b\sqrt{2}d}{16(ad-bc)^2a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) + \frac{3b^2\sqrt{2}c}{16(ad-bc)^2a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) \\
& - \frac{7b\sqrt{2}d}{32(ad-bc)^2a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \\
& + \frac{3b^2\sqrt{2}c}{32(ad-bc)^2a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2 + \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x^2 - \sqrt[4]{\frac{a}{b}}x\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right)
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^2/(d\*x^4+c), x)

[Out] 1/8\*d^2/(a\*d-b\*c)^2\*(c/d)^(1/4)/c^2^(1/2)\*ln((x^2+(c/d)^(1/4)\*x^2)^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)\*x^2^(1/2)+(c/d)^(1/2))+1/4\*d^2/(a\*d-b\*c)^2\*(c/d)^(1/4)/c^2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x+1)+1/4\*d^2/(a\*d-b\*c)^2\*(c/d)^(1/4)/c^2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x-1)-1/4\*b/(a\*d-b\*c)^2\*x/(b\*x^4+a)\*d+1/4\*b^2/(a\*d-b\*c)^2\*x/a/(b\*x^4+a)\*c-7/16\*b/(a\*d-b\*c)^2/a\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)\*d+3/16\*b^2/(a\*d-b\*c)^2/a^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)\*c-7/16\*b/(a\*d-b\*c)^2/a\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)\*d+3/16\*b^2/(a\*d-b\*c)^2/a^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)\*c-7/32\*b/(a\*d-b\*c)^2/a\*(a/b)^(1/4)\*2^(1/2)\*ln((x^2+(a/b)^(1/4)\*x^2)^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*x^2^(1/2)+(a/b)^(1/2))\*d+3/32\*b^2/(a\*d-b\*c)^2/a^2\*(a/b)^(1/4)\*2^(1/2)\*ln((x^2+(a/b)^(1/4)\*x^2)^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*x^2^(1/2)+(a/b)^(1/2))\*c

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4 + a)^2*(d*x^4 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

```
Fricas [A] time = 22.3349, size = 3665, normalized size = 7.14
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4 + a)^2*(d*x^4 + c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 -
56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 2
8*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2
*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*arctan((-d^7/(b^8*c^11 - 8*a
*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^
4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4
*d^7 + a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)/(d
^2*x + d^2*sqrt((d^4*x^2 + (b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c
^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(-d^7/(b^8*c^11 - 8*a
*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^
4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4
*d^7 + a^8*c^3*d^8))))/d^4))) - 4*((a*b^2*c - a^2*b*d)*x^4 + a^2*b
*c - a^3*d)*(-81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^
2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b
^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4
*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c
*d^7 + a^15*d^8))^(1/4)*arctan(-(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*
c*d^2)*(-81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 411
6*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*
d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^
4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 +
a^15*d^8))^(1/4)/((3*b^2*c - 7*a*b*d)*x + (3*b^2*c - 7*a*b*d)*sq
rt((9*b^4*c^2 - 42*a*b^3*c*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 +
a^8*d^4)*sqrt(-81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2
- 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*
c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4
*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 +
a^15*d^8)))/(9*b^4*c^2 - 42*a*b^3*c*d + 49*a^2*b^2*d^2))) - 4*(-
d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*
c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^
5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d
)*x^4 + a^2*b*c - a^3*d)*log(d^2*x + (-d^7/(b^8*c^11 - 8*a*b^7*c^
10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^
4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 +
a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)) + 4*(-d^
7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^
8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*
d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*
x^4 + a^2*b*c - a^3*d)*log(d^2*x - (-d^7/(b^8*c^11 - 8*a*b^7*c^10
*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4
- 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^
8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)) + ((a*b^2*
c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*(-81*b^7*c^4 - 756*a*b^6*c^3
*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4
)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b
^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*
b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*log(-(3*b^2*c - 7
*a*b*d)*x + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*(-81*b^7*c^4 -
756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 24
01*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^
2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*
d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)) -
((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*(-81*b^7*c^4 - 756*a
*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4
*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 5
6*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 +
```

$$28*a^{13}*b^2*c^2*d^6 - 8*a^{14}*b*c*d^7 + a^{15}*d^8)^{(1/4)}*\log(-(3*b^2*c - 7*a*b*d)*x - (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^{10}*b^5*c^5*d^3 + 70*a^{11}*b^4*c^4*d^4 - 56*a^{12}*b^3*c^3*d^5 + 28*a^{13}*b^2*c^2*d^6 - 8*a^{14}*b*c*d^7 + a^{15}*d^8))^{(1/4)}) - 4*b*x)/((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)\*\*2/(d\*x\*\*4+c),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.227731, size = 900, normalized size = 1.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)^2\*(d\*x^4 + c)),x, algorithm="giac")

[Out]  $\frac{1}{2}*(c*d^3)^{(1/4)}*d*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(\sqrt{2}*b^2*c^3 - 2*\sqrt{2}*a*b*c^2*d + \sqrt{2}*a^2*c*d^2) + 1/2*(c*d^3)^{(1/4)}*d*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(\sqrt{2}*b^2*c^3 - 2*\sqrt{2}*a*b*c^2*d + \sqrt{2}*a^2*c*d^2) + 1/4*(c*d^3)^{(1/4)}*d*\ln(x^2 + \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b^2*c^3 - 2*\sqrt{2}*a*b*c^2*d + \sqrt{2}*a^2*c*d^2) - 1/4*(c*d^3)^{(1/4)}*d*\ln(x^2 - \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b^2*c^3 - 2*\sqrt{2}*a*b*c^2*d + \sqrt{2}*a^2*c*d^2) + 1/8*(3*(a*b^3)^{(1/4)}*b*c - 7*(a*b^3)^{(1/4)}*a*d)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(\sqrt{2}*a^2*b^2*c^2 - 2*\sqrt{2}*a^3*b*c*d + \sqrt{2}*a^4*d^2) + 1/8*(3*(a*b^3)^{(1/4)}*b*c - 7*(a*b^3)^{(1/4)}*a*d)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(\sqrt{2}*a^2*b^2*c^2 - 2*\sqrt{2}*a^3*b*c*d + \sqrt{2}*a^4*d^2) + 1/16*(3*(a*b^3)^{(1/4)}*b*c - 7*(a*b^3)^{(1/4)}*a*d)*\ln(x^2 + \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*a^2*b^2*c^2 - 2*\sqrt{2}*a^3*b*c*d + \sqrt{2}*a^4*d^2) - 1/16*(3*(a*b^3)^{(1/4)}*b*c - 7*(a*b^3)^{(1/4)}*a*d)*\ln(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*a^2*b^2*c^2 - 2*\sqrt{2}*a^3*b*c*d + \sqrt{2}*a^4*d^2) + 1/4*b*x/((b*x^4 + a)*(a*b*c - a^2*d))$

$$3.73 \quad \int \frac{1}{(a+bx^4)^2(c+dx^4)^2} dx$$

**Optimal.** Leaf size=596

$$\begin{aligned} & \frac{b^{7/4}(3bc - 11ad) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}(bc - ad)^3} \\ & + \frac{b^{7/4}(3bc - 11ad) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}(bc - ad)^3} - \frac{b^{7/4}(3bc - 11ad) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc - ad)^3} \\ & + \frac{b^{7/4}(3bc - 11ad) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2}a^{7/4}(bc - ad)^3} - \frac{d^{7/4}(11bc - 3ad) \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{16\sqrt{2}c^{7/4}(bc - ad)^3} \\ & + \frac{d^{7/4}(11bc - 3ad) \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{16\sqrt{2}c^{7/4}(bc - ad)^3} \\ & - \frac{d^{7/4}(11bc - 3ad) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc - ad)^3} + \frac{d^{7/4}(11bc - 3ad) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}} + 1\right)}{8\sqrt{2}c^{7/4}(bc - ad)^3} \\ & + \frac{bx}{4a(a + bx^4)(c + dx^4)(bc - ad)} + \frac{dx(ad + bc)}{4ac(c + dx^4)(bc - ad)^2} \end{aligned}$$

[Out] (d\*(b\*c + a\*d)\*x)/(4\*a\*c\*(b\*c - a\*d)^2\*(c + d\*x^4)) + (b\*x)/(4\*a\*(b\*c - a\*d)\*(a + b\*x^4)\*(c + d\*x^4)) - (b^(7/4)\*(3\*b\*c - 11\*a\*d)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(8\*Sqrt[2]\*a^(7/4)\*(b\*c - a\*d)^3) + (b^(7/4)\*(3\*b\*c - 11\*a\*d)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(8\*Sqrt[2]\*a^(7/4)\*(b\*c - a\*d)^3) - (d^(7/4)\*(11\*b\*c - 3\*a\*d)\*ArcTan[1 - (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)])/(8\*Sqrt[2]\*c^(7/4)\*(b\*c - a\*d)^3) + (d^(7/4)\*(11\*b\*c - 3\*a\*d)\*ArcTan[1 + (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)])/(8\*Sqrt[2]\*c^(7/4)\*(b\*c - a\*d)^3) - (b^(7/4)\*(3\*b\*c - 11\*a\*d)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(16\*Sqrt[2]\*a^(7/4)\*(b\*c - a\*d)^3) + (b^(7/4)\*(3\*b\*c - 11\*a\*d)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(16\*Sqrt[2]\*a^(7/4)\*(b\*c - a\*d)^3) - (d^(7/4)\*(11\*b\*c - 3\*a\*d)\*Log[Sqrt[c] - Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2])/(16\*Sqrt[2]\*c^(7/4)\*(b\*c - a\*d)^3) + (d^(7/4)\*(11\*b\*c - 3\*a\*d)\*Log[Sqrt[c] + Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2])/(16\*Sqrt[2]\*c^(7/4)\*(b\*c - a\*d)^3)

**Rubi [A]** time = 1.46784, antiderivative size = 596, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$

$$\begin{aligned} & \frac{b^{7/4}(3bc - 11ad) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}(bc - ad)^3} \\ & + \frac{b^{7/4}(3bc - 11ad) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{16\sqrt{2}a^{7/4}(bc - ad)^3} - \frac{b^{7/4}(3bc - 11ad) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc - ad)^3} \\ & + \frac{b^{7/4}(3bc - 11ad) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2}a^{7/4}(bc - ad)^3} - \frac{d^{7/4}(11bc - 3ad) \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{16\sqrt{2}c^{7/4}(bc - ad)^3} \\ & + \frac{d^{7/4}(11bc - 3ad) \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{16\sqrt{2}c^{7/4}(bc - ad)^3} \\ & - \frac{d^{7/4}(11bc - 3ad) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc - ad)^3} + \frac{d^{7/4}(11bc - 3ad) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}} + 1\right)}{8\sqrt{2}c^{7/4}(bc - ad)^3} \\ & + \frac{bx}{4a(a + bx^4)(c + dx^4)(bc - ad)} + \frac{dx(ad + bc)}{4ac(c + dx^4)(bc - ad)^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^2\*(c + d\*x^4)^2),x]

[Out]  $(d*(b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^4)) + (b*x)/(4*a*(b*c - a*d)*(a + b*x^4)*(c + d*x^4)) - (b^{7/4}*(3*b*c - 11*a*d)*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{1/4}*x)/a^{1/4}])/(8*\text{Sqrt}[2]*a^{7/4}*(b*c - a*d)^3) + (b^{7/4}*(3*b*c - 11*a*d)*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{1/4}*x)/a^{1/4}])/(8*\text{Sqrt}[2]*a^{7/4}*(b*c - a*d)^3) - (d^{7/4}*(11*b*c - 3*a*d)*\text{ArcTan}[1 - (\text{Sqrt}[2]*d^{1/4}*x)/c^{1/4}])/(8*\text{Sqrt}[2]*c^{7/4}*(b*c - a*d)^3) + (d^{7/4}*(11*b*c - 3*a*d)*\text{ArcTan}[1 + (\text{Sqrt}[2]*d^{1/4}*x)/c^{1/4}])/(8*\text{Sqrt}[2]*c^{7/4}*(b*c - a*d)^3) - (b^{7/4}*(3*b*c - 11*a*d)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{7/4}*(b*c - a*d)^3) + (b^{7/4}*(3*b*c - 11*a*d)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{7/4}*(b*c - a*d)^3) - (d^{7/4}*(11*b*c - 3*a*d)*\text{Log}[\text{Sqrt}[c] - \text{Sqrt}[2]*c^{1/4}*d^{1/4}*x + \text{Sqrt}[d]*x^2])/(16*\text{Sqrt}[2]*c^{7/4}*(b*c - a*d)^3) + (d^{7/4}*(11*b*c - 3*a*d)*\text{Log}[\text{Sqrt}[c] + \text{Sqrt}[2]*c^{1/4}*d^{1/4}*x + \text{Sqrt}[d]*x^2])/(16*\text{Sqrt}[2]*c^{7/4}*(b*c - a*d)^3)$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*4+a)\*\*2/(d\*x\*\*4+c)\*\*2,x)

[Out] Timed out

**Mathematica [A]** time = 3.49316, size = 561, normalized size = 0.94

$$\frac{1}{32} \left( \frac{\sqrt{2}b^{7/4}(11ad - 3bc) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{a^{7/4}(bc - ad)^3} + \frac{\sqrt{2}b^{7/4}(11ad - 3bc) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx} + \sqrt{a} + \sqrt{bx^2}\right)}{a^{7/4}(ad - bc)^3} + \frac{2\sqrt{2}b^{7/4}(11ad - 3bc) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}}\right)}{a^{7/4}(bc - ad)^3} + \frac{2\sqrt{2}b^{7/4}(11ad - 3bc) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{bx}}{\sqrt[4]{a}} + 1\right)}{a^{7/4}(ad - bc)^3} + \frac{8b^2x}{a(a + bx^4)(bc - ad)^2} + \frac{\sqrt{2}d^{7/4}(11bc - 3ad) \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{c^{7/4}(ad - bc)^3} + \frac{\sqrt{2}d^{7/4}(11bc - 3ad) \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{dx} + \sqrt{c} + \sqrt{dx^2}\right)}{c^{7/4}(bc - ad)^3} + \frac{2\sqrt{2}d^{7/4}(3ad - 11bc) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}}\right)}{c^{7/4}(bc - ad)^3} + \frac{2\sqrt{2}d^{7/4}(11bc - 3ad) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{dx}}{\sqrt[4]{c}} + 1\right)}{c^{7/4}(bc - ad)^3} + \frac{8d^2x}{c(c + dx^4)(bc - ad)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^4)^2\*(c + d\*x^4)^2),x]

[Out]  $((8*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^4)) + (8*d^2*x)/(c*(b*c - a*d)^2*(c + d*x^4)) + (2*\text{Sqrt}[2]*b^{7/4}*(-3*b*c + 11*a*d)*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{1/4}*x)/a^{1/4}])/(a^{7/4}*(b*c - a*d)^3) + (2*\text{Sqrt}[2]*b^{7/4}*(-3*b*c + 11*a*d)*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{1/4}*x)/a^{1/4}])/(a^{7/4}*(b*c - a*d)^3) - (d^{7/4}*(11*b*c - 3*a*d)*\text{ArcTan}[1 - (\text{Sqrt}[2]*d^{1/4}*x)/c^{1/4}])/(c^{7/4}*(b*c - a*d)^3) + (d^{7/4}*(11*b*c - 3*a*d)*\text{ArcTan}[1 + (\text{Sqrt}[2]*d^{1/4}*x)/c^{1/4}])/(c^{7/4}*(b*c - a*d)^3) - (b^{7/4}*(3*b*c - 11*a*d)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{7/4}*(b*c - a*d)^3) + (b^{7/4}*(3*b*c - 11*a*d)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{7/4}*(b*c - a*d)^3) - (d^{7/4}*(11*b*c - 3*a*d)*\text{Log}[\text{Sqrt}[c] - \text{Sqrt}[2]*c^{1/4}*d^{1/4}*x + \text{Sqrt}[d]*x^2])/(16*\text{Sqrt}[2]*c^{7/4}*(b*c - a*d)^3) + (d^{7/4}*(11*b*c - 3*a*d)*\text{Log}[\text{Sqrt}[c] + \text{Sqrt}[2]*c^{1/4}*d^{1/4}*x + \text{Sqrt}[d]*x^2])/(16*\text{Sqrt}[2]*c^{7/4}*(b*c - a*d)^3)$



$$\begin{aligned} & 1/4) ])/(a^{(7/4)} * (-b*c) + a*d)^3) + (2*sqrt[2]*d^{(7/4)} * (-11*b*c + \\ & 3*a*d)*ArcTan[1 - (sqrt[2]*d^{(1/4)}*x)/c^{(1/4)}])/(c^{(7/4)} * (b*c - \\ & a*d)^3) + (2*sqrt[2]*d^{(7/4)} * (11*b*c - 3*a*d)*ArcTan[1 + (sqrt[2] \\ & *d^{(1/4)}*x)/c^{(1/4)}])/(c^{(7/4)} * (b*c - a*d)^3) + (sqrt[2]*b^{(7/4)} * \\ & (-3*b*c + 11*a*d)*Log[sqrt[a] - sqrt[2]*a^{(1/4)}*b^{(1/4)}*x + sqrt[ \\ & b]*x^2])/(a^{(7/4)} * (b*c - a*d)^3) + (sqrt[2]*b^{(7/4)} * (-3*b*c + 11* \\ & a*d)*Log[sqrt[a] + sqrt[2]*a^{(1/4)}*b^{(1/4)}*x + sqrt[b]*x^2])/(a^{( \\ & 7/4)} * (-b*c) + a*d)^3) + (sqrt[2]*d^{(7/4)} * (11*b*c - 3*a*d)*Log[Sq \\ & rt[c] - sqrt[2]*c^{(1/4)}*d^{(1/4)}*x + sqrt[d]*x^2])/(c^{(7/4)} * (-b*c \\ & ) + a*d)^3) + (sqrt[2]*d^{(7/4)} * (11*b*c - 3*a*d)*Log[sqrt[c] + Sqr \\ & t[2]*c^{(1/4)}*d^{(1/4)}*x + sqrt[d]*x^2])/(c^{(7/4)} * (b*c - a*d)^3))/3 \\ & 2 \end{aligned}$$

**Maple [A]** time = 0.003, size = 784, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^2/(d\*x^4+c)^2, x)

[Out]  $\frac{1}{4}d^3/(a^3d-b^3c)^3/c^3x/(d^3x^4+c)^3 - \frac{1}{4}d^2/(a^3d-b^3c)^3x/(d^3x^4+c)^3 + \frac{3}{16}d^3/(a^3d-b^3c)^3/c^2 * (c/d)^{1/4} * 2^{1/2} * \arctan(2^{1/2}/(c/d)^{1/4} * x + 1) * a - \frac{11}{16}d^2/(a^3d-b^3c)^3/c * (c/d)^{1/4} * 2^{1/2} * \arctan(2^{1/2}/(c/d)^{1/4} * x + 1) * b + \frac{3}{16}d^3/(a^3d-b^3c)^3/c^2 * (c/d)^{1/4} * 2^{1/2} * \arctan(2^{1/2}/(c/d)^{1/4} * x - 1) * a - \frac{11}{16}d^2/(a^3d-b^3c)^3/c * (c/d)^{1/4} * 2^{1/2} * \arctan(2^{1/2}/(c/d)^{1/4} * x - 1) * b + \frac{3}{32}d^3/(a^3d-b^3c)^3/c^2 * (c/d)^{1/4} * 2^{1/2} * \ln((x^2+(c/d)^{1/4} * x)^{2^{1/2}} + (c/d)^{1/4}) / (x^2 - (c/d)^{1/4} * x)^{2^{1/2}} + (c/d)^{1/4}) * a - \frac{11}{32}d^2/(a^3d-b^3c)^3/c * (c/d)^{1/4} * 2^{1/2} * \ln((x^2+(c/d)^{1/4} * x)^{2^{1/2}} + (c/d)^{1/4}) / (x^2 - (c/d)^{1/4} * x)^{2^{1/2}} + (c/d)^{1/4}) * b + \frac{1}{4} * b^2/(a^3d-b^3c)^3 * x / (b^3x^4+a)^3 * d - \frac{1}{4} * b^3/(a^3d-b^3c)^3 * x / a / (b^3x^4+a)^3 * c + \frac{11}{16} * b^2/(a^3d-b^3c)^3 * a^{1/4} * 2^{1/2} * \arctan(2^{1/2}/(a/b)^{1/4} * x - 1) * d - \frac{3}{16} * b^3/(a^3d-b^3c)^3 * a^{1/4} * 2^{1/2} * \arctan(2^{1/2}/(a/b)^{1/4} * x - 1) * c + \frac{11}{32} * b^2/(a^3d-b^3c)^3 * a^{1/4} * 2^{1/2} * \ln((x^2+(a/b)^{1/4} * x)^{2^{1/2}} + (a/b)^{1/4}) / (x^2 - (a/b)^{1/4} * x)^{2^{1/2}} + (a/b)^{1/4}) * d - \frac{3}{32} * b^3/(a^3d-b^3c)^3 * a^{1/4} * 2^{1/2} * \ln((x^2+(a/b)^{1/4} * x)^{2^{1/2}} + (a/b)^{1/4}) / (x^2 - (a/b)^{1/4} * x)^{2^{1/2}} + (a/b)^{1/4}) * c + \frac{11}{16} * b^2/(a^3d-b^3c)^3 * a^{1/4} * 2^{1/2} * \arctan(2^{1/2}/(a/b)^{1/4} * x + 1) * d - \frac{3}{16} * b^3/(a^3d-b^3c)^3 * a^{1/4} * 2^{1/2} * \arctan(2^{1/2}/(a/b)^{1/4} * x + 1) * c$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)^2\*(d\*x^4 + c)^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)^2\*(d\*x^4 + c)^2), x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)**2/(d*x**4+c)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^2(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^2*(d*x^4 + c)^2),x, algorithm="giac")`

[Out] `integrate(1/((b*x^4 + a)^2*(d*x^4 + c)^2), x)`

$$3.74 \quad \int \frac{(a-bx^4)^{5/2}}{c-dx^4} dx$$

**Optimal.** Leaf size=321

$$\frac{\sqrt[4]{ab^{3/4}} \sqrt{1 - \frac{bx^4}{a}} (47a^2d^2 - 56abcd + 21b^2c^2) F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{21d^3 \sqrt{a - bx^4}} - \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad)^3 \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd^3} \sqrt{a - bx^4}} - \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad)^3 \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd^3} \sqrt{a - bx^4}} - \frac{bx\sqrt{a - bx^4}(7bc - 13ad)}{21d^2} + \frac{bx(a - bx^4)^{3/2}}{7d}$$

[Out]  $-(b*(7*b*c - 13*a*d)*x*\text{Sqrt}[a - b*x^4])/(21*d^2) + (b*x*(a - b*x^4)^{(3/2)})/(7*d) + (a^{(1/4)}*b^{(3/4)}*(21*b^2*c^2 - 56*a*b*c*d + 47*a^2*d^2)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticF}[\text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(21*d^3*\text{Sqrt}[a - b*x^4]) - (a^{(1/4)}*(b*c - a*d)^3*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[-((\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c])), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(2*b^{(1/4)}*c*d^3*\text{Sqrt}[a - b*x^4]) - (a^{(1/4)}*(b*c - a*d)^3*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(2*b^{(1/4)}*c*d^3*\text{Sqrt}[a - b*x^4])$

**Rubi [A]** time = 0.914184, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$

$$\frac{\sqrt[4]{ab^{3/4}} \sqrt{1 - \frac{bx^4}{a}} (47a^2d^2 - 56abcd + 21b^2c^2) F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{21d^3 \sqrt{a - bx^4}} - \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad)^3 \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd^3} \sqrt{a - bx^4}} - \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad)^3 \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd^3} \sqrt{a - bx^4}} - \frac{bx\sqrt{a - bx^4}(7bc - 13ad)}{21d^2} + \frac{bx(a - bx^4)^{3/2}}{7d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a - b*x^4)^{(5/2)}/(c - d*x^4), x]$

[Out]  $-(b*(7*b*c - 13*a*d)*x*\text{Sqrt}[a - b*x^4])/(21*d^2) + (b*x*(a - b*x^4)^{(3/2)})/(7*d) + (a^{(1/4)}*b^{(3/4)}*(21*b^2*c^2 - 56*a*b*c*d + 47*a^2*d^2)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticF}[\text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(21*d^3*\text{Sqrt}[a - b*x^4]) - (a^{(1/4)}*(b*c - a*d)^3*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[-((\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c])), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(2*b^{(1/4)}*c*d^3*\text{Sqrt}[a - b*x^4]) - (a^{(1/4)}*(b*c - a*d)^3*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(2*b^{(1/4)}*c*d^3*\text{Sqrt}[a - b*x^4])$

**Rubi in Sympy [A]** time = 151.844, size = 292, normalized size = 0.91

$$\frac{\sqrt[4]{ab^3} \sqrt{1 - \frac{bx^4}{a}} (47a^2d^2 - 56abcd + 21b^2c^2) F\left(\operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{21d^3 \sqrt{a - bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (ad - bc)^3 \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd^3} \sqrt{a - bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (ad - bc)^3 \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd^3} \sqrt{a - bx^4}} + \frac{bx(a - bx^4)^{\frac{3}{2}}}{7d} + \frac{bx\sqrt{a - bx^4}(13ad - 7bc)}{21d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((-b*x**4+a)**(5/2)/(-d*x**4+c), x)`

[Out] `a**(1/4)*b**(3/4)*sqrt(1 - b*x**4/a)*(47*a**2*d**2 - 56*a*b*c*d + 21*b**2*c**2)*elliptic_f(asin(b**(1/4)*x/a**(1/4)), -1)/(21*d**3*sqrt(a - b*x**4)) + a**(1/4)*sqrt(1 - b*x**4/a)*(a*d - b*c)**3*elliptic_pi(-sqrt(a)*sqrt(d)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(2*b**(1/4)*c*d**3*sqrt(a - b*x**4)) + a**(1/4)*sqrt(1 - b*x**4/a)*(a*d - b*c)**3*elliptic_pi(sqrt(a)*sqrt(d)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(2*b**(1/4)*c*d**3*sqrt(a - b*x**4)) + b*x*(a - b*x**4)**(3/2)/(7*d) + b*x*sqrt(a - b*x**4)*(13*a*d - 7*b*c)/(21*d**2)`

**Mathematica [C]** time = 1.72882, size = 385, normalized size = 1.2

$$x \left( \frac{9abcx^4(47a^2d^2 - 56abcd + 21b^2c^2) F_1\left(\frac{5}{4}, \frac{1}{2}, 1, \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{(c-dx^4)\left(2x^4\left(2adF_1\left(\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right) + 9acF_1\left(\frac{5}{4}, \frac{1}{2}, 1, \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right)} + \frac{25a^2c(21a^2d^2 - 16abcd + 7b^2c^2)}{(c-dx^4)\left(2x^4\left(2adF_1\left(\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right) + 9acF_1\left(\frac{5}{4}, \frac{1}{2}, 1, \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right)} \right) / (105d^2 \sqrt{a - bx^4})$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(a - b*x^4)^(5/2)/(c - d*x^4), x]`

[Out] `(x*(5*b*(-a + b*x^4)*(7*b*c - 16*a*d + 3*b*d*x^4) + (25*a^2*c*(7*b^2*c^2 - 16*a*b*c*d + 21*a^2*d^2)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/(c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])) - (9*a*b*c*(21*b^2*c^2 - 56*a*b*c*d + 47*a^2*d^2)*x^4*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/(c - d*x^4)*(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c])))/(105*d^2*sqrt[a - b*x^4])`

**Maple [C]** time = 0.069, size = 408, normalized size = 1.3

$$-\frac{b^2x^5}{7d}\sqrt{-bx^4+a} + \frac{x}{3b}\left(\frac{b^2(3ad-bc)}{d^2} - \frac{5ab^2}{7d}\right)\sqrt{-bx^4+a} - 1\left(-\frac{b(3a^2d^2-3cabd+b^2c^2)}{d^3} + \frac{a}{3b}\left(\frac{b^2(3ad-bc)}{d^2} - \frac{5ab^2}{7d}\right)\right)\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}\operatorname{EllipticF}\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}, i\right) + \frac{1}{8d^4}\sum_{\alpha=\operatorname{RootOf}(dZ^4-c)}\frac{-a^3d^3+3a^2cd^2b-3ac^2db^2+c^3b^3}{-\alpha^3}\left(-1\operatorname{Artanh}\left(\frac{-2\alpha^2bx^2+2a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{ad-bc}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^4+a)^(5/2)/(-d*x^4+c), x)`

[Out] 
$$-1/7*b^2/d*x^5*(-b*x^4+a)^{1/2}+1/3*(b^2/d^2*(3*a*d-b*c)-5/7*b^2/d^2*a)/b*x*(-b*x^4+a)^{1/2}-(-b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)/d^3+1/3*(b^2/d^2*(3*a*d-b*c)-5/7*b^2/d^2*a)/b*a)/(1/a^{1/2}*b^{1/2})^{1/2}*(1-b^{1/2}*x^2/a^{1/2})^{1/2}*(1+b^{1/2}*x^2/a^{1/2})^{1/2}/(-b*x^4+a)^{1/2}*EllipticF(x*(1/a^{1/2}*b^{1/2})^{1/2}, I)+1/8/d^4*sum((-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/_alpha^3*(-1/((a*d-b*c)/d)^{1/2}*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^{1/2})/(-b*x^4+a)^{1/2})-2/(1/a^{1/2}*b^{1/2})^{1/2}*_alpha^3*d/c*(1-b^{1/2}*x^2/a^{1/2})^{1/2}*(1+b^{1/2}*x^2/a^{1/2})^{1/2}/(-b*x^4+a)^{1/2}*EllipticPi(x*(1/a^{1/2}*b^{1/2})^{1/2}, a^{1/2}/b^{1/2}*_alpha^2/c*d, (-1/a^{1/2}*b^{1/2})^{1/2}/(1/a^{1/2}*b^{1/2})^{1/2}), _alpha=RootOf(_Z^4*d-c))$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{(-bx^4 + a)^{\frac{5}{2}}}{dx^4 - c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(-b*x^4 + a)^(5/2)/(d*x^4 - c), x, algorithm="maxima")`

[Out] `-integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(-b*x^4 + a)^(5/2)/(d*x^4 - c), x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^2\sqrt{a-bx^4}}{-c+dx^4} dx - \int \frac{b^2x^8\sqrt{a-bx^4}}{-c+dx^4} dx - \int \left( \frac{2abx^4\sqrt{a-bx^4}}{-c+dx^4} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**4+a)**(5/2)/(-d*x**4+c), x)`

[Out] `-Integral(a**2*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(b**2*x**8*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(-2*a*b*x**4*sqrt(a - b*x**4)/(-c + d*x**4), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{(-bx^4 + a)^{\frac{5}{2}}}{dx^4 - c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-b*x^4 + a)^(5/2)/(d*x^4 - c),x, algorithm="giac")
```

```
[Out] integrate(-(-b*x^4 + a)^(5/2)/(d*x^4 - c), x)
```

$$3.75 \quad \int \frac{(a-bx^4)^{3/2}}{c-dx^4} dx$$

**Optimal.** Leaf size=277

$$\frac{\sqrt[4]{ab^{3/4}} \sqrt{1 - \frac{bx^4}{a}} (3bc - 5ad) F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{3d^2 \sqrt{a - bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad)^2 \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd^2} \sqrt{a - bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad)^2 \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd^2} \sqrt{a - bx^4}} + \frac{bx\sqrt{a - bx^4}}{3d}$$

[Out] (b\*x\*Sqrt[a - b\*x^4])/(3\*d) - (a^(1/4)\*b^(3/4)\*(3\*b\*c - 5\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticF[ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(3\*d^2\*Sqrt[a - b\*x^4]) + (a^(1/4)\*(b\*c - a\*d)^2\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*b^(1/4)\*c\*d^2\*Sqrt[a - b\*x^4]) + (a^(1/4)\*(b\*c - a\*d)^2\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*b^(1/4)\*c\*d^2\*Sqrt[a - b\*x^4])

**Rubi [A]** time = 0.679383, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$

$$\frac{\sqrt[4]{ab^{3/4}} \sqrt{1 - \frac{bx^4}{a}} (3bc - 5ad) F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{3d^2 \sqrt{a - bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad)^2 \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd^2} \sqrt{a - bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad)^2 \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd^2} \sqrt{a - bx^4}} + \frac{bx\sqrt{a - bx^4}}{3d}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^4)^(3/2)/(c - d\*x^4), x]

[Out] (b\*x\*Sqrt[a - b\*x^4])/(3\*d) - (a^(1/4)\*b^(3/4)\*(3\*b\*c - 5\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticF[ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(3\*d^2\*Sqrt[a - b\*x^4]) + (a^(1/4)\*(b\*c - a\*d)^2\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*b^(1/4)\*c\*d^2\*Sqrt[a - b\*x^4]) + (a^(1/4)\*(b\*c - a\*d)^2\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*b^(1/4)\*c\*d^2\*Sqrt[a - b\*x^4])

**Rubi in Sympy [A]** time = 110.42, size = 248, normalized size = 0.9

$$\frac{\sqrt[4]{ab^{3/4}} \sqrt{1 - \frac{bx^4}{a}} (5ad - 3bc) F\left(\operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{3d^2 \sqrt{a - bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (ad - bc)^2 \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd^2} \sqrt{a - bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (ad - bc)^2 \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd^2} \sqrt{a - bx^4}} + \frac{bx\sqrt{a - bx^4}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((-b*x**4+a)**(3/2)/(-d*x**4+c),x)`

[Out]  $a^{1/4} b^{3/4} \sqrt{1 - b x^4/a} (5 a^d - 3 b^c) \operatorname{elliptic}_f(a \sin(b^{1/4} x/a^{1/4}), -1)/(3 d^2 \sqrt{a - b x^4}) + a^{1/4} \sqrt{1 - b x^4/a} (a^d - b^c)^2 \operatorname{elliptic}_\pi(-\sqrt{a} \sqrt{d}/(\sqrt{b} \sqrt{c}), \operatorname{asin}(b^{1/4} x/a^{1/4}), -1)/(2 b^{1/4} c^d \sqrt{a - b x^4}) + a^{1/4} \sqrt{1 - b x^4/a} (a^d - b^c)^2 \operatorname{elliptic}_\pi(\sqrt{a} \sqrt{d}/(\sqrt{b} \sqrt{c}), \operatorname{asin}(b^{1/4} x/a^{1/4}), -1)/(2 b^{1/4} c^d \sqrt{a - b x^4}) + b x \sqrt{a - b x^4}/(3 d)$

**Mathematica [C]** time = 0.743145, size = 419, normalized size = 1.51

$$x \left( \frac{b(-10x^4(a-bx^4)(c-dx^4)(2adF_1\left(\frac{9}{4}; \frac{1}{2}, 2; \frac{13}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{9}{4}; \frac{3}{2}, 1; \frac{13}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)) - 9ac(5a(c-2dx^4) - 2bcx^4 + 5bdx^8)F_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{2x^4(2adF_1\left(\frac{9}{4}; \frac{1}{2}, 2; \frac{13}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{9}{4}; \frac{3}{2}, 1; \frac{13}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)) + 9acF_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)} - \frac{2x^4(2adF_1\left(\frac{9}{4}; \frac{1}{2}, 2; \frac{13}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{9}{4}; \frac{3}{2}, 1; \frac{13}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right))}{15d\sqrt{a - bx^4}(dx^4 - c)} \right)$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(a - b*x^4)^(3/2)/(c - d*x^4),x]`

[Out]  $(x^* ((-25*a^2*c*(-(b*c) + 3*a*d)*\operatorname{AppellF1}[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/(5*a*c*\operatorname{AppellF1}[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*\operatorname{AppellF1}[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*\operatorname{AppellF1}[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])) + (b*(-9*a*c*(-2*b*c*x^4 + 5*b*d*x^8 + 5*a*(c - 2*d*x^4))*\operatorname{AppellF1}[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] - 10*x^4*(a - b*x^4)*(c - d*x^4)*(2*a*d*\operatorname{AppellF1}[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*\operatorname{AppellF1}[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c])))/(9*a*c*\operatorname{AppellF1}[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*\operatorname{AppellF1}[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*\operatorname{AppellF1}[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c])))/(15*d*\operatorname{Sqrt}[a - b*x^4])^*(-c + d*x^4)$

**Maple [C]** time = 0.027, size = 311, normalized size = 1.1

$$\frac{bx}{3d} \sqrt{-bx^4 + a} - 1 \left( -\frac{b(2ad - bc)}{d^2} + \frac{ab}{3d} \right) \sqrt{1 - x^2 \sqrt{b} \frac{1}{\sqrt{a}}} \sqrt{1 + x^2 \sqrt{b} \frac{1}{\sqrt{a}}} \operatorname{EllipticF} \left( x \sqrt{1 \sqrt{b} \frac{1}{\sqrt{a}}}, i \right) \frac{1}{\sqrt{1 \sqrt{b} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{-bx^4 + a}} + \frac{1}{8d^3} \sum_{\alpha = \operatorname{RootOf}(\_Z^4 d - c)} \frac{-a^2 d^2 + 2cabd - b^2 c^2}{\alpha^3} \left( -1 \operatorname{Artanh} \left( \frac{-2 \alpha^2 b x^2 + 2a}{2} \frac{1}{\sqrt{\frac{ad-bc}{d}}} \frac{1}{\sqrt{-bx^4 + a}} \right) \frac{1}{\sqrt{\frac{ad-bc}{d}}} - 2 \frac{\alpha d}{c \sqrt{-bx^4 + a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^4+a)^(3/2)/(-d*x^4+c),x)`

[Out]  $1/3*b*x*(-b*x^4+a)^{(1/2)}/d - (-b*(2*a*d-b*c)/d^2 + 1/3*b/d*a)/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-b^{(1/2)}*x^2/a^{(1/2)})^{(1/2)}*(1+b^{(1/2)}*x^2/a^{(1/2)})^{(1/2)}/(-b*x^4+a)^{(1/2)}*\operatorname{EllipticF}(x*(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}, I) + 1/8/d^3*\operatorname{sum}((-a^2*d^2+2*a*b*c*d-b^2*c^2)/\alpha^3*(-1/((a*d-b*c)/d)^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^{(1/2)})/(-b*x^4+a)^{(1/2)})-2/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*\alpha^3*d/c*(1-b^{(1/2)}*x^2/a^{(1/2)})^{(1/2)}*(1+b^{(1/2)}*x^2/a^{(1/2)})^{(1/2)}/(-b*x^4+a)^{(1/2)}*\operatorname{EllipticPi}(x*(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}, a^{(1/2)}/b^{(1/2)})^{(1/2)}*\alpha^2/c*d, (-1/a^{(1/2)}*b^{(1/2)})^{(1/2)}/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)})$



/2)), \_alpha=RootOf(\_Z^4\*d-c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{(-bx^4 + a)^{\frac{3}{2}}}{dx^4 - c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(-b\*x^4 + a)^(3/2)/(d\*x^4 - c),x, algorithm="maxima")

[Out] -integrate((-b\*x^4 + a)^(3/2)/(d\*x^4 - c), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(-b\*x^4 + a)^(3/2)/(d\*x^4 - c),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{a\sqrt{a - bx^4}}{-c + dx^4} dx - \int \left( -\frac{bx^4\sqrt{a - bx^4}}{-c + dx^4} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*4+a)\*\*(3/2)/(-d\*x\*\*4+c),x)

[Out] -Integral(a\*sqrt(a - b\*x\*\*4)/(-c + d\*x\*\*4), x) - Integral(-b\*x\*\*4\*sqrt(a - b\*x\*\*4)/(-c + d\*x\*\*4), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{(-bx^4 + a)^{\frac{3}{2}}}{dx^4 - c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(-b\*x^4 + a)^(3/2)/(d\*x^4 - c),x, algorithm="giac")

[Out] integrate(-(-b\*x^4 + a)^(3/2)/(d\*x^4 - c), x)

### 3.76 $\int \frac{\sqrt{a-bx^4}}{c-dx^4} dx$

**Optimal.** Leaf size=240

$$\frac{\sqrt[4]{ab^3} \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad) \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd}\sqrt{a-bx^4}}$$

$$- \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad) \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd}\sqrt{a-bx^4}}$$

[Out] (a^(1/4)\*b^(3/4)\*Sqrt[1 - (b\*x^4)/a]\*EllipticF[ArcSin[(b^(1/4)\*x/a^(1/4)], -1)]/(d\*Sqrt[a - b\*x^4]) - (a^(1/4)\*(b\*c - a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x/a^(1/4)], -1)]/(2\*b^(1/4)\*c\*d\*Sqrt[a - b\*x^4]) - (a^(1/4)\*(b\*c - a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x/a^(1/4)], -1)]/(2\*b^(1/4)\*c\*d\*Sqrt[a - b\*x^4])

**Rubi [A]** time = 0.448504, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$\frac{\sqrt[4]{ab^3} \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad) \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd}\sqrt{a-bx^4}}$$

$$- \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad) \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd}\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - b\*x^4]/(c - d\*x^4), x]

[Out] (a^(1/4)\*b^(3/4)\*Sqrt[1 - (b\*x^4)/a]\*EllipticF[ArcSin[(b^(1/4)\*x/a^(1/4)], -1)]/(d\*Sqrt[a - b\*x^4]) - (a^(1/4)\*(b\*c - a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x/a^(1/4)], -1)]/(2\*b^(1/4)\*c\*d\*Sqrt[a - b\*x^4]) - (a^(1/4)\*(b\*c - a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x/a^(1/4)], -1)]/(2\*b^(1/4)\*c\*d\*Sqrt[a - b\*x^4])

**Rubi in Sympy [A]** time = 73.1836, size = 211, normalized size = 0.88

$$\frac{\sqrt[4]{ab^3} \sqrt{1 - \frac{bx^4}{a}} F\left(\operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{d\sqrt{a-bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (ad - bc) \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd}\sqrt{a-bx^4}}$$

$$+ \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (ad - bc) \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd}\sqrt{a-bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*4+a)\*\*(1/2)/(-d\*x\*\*4+c), x)

[Out] a\*\*(1/4)\*b\*\*(3/4)\*sqrt(1 - b\*x\*\*4/a)\*elliptic\_f(asin(b\*\*(1/4)\*x/a\*\*(1/4)), -1)/(d\*sqrt(a - b\*x\*\*4)) + a\*\*(1/4)\*sqrt(1 - b\*x\*\*4/a)\*(a\*d - b\*c)\*elliptic\_pi(-sqrt(a)\*sqrt(d)/(sqrt(b)\*sqrt(c)), asin(b\*\*(1/4)\*x/a\*\*(1/4)), -1)/(2\*b\*\*(1/4)\*c\*d\*sqrt(a - b\*x\*\*4)) + a\*\*

$$(1/4) \cdot \sqrt{1 - b \cdot x^{**4}/a} \cdot (a \cdot d - b \cdot c) \cdot \text{elliptic\_pi}(\sqrt{a} \cdot \sqrt{d} / (\sqrt{b} \cdot \sqrt{c}), \text{asin}(b^{**}(1/4) \cdot x/a^{**}(1/4)), -1) / (2 \cdot b^{**}(1/4) \cdot c \cdot d \cdot \sqrt{a - b \cdot x^{**4}})$$

**Mathematica [C]** time = 0.276908, size = 155, normalized size = 0.65

$$\frac{5acx\sqrt{a-bx^4}F_1\left(\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{(c-dx^4)\left(2x^4\left(bcF_1\left(\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right) - 2adF_1\left(\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right) - 5acF_1\left(\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a - b\*x^4]/(c - d\*x^4), x]

[Out] (-5\*a\*c\*x\*Sqrt[a - b\*x^4]\*AppellF1[1/4, -1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c])/((c - d\*x^4)\*(-5\*a\*c\*AppellF1[1/4, -1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c] + 2\*x^4\*(-2\*a\*d\*AppellF1[5/4, -1/2, 2, 9/4, (b\*x^4)/a, (d\*x^4)/c] + b\*c\*AppellF1[5/4, 1/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c]))

**Maple [C]** time = 0.025, size = 259, normalized size = 1.1

$$\frac{b}{d} \sqrt{1 - x^2 \sqrt{b} \frac{1}{\sqrt{a}}} \sqrt{1 + x^2 \sqrt{b} \frac{1}{\sqrt{a}}} \text{EllipticF}\left(x \sqrt{1 \sqrt{b} \frac{1}{\sqrt{a}}}, i\right) \frac{1}{\sqrt{1 \sqrt{b} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{-bx^4 + a}}$$

$$+ \frac{1}{8d^2} \sum_{\alpha = \text{RootOf}(\_Z^4 d - c)} \frac{-ad + bc}{-\alpha^3} \left( -1 \text{Artanh}\left(\frac{-2 \alpha^2 bx^2 + 2a}{2} \frac{1}{\sqrt{\frac{ad-bc}{d}}} \frac{1}{\sqrt{-bx^4 + a}}\right) \frac{1}{\sqrt{\frac{ad-bc}{d}}} - 2 \frac{\alpha^3 d}{c \sqrt{-bx^4 + a}} \sqrt{1 - \frac{bx^4}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^4+a)^(1/2)/(-d\*x^4+c), x)

[Out] b/d/(1/a^(1/2)\*b^(1/2))^(1/2)\*(1-b^(1/2)\*x^2/a^(1/2))^(1/2)\*(1+b^(1/2)\*x^2/a^(1/2))^(1/2)/(-b\*x^4+a)^(1/2)\*EllipticF(x\*(1/a^(1/2)\*b^(1/2))^(1/2), I)+1/8/d^2\*sum((-a\*d+b\*c)/\_alpha^3\*(-1/((a\*d-b\*c)/d)^(1/2)\*arctanh(1/2\*(-2\*\_alpha^2\*b\*x^2+2\*a)/((a\*d-b\*c)/d)^(1/2)/(-b\*x^4+a)^(1/2))-2/(1/a^(1/2)\*b^(1/2))^(1/2)\*\_alpha^3\*d/c\*(1-b^(1/2)\*x^2/a^(1/2))^(1/2)\*(1+b^(1/2)\*x^2/a^(1/2))^(1/2)/(-b\*x^4+a)^(1/2)\*EllipticPi(x\*(1/a^(1/2)\*b^(1/2))^(1/2), a^(1/2)/b^(1/2)\*\_alpha^2/c\*d, (-1/a^(1/2)\*b^(1/2))^(1/2)/(1/a^(1/2)\*b^(1/2))^(1/2)), \_alpha=RootOf(\_Z^4\*d-c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{\sqrt{-bx^4 + a}}{dx^4 - c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sqrt(-b\*x^4 + a)/(d\*x^4 - c), x, algorithm="maxima")

[Out] -integrate(sqrt(-b\*x^4 + a)/(d\*x^4 - c), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-sqrt(-b*x^4 + a)/(d*x^4 - c),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{a - bx^4}}{-c + dx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**4+a)**(1/2)/(-d*x**4+c),x)`

[Out] `-Integral(sqrt(a - b*x**4)/(-c + d*x**4), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-bx^4 + a}}{dx^4 - c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-sqrt(-b*x^4 + a)/(d*x^4 - c),x, algorithm="giac")`

[Out] `integrate(-sqrt(-b*x^4 + a)/(d*x^4 - c), x)`

$$3.77 \quad \int \frac{1}{\sqrt{a-bx^4(c-dx^4)}} dx$$

**Optimal.** Leaf size=162

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle| -1\right)}{2\sqrt[4]{bc}\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle| -1\right)}{2\sqrt[4]{bc}\sqrt{a-bx^4}}$$

[Out] (a^(1/4)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*b^(1/4)\*c\*Sqrt[a - b\*x^4]) + (a^(1/4)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*b^(1/4)\*c\*Sqrt[a - b\*x^4])

**Rubi [A]** time = 0.326382, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle| -1\right)}{2\sqrt[4]{bc}\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle| -1\right)}{2\sqrt[4]{bc}\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a - b\*x^4]\*(c - d\*x^4)),x]

[Out] (a^(1/4)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*b^(1/4)\*c\*Sqrt[a - b\*x^4]) + (a^(1/4)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*b^(1/4)\*c\*Sqrt[a - b\*x^4])

**Rubi in Sympy [A]** time = 52.4592, size = 143, normalized size = 0.88

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle| -1\right)}{2\sqrt[4]{bc}\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle| -1\right)}{2\sqrt[4]{bc}\sqrt{a-bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*4+a)\*\*(1/2)/(-d\*x\*\*4+c),x)

[Out] a\*\*(1/4)\*sqrt(1 - b\*x\*\*4/a)\*elliptic\_pi(-sqrt(a)\*sqrt(d)/(sqrt(b)\*sqrt(c)), asin(b\*\*(1/4)\*x/a\*\*(1/4)), -1)/(2\*b\*\*(1/4)\*c\*sqrt(a - b\*x\*\*4)) + a\*\*(1/4)\*sqrt(1 - b\*x\*\*4/a)\*elliptic\_pi(sqrt(a)\*sqrt(d)/(sqrt(b)\*sqrt(c)), asin(b\*\*(1/4)\*x/a\*\*(1/4)), -1)/(2\*b\*\*(1/4)\*c\*sqrt(a - b\*x\*\*4))

**Mathematica [C]** time = 0.234785, size = 156, normalized size = 0.96

$$\frac{5acx F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{\sqrt{a-bx^4}(dx^4-c)\left(2x^4\left(2adF_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right) + 5acF_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[a - b\*x^4]\*(c - d\*x^4)),x]

[Out]  $(-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/(Sqrt[a - b*x^4]*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))$

**Maple [C]** time = 0.023, size = 183, normalized size = 1.1

$$-\frac{1}{8d} \sum_{\alpha = \text{RootOf}(\_Z^4 d - c)} \frac{1}{\alpha^3} \left( -1 \operatorname{Arctanh} \left( \frac{-2 \alpha^2 b x^2 + 2 a}{2} \frac{1}{\sqrt{\frac{ad-bc}{d}}} \frac{1}{\sqrt{-bx^4 + a}} \right) \frac{1}{\sqrt{\frac{ad-bc}{d}}} - 2 \frac{d \alpha^3}{c \sqrt{-bx^4 + a}} \sqrt{1 - \frac{\alpha^2}{d}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c), x)`

[Out]  $-1/8/d*\text{sum}(1/\alpha^3*(-1/((a*d-b*c)/d)^(1/2)*\text{arctanh}(1/2*(-2*\alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*\alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*\text{EllipticPi}(x*(1/a^(1/2)*b^(1/2))^(1/2), a^(1/2)/b^(1/2)*\alpha^2/c*d, (-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))), \alpha = \text{RootOf}(\_Z^4*d-c))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{-bx^4 + a}(dx^4 - c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x, algorithm="maxima")`

[Out] `-integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-c\sqrt{a - bx^4} + dx^4\sqrt{a - bx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c), x)`

[Out] -Integral(1/(-c\*sqrt(a - b\*x\*\*4) + d\*x\*\*4\*sqrt(a - b\*x\*\*4)), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sqrt{-bx^4 + a}(dx^4 - c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/(sqrt(-b\*x^4 + a)\*(d\*x^4 - c)),x, algorithm="giac")

[Out] integrate(-1/(sqrt(-b\*x^4 + a)\*(d\*x^4 - c)), x)

$$3.78 \quad \int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)} dx$$

**Optimal.** Leaf size=281

$$\frac{b^{3/4} \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2a^{3/4} \sqrt{a - bx^4}(bc - ad)} + \frac{bx}{2a \sqrt{a - bx^4}(bc - ad)}$$

$$- \frac{\sqrt[4]{ad} \sqrt{1 - \frac{bx^4}{a}} \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bc} \sqrt{a - bx^4}(bc - ad)} - \frac{\sqrt[4]{ad} \sqrt{1 - \frac{bx^4}{a}} \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bc} \sqrt{a - bx^4}(bc - ad)}$$

[Out] (b\*x)/(2\*a\*(b\*c - a\*d)\*Sqrt[a - b\*x^4]) + (b^(3/4)\*Sqrt[1 - (b\*x^4)/a]\*EllipticF[ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*a^(3/4)\*(b\*c - a\*d)\*Sqrt[a - b\*x^4]) - (a^(1/4)\*d\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*b^(1/4)\*c\*(b\*c - a\*d)\*Sqrt[a - b\*x^4]) - (a^(1/4)\*d\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*b^(1/4)\*c\*(b\*c - a\*d)\*Sqrt[a - b\*x^4])

**Rubi [A]** time = 0.609153, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$

$$\frac{b^{3/4} \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2a^{3/4} \sqrt{a - bx^4}(bc - ad)} + \frac{bx}{2a \sqrt{a - bx^4}(bc - ad)}$$

$$- \frac{\sqrt[4]{ad} \sqrt{1 - \frac{bx^4}{a}} \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bc} \sqrt{a - bx^4}(bc - ad)} - \frac{\sqrt[4]{ad} \sqrt{1 - \frac{bx^4}{a}} \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bc} \sqrt{a - bx^4}(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a - b\*x^4)^(3/2)\*(c - d\*x^4)), x]

[Out] (b\*x)/(2\*a\*(b\*c - a\*d)\*Sqrt[a - b\*x^4]) + (b^(3/4)\*Sqrt[1 - (b\*x^4)/a]\*EllipticF[ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*a^(3/4)\*(b\*c - a\*d)\*Sqrt[a - b\*x^4]) - (a^(1/4)\*d\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*b^(1/4)\*c\*(b\*c - a\*d)\*Sqrt[a - b\*x^4]) - (a^(1/4)\*d\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*b^(1/4)\*c\*(b\*c - a\*d)\*Sqrt[a - b\*x^4])

**Rubi in Sympy [A]** time = 106.733, size = 241, normalized size = 0.86

$$\frac{\sqrt[4]{ad} \sqrt{1 - \frac{bx^4}{a}} \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bc} \sqrt{a - bx^4}(ad - bc)} + \frac{\sqrt[4]{ad} \sqrt{1 - \frac{bx^4}{a}} \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bc} \sqrt{a - bx^4}(ad - bc)}$$

$$- \frac{bx}{2a \sqrt{a - bx^4}(ad - bc)} - \frac{b^{3/4} \sqrt{1 - \frac{bx^4}{a}} F\left(\operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2a^{3/4} \sqrt{a - bx^4}(ad - bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*4+a)\*\*(3/2)/(-d\*x\*\*4+c), x)

[Out] a\*\*(1/4)\*d\*sqrt(1 - b\*x\*\*4/a)\*elliptic\_pi(-sqrt(a)\*sqrt(d)/(sqrt(b)\*sqrt(c)), asin(b\*\*(1/4)\*x/a\*\*(1/4)), -1)/(2\*b\*\*(1/4)\*c\*sqrt(a



- b\*x\*\*4)\*(a\*d - b\*c)) + a\*\*(1/4)\*d\*sqrt(1 - b\*x\*\*4/a)\*elliptic\_p  
i(sqrt(a)\*sqrt(d)/(sqrt(b)\*sqrt(c)), asin(b\*\*(1/4)\*x/a\*\*(1/4)), -  
1)/(2\*b\*\*(1/4)\*c\*sqrt(a - b\*x\*\*4)\*(a\*d - b\*c)) - b\*x/(2\*a\*sqrt(a  
- b\*x\*\*4)\*(a\*d - b\*c)) - b\*\*(3/4)\*sqrt(1 - b\*x\*\*4/a)\*elliptic\_f(a  
sin(b\*\*(1/4)\*x/a\*\*(1/4)), -1)/(2\*a\*\*(3/4)\*sqrt(a - b\*x\*\*4)\*(a\*d -  
b\*c))

**Mathematica [C]** time = 0.603338, size = 329, normalized size = 1.17

$$x \left( \frac{9bcdx^4 F_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{(c-dx^4)\left(2x^4\left(2adF_1\left(\frac{9}{4}; \frac{1}{2}, 2; \frac{13}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{9}{4}; \frac{3}{2}, 1; \frac{13}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right) + 9acF_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)} - \frac{25c(bc-2ad)F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{(c-dx^4)\left(2x^4\left(2adF_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right) + 9acF_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)} \right) \frac{1}{10\sqrt{a-bx^4}(ad-bc)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a - b\*x^4)^(3/2)\*(c - d\*x^4)), x]

[Out] (x\*((-5\*b)/a - (25\*c\*(b\*c - 2\*a\*d)\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c])/((c - d\*x^4)\*(5\*a\*c\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c] + 2\*x^4\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, (b\*x^4)/a, (d\*x^4)/c] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c]))) + (9\*b\*c\*d\*x^4\*AppellF1[5/4, 1/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c])/((c - d\*x^4)\*(9\*a\*c\*AppellF1[5/4, 1/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c] + 2\*x^4\*(2\*a\*d\*AppellF1[9/4, 1/2, 2, 13/4, (b\*x^4)/a, (d\*x^4)/c] + b\*c\*AppellF1[9/4, 3/2, 1, 13/4, (b\*x^4)/a, (d\*x^4)/c]))))/(10\*(-(b\*c) + a\*d)\*Sqrt[a - b\*x^4])

**Maple [C]** time = 0.043, size = 301, normalized size = 1.1

$$\frac{bx}{2a(ad-bc)} \frac{1}{\sqrt{-(x^4 - \frac{a}{b})b}} - \frac{b}{2a(ad-bc)} \sqrt{1-x^2\sqrt{b}} \frac{1}{\sqrt{a}} \sqrt{1+x^2\sqrt{b}} \frac{1}{\sqrt{a}} \text{EllipticF}\left(x\sqrt{1\sqrt{b}} \frac{1}{\sqrt{a}}, i\right) \frac{1}{\sqrt{1\sqrt{b}} \frac{1}{\sqrt{a}}} \frac{1}{\sqrt{-bx^4+a}}$$

$$-\frac{1}{8} \sum_{\alpha=\text{RootOf}(-Z^4-d-c)} \frac{1}{(ad-bc)\alpha^3} \left( -1 \text{Artanh}\left(\frac{-2\alpha^2 bx^2 + 2a}{2} \frac{1}{\sqrt{\frac{ad-bc}{d}}} \frac{1}{\sqrt{-bx^4+a}}\right) \frac{1}{\sqrt{\frac{ad-bc}{d}}} - 2 \frac{\alpha^3 d}{c\sqrt{-bx^4+a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^4+a)^(3/2)/(-d\*x^4+c), x)

[Out] -1/2\*b/a\*x/(a\*d-b\*c)/(-(x^4-a/b)\*b)^(1/2)-1/2\*b/a/(a\*d-b\*c)/(1/a^(1/2)\*b^(1/2))^(1/2)\*(1-b^(1/2)\*x^2/a^(1/2))^(1/2)\*(1+b^(1/2)\*x^2/a^(1/2))^(1/2)/(-b\*x^4+a)^(1/2)\*EllipticF(x\*(1/a^(1/2)\*b^(1/2))^(1/2), I)-1/8\*sum(1/(a\*d-b\*c)/\_alpha^3\*(-1/((a\*d-b\*c)/d)^(1/2)\*arc tanh(1/2\*(-2\*\_alpha^2\*b\*x^2+2\*a)/((a\*d-b\*c)/d)^(1/2)/(-b\*x^4+a)^(1/2))-2/(1/a^(1/2)\*b^(1/2))^(1/2)\*\_alpha^3\*d/c\*(1-b^(1/2)\*x^2/a^(1/2))^(1/2)\*(1+b^(1/2)\*x^2/a^(1/2))^(1/2)/(-b\*x^4+a)^(1/2)\*EllipticPi(x\*(1/a^(1/2)\*b^(1/2))^(1/2), a^(1/2)/b^(1/2)\*\_alpha^2/c\*d, (-1/a^(1/2)\*b^(1/2))^(1/2)/(1/a^(1/2)\*b^(1/2))^(1/2)), \_alpha=RootOf(-Z^4\*d-c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(-bx^4+a)^{\frac{3}{2}}(dx^4-c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)),x, algorithm="maxima")`

[Out] `-integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)), x)`

**Fricas** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)),x, algorithm="fricas")`

[Out] Timed out

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-ac\sqrt{a-bx^4} + adx^4\sqrt{a-bx^4} + bcx^4\sqrt{a-bx^4} - bdx^8\sqrt{a-bx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c),x)`

[Out] `-Integral(1/(-a*c*sqrt(a - b*x**4) + a*d*x**4*sqrt(a - b*x**4) + b*c*x**4*sqrt(a - b*x**4) - b*d*x**8*sqrt(a - b*x**4)), x)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(-bx^4 + a)^{\frac{3}{2}}(dx^4 - c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)),x, algorithm="giac")`

[Out] `integrate(-1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)), x)`

$$3.79 \quad \int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)} dx$$

**Optimal.** Leaf size=334

$$\frac{b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (5bc - 11ad) F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx^4}}{\sqrt[4]{a}}\right) \middle| -1\right)}{12a^{7/4} \sqrt{a - bx^4} (bc - ad)^2} + \frac{bx(5bc - 11ad)}{12a^2 \sqrt{a - bx^4} (bc - ad)^2} + \frac{\sqrt[4]{ad^2} \sqrt{1 - \frac{bx^4}{a}} \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^4}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bc} \sqrt{a - bx^4} (bc - ad)^2} + \frac{\sqrt[4]{ad^2} \sqrt{1 - \frac{bx^4}{a}} \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^4}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bc} \sqrt{a - bx^4} (bc - ad)^2} + \frac{bx}{6a(a - bx^4)^{3/2} (bc - ad)}$$

[Out] (b\*x)/(6\*a\*(b\*c - a\*d)\*(a - b\*x^4)^(3/2)) + (b\*(5\*b\*c - 11\*a\*d)\*x)/(12\*a^2\*(b\*c - a\*d)^2\*Sqrt[a - b\*x^4]) + (b^(3/4)\*(5\*b\*c - 11\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticF[ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(12\*a^(7/4)\*(b\*c - a\*d)^2\*Sqrt[a - b\*x^4]) + (a^(1/4)\*d^2\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*b^(1/4)\*c\*(b\*c - a\*d)^2\*Sqrt[a - b\*x^4]) + (a^(1/4)\*d^2\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*b^(1/4)\*c\*(b\*c - a\*d)^2\*Sqrt[a - b\*x^4])

**Rubi [A]** time = 0.961753, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$

$$\frac{b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (5bc - 11ad) F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx^4}}{\sqrt[4]{a}}\right) \middle| -1\right)}{12a^{7/4} \sqrt{a - bx^4} (bc - ad)^2} + \frac{bx(5bc - 11ad)}{12a^2 \sqrt{a - bx^4} (bc - ad)^2} + \frac{\sqrt[4]{ad^2} \sqrt{1 - \frac{bx^4}{a}} \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^4}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bc} \sqrt{a - bx^4} (bc - ad)^2} + \frac{\sqrt[4]{ad^2} \sqrt{1 - \frac{bx^4}{a}} \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^4}}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{bc} \sqrt{a - bx^4} (bc - ad)^2} + \frac{bx}{6a(a - bx^4)^{3/2} (bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a - b\*x^4)^(5/2)\*(c - d\*x^4)),x]

[Out] (b\*x)/(6\*a\*(b\*c - a\*d)\*(a - b\*x^4)^(3/2)) + (b\*(5\*b\*c - 11\*a\*d)\*x)/(12\*a^2\*(b\*c - a\*d)^2\*Sqrt[a - b\*x^4]) + (b^(3/4)\*(5\*b\*c - 11\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticF[ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(12\*a^(7/4)\*(b\*c - a\*d)^2\*Sqrt[a - b\*x^4]) + (a^(1/4)\*d^2\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*b^(1/4)\*c\*(b\*c - a\*d)^2\*Sqrt[a - b\*x^4]) + (a^(1/4)\*d^2\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(2\*b^(1/4)\*c\*(b\*c - a\*d)^2\*Sqrt[a - b\*x^4])

**Rubi in Sympy [A]** time = 165.584, size = 298, normalized size = 0.89

$$\frac{\sqrt[4]{ad^2} \sqrt{1 - \frac{bx^4}{a}} \left( -\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| -1 \right)}{2\sqrt[4]{bc} \sqrt{a - bx^4} (ad - bc)^2} + \frac{\sqrt[4]{ad^2} \sqrt{1 - \frac{bx^4}{a}} \left( \frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| -1 \right)}{2\sqrt[4]{bc} \sqrt{a - bx^4} (ad - bc)^2}$$

$$- \frac{bx}{6a(a - bx^4)^{\frac{3}{2}} (ad - bc)} - \frac{bx(11ad - 5bc)}{12a^2 \sqrt{a - bx^4} (ad - bc)^2}$$

$$- \frac{b^{\frac{3}{4}} \sqrt{1 - \frac{bx^4}{a}} (11ad - 5bc) F \left( \operatorname{asin} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| -1 \right)}{12a^{\frac{7}{4}} \sqrt{a - bx^4} (ad - bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c), x)`

[Out] `a**(1/4)*d**2*sqrt(1 - b*x**4/a)*elliptic_pi(-sqrt(a)*sqrt(d)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(2*b**(1/4)*c*sqrt(a - b*x**4)*(a*d - b*c)**2) + a**(1/4)*d**2*sqrt(1 - b*x**4/a)*elliptic_pi(sqrt(a)*sqrt(d)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(2*b**(1/4)*c*sqrt(a - b*x**4)*(a*d - b*c)**2) - b*x/(6*a*(a - b*x**4)**(3/2)*(a*d - b*c)) - b*x*(11*a*d - 5*b*c)/(12*a**2*sqrt(a - b*x**4)*(a*d - b*c)**2) - b**(3/4)*sqrt(1 - b*x**4/a)*(11*a*d - 5*b*c)*elliptic_f(asin(b**(1/4)*x/a**(1/4)), -1)/(12*a**(7/4)*sqrt(a - b*x**4)*(a*d - b*c)**2)`

**Mathematica [C]** time = 1.56833, size = 396, normalized size = 1.19

$$x \left( \frac{25ac(12a^2d^2 - 11abcd + 5b^2c^2) F_1 \left( \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \frac{bx^4}{a}, \frac{dx^4}{c} \right)}{(c-dx^4) \left( 2x^4 \left( 2adF_1 \left( \frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c} \right) + bcF_1 \left( \frac{3}{4}, \frac{3}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c} \right) \right) + 5acF_1 \left( \frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \frac{bx^4}{a}, \frac{dx^4}{c} \right) \right)} + \frac{5b(13a^2d - ab(7c + 11dx^4) + 5b^2cx^4)}{bx^4 - a} + \frac{1}{(c-dx^4)(2x^4} \right)$$

$$60a^2 \sqrt{a - bx^4} (bc - ad)^2$$

Warning: Unable to verify antiderivative.

[In] `Integrate[1/((a - b*x^4)^(5/2)*(c - d*x^4)), x]`

[Out] `(x*((5*b*(13*a^2*d + 5*b^2*c*x^4 - a*b*(7*c + 11*d*x^4)))/(-a + b*x^4) + (25*a*c*(5*b^2*c^2 - 11*a*b*c*d + 12*a^2*d^2)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/((c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])) + (9*a*b*c*d*(-5*b*c + 11*a*d)*x^4*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/((c - d*x^4)*(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c]))))/(60*a^2*(b*c - a*d)^2*sqrt[a - b*x^4])`

**Maple [C]** time = 0.047, size = 361, normalized size = 1.1

$$- \frac{x}{6ab(ad - bc)} \sqrt{-bx^4 + a} \left( x^4 - \frac{a}{b} \right)^{-2} - \frac{bx(11ad - 5bc)}{12a^2(ad - bc)^2} \frac{1}{\sqrt{-(x^4 - \frac{a}{b})b}}$$

$$- \frac{b(11ad - 5bc)}{12a^2(ad - bc)^2} \sqrt{1 - x^2\sqrt{b}} \frac{1}{\sqrt{a}} \sqrt{1 + x^2\sqrt{b}} \frac{1}{\sqrt{a}} \operatorname{EllipticF} \left( x \sqrt{1\sqrt{b}} \frac{1}{\sqrt{a}}, i \right) \frac{1}{\sqrt{1\sqrt{b}} \frac{1}{\sqrt{a}}} \frac{1}{\sqrt{-bx^4 + a}}$$

$$- \frac{d}{8} \sum_{\alpha = \operatorname{RootOf}(-Z^4 - c)} \frac{1}{(ad - bc)^2 - \alpha^3} \left( -1 \operatorname{Artanh} \left( \frac{-2\alpha^2 bx^2 + 2a}{2} \frac{1}{\sqrt{\frac{ad - bc}{d}}} \frac{1}{\sqrt{-bx^4 + a}} \right) \frac{1}{\sqrt{\frac{ad - bc}{d}}} - 2 \frac{\alpha^3 d}{c\sqrt{-bx^4 + a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x)`

[Out] 
$$-1/6/a*x/b/(a*d-b*c)*(-b*x^4+a)^{(1/2)}/(x^4-a/b)^2-1/12*b/a^2*x*(11*a*d-5*b*c)/(a*d-b*c)^2/(-x^4-a/b)*b^{(1/2)}-1/12*b/a^2*(11*a*d-5*b*c)/(a*d-b*c)^2/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-b^{(1/2)}*x^2/a^{(1/2)})^{(1/2)}*(1+b^{(1/2)}*x^2/a^{(1/2)})^{(1/2)}/(-b*x^4+a)^{(1/2)}*EllipticF(x*(1/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)-1/8*d*sum(1/(a*d-b*c)^2/_alpha^3*(-1/((a*d-b*c)/d)^{(1/2)}*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^{(1/2)}/(-b*x^4+a)^{(1/2)})-2/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*_alpha^3*d/c*(1-b^{(1/2)}*x^2/a^{(1/2)})^{(1/2)}*(1+b^{(1/2)}*x^2/a^{(1/2)})^{(1/2)}/(-b*x^4+a)^{(1/2)}*EllipticPi(x*(1/a^{(1/2)}*b^{(1/2)})^{(1/2)},a^{(1/2)}/b^{(1/2)}*_alpha^2/c*d,(-1/a^{(1/2)}*b^{(1/2)})^{(1/2)}/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}),_alpha=RootOf(_Z^4*d-c))$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(-bx^4 + a)^{\frac{5}{2}}(dx^4 - c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)),x, algorithm="maxima")`

[Out] `-integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(-bx^4 + a)^{\frac{5}{2}}(dx^4 - c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)),x, algorithm="giac")`

```
[Out] integrate(-1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)), x)
```

$$3.80 \quad \int \frac{(a+bx^4)^{3/2}}{c+dx^4} dx$$

**Optimal.** Leaf size=973

result too large to display

```
[Out] (b*x*Sqrt[a + b*x^4])/(3*d) + ((b*c - a*d)^2*ArcTan[(Sqrt[-((Sqrt
[-c]*(b - (a*d)/c))/Sqrt[d]])*x]/Sqrt[a + b*x^4])/(4*c*d^2*Sqrt[
(b*c - a*d)/(Sqrt[-c]*Sqrt[d])]) + ((b*c - a*d)^2*ArcTan[(Sqrt[(S
qrt[-c]*(b - (a*d)/c))/Sqrt[d])*x]/Sqrt[a + b*x^4])/(4*c*d^2*Sqr
t[-((b*c - a*d)/(Sqrt[-c]*Sqrt[d])])) - (b^(3/4)*(3*b*c - 5*a*d)*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*d^2*
Sqrt[a + b*x^4]) + (b^(1/4)*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c - Sqrt[a]*Sqrt[-
c]*Sqrt[d])*d^2*Sqrt[a + b*x^4]) + (b^(1/4)*(b*c - a*d)^2*(Sqrt[a
] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c
+ Sqrt[a]*Sqrt[-c]*Sqrt[d])*d^2*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqr
t[-c] + Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqr
t[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]),
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b
]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*d^2*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqr
t[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqr
t[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]),
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[
b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*d^2*Sqrt[a + b*x^4])
```

**Rubi [A]** time = 3.70666, antiderivative size = 973, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{-c}(b-ad)}{c}}x}{\sqrt{bx^4+a}}\right)(bc-ad)^2}{4cd^2\sqrt{\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{-c}(b-ad)}{c}}x}{\sqrt{bx^4+a}}\right)(bc-ad)^2}{4cd^2\sqrt{-\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}}$$

$$+ \frac{\sqrt[4]{b}(\sqrt{bx^2+\sqrt{a}})\sqrt{\frac{bx^4+a}{(\sqrt{bx^2+\sqrt{a}})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)(bc-ad)^2}{4\sqrt[4]{a}(\sqrt{bc}-\sqrt{a}\sqrt{-c}\sqrt{d})d^2\sqrt{bx^4+a}}$$

$$+ \frac{\sqrt[4]{b}(\sqrt{bx^2+\sqrt{a}})\sqrt{\frac{bx^4+a}{(\sqrt{bx^2+\sqrt{a}})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)(bc-ad)^2}{4\sqrt[4]{a}(\sqrt{bc}+\sqrt{a}\sqrt{-c}\sqrt{d})d^2\sqrt{bx^4+a}}$$

$$- \frac{(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d})(\sqrt{bx^2+\sqrt{a}})\sqrt{\frac{bx^4+a}{(\sqrt{bx^2+\sqrt{a}})^2}}\left(-\frac{(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d})^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)(bc-ad)^2}{8\sqrt[4]{a}\sqrt[4]{bc}(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d})d^2\sqrt{bx^4+a}}$$

$$- \frac{(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d})(\sqrt{bx^2+\sqrt{a}})\sqrt{\frac{bx^4+a}{(\sqrt{bx^2+\sqrt{a}})^2}}\left(\frac{(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d})^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)(bc-ad)^2}{8\sqrt[4]{a}\sqrt[4]{bc}(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d})d^2\sqrt{bx^4+a}}$$

$$- \frac{b^{3/4}(3bc-5ad)(\sqrt{bx^2+\sqrt{a}})\sqrt{\frac{bx^4+a}{(\sqrt{bx^2+\sqrt{a}})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{6\sqrt[4]{ad^2}\sqrt{bx^4+a}} + \frac{bx\sqrt{bx^4+a}}{3d}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*x^4)^(3/2)/(c + d\*x^4), x]

```
[Out] (b*x*Sqrt[a + b*x^4])/(3*d) + ((b*c - a*d)^2*ArcTan[(Sqrt[-((Sqrt
[-c]*(b - (a*d)/c))/Sqrt[d]])*x]/Sqrt[a + b*x^4])/(4*c*d^2*Sqrt[
(b*c - a*d)/(Sqrt[-c]*Sqrt[d])]) + ((b*c - a*d)^2*ArcTan[(Sqrt[(S
qrt[-c]*(b - (a*d)/c))/Sqrt[d]]*x]/Sqrt[a + b*x^4])/(4*c*d^2*Sqr
t[-((b*c - a*d)/(Sqrt[-c]*Sqrt[d])])) - (b^(3/4)*(3*b*c - 5*a*d)*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*d^2*
Sqrt[a + b*x^4]) + (b^(1/4)*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c - Sqrt[a]*Sqrt[-
c]*Sqrt[d])*d^2*Sqrt[a + b*x^4]) + (b^(1/4)*(b*c - a*d)^2*(Sqrt[a
] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c
+ Sqrt[a]*Sqrt[-c]*Sqrt[d])*d^2*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqr
t[-c] + Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqr
t[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]),
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b
]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*d^2*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sq
rt[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sq
rt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]),
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[
b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*d^2*Sqrt[a + b*x^4])
```

---

**Rubi in Sympy [F-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**4+a)**(3/2)/(d*x**4+c),x)
```

```
[Out] Timed out
```

---

**Mathematica [C]** time = 1.14671, size = 435, normalized size = 0.45

$$x \frac{\left( \frac{25a^2c(3ad-bc)F_1\left(\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{5acF_1\left(\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)} - 2x^4 \left( 2adF_1\left(\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right) \right)}{15d\sqrt{a+bx^4}(c+dx^4)} + \frac{b(10x^4(a+bx^4)(c+dx^4) \left( 2adF_1\left(\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right))}{2x^4 \left( 2adF_1\left(\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(3/2)/(c + d*x^4),x]
```

```
[Out] (x*((25*a^2*c*(-(b*c) + 3*a*d)*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^
4)/a), -((d*x^4)/c)])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)
/a), -((d*x^4)/c)] - 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b
*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4
)/a), -((d*x^4)/c)])) + (b*(-9*a*c*(5*a*(c + 2*d*x^4) + b*x^4*(2*
c + 5*d*x^4))*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/
c)] + 10*x^4*(a + b*x^4)*(c + d*x^4)*(2*a*d*AppellF1[9/4, 1/2, 2,
13/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[9/4, 3/2, 1, 13
/4, -((b*x^4)/a), -((d*x^4)/c)])))/(-9*a*c*AppellF1[5/4, 1/2, 1,
9/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*x^4*(2*a*d*AppellF1[9/4, 1/2
, 2, 13/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[9/4, 3/2, 1
, 13/4, -((b*x^4)/a), -((d*x^4)/c)])))/(15*d*Sqrt[a + b*x^4]*(c
+ d*x^4))
```



**Maple [C]** time = 0.036, size = 322, normalized size = 0.3

$$\frac{bx}{3d}\sqrt{bx^4+a} + 1\left(\frac{b(2ad-bc)}{d^2} - \frac{ab}{3d}\right)\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}, i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

$$- \frac{1}{8d^3}\sum_{\alpha=\text{RootOf}(dZ^4+c)}\frac{-a^2d^2+2cabd-b^2c^2}{-\alpha^3}\left(-1\text{Artanh}\left(\frac{2-\alpha^2bx^2+2a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{bx^4+a}}\right)\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}+2\frac{\alpha^3}{c\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(3/2)/(d\*x^4+c), x)

[Out] 1/3\*b\*x\*(b\*x^4+a)^(1/2)/d+(b\*(2\*a\*d-b\*c)/d^2-1/3\*b/d\*a)/(I/a^(1/2)\*b^(1/2))^(1/2)\*(1-I/a^(1/2)\*b^(1/2)\*x^2)^(1/2)\*(1+I/a^(1/2)\*b^(1/2)\*x^2)^(1/2)/(b\*x^4+a)^(1/2)\*EllipticF(x\*(I/a^(1/2)\*b^(1/2))^(1/2), I)-1/8/d^3\*sum((-a^2\*d^2+2\*a\*b\*c\*d-b^2\*c^2)/\_alpha^3\*(-1/((a\*d-b\*c)/d)^(1/2)\*arctanh(1/2\*(2\*\_alpha^2\*b\*x^2+2\*a)/((a\*d-b\*c)/d)^(1/2))/(b\*x^4+a)^(1/2))+2/(I/a^(1/2)\*b^(1/2))^(1/2)\*\_alpha^3\*d/c\*(1-I/a^(1/2)\*b^(1/2)\*x^2)^(1/2)\*(1+I/a^(1/2)\*b^(1/2)\*x^2)^(1/2)/(b\*x^4+a)^(1/2)\*EllipticPi(x\*(I/a^(1/2)\*b^(1/2))^(1/2), I\*a^(1/2)/b^(1/2)\*\_alpha^2/c\*d, (-I/a^(1/2)\*b^(1/2))^(1/2)/(I/a^(1/2)\*b^(1/2))^(1/2)), \_alpha=RootOf(\_Z^4\*d+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+a)^{\frac{3}{2}}}{dx^4+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(3/2)/(d\*x^4 + c), x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(3/2)/(d\*x^4 + c), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(3/2)/(d\*x^4 + c), x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^4)^{\frac{3}{2}}}{c+dx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(3/2)/(d\*x\*\*4+c), x)

[Out] Integral((a + b\*x\*\*4)\*\*(3/2)/(c + d\*x\*\*4), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(3/2)/(d\*x^4 + c),x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(3/2)/(d\*x^4 + c), x)

$$3.81 \quad \int \frac{\sqrt{a+bx^4}}{c+dx^4} dx$$

**Optimal.** Leaf size=932

result too large to display

```
[Out] -((b*c - a*d)*ArcTan[(Sqrt[-((Sqrt[-c]*(b - (a*d)/c))/Sqrt[d]])*x
)/Sqrt[a + b*x^4]])/(4*c*d*Sqrt[(b*c - a*d)/(Sqrt[-c]*Sqrt[d])])
- ((b*c - a*d)*ArcTan[(Sqrt[(Sqrt[-c]*(b - (a*d)/c))/Sqrt[d]]*x)/
Sqrt[a + b*x^4]])/(4*c*d*Sqrt[-((b*c - a*d)/(Sqrt[-c]*Sqrt[d])]))
+ (b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)]^2)*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*
a^(1/4)*d*Sqrt[a + b*x^4]) - (b^(1/4)*(b*c - a*d)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)]^2)*EllipticF[2*
ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c - Sqrt[a
]*Sqrt[-c]*Sqrt[d])*d*Sqrt[a + b*x^4]) - (b^(1/4)*(b*c - a*d)*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)]^2)*
EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b
]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d])*d*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqr
t[-c] + Sqrt[a]*Sqrt[d])*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)]^2)*EllipticPi[-(Sqrt[b]*Sqr
t[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]),
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b
]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*d*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqr
t[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)]^2)*EllipticPi[(Sqrt[b]*Sqr
t[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*A
rcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b]*S
qrt[-c] + Sqrt[a]*Sqrt[d])*d*Sqrt[a + b*x^4])
```

**Rubi [A]** time = 1.76572, antiderivative size = 932, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{(bc - ad) \tan^{-1} \left( \frac{\sqrt{\frac{\sqrt{-c}(b-ad)}{c}} x}{\sqrt{bx^4+a}} \right) - (bc - ad) \tan^{-1} \left( \frac{\sqrt{\frac{\sqrt{-c}(b-ad)}{c}} x}{\sqrt{bx^4+a}} \right)}{4cd \sqrt{\frac{bc-ad}{\sqrt{-c}\sqrt{d}}} - 4cd \sqrt{-\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}}$$

$$\frac{\sqrt[4]{b}(bc - ad) (\sqrt{bx^2 + \sqrt{a}}) \sqrt{\frac{bx^4+a}{(\sqrt{bx^2+\sqrt{a}})^2}} F \left( 2 \tan^{-1} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| \frac{1}{2} \right)}{4\sqrt[4]{a} (\sqrt{bc} - \sqrt{a}\sqrt{-c}\sqrt{d}) d\sqrt{bx^4 + a}}$$

$$\frac{\sqrt[4]{b}(bc - ad) (\sqrt{bx^2 + \sqrt{a}}) \sqrt{\frac{bx^4+a}{(\sqrt{bx^2+\sqrt{a}})^2}} F \left( 2 \tan^{-1} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| \frac{1}{2} \right)}{4\sqrt[4]{a} (\sqrt{bc} + \sqrt{a}\sqrt{-c}\sqrt{d}) d\sqrt{bx^4 + a}}$$

$$+ \frac{b^{3/4} (\sqrt{bx^2 + \sqrt{a}}) \sqrt{\frac{bx^4+a}{(\sqrt{bx^2+\sqrt{a}})^2}} F \left( 2 \tan^{-1} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| \frac{1}{2} \right)}{2\sqrt[4]{ad}\sqrt{bx^4 + a}}$$

$$+ \frac{(\sqrt{b}\sqrt{-c} + \sqrt{a}\sqrt{d}) (bc - ad) (\sqrt{bx^2 + \sqrt{a}}) \sqrt{\frac{bx^4+a}{(\sqrt{bx^2+\sqrt{a}})^2}} \left( -\frac{(\sqrt{b}\sqrt{-c} - \sqrt{a}\sqrt{d})^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}}; 2 \tan^{-1} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| \frac{1}{2} \right)}{8\sqrt[4]{a}\sqrt[4]{bc} (\sqrt{b}\sqrt{-c} - \sqrt{a}\sqrt{d}) d\sqrt{bx^4 + a}}$$

$$+ \frac{(\sqrt{b}\sqrt{-c} - \sqrt{a}\sqrt{d}) (bc - ad) (\sqrt{bx^2 + \sqrt{a}}) \sqrt{\frac{bx^4+a}{(\sqrt{bx^2+\sqrt{a}})^2}} \left( \frac{(\sqrt{b}\sqrt{-c} + \sqrt{a}\sqrt{d})^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}}; 2 \tan^{-1} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a}} \right) \middle| \frac{1}{2} \right)}{8\sqrt[4]{a}\sqrt[4]{bc} (\sqrt{b}\sqrt{-c} + \sqrt{a}\sqrt{d}) d\sqrt{bx^4 + a}}$$

Warning: Unable to verify antiderivative.

[In] Int[Sqrt[a + b\*x^4]/(c + d\*x^4), x]

```
[Out] -((b*c - a*d)*ArcTan[(Sqrt[-((Sqrt[-c]*(b - (a*d)/c))/Sqrt[d]])*x
)/Sqrt[a + b*x^4]])/(4*c*d*Sqrt[(b*c - a*d)/(Sqrt[-c]*Sqrt[d])])
- ((b*c - a*d)*ArcTan[(Sqrt[(Sqrt[-c]*(b - (a*d)/c))/Sqrt[d]]*x)/
Sqrt[a + b*x^4]])/(4*c*d*Sqrt[-((b*c - a*d)/(Sqrt[-c]*Sqrt[d])]))
+ (b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*
a^(1/4)*d*Sqrt[a + b*x^4]) - (b^(1/4)*(b*c - a*d)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*
ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c - Sqrt[a
]*Sqrt[-c]*Sqrt[d])*d*Sqrt[a + b*x^4]) - (b^(1/4)*(b*c - a*d)*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*
EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b
]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d])*d*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sq
rt[-c] + Sqrt[a]*Sqrt[d])*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqr
t[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]),
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b
]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*d*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt
[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-
c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*A
rcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b]*S
qrt[-c] + Sqrt[a]*Sqrt[d])*d*Sqrt[a + b*x^4])
```

---

**Rubi in Sympy [A]** time = 149.324, size = 814, normalized size = 0.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**4+a)**(1/2)/(d*x**4+c),x)
```

```
[Out] (a*d - b*c)*atan(x*sqrt(sqrt(-c)*(-a*d + b*c)/(c*sqrt(d)))/sqrt(a
+ b*x**4))/(4*c*d*sqrt(-sqrt(-c)*(a*d - b*c)/(c*sqrt(d)))) + (a*
d - b*c)*atan(x*sqrt(sqrt(-c)*(a*d - b*c)/(c*sqrt(d)))/sqrt(a + b
*x**4))/(4*c*d*sqrt(sqrt(-c)*(a*d - b*c)/(c*sqrt(d)))) + b**(3/4)
*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b
)*x**2)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(2*a**(1/4)*
d*sqrt(a + b*x**4)) + b**(1/4)*sqrt((a + b*x**4)/(sqrt(a) + sqrt(
b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(a*d - b*c)*elliptic_f(2*at
an(b**(1/4)*x/a**(1/4)), 1/2)/(4*a**(1/4)*d*sqrt(a + b*x**4)*(sqr
t(a)*sqrt(d)*sqrt(-c) + sqrt(b)*c)) + b**(1/4)*sqrt((a + b*x**4)/
(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(a*d - b*c)
*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(4*a**(1/4)*d*sqrt(
a + b*x**4)*(-sqrt(a)*sqrt(d)*sqrt(-c) + sqrt(b)*c)) + sqrt((a +
b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(sq
rt(a)*sqrt(d) - sqrt(b)*sqrt(-c))*(a*d - b*c)*elliptic_pi((sqrt(a
)*sqrt(d) + sqrt(b)*sqrt(-c))**2/(4*sqrt(a)*sqrt(b)*sqrt(d)*sqrt(
-c)), 2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(8*a**(1/4)*b**(1/4)*c*d*
sqrt(a + b*x**4)*(sqrt(a)*sqrt(d) + sqrt(b)*sqrt(-c))) + sqrt((a
+ b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*
(sqrt(a)*sqrt(d) + sqrt(b)*sqrt(-c))*(a*d - b*c)*elliptic_pi(-(sqr
t(a)*sqrt(d) - sqrt(b)*sqrt(-c))**2/(4*sqrt(a)*sqrt(b)*sqrt(d)*sq
rt(-c)), 2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(8*a**(1/4)*b**(1/4)*c
*d*sqrt(a + b*x**4)*(sqrt(a)*sqrt(d) - sqrt(b)*sqrt(-c)))
```

---

**Mathematica [C]** time = 0.237976, size = 161, normalized size = 0.17

$$\frac{5acx\sqrt{a+bx^4}F_1\left(\frac{1}{4}, -\frac{1}{2}, 1; \frac{5}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{(c+dx^4)\left(2x^4\left(bcF_1\left(\frac{5}{4}, \frac{1}{2}, 1; \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) - 2adF_1\left(\frac{5}{4}, -\frac{1}{2}, 2; \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) + 5acF_1\left(\frac{1}{4}, -\frac{1}{2}, 1; \frac{5}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^4]/(c + d*x^4),x]
```

[Out]  $(5*a*c*x*\text{Sqrt}[a + b*x^4]*\text{AppellF1}[1/4, -1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/((c + d*x^4)*(5*a*c*\text{AppellF1}[1/4, -1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*x^4*(-2*a*d*\text{AppellF1}[5/4, -1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*\text{AppellF1}[5/4, 1/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))$

**Maple [C]** time = 0.023, size = 273, normalized size = 0.3

$$\frac{b}{d} \sqrt{1 - ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \sqrt{1 + ix^2 \sqrt{b} \frac{1}{\sqrt{a}}} \text{EllipticF} \left( x \sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}, i \right) \frac{1}{\sqrt{i \sqrt{b} \frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{bx^4 + a}}$$

$$- \frac{1}{8d^2} \sum_{\alpha = \text{RootOf}(-Z^4 d + c)} \frac{-ad + bc}{-\alpha^3} \left( -1 \text{Artanh} \left( \frac{2 - \alpha^2 bx^2 + 2a}{2} \frac{1}{\sqrt{\frac{ad-bc}{d}}} \frac{1}{\sqrt{bx^4 + a}} \right) \frac{1}{\sqrt{\frac{ad-bc}{d}}} + 2 \frac{\alpha^3 d}{c \sqrt{bx^4 + a}} \sqrt{1 - \frac{i \sqrt{b}}{\sqrt{a}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+a)^(1/2)/(d*x^4+c), x)`

[Out]  $b/d/(I/a^{(1/2)*b^{(1/2)}})^{(1/2)*(1-I/a^{(1/2)*b^{(1/2)*x^2}})^{(1/2)*(1+I/a^{(1/2)*b^{(1/2)*x^2}})^{(1/2)/(b*x^4+a)^{(1/2)*\text{EllipticF}(x*(I/a^{(1/2)*b^{(1/2)}})^{(1/2)}, I)-1/8/d^2*\text{sum}((-a*d+b*c)/\alpha^3*(-1/((a*d-b*c)/d)^{(1/2)*\text{arctanh}(1/2*(2*\alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^{(1/2)/(b*x^4+a)^{(1/2)})+2/(I/a^{(1/2)*b^{(1/2)}})^{(1/2)*\alpha^3*d/c*(1-I/a^{(1/2)*b^{(1/2)*x^2}})^{(1/2)*(1+I/a^{(1/2)*b^{(1/2)*x^2}})^{(1/2)/(b*x^4+a)^{(1/2)*\text{EllipticPi}(x*(I/a^{(1/2)*b^{(1/2)}})^{(1/2)}, I*a^{(1/2)/b^{(1/2)})*\alpha^2/c*d, (-I/a^{(1/2)*b^{(1/2)}})^{(1/2)/(I/a^{(1/2)*b^{(1/2)}})^{(1/2)}))}, \alpha = \text{RootOf}(-Z^4*d+c))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^4 + a)/(d*x^4 + c), x, algorithm="maxima")`

[Out] `integrate(sqrt(b*x^4 + a)/(d*x^4 + c), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^4 + a)/(d*x^4 + c), x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^4}}{c + dx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**4+a)**(1/2)/(d*x**4+c),x)`

[Out] `Integral(sqrt(a + b*x**4)/(c + d*x**4), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^4 + a)/(d*x^4 + c),x, algorithm="giac")`

[Out] `integrate(sqrt(b*x^4 + a)/(d*x^4 + c), x)`

$$3.82 \quad \int \frac{1}{\sqrt{a+bx^4}(c+dx^4)} dx$$

**Optimal.** Leaf size=775

$$\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-c}(b-\frac{ad}{c})}}{\sqrt{d}}}}{\sqrt{a+bx^4}}\right)}{4c\sqrt{\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}} + \frac{\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-c}(b-\frac{ad}{c})}}{\sqrt{d}}}}{\sqrt{a+bx^4}}\right)}{4c\sqrt{-\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}}$$

$$+ \frac{\sqrt[4]{b}\left(\sqrt{a} + \sqrt{bx^2}\right) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{bc} - \sqrt{a}\sqrt{-c}\sqrt{d}\right)}$$

$$+ \frac{\sqrt[4]{b}\left(\sqrt{a} + \sqrt{bx^2}\right) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{a}\sqrt{-c}\sqrt{d} + \sqrt{bc}\right)}$$

$$- \frac{\left(\sqrt{a} + \sqrt{bx^2}\right) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} \left(\sqrt{a}\sqrt{d} + \sqrt{b}\sqrt{-c}\right) \left(-\frac{(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d})^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}}; 2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{bc}\sqrt{a+bx^4}\left(\sqrt{b}\sqrt{-c} - \sqrt{a}\sqrt{d}\right)}$$

$$- \frac{\left(\sqrt{a} + \sqrt{bx^2}\right) \sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}} \left(\sqrt{b}\sqrt{-c} - \sqrt{a}\sqrt{d}\right) \left(\frac{(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d})^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}}; 2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{bc}\sqrt{a+bx^4}\left(\sqrt{a}\sqrt{d} + \sqrt{b}\sqrt{-c}\right)}$$

[Out] ArcTan[(Sqrt[-((Sqrt[-c]\*(b - (a\*d)/c)))/Sqrt[d]])\*x]/Sqrt[a + b\*x^4]/(4\*c\*Sqrt[(b\*c - a\*d)/(Sqrt[-c]\*Sqrt[d])]) + ArcTan[(Sqrt[(Sqrt[-c]\*(b - (a\*d)/c))/Sqrt[d]])\*x]/Sqrt[a + b\*x^4]/(4\*c\*Sqrt[-((b\*c - a\*d)/(Sqrt[-c]\*Sqrt[d]))]) + (b^(1/4)\*(Sqrt[a] + Sqrt[b]\*x^2)\*Sqrt[(a + b\*x^4)/(Sqrt[a] + Sqrt[b]\*x^2)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*x)/a^(1/4)], 1/2])/(4\*a^(1/4)\*(Sqrt[b]\*c - Sqrt[a]\*Sqrt[-c]\*Sqrt[d])\*Sqrt[a + b\*x^4]) + (b^(1/4)\*(Sqrt[a] + Sqrt[b]\*x^2)\*Sqrt[(a + b\*x^4)/(Sqrt[a] + Sqrt[b]\*x^2)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*x)/a^(1/4)], 1/2])/(4\*a^(1/4)\*(Sqrt[b]\*c + Sqrt[a]\*Sqrt[-c]\*Sqrt[d])\*Sqrt[a + b\*x^4]) - ((Sqrt[b]\*Sqrt[-c] + Sqrt[a]\*Sqrt[d])\*(Sqrt[a] + Sqrt[b]\*x^2)\*Sqrt[(a + b\*x^4)/(Sqrt[a] + Sqrt[b]\*x^2)^2]\*EllipticPi[-(Sqrt[b]\*Sqrt[-c] - Sqrt[a]\*Sqrt[d])^2/(4\*Sqrt[a]\*Sqrt[b]\*Sqrt[-c]\*Sqrt[d]), 2\*ArcTan[(b^(1/4)\*x)/a^(1/4)], 1/2])/(8\*a^(1/4)\*b^(1/4)\*c\*(Sqrt[b]\*Sqrt[-c] - Sqrt[a]\*Sqrt[d])\*Sqrt[a + b\*x^4]) - ((Sqrt[b]\*Sqrt[-c] - Sqrt[a]\*Sqrt[d])\*(Sqrt[a] + Sqrt[b]\*x^2)\*Sqrt[(a + b\*x^4)/(Sqrt[a] + Sqrt[b]\*x^2)^2]\*EllipticPi[(Sqrt[b]\*Sqrt[-c] + Sqrt[a]\*Sqrt[d])^2/(4\*Sqrt[a]\*Sqrt[b]\*Sqrt[-c]\*Sqrt[d]), 2\*ArcTan[(b^(1/4)\*x)/a^(1/4)], 1/2])/(8\*a^(1/4)\*b^(1/4)\*c\*(Sqrt[b]\*Sqrt[-c] + Sqrt[a]\*Sqrt[d])\*Sqrt[a + b\*x^4])

**Rubi [A]** time = 1.23431, antiderivative size = 775, normalized size of antiderivative = 1., number of

steps used = 5, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-c}(b-\frac{ad}{c})}}{\sqrt{d}}}}{\sqrt{a+bx^4}}\right)}{4c\sqrt{\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}} + \frac{\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-c}(b-\frac{ad}{c})}}{\sqrt{d}}}}{\sqrt{a+bx^4}}\right)}{4c\sqrt{-\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}}$$

$$+ \frac{\sqrt[4]{b}(\sqrt{a} + \sqrt{bx^2})\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{a+bx^4}(\sqrt{bc} - \sqrt{a}\sqrt{-c}\sqrt{d})}$$

$$+ \frac{\sqrt[4]{b}(\sqrt{a} + \sqrt{bx^2})\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{a+bx^4}(\sqrt{a}\sqrt{-c}\sqrt{d} + \sqrt{bc})}$$

$$- \frac{(\sqrt{a} + \sqrt{bx^2})\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}}(\sqrt{a}\sqrt{d} + \sqrt{b}\sqrt{-c})\left(-\frac{(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d})^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}}; 2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{bc}\sqrt{a+bx^4}(\sqrt{b}\sqrt{-c} - \sqrt{a}\sqrt{d})}$$

$$- \frac{(\sqrt{a} + \sqrt{bx^2})\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{bx^2})^2}}(\sqrt{b}\sqrt{-c} - \sqrt{a}\sqrt{d})\left(\frac{(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d})^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}}; 2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{bc}\sqrt{a+bx^4}(\sqrt{a}\sqrt{d} + \sqrt{b}\sqrt{-c})}$$

Warning: Unable to verify antiderivative.

[In] Int[1/(Sqrt[a + b\*x^4]\*(c + d\*x^4)), x]

[Out] ArcTan[(Sqrt[-((Sqrt[-c]\*(b - (a\*d)/c))/Sqrt[d]])\*x)/Sqrt[a + b\*x^4]]/(4\*c\*Sqrt[(b\*c - a\*d)/(Sqrt[-c]\*Sqrt[d])]) + ArcTan[(Sqrt[(Sqrt[-c]\*(b - (a\*d)/c))/Sqrt[d]])\*x)/Sqrt[a + b\*x^4]]/(4\*c\*Sqrt[-((b\*c - a\*d)/(Sqrt[-c]\*Sqrt[d])])) + (b^(1/4)\*(Sqrt[a] + Sqrt[b]\*x^2)\*Sqrt[(a + b\*x^4)/(Sqrt[a] + Sqrt[b]\*x^2)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*x)/a^(1/4)], 1/2])/(4\*a^(1/4)\*(Sqrt[b]\*c - Sqrt[a]\*Sqrt[-c]\*Sqrt[d])\*Sqrt[a + b\*x^4]) + (b^(1/4)\*(Sqrt[a] + Sqrt[b]\*x^2)\*Sqrt[(a + b\*x^4)/(Sqrt[a] + Sqrt[b]\*x^2)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*x)/a^(1/4)], 1/2])/(4\*a^(1/4)\*(Sqrt[b]\*c + Sqrt[a]\*Sqrt[-c]\*Sqrt[d])\*Sqrt[a + b\*x^4]) - ((Sqrt[b]\*Sqrt[-c] + Sqrt[a]\*Sqrt[d])\*(Sqrt[a] + Sqrt[b]\*x^2)\*Sqrt[(a + b\*x^4)/(Sqrt[a] + Sqrt[b]\*x^2)^2]\*EllipticPi[-(Sqrt[b]\*Sqrt[-c] - Sqrt[a]\*Sqrt[d])^2/(4\*Sqrt[a]\*Sqrt[b]\*Sqrt[-c]\*Sqrt[d]), 2\*ArcTan[(b^(1/4)\*x)/a^(1/4)], 1/2])/(8\*a^(1/4)\*b^(1/4)\*c\*(Sqrt[b]\*Sqrt[-c] - Sqrt[a]\*Sqrt[d])\*Sqrt[a + b\*x^4]) - ((Sqrt[b]\*Sqrt[-c] - Sqrt[a]\*Sqrt[d])\*(Sqrt[a] + Sqrt[b]\*x^2)\*Sqrt[(a + b\*x^4)/(Sqrt[a] + Sqrt[b]\*x^2)^2]\*EllipticPi[(Sqrt[b]\*Sqrt[-c] + Sqrt[a]\*Sqrt[d])^2/(4\*Sqrt[a]\*Sqrt[b]\*Sqrt[-c]\*Sqrt[d]), 2\*ArcTan[(b^(1/4)\*x)/a^(1/4)], 1/2])/(8\*a^(1/4)\*b^(1/4)\*c\*(Sqrt[b]\*Sqrt[-c] + Sqrt[a]\*Sqrt[d])\*Sqrt[a + b\*x^4])

**Rubi in Sympy [A]** time = 100.558, size = 690, normalized size = 0.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*4+a)\*\*(1/2)/(d\*x\*\*4+c), x)

[Out] atan(x\*sqrt(sqrt(-c)\*(a\*d - b\*c)/(c\*sqrt(d)))/sqrt(a + b\*x\*\*4))/(4\*c\*sqrt(sqrt(-c)\*(a\*d - b\*c)/(c\*sqrt(d)))) + atan(x\*sqrt(sqrt(-c)\*(-a\*d + b\*c)/(c\*sqrt(d)))/sqrt(a + b\*x\*\*4))/(4\*c\*sqrt(sqrt(-c)\*(-a\*d + b\*c)/(c\*sqrt(d)))) - b\*\*(1/4)\*sqrt((a + b\*x\*\*4)/(sqrt(a) + sqrt(b)\*x\*\*2)\*\*2)\*(sqrt(a) + sqrt(b)\*x\*\*2)\*elliptic\_f(2\*atan(b\*\*(1/4)\*x/a\*\*(1/4)), 1/2)/(4\*a\*\*(1/4)\*sqrt(-c)\*sqrt(a + b\*x\*\*4)\*(sqrt(a)\*sqrt(d) + sqrt(b)\*sqrt(-c))) + b\*\*(1/4)\*sqrt((a + b\*x\*\*4)/



$(\sqrt{a} + \sqrt{b}x^2)^2 (\sqrt{a} + \sqrt{b}x^2) \operatorname{elliptic}_f(2 \operatorname{atan}(b^{1/4}x/a^{1/4}), 1/2)/(4a^{1/4}\sqrt{-c}\sqrt{a+bx^4}) (\sqrt{a}\sqrt{d} - \sqrt{b}\sqrt{-c}) + \sqrt{(a+bx^4)}/(\sqrt{a} + \sqrt{b}x^2)^2 (\sqrt{a} + \sqrt{b}x^2) (\sqrt{a}\sqrt{d} - \sqrt{b}\sqrt{-c}) \operatorname{elliptic}_\pi((\sqrt{a}\sqrt{d} + \sqrt{b}\sqrt{-c})^2/(4\sqrt{a}\sqrt{b}\sqrt{d}\sqrt{-c}), 2 \operatorname{atan}(b^{1/4}x/a^{1/4}), 1/2)/(8a^{1/4}b^{1/4}c\sqrt{a+bx^4}) (\sqrt{a}\sqrt{d} + \sqrt{b}\sqrt{-c}) + \sqrt{(a+bx^4)}/(\sqrt{a} + \sqrt{b}x^2)^2 (\sqrt{a} + \sqrt{b}x^2) (\sqrt{a}\sqrt{d} + \sqrt{b}\sqrt{-c}) \operatorname{elliptic}_\pi(-(\sqrt{a}\sqrt{d} - \sqrt{b}\sqrt{-c})^2/(4\sqrt{a}\sqrt{b}\sqrt{d}\sqrt{-c}), 2 \operatorname{atan}(b^{1/4}x/a^{1/4}), 1/2)/(8a^{1/4}b^{1/4}c\sqrt{a+bx^4}) (\sqrt{a}\sqrt{d} - \sqrt{b}\sqrt{-c}))$

**Mathematica [C]** time = 0.0810482, size = 161, normalized size = 0.21

$$5acx F_1\left(\frac{1}{4}, \frac{1}{2}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)$$

$$\sqrt{a+bx^4}(c+dx^4) \left(2x^4 \left(2ad F_1\left(\frac{5}{4}, \frac{1}{2}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + bc F_1\left(\frac{5}{4}, \frac{3}{2}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) - 5ac F_1\left(\frac{1}{4}, \frac{1}{2}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[a + b\*x^4]\*(c + d\*x^4)), x]

[Out] (-5\*a\*c\*x\*AppellF1[1/4, 1/2, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)])/(Sqrt[a + b\*x^4]\*(c + d\*x^4)\*(-5\*a\*c\*AppellF1[1/4, 1/2, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 2\*x^4\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)]))

**Maple [C]** time = 0.021, size = 191, normalized size = 0.3

$$\frac{1}{8d} \sum_{\alpha = \operatorname{RootOf}(\_Z^4 d + c)} \frac{1}{-\alpha^3} \left( -1 \operatorname{Artanh} \left( \frac{2\_alpha^2 b x^2 + 2a}{2} \frac{1}{\sqrt{\frac{ad-bc}{d}}} \frac{1}{\sqrt{bx^4 + a}} \right) \frac{1}{\sqrt{\frac{ad-bc}{d}}} + 2 \frac{d\_alpha^3}{c\sqrt{bx^4 + a}} \sqrt{1 - \frac{i\sqrt{bx^2}}{\sqrt{a}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(1/2)/(d\*x^4+c), x)

[Out] 1/8/d\*sum(1/\_alpha^3\*(-1/((a\*d-b\*c)/d)^(1/2)\*arctanh(1/2\*(2\*\_alpha^2\*b\*x^2+2\*a)/((a\*d-b\*c)/d)^(1/2)/(b\*x^4+a)^(1/2))+2/(I/a^(1/2)\*b^(1/2))^^(1/2)\*\_alpha^3\*d/c\*(1-I/a^(1/2)\*b^(1/2)\*x^2)^(1/2)\*(1+I/a^(1/2)\*b^(1/2)\*x^2)^(1/2)/(b\*x^4+a)^(1/2)\*EllipticPi(x\*(I/a^(1/2)\*b^(1/2))^^(1/2), I\*a^(1/2)/b^(1/2)\*\_alpha^2/c\*d, (-I/a^(1/2)\*b^(1/2))^^(1/2)/(I/a^(1/2)\*b^(1/2))^^(1/2)), \_alpha=RootOf(\_Z^4\*d+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + a}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(b\*x^4 + a)\*(d\*x^4 + c)), x, algorithm="maxima")

[Out] integrate(1/(sqrt(b\*x^4 + a)\*(d\*x^4 + c)), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)),x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bx^4}(c + dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)**(1/2)/(d*x**4+c),x)`

[Out] `Integral(1/(sqrt(a + b*x**4)*(c + d*x**4)), x)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + a}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)), x)`

$$3.83 \quad \int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)} dx$$

**Optimal.** Leaf size=969

result too large to display

```
[Out] (b*x)/(2*a*(b*c - a*d)*Sqrt[a + b*x^4]) - (d*ArcTan[(Sqrt[-((Sqrt
[-c]*(b - (a*d)/c))/Sqrt[d]])*x]/Sqrt[a + b*x^4])]/(4*c*(b*c - a*
d)*Sqrt[(b*c - a*d)/(Sqrt[-c]*Sqrt[d])]) - (d*ArcTan[(Sqrt[(Sqrt[
-c]*(b - (a*d)/c))/Sqrt[d])*x]/Sqrt[a + b*x^4])]/(4*c*(b*c - a*d)
*Sqrt[-((b*c - a*d)/(Sqrt[-c]*Sqrt[d])])) + (b^(3/4)*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF
[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/((4*a^(5/4)*(b*c - a*d)*Sqrt
[a + b*x^4]) - (b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4
)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/
4)], 1/2])/((4*a^(1/4)*(Sqrt[b]*c - Sqrt[a]*Sqrt[-c]*Sqrt[d])*(b*c
- a*d)*Sqrt[a + b*x^4]) - (b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/((4*a^(1/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]*S
qrt[d])*(b*c - a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] + Sqrt[
a]*Sqrt[d])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])
^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/((8*a^(1/4)*b^(1/4)*c*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqr
t[d])*(b*c - a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] - Sqrt[a]
*Sqrt[d])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/
(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/((8*a^(1/4)*b^(1/4)*c*(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d
])*(b*c - a*d)*Sqrt[a + b*x^4]))
```

**Rubi [A]** time = 2.40571, antiderivative size = 969, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\begin{aligned} & \frac{bx}{2a(bc - ad)\sqrt{bx^4 + a}} - \frac{d \tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{-c}(b - \frac{ad}{c})}{\sqrt{d}}}x}{\sqrt{bx^4 + a}}\right)}{4c(bc - ad)\sqrt{\frac{bc - ad}{\sqrt{-c}\sqrt{d}}}} - \frac{d \tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{-c}(b - \frac{ad}{c})}{\sqrt{d}}}x}{\sqrt{bx^4 + a}}\right)}{4c(bc - ad)\sqrt{-\frac{bc - ad}{\sqrt{-c}\sqrt{d}}}} \\ & - \frac{\sqrt[4]{bd}(\sqrt{bx^2 + \sqrt{a}})\sqrt{\frac{bx^4 + a}{(\sqrt{bx^2 + \sqrt{a}})^2}}F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4\sqrt[4]{a}(\sqrt{bc} - \sqrt{a}\sqrt{-c}\sqrt{d})(bc - ad)\sqrt{bx^4 + a}} \\ & - \frac{\sqrt[4]{bd}(\sqrt{bx^2 + \sqrt{a}})\sqrt{\frac{bx^4 + a}{(\sqrt{bx^2 + \sqrt{a}})^2}}F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4\sqrt[4]{a}(\sqrt{bc} + \sqrt{a}\sqrt{-c}\sqrt{d})(bc - ad)\sqrt{bx^4 + a}} \\ & + \frac{b^{3/4}(\sqrt{bx^2 + \sqrt{a}})\sqrt{\frac{bx^4 + a}{(\sqrt{bx^2 + \sqrt{a}})^2}}F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4a^{5/4}(bc - ad)\sqrt{bx^4 + a}} \\ & + \frac{(\sqrt{b}\sqrt{-c} + \sqrt{a}\sqrt{d})d(\sqrt{bx^2 + \sqrt{a}})\sqrt{\frac{bx^4 + a}{(\sqrt{bx^2 + \sqrt{a}})^2}}\left(-\frac{(\sqrt{b}\sqrt{-c} - \sqrt{a}\sqrt{d})^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}}; 2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{bc}(\sqrt{b}\sqrt{-c} - \sqrt{a}\sqrt{d})(bc - ad)\sqrt{bx^4 + a}} \\ & + \frac{(\sqrt{b}\sqrt{-c} - \sqrt{a}\sqrt{d})d(\sqrt{bx^2 + \sqrt{a}})\sqrt{\frac{bx^4 + a}{(\sqrt{bx^2 + \sqrt{a}})^2}}\left(\frac{(\sqrt{b}\sqrt{-c} + \sqrt{a}\sqrt{d})^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}}; 2 \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{bc}(\sqrt{b}\sqrt{-c} + \sqrt{a}\sqrt{d})(bc - ad)\sqrt{bx^4 + a}} \end{aligned}$$

Warning: Unable to verify antiderivative.

[In] Int[1/((a + b\*x^4)^(3/2)\*(c + d\*x^4)), x]

```
[Out] (b*x)/(2*a*(b*c - a*d)*Sqrt[a + b*x^4]) - (d*ArcTan[(Sqrt[-((Sqrt
[-c]*(b - (a*d)/c))/Sqrt[d]])*x]/Sqrt[a + b*x^4])]/(4*c*(b*c - a*
d)*Sqrt[(b*c - a*d)/(Sqrt[-c]*Sqrt[d])]) - (d*ArcTan[(Sqrt[(Sqrt[
-c]*(b - (a*d)/c))/Sqrt[d]]*x]/Sqrt[a + b*x^4])]/(4*c*(b*c - a*d)
*Sqrt[-((b*c - a*d)/(Sqrt[-c]*Sqrt[d])])) + (b^(3/4)*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF
[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*(b*c - a*d)*Sqrt
[a + b*x^4]) - (b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4
)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/
4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c - Sqrt[a]*Sqrt[-c]*Sqrt[d])*(b*c
- a*d)*Sqrt[a + b*x^4]) - (b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1
/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]*S
qrt[d])*(b*c - a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] + Sqrt[
a]*Sqrt[d])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])
^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqr
t[d])*(b*c - a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] - Sqrt[a]
*Sqrt[d])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/
(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d
])*(b*c - a*d)*Sqrt[a + b*x^4])
```

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b*x**4+a)**(3/2)/(d*x**4+c), x)
```

```
[Out] Timed out
```

**Mathematica [C]** time = 0.495144, size = 342, normalized size = 0.35

$$x \left( \frac{9bcdx^4 F_1\left(\frac{5}{4}, \frac{1}{2}, \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{(c+dx^4)\left(2x^4\left(2adF_1\left(\frac{9}{4}, \frac{1}{2}, \frac{13}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + bcF_1\left(\frac{9}{4}, \frac{3}{2}, \frac{13}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) - 9acF_1\left(\frac{5}{4}, \frac{1}{2}, \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right)} + \frac{25c(bc - dx^4)}{(c+dx^4)\left(2x^4\left(2adF_1\left(\frac{5}{4}, \frac{1}{2}, \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) - 9acF_1\left(\frac{5}{4}, \frac{1}{2}, \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right)} \right) \frac{1}{10\sqrt{a+bx^4}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(3/2)*(c + d*x^4)), x]
```

```
[Out] (x*((-5*b)/a + (25*c*(b*c - 2*a*d)*AppellF1[1/4, 1/2, 1, 5/4, -((
b*x^4)/a), -((d*x^4)/c)])/((c + d*x^4)*(-5*a*c*AppellF1[1/4, 1/2,
1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*x^4*(2*a*d*AppellF1[5/4,
1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2,
1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])) + (9*b*c*d*x^4*AppellF1[
5/4, 1/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/((c + d*x^4)*(-9*a
*c*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*x^4
*(2*a*d*AppellF1[9/4, 1/2, 2, 13/4, -((b*x^4)/a), -((d*x^4)/c)] +
b*c*AppellF1[9/4, 3/2, 1, 13/4, -((b*x^4)/a), -((d*x^4)/c)])))/
/(10*(-(b*c) + a*d)*Sqrt[a + b*x^4])
```

**Maple [C]** time = 0.043, size = 313, normalized size = 0.3

$$\begin{aligned}
 & -\frac{bx}{2a(ad-bc)} \frac{1}{\sqrt{\left(x^4 + \frac{a}{b}\right)b}} \\
 & -\frac{b}{2a(ad-bc)} \sqrt{1-ix^2\sqrt{b}} \frac{1}{\sqrt{a}} \sqrt{1+ix^2\sqrt{b}} \frac{1}{\sqrt{a}} \operatorname{EllipticF}\left(x\sqrt{i\sqrt{b}} \frac{1}{\sqrt{a}}, i\right) \frac{1}{\sqrt{i\sqrt{b}} \frac{1}{\sqrt{a}}} \frac{1}{\sqrt{bx^4+a}} \\
 & +\frac{1}{8} \sum_{\text{alpha}=\operatorname{RootOf}(\_Z^4+d+c)} \frac{1}{(ad-bc)\_alpha^3} \left( -1 \operatorname{Artanh}\left(\frac{2\_alpha^2 bx^2+2a}{2} \frac{1}{\sqrt{\frac{ad-bc}{d}}} \frac{1}{\sqrt{bx^4+a}}\right) \frac{1}{\sqrt{\frac{ad-bc}{d}}} + 2 \frac{-\_alpha^3 d}{c\sqrt{bx^4+a}} \sqrt{1-} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(3/2)/(d\*x^4+c), x)

[Out] 
$$\begin{aligned}
 & -1/2*b/a*x/(a*d-b*c)/((x^4+a/b)*b)^(1/2)-1/2*b/a/(a*d-b*c)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*\operatorname{EllipticF}(x*(I/a^(1/2)*b^(1/2))^(1/2), I)+1/8*\sum(1/(a*d-b*c)/\_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*\operatorname{arctanh}(1/2*(2*\_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*\_alpha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*\operatorname{EllipticPi}(x*(I/a^(1/2)*b^(1/2))^(1/2), I*a^(1/2)/b^(1/2)*\_alpha^2/c*d, (-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)), \_alpha=\operatorname{RootOf}(\_Z^4*d+c))
 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{3}{2}}(dx^4+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4+a)^(3/2)\*(d\*x^4+c)), x, algorithm="maxima")

[Out] integrate(1/((b\*x^4+a)^(3/2)\*(d\*x^4+c)), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4+a)^(3/2)\*(d\*x^4+c)), x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^4)^{\frac{3}{2}}(c+dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)**(3/2)/(d*x**4+c),x)`

[Out] `Integral(1/((a + b*x**4)**(3/2)*(c + d*x**4)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{3}{2}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)), x)`

$$3.84 \quad \int \frac{1}{(a+bx^4)^{5/2}(c+dx^4)} dx$$

**Optimal.** Leaf size=1029

result too large to display

```
[Out] (b*x)/(6*a*(b*c - a*d)*(a + b*x^4)^(3/2)) + (b*(5*b*c - 11*a*d)*x
)/(12*a^2*(b*c - a*d)^2*Sqrt[a + b*x^4]) + (d^2*ArcTan[(Sqrt[-((S
qrt[-c]*(b - (a*d)/c))/Sqrt[d]])*x]/Sqrt[a + b*x^4])]/(4*c*(b*c -
a*d)^2*Sqrt[(b*c - a*d)/(Sqrt[-c]*Sqrt[d])]) + (d^2*ArcTan[(Sqrt
[(Sqrt[-c]*(b - (a*d)/c))/Sqrt[d]]*x)/Sqrt[a + b*x^4])]/(4*c*(b*c
- a*d)^2*Sqrt[-((b*c - a*d)/(Sqrt[-c]*Sqrt[d])])) + (b^(1/4)*d^2
*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)
^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]/(4*a^(1/4)*(Sqr
t[b]*c - Sqrt[a]*Sqrt[-c]*Sqrt[d])*(b*c - a*d)^2*Sqrt[a + b*x^4]
) + (b^(1/4)*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]
)/(4*a^(1/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d])*(b*c - a*d)^2
*Sqrt[a + b*x^4]) + (b^(3/4)*(5*b*c - 11*a*d)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcT
an[(b^(1/4)*x)/a^(1/4)], 1/2])/(24*a^(9/4)*(b*c - a*d)^2*Sqrt[a +
b*x^4]) - ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*d^2*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticP
i[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqr
t[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b
^(1/4)*c*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*Sqrt[a
+ b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*d^2*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqr
t[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b
^(1/4)*c*(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*Sqrt[a
+ b*x^4])
```

**Rubi [A]** time = 4.38356, antiderivative size = 1029, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{-c}(b-ad)}{c}}x}{\sqrt{bx^4+a}}\right)d^2}{4c(bc-ad)^2\sqrt{\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{-c}(b-ad)}{c}}x}{\sqrt{bx^4+a}}\right)d^2}{4c(bc-ad)^2\sqrt{-\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}}$$

$$+ \frac{\sqrt[4]{b}\left(\sqrt{bx^2+\sqrt{a}}\right)\sqrt{\frac{bx^4+a}{(\sqrt{bx^2+\sqrt{a}})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)d^2}{4\sqrt[4]{a}\left(\sqrt{bc}-\sqrt{a}\sqrt{-c}\sqrt{d}\right)(bc-ad)^2\sqrt{bx^4+a}}$$

$$+ \frac{\sqrt[4]{b}\left(\sqrt{bx^2+\sqrt{a}}\right)\sqrt{\frac{bx^4+a}{(\sqrt{bx^2+\sqrt{a}})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)d^2}{4\sqrt[4]{a}\left(\sqrt{bc}+\sqrt{a}\sqrt{-c}\sqrt{d}\right)(bc-ad)^2\sqrt{bx^4+a}}$$

$$- \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)\left(\sqrt{bx^2+\sqrt{a}}\right)\sqrt{\frac{bx^4+a}{(\sqrt{bx^2+\sqrt{a}})^2}}\left(-\frac{(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d})^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)d^2}{8\sqrt[4]{a}\sqrt[4]{bc}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)^2\sqrt{bx^4+a}}$$

$$- \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\left(\sqrt{bx^2+\sqrt{a}}\right)\sqrt{\frac{bx^4+a}{(\sqrt{bx^2+\sqrt{a}})^2}}\left(\frac{(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d})^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)d^2}{8\sqrt[4]{a}\sqrt[4]{bc}\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)(bc-ad)^2\sqrt{bx^4+a}}$$

$$+ \frac{b^{3/4}(5bc-11ad)\left(\sqrt{bx^2+\sqrt{a}}\right)\sqrt{\frac{bx^4+a}{(\sqrt{bx^2+\sqrt{a}})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{24a^{9/4}(bc-ad)^2\sqrt{bx^4+a}}$$

$$+ \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{bx^4+a}} + \frac{bx}{6a(bc-ad)(bx^4+a)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Int[1/((a + b\*x^4)^(5/2)\*(c + d\*x^4)),x]

[Out] (b\*x)/(6\*a\*(b\*c - a\*d)\*(a + b\*x^4)^(3/2)) + (b\*(5\*b\*c - 11\*a\*d)\*x)/(12\*a^2\*(b\*c - a\*d)^2\*Sqrt[a + b\*x^4]) + (d^2\*ArcTan[(Sqrt[-((Sqrt[-c]\*(b - (a\*d)/c))/Sqrt[d]])\*x)/Sqrt[a + b\*x^4]])/(4\*c\*(b\*c - a\*d)^2\*Sqrt[(b\*c - a\*d)/(Sqrt[-c]\*Sqrt[d])]) + (d^2\*ArcTan[(Sqrt[(Sqrt[-c]\*(b - (a\*d)/c))/Sqrt[d]]\*x)/Sqrt[a + b\*x^4]])/(4\*c\*(b\*c - a\*d)^2\*Sqrt[-((b\*c - a\*d)/(Sqrt[-c]\*Sqrt[d])])]) + (b^(1/4)\*d^2\*(Sqrt[a] + Sqrt[b]\*x^2)\*Sqrt[(a + b\*x^4)/(Sqrt[a] + Sqrt[b]\*x^2)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*x)/a^(1/4)], 1/2])/(4\*a^(1/4)\*(Sqrt[b]\*c - Sqrt[a]\*Sqrt[-c]\*Sqrt[d])\*(b\*c - a\*d)^2\*Sqrt[a + b\*x^4]) + (b^(1/4)\*d^2\*(Sqrt[a] + Sqrt[b]\*x^2)\*Sqrt[(a + b\*x^4)/(Sqrt[a] + Sqrt[b]\*x^2)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*x)/a^(1/4)], 1/2])/(4\*a^(1/4)\*(Sqrt[b]\*c + Sqrt[a]\*Sqrt[-c]\*Sqrt[d])\*(b\*c - a\*d)^2\*Sqrt[a + b\*x^4]) + (b^(3/4)\*(5\*b\*c - 11\*a\*d)\*(Sqrt[a] + Sqrt[b]\*x^2)\*Sqrt[(a + b\*x^4)/(Sqrt[a] + Sqrt[b]\*x^2)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*x)/a^(1/4)], 1/2])/(24\*a^(9/4)\*(b\*c - a\*d)^2\*Sqrt[a + b\*x^4]) - ((Sqrt[b]\*Sqrt[-c] + Sqrt[a]\*Sqrt[d])\*d^2\*(Sqrt[a] + Sqrt[b]\*x^2)\*Sqrt[(a + b\*x^4)/(Sqrt[a] + Sqrt[b]\*x^2)^2]\*EllipticPi[-(Sqrt[b]\*Sqrt[-c] - Sqrt[a]\*Sqrt[d])^2/(4\*Sqrt[a]\*Sqrt[b]\*Sqrt[-c]\*Sqrt[d]), 2\*ArcTan[(b^(1/4)\*x)/a^(1/4)], 1/2])/(8\*a^(1/4)\*b^(1/4)\*c\*(Sqrt[b]\*Sqrt[-c] - Sqrt[a]\*Sqrt[d])\*(b\*c - a\*d)^2\*Sqrt[a + b\*x^4]) - ((Sqrt[b]\*Sqrt[-c] - Sqrt[a]\*Sqrt[d])\*d^2\*(Sqrt[a] + Sqrt[b]\*x^2)\*Sqrt[(a + b\*x^4)/(Sqrt[a] + Sqrt[b]\*x^2)^2]\*EllipticPi[(Sqrt[b]\*Sqrt[-c] + Sqrt[a]\*Sqrt[d])^2/(4\*Sqrt[a]\*Sqrt[b]\*Sqrt[-c]\*Sqrt[d]), 2\*ArcTan[(b^(1/4)\*x)/a^(1/4)], 1/2])/(8\*a^(1/4)\*b^(1/4)\*c\*(Sqrt[b]\*Sqrt[-c] + Sqrt[a]\*Sqrt[d])\*(b\*c - a\*d)^2\*Sqrt[a + b\*x^4])

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*4+a)\*\*(5/2)/(d\*x\*\*4+c),x)

[Out] Timed out

**Mathematica [C]** time = 1.64098, size = 406, normalized size = 0.39

$$x \left( \frac{25ac(12a^2d^2 - 11abcd + 5b^2c^2)F_1\left(\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) - 2x^4(2adF_1\left(\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right))}{(c+dx^4)\left(5acF_1\left(\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) - 2x^4(2adF_1\left(\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right))\right)} + \frac{5b(-13a^2d+ab(7c-11dx^4)+5b^2cx^4)}{a+bx^4} + \frac{c}{(c+dx^4)} \right) \frac{1}{60a^2\sqrt{a+bx^4}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(5/2)\*(c + d\*x^4)),x]

[Out] (x\*((5\*b\*(-13\*a^2\*d + 5\*b^2\*c\*x^4 + a\*b\*(7\*c - 11\*d\*x^4)))/(a + b\*x^4) + (25\*a\*c\*(5\*b^2\*c^2 - 11\*a\*b\*c\*d + 12\*a^2\*d^2)\*AppellF1[1/4, 1/2, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)]/(c + d\*x^4)\*(5\*a\*c\*AppellF1[1/4, 1/2, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] - 2\*x^4\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])))/(c + d\*x^4)\*(-9\*a\*c\*AppellF1[5/4, 1/2, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)]/(c + d\*x^4)\*(9\*a\*c\*AppellF1[5/4, 1/2, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 2\*x^4\*(2\*a\*d\*AppellF1[9/4, 1/2, 2, 13/4, -((b\*x^4)/a), -((d\*x^4)/c)] + b\*c\*AppellF1[9/4, 3/2, 1, 13/4, -((b\*x^4)/a), -((d\*x^4)/c)])))/(60\*a^2\*(b\*c - a\*d)^2\*Sqrt[a + b\*x^4])



**Maple [C]** time = 0.047, size = 371, normalized size = 0.4

$$\begin{aligned}
 & -\frac{x}{6ab(ad-bc)}\sqrt{bx^4+a}\left(x^4+\frac{a}{b}\right)^{-2}-\frac{bx(11ad-5bc)}{12a^2(ad-bc)^2}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}} \\
 & -\frac{b(11ad-5bc)}{12a^2(ad-bc)^2}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\operatorname{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} \\
 & +\frac{d}{8}\sum_{\text{\_alpha}=\operatorname{RootOf}(\text{\_Z}^4d+c)}\frac{1}{(ad-bc)^2\text{\_alpha}^3}\left(-1\operatorname{Artanh}\left(\frac{2\text{\_alpha}^2bx^2+2a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}+2\frac{\text{\_alpha}^3d}{c\sqrt{bx^4+a}}\sqrt{1}\right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(5/2)/(d\*x^4+c),x)

[Out] 
$$\begin{aligned}
 & -1/6/a*x/b/(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2-1/12*b/a^2*x*(11 \\
 & *a*d-5*b*c)/(a*d-b*c)^2/((x^4+a/b)*b)^(1/2)-1/12*b/a^2*(11*a*d-5* \\
 & b*c)/(a*d-b*c)^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x \\
 & ^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*\operatorname{EllipticF}(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/8*d*\operatorname{sum}(1/(a*d-b*c)^2/\text{\_alpha}^3 \\
 & ^3*(-1/((a*d-b*c)/d)^(1/2)*\operatorname{arctanh}(1/2*(2*\text{\_alpha}^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*\text{\_alpha}^3 \\
 & *d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*\operatorname{EllipticPi}(x*(I/a^(1/2)*b^(1/2))^(1/2),I* \\
 & a^(1/2)/b^(1/2)*\text{\_alpha}^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)),\text{\_alpha}=\operatorname{RootOf}(\text{\_Z}^4*d+c))
 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{5}{2}}(dx^4+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4+a)^(5/2)\*(d\*x^4+c)),x, algorithm="maxima")

[Out] integrate(1/((b\*x^4+a)^(5/2)\*(d\*x^4+c)),x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4+a)^(5/2)\*(d\*x^4+c)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^4)^{\frac{5}{2}}(c+dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)**(5/2)/(d*x**4+c),x)`

[Out] `Integral(1/((a + b*x**4)**(5/2)*(c + d*x**4)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{5}{2}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 + c)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 + c)), x)`

$$3.85 \quad \int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2} dx$$

**Optimal.** Leaf size=426

$$\begin{aligned} & \frac{bx\sqrt{a-bx^4}(21a^2d^2-122abcd+77b^2c^2)}{84cd^3} \\ & + \frac{\sqrt[4]{ab^{3/4}}\sqrt{1-\frac{bx^4}{a}}(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3)F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle| -1\right)}{84cd^4\sqrt{a-bx^4}} \\ & - \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+11bc)(bc-ad)^3\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle| -1\right)}{8\sqrt[4]{bc^2d^4}\sqrt{a-bx^4}} \\ & - \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+11bc)(bc-ad)^3\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle| -1\right)}{8\sqrt[4]{bc^2d^4}\sqrt{a-bx^4}} \\ & + \frac{bx(a-bx^4)^{3/2}(11bc-7ad)}{28cd^2} - \frac{x(a-bx^4)^{5/2}(bc-ad)}{4cd(c-dx^4)} \end{aligned}$$

[Out]  $-(b*(77*b^2*c^2 - 122*a*b*c*d + 21*a^2*d^2)*x*\text{Sqrt}[a - b*x^4])/(84*c*d^3) + (b*(11*b*c - 7*a*d)*x*(a - b*x^4)^{(3/2)})/(28*c*d^2) - ((b*c - a*d)*x*(a - b*x^4)^{(5/2)})/(4*c*d*(c - d*x^4)) + (a^{(1/4)}*b^{(3/4)}*(231*b^3*c^3 - 553*a*b^2*c^2*d + 349*a^2*b*c*d^2 + 21*a^3*d^3)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticF}[\text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(84*c*d^4*\text{Sqrt}[a - b*x^4]) - (a^{(1/4)}*(b*c - a*d)^3*(11*b*c + 3*a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[ -((\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]))], \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*d^4*\text{Sqrt}[a - b*x^4]) - (a^{(1/4)}*(b*c - a*d)^3*(11*b*c + 3*a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c])], \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*d^4*\text{Sqrt}[a - b*x^4])$

**Rubi [A]** time = 1.39091, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$

$$\begin{aligned} & \frac{bx\sqrt{a-bx^4}(21a^2d^2-122abcd+77b^2c^2)}{84cd^3} \\ & + \frac{\sqrt[4]{ab^{3/4}}\sqrt{1-\frac{bx^4}{a}}(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3)F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle| -1\right)}{84cd^4\sqrt{a-bx^4}} \\ & - \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+11bc)(bc-ad)^3\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle| -1\right)}{8\sqrt[4]{bc^2d^4}\sqrt{a-bx^4}} \\ & - \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+11bc)(bc-ad)^3\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle| -1\right)}{8\sqrt[4]{bc^2d^4}\sqrt{a-bx^4}} \\ & + \frac{bx(a-bx^4)^{3/2}(11bc-7ad)}{28cd^2} - \frac{x(a-bx^4)^{5/2}(bc-ad)}{4cd(c-dx^4)} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a - b*x^4)^{(7/2)}/(c - d*x^4)^2, x]$

[Out]  $-(b*(77*b^2*c^2 - 122*a*b*c*d + 21*a^2*d^2)*x*\text{Sqrt}[a - b*x^4])/(84*c*d^3) + (b*(11*b*c - 7*a*d)*x*(a - b*x^4)^{(3/2)})/(28*c*d^2) - ((b*c - a*d)*x*(a - b*x^4)^{(5/2)})/(4*c*d*(c - d*x^4)) + (a^{(1/4)}*b^{(3/4)}*(231*b^3*c^3 - 553*a*b^2*c^2*d + 349*a^2*b*c*d^2 + 21*a^3*d^3)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticF}[\text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(84*c*d^4*\text{Sqrt}[a - b*x^4]) - (a^{(1/4)}*(b*c - a*d)^3*(11*b*c + 3*a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[ -((\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]))], \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*d^4*\text{Sqrt}[a - b*x^4]) - (a^{(1/4)}*(b*c - a*d)^3*(11*b*c + 3*a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c])], \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*d^4*\text{Sqrt}[a - b*x^4])$

$$^4 \sqrt{a - b^2 x^4} - (a^{1/4}) (b^2 c - a^2 d)^3 (11 b^2 c + 3 a^2 d) \sqrt{\text{EllipticPi}\left[\frac{\sqrt{a} \sqrt{d}}{\sqrt{b} \sqrt{c}}, \text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]} / (8 b^{1/4} c^2 d^4 \sqrt{a - b^2 x^4})$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((-b*x**4+a)**(7/2)/(-d*x**4+c)**2,x)`

[Out] Timed out

**Mathematica [C]** time = 1.82599, size = 580, normalized size = 1.36

$$x \left( \frac{25a^2(63a^3d^3+63a^2bcd^2-155ab^2c^2d+77b^3c^3)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{2x^4\left(2adF_1\left(\frac{5}{4};\frac{3}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)} + \frac{9ac(105a^4d^3-63a^3bd^2(5c+2dx^4)+a^2b^2cd(775c-494dx^4)+ab^3c^2d^2)}{4cd^3(dx^4-c)} \right) \sqrt{-bx^4+a}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(a - b*x^4)^(7/2)/(c - d*x^4)^2,x]`

[Out]  $(x^5 \left( (25 a^2 (77 b^3 c^3 - 155 a b^2 c^2 d + 63 a^2 b^2 c d^2 + 63 a^3 d^3) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right] / (5 a^2 c^2 \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right] + 2 x^4 (2 a^2 d \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right] + b^2 c \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]) \right) + (9 a^2 c^2 (105 a^4 d^3 + a^2 b^2 c^2 d (775 c - 494 d x^4) - 63 a^3 b^2 d^2 (5 c + 2 d x^4) + 2 b^4 c^2 x^4 (77 c^2 - 110 c d x^4 - 30 d^2 x^8) + a^2 b^3 c^2 (-385 c^2 - 2 c d x^4 + 520 d^2 x^8)) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right] - 10 x^4 (-a + b x^4) (-63 a^2 b^2 c^2 d^2 + 21 a^3 d^3 + a^2 b^2 c^2 d (155 c - 92 d x^4) + b^3 c^2 (-77 c^2 + 44 c d x^4 + 12 d^2 x^8)) (2 a^2 d \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right] + b^2 c \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]) \right) / (c^2 (9 a^2 c^2 \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right] + 2 x^4 (2 a^2 d \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right] + b^2 c \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right])) \right) \right) / (420 d^3 \sqrt{a - b^2 x^4} (c - d x^4))$

**Maple [C]** time = 0.041, size = 540, normalized size = 1.3

$$\begin{aligned} & - \frac{(a^3 d^3 - 3 a^2 c d^2 b + 3 a c^2 d b^2 - c^3 b^3) x \sqrt{-b x^4 + a}}{4 c d^3 (d x^4 - c)} \\ & - \frac{b^3 x^5 \sqrt{-b x^4 + a} - \frac{x}{3 b} \left( -2 \frac{b^3 (2 a d - b c)}{d^3} + \frac{5 a b^3}{7 d^2} \right) \sqrt{-b x^4 + a}}{7 d^2} \\ & + 1 \left( \frac{b^2 (6 a^2 d^2 - 8 c a b d + 3 b^2 c^2)}{d^4} + \frac{b (a^3 d^3 - 3 a^2 c d^2 b + 3 a c^2 d b^2 - c^3 b^3)}{4 d^4 c} + \frac{a}{3 b} \left( -2 \frac{b^3 (2 a d - b c)}{d^3} + \frac{5 a b^3}{7 d^2} \right) \right) \sqrt{1 - x^2 \sqrt{b}} \\ & - \frac{1}{32 d^5 c} \sum_{\alpha = \text{RootOf}(\_Z^4 d - c)} \frac{3 a^4 d^4 + 2 a^3 b d^3 c - 24 a^2 b^2 c^2 d^2 + 30 a b^3 c^3 d - 11 b^4 c^4}{\alpha^3} \left( -1 \text{Artanh} \left( \frac{-2 \alpha b x^2 + 2 a}{2} \frac{1}{\sqrt{\frac{a d - c}{d}}} \right) \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x)`

[Out] 
$$-1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/c/d^3*x*(-b*x^4+a)^{1/2}/(d*x^4-c)-1/7*b^3/d^2*x^5*(-b*x^4+a)^{1/2}-1/3*(-2*b^3/d^3*(2*a*d-b*c)+5/7*b^3/d^2*a)/b*x*(-b*x^4+a)^{1/2}+(b^2*(6*a^2*d^2-8*a*b*c*d+3*b^2*c^2)/d^4+1/4*b/d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/c+1/3*(-2*b^3/d^3*(2*a*d-b*c)+5/7*b^3/d^2*a)/b*a/(1/a^{1/2}*b^{1/2})^{1/2}*(1-b^{1/2}*x^2/a^{1/2})^{1/2}*(1+b^{1/2}*x^2/a^{1/2})^{1/2}/(-b*x^4+a)^{1/2}*EllipticF(x*(1/a^{1/2})^{1/2}*b^{1/2})^{1/2}, I)-1/32/d^5/c*\text{sum}((3*a^4*d^4+2*a^3*b*c*d^3-24*a^2*b^2*c^2*d^2+30*a*b^3*c^3*d-11*b^4*c^4)/\_alpha^3*(-1/((a*d-b*c)/d)^{1/2}*\text{arctanh}(1/2*(-2*\_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^{1/2})/(-b*x^4+a)^{1/2})-2/(1/a^{1/2}*b^{1/2})^{1/2}*\_alpha^3*d/c*(1-b^{1/2}*x^2/a^{1/2})^{1/2}*(1+b^{1/2}*x^2/a^{1/2})^{1/2}/(-b*x^4+a)^{1/2}*EllipticPi(x*(1/a^{1/2})^{1/2}*b^{1/2})^{1/2}, a^{1/2}/b^{1/2}*\_alpha^2/c*d, (-1/a^{1/2}*b^{1/2})^{1/2}/(1/a^{1/2}*b^{1/2})^{1/2}), \_alpha=\text{RootOf}(\_Z^4*d-c)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^4 + a)^{\frac{7}{2}}}{(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2,x, algorithm="maxima")`

[Out] `integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2, x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2,x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**4+a)**(7/2)/(-d*x**4+c)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^4 + a)^{\frac{7}{2}}}{(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2, x)
```

$$3.86 \quad \int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2} dx$$

Optimal. Leaf size=365

$$\begin{aligned} & \frac{\sqrt[4]{ab}^{3/4} \sqrt{1 - \frac{bx^4}{a}} (-3a^2d^2 - 26abcd + 21b^2c^2) F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{12cd^3\sqrt{a-bx^4}} \\ & + \frac{\sqrt[4]{a}\sqrt{1 - \frac{bx^4}{a}}(3ad + 7bc)(bc - ad)^2 \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc}^2d^3\sqrt{a-bx^4}} \\ & + \frac{\sqrt[4]{a}\sqrt{1 - \frac{bx^4}{a}}(3ad + 7bc)(bc - ad)^2 \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc}^2d^3\sqrt{a-bx^4}} \\ & + \frac{bx\sqrt{a-bx^4}(7bc - 3ad)}{12cd^2} - \frac{x(a-bx^4)^{3/2}(bc - ad)}{4cd(c-dx^4)} \end{aligned}$$

[Out] (b\*(7\*b\*c - 3\*a\*d)\*x\*Sqrt[a - b\*x^4])/(12\*c\*d^2) - ((b\*c - a\*d)\*x\*(a - b\*x^4)^(3/2))/(4\*c\*d\*(c - d\*x^4)) - (a^(1/4)\*b^(3/4)\*(21\*b^2\*c^2 - 26\*a\*b\*c\*d - 3\*a^2\*d^2)\*Sqrt[1 - (b\*x^4)/a]\*EllipticF[ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(12\*c\*d^3\*Sqrt[a - b\*x^4]) + (a^(1/4)\*(b\*c - a\*d)^2\*(7\*b\*c + 3\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d^3\*Sqrt[a - b\*x^4]) + (a^(1/4)\*(b\*c - a\*d)^2\*(7\*b\*c + 3\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d^3\*Sqrt[a - b\*x^4])

Rubi [A] time = 1.04876, antiderivative size = 365, normalized size of antiderivative = 1., number of rules used = 10, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$

$$\begin{aligned} & \frac{\sqrt[4]{ab}^{3/4} \sqrt{1 - \frac{bx^4}{a}} (-3a^2d^2 - 26abcd + 21b^2c^2) F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{12cd^3\sqrt{a-bx^4}} \\ & + \frac{\sqrt[4]{a}\sqrt{1 - \frac{bx^4}{a}}(3ad + 7bc)(bc - ad)^2 \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc}^2d^3\sqrt{a-bx^4}} \\ & + \frac{\sqrt[4]{a}\sqrt{1 - \frac{bx^4}{a}}(3ad + 7bc)(bc - ad)^2 \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc}^2d^3\sqrt{a-bx^4}} \\ & + \frac{bx\sqrt{a-bx^4}(7bc - 3ad)}{12cd^2} - \frac{x(a-bx^4)^{3/2}(bc - ad)}{4cd(c-dx^4)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^4)^(5/2)/(c - d\*x^4)^2, x]

[Out] (b\*(7\*b\*c - 3\*a\*d)\*x\*Sqrt[a - b\*x^4])/(12\*c\*d^2) - ((b\*c - a\*d)\*x\*(a - b\*x^4)^(3/2))/(4\*c\*d\*(c - d\*x^4)) - (a^(1/4)\*b^(3/4)\*(21\*b^2\*c^2 - 26\*a\*b\*c\*d - 3\*a^2\*d^2)\*Sqrt[1 - (b\*x^4)/a]\*EllipticF[ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(12\*c\*d^3\*Sqrt[a - b\*x^4]) + (a^(1/4)\*(b\*c - a\*d)^2\*(7\*b\*c + 3\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d^3\*Sqrt[a - b\*x^4]) + (a^(1/4)\*(b\*c - a\*d)^2\*(7\*b\*c + 3\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d^3\*Sqrt[a - b\*x^4])

**Rubi in Sympy [A]** time = 168.627, size = 333, normalized size = 0.91

$$\frac{\sqrt[4]{ab^3} \sqrt{1 - \frac{bx^4}{a}} (3a^2d^2 + 26abcd - 21b^2c^2) F\left(\operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{12cd^3 \sqrt{a - bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (ad - bc)^2 (3ad + 7bc) \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2d^3} \sqrt{a - bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (ad - bc)^2 (3ad + 7bc) \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2d^3} \sqrt{a - bx^4}} - \frac{bx\sqrt{a - bx^4} (3ad - 7bc)}{12cd^2} + \frac{x(a - bx^4)^{\frac{3}{2}} (ad - bc)}{4cd(c - dx^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)`

[Out]  $a^{1/4} b^{3/4} \sqrt{1 - bx^4/a} (3a^2d^2 + 26abcd - 21b^2c^2) \operatorname{elliptic\_f}(\operatorname{asin}(b^{1/4}x/a^{1/4}), -1)/(12c^3d^3 \sqrt{a - bx^4}) + a^{1/4} \sqrt{1 - bx^4/a} (ad - bc)^2 (3ad + 7bc) \operatorname{elliptic\_pi}(-\sqrt{a}\sqrt{d}/(\sqrt{b}\sqrt{c}), \operatorname{asin}(b^{1/4}x/a^{1/4}), -1)/(8b^{1/4}c^2d^3 \sqrt{a - bx^4}) + a^{1/4} \sqrt{1 - bx^4/a} (ad - bc)^2 (3ad + 7bc) \operatorname{elliptic\_pi}(\sqrt{a}\sqrt{d}/(\sqrt{b}\sqrt{c}), \operatorname{asin}(b^{1/4}x/a^{1/4}), -1)/(8b^{1/4}c^2d^3 \sqrt{a - bx^4}) - bx\sqrt{a - bx^4} (3ad - 7bc)/(12c^3d^3) + x(a - bx^4)^{3/2} (ad - bc)/(4cd(c - dx^4))$

**Mathematica [C]** time = 1.3619, size = 491, normalized size = 1.35

$$x \frac{\left(-10x^4(a-bx^4)(3a^2d^2-6abcd+b^2c(7c-4dx^4))\left(2adF_1\left(\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right) - 9ac(15a^3d^2 - 6a^2bd(5c+3dx^4) + ab^2c(35c-16dx^4)) - 6a^2b^2d^2(5c+3dx^4) + 2b^3c^2x^4(-7c+10dx^4)\right) \operatorname{AppellF1}\left[\frac{1}{4}, 1, \frac{5}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] + 2x^4(2a^2d^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] + b^2c \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right]) + (-9a^2c(15a^3d^2 + a^2b^2c(35c - 16dx^4)) - 6a^2b^2d^2(5c + 3dx^4) + 2b^3c^2x^4(-7c + 10dx^4)) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] - 10x^4(a - bx^4)(-6a^2b^2c^2d + 3a^2d^2 + b^2c(7c - 4dx^4)) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] + b^2c \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right])}{c(2x^4(2adF_1\left(\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)) + 9acF_1\left(\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right))} 60d^2 \sqrt{a - bx^4} (dx^4)$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(a - b*x^4)^(5/2)/(c - d*x^4)^2,x]`

[Out]  $(x^4((-25a^2(-7b^2c^2 + 6abc^2d + 9a^2d^2) \operatorname{AppellF1}\left[\frac{1}{4}, 1, \frac{5}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right]) / (5a^2c \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] + 2x^4(2a^2d^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] + b^2c \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right]) + (-9a^2c(15a^3d^2 + a^2b^2c(35c - 16dx^4)) - 6a^2b^2d^2(5c + 3dx^4) + 2b^3c^2x^4(-7c + 10dx^4)) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] - 10x^4(a - bx^4)(-6a^2b^2c^2d + 3a^2d^2 + b^2c(7c - 4dx^4)) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] + b^2c \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right]) / (c(9a^2c \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{5}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] + 2x^4(2a^2d^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] + b^2c \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right]) + (-9a^2c(15a^3d^2 + a^2b^2c(35c - 16dx^4)) - 6a^2b^2d^2(5c + 3dx^4) + 2b^3c^2x^4(-7c + 10dx^4)) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] - 10x^4(a - bx^4)(-6a^2b^2c^2d + 3a^2d^2 + b^2c(7c - 4dx^4)) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] + b^2c \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right]) / (60d^2 \sqrt{a - bx^4} (-c + dx^4))$



**Maple [C]** time = 0.04, size = 412, normalized size = 1.1

$$\begin{aligned}
 & -\frac{(a^2d^2 - 2cabd + b^2c^2)x}{4cd^2(dx^4 - c)}\sqrt{-bx^4 + a} + \frac{b^2x}{3d^2}\sqrt{-bx^4 + a} \\
 & + 1\left(\frac{b^2(3ad - 2bc)}{d^3} + \frac{b(a^2d^2 - 2cabd + b^2c^2)}{4cd^3} - \frac{ab^2}{3d^2}\right)\sqrt{1 - x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1 + x^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}, i\right)\frac{1}{\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}} \\
 & - \frac{1}{32cd^4}\sum_{\alpha=\text{RootOf}(\_Z^4d-c)}\frac{3a^3d^3 + a^2cd^2b - 11ac^2db^2 + 7c^3b^3}{-\alpha^3}\left(-1\text{Artanh}\left(\frac{-2\_alpha^2bx^2 + 2a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4 + a}}\right)\right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^4+a)^(5/2)/(-d\*x^4+c)^2, x)

[Out] 
$$\begin{aligned}
 & -1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c/d^2*x*(-b*x^4+a)^(1/2)/(d*x^4-c) \\
 & +1/3*b^2/d^2*x*(-b*x^4+a)^(1/2)+(b^2*(3*a*d-2*b*c)/d^3+1/4*b/d^4 \\
 & * (a^2*d^2-2*a*b*c*d+b^2*c^2)/c-1/3*b^2/d^2*a)/(1/a^(1/2)*b^(1/2)) \\
 & )^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2) \\
 & /(-b*x^4+a)^(1/2)*\text{EllipticF}(x*(1/a^(1/2)*b^(1/2))^(1/2), I)-1/32 \\
 & /c/d^4*\text{sum}((3*a^3*d^3+a^2*b*c*d^2-11*a*b^2*c^2*d+7*b^3*c^3)/\_alpha \\
 & a^3*(-1/((a*d-b*c)/d)^(1/2)*\text{arctanh}(1/2*(-2*\_alpha^2*b*x^2+2*a)/( \\
 & (a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)* \\
 & \_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2)) \\
 & )^(1/2)/(-b*x^4+a)^(1/2)*\text{EllipticPi}(x*(1/a^(1/2)*b^(1/2))^(1/2), a \\
 & ^{(1/2)}/b^{(1/2)*\_alpha^2/c*d}, (-1/a^{(1/2)*b^{(1/2)}})^{(1/2)}/(1/a^{(1/2)*b^{(1/2)}})^{(1/2)}), \\
 & \_alpha=\text{RootOf}(\_Z^4*d-c))
 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^4 + a)^{\frac{5}{2}}}{(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4 + a)^(5/2)/(d\*x^4 - c)^2, x, algorithm="maxima")

[Out] integrate((-b\*x^4 + a)^(5/2)/(d\*x^4 - c)^2, x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4 + a)^(5/2)/(d\*x^4 - c)^2, x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^4 + a)^{\frac{5}{2}}}{(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2,x, algorithm="giac")`

[Out] `integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2, x)`

$$3.87 \quad \int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2} dx$$

**Optimal.** Leaf size=309

$$\frac{\sqrt[4]{ab}^{3/4} \sqrt{1 - \frac{bx^4}{a}} (ad + 3bc) F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{4cd^2 \sqrt{a - bx^4}} - \frac{3\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad)(ad + bc) \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2 d^2} \sqrt{a - bx^4}} - \frac{3\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad)(ad + bc) \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2 d^2} \sqrt{a - bx^4}} - \frac{x\sqrt{a - bx^4}(bc - ad)}{4cd(c - dx^4)}$$

[Out]  $-\left(\frac{(b^*c - a^*d)*x*\text{Sqrt}[a - b^*x^4]}{(4^*c^*d^*(c - d^*x^4))} + (a^{(1/4)}*b^{(3/4)}*(3*b^*c + a^*d)*\text{Sqrt}[1 - (b^*x^4)/a]*\text{EllipticF}[\text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(4^*c^*d^2*\text{Sqrt}[a - b^*x^4]) - (3*a^{(1/4)}*(b^*c - a^*d)*(b^*c + a^*d)*\text{Sqrt}[1 - (b^*x^4)/a]*\text{EllipticPi}[-((\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c])), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8^*b^{(1/4)}*c^2*d^2*\text{Sqrt}[a - b^*x^4]) - (3*a^{(1/4)}*(b^*c - a^*d)*(b^*c + a^*d)*\text{Sqrt}[1 - (b^*x^4)/a]*\text{EllipticPi}[(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8^*b^{(1/4)}*c^2*d^2*\text{Sqrt}[a - b^*x^4])$

**Rubi [A]** time = 0.745897, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$

$$\frac{\sqrt[4]{ab}^{3/4} \sqrt{1 - \frac{bx^4}{a}} (ad + 3bc) F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{4cd^2 \sqrt{a - bx^4}} - \frac{3\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad)(ad + bc) \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2 d^2} \sqrt{a - bx^4}} - \frac{3\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad)(ad + bc) \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2 d^2} \sqrt{a - bx^4}} - \frac{x\sqrt{a - bx^4}(bc - ad)}{4cd(c - dx^4)}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^4)^(3/2)/(c - d\*x^4)^2, x]

[Out]  $-\left(\frac{(b^*c - a^*d)*x*\text{Sqrt}[a - b^*x^4]}{(4^*c^*d^*(c - d^*x^4))} + (a^{(1/4)}*b^{(3/4)}*(3*b^*c + a^*d)*\text{Sqrt}[1 - (b^*x^4)/a]*\text{EllipticF}[\text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(4^*c^*d^2*\text{Sqrt}[a - b^*x^4]) - (3*a^{(1/4)}*(b^*c - a^*d)*(b^*c + a^*d)*\text{Sqrt}[1 - (b^*x^4)/a]*\text{EllipticPi}[-((\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c])), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8^*b^{(1/4)}*c^2*d^2*\text{Sqrt}[a - b^*x^4]) - (3*a^{(1/4)}*(b^*c - a^*d)*(b^*c + a^*d)*\text{Sqrt}[1 - (b^*x^4)/a]*\text{EllipticPi}[(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8^*b^{(1/4)}*c^2*d^2*\text{Sqrt}[a - b^*x^4])$

**Rubi in Sympy [A]** time = 107.566, size = 279, normalized size = 0.9

$$\frac{\sqrt[4]{ab^3} \sqrt{1 - \frac{bx^4}{a}} (ad + 3bc) F\left(\operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{4cd^2 \sqrt{a - bx^4}} + \frac{3\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (ad - bc)(ad + bc) \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2 d^2} \sqrt{a - bx^4}} + \frac{3\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (ad - bc)(ad + bc) \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2 d^2} \sqrt{a - bx^4}} + \frac{x\sqrt{a - bx^4} (ad - bc)}{4cd(c - dx^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)`

[Out] `a**(1/4)*b**(3/4)*sqrt(1 - b*x**4/a)*(a*d + 3*b*c)*elliptic_f(asin(b**(1/4)*x/a**(1/4)), -1)/(4*c*d**2*sqrt(a - b*x**4)) + 3*a**(1/4)*sqrt(1 - b*x**4/a)*(a*d - b*c)*(a*d + b*c)*elliptic_pi(-sqrt(a)*sqrt(d)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(8*b**(1/4)*c**2*d**2*sqrt(a - b*x**4)) + 3*a**(1/4)*sqrt(1 - b*x**4/a)*(a*d - b*c)*(a*d + b*c)*elliptic_pi(sqrt(a)*sqrt(d)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(8*b**(1/4)*c**2*d**2*sqrt(a - b*x**4)) + x*sqrt(a - b*x**4)*(a*d - b*c)/(4*c*d*(c - d*x**4))`

**Mathematica [C]** time = 0.593966, size = 423, normalized size = 1.37

$$x \left( \frac{-9ac(5a^2d - ab(5c + 6dx^4)) + 2b^2cx^4 F_1\left(\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right) - 10x^4(a - bx^4)(ad - bc) \left(2adF_1\left(\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right)}{c \left(2x^4 \left(2adF_1\left(\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right) + 9acF_1\left(\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right)} \right) - \frac{2x^4(2adF_1\left(\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right))}{20d\sqrt{a - bx^4}(dx^4 - c)}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(a - b*x^4)^(3/2)/(c - d*x^4)^2,x]`

[Out] `(x*((-25*a^2*(b*c + 3*a*d)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])) + (-9*a*c*(5*a^2*d + 2*b^2*c*x^4 - a*b*(5*c + 6*d*x^4))*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] - 10*(-(b*c) + a*d)*x^4*(a - b*x^4)*(2*a*d*AppellF1[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c]))/(c*(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c])))`

**Maple [C]** time = 0.036, size = 329, normalized size = 1.1

$$-\frac{(ad - bc)x}{4cd(dx^4 - c)} \sqrt{-bx^4 + a} + 1 \left( \frac{b^2}{d^2} + \frac{(ad - bc)b}{4cd^2} \right) \sqrt{1 - x^2\sqrt{b}\frac{1}{\sqrt{a}}} \sqrt{1 + x^2\sqrt{b}\frac{1}{\sqrt{a}}} \operatorname{EllipticF}\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}, i\right) \frac{1}{\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}} \frac{1}{\sqrt{-bx^4 + a}} - \frac{3}{32cd^3} \sum_{\alpha = \operatorname{RootOf}(\_Z^4d - c)} \frac{a^2d^2 - b^2c^2}{-\alpha^3} \left( -1 \operatorname{Artanh}\left(\frac{-2\alpha^2bx^2 + 2a}{2} \frac{1}{\sqrt{\frac{ad-bc}{d}}} \frac{1}{\sqrt{-bx^4 + a}}\right) \frac{1}{\sqrt{\frac{ad-bc}{d}}} - 2 \frac{\alpha^3d}{c\sqrt{-bx^4 + a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x)`

[Out] 
$$-1/4*(a*d-b*c)/c/d*x*(-b*x^4+a)^{(1/2)}/(d*x^4-c)+(b^2/d^2+1/4*b/d^2*(a*d-b*c)/c)/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-b^{(1/2)}*x^2/a^{(1/2)})^{(1/2)}*(1+b^{(1/2)}*x^2/a^{(1/2)})^{(1/2)}/(-b*x^4+a)^{(1/2)}*EllipticF(x*(1/a^{(1/2)}*b^{(1/2)})^{(1/2)},I)-3/32/c/d^3*\sum((a^2*d^2-b^2*c^2)/\_alpha^3*(-1/((a*d-b*c)/d)^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^{(1/2)}/(-b*x^4+a)^{(1/2)})-2/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)})*\_alpha^3*d/c*(1-b^{(1/2)}*x^2/a^{(1/2)})^{(1/2)}*(1+b^{(1/2)}*x^2/a^{(1/2)})^{(1/2)}/(-b*x^4+a)^{(1/2)}*EllipticPi(x*(1/a^{(1/2)}*b^{(1/2)})^{(1/2)},a^{(1/2)}/b^{(1/2)}*\_alpha^2/c*d,(-1/a^{(1/2)}*b^{(1/2)})^{(1/2)}/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}),\_alpha=\operatorname{RootOf}(\_Z^4*d-c))$$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^4 + a)^{\frac{3}{2}}}{(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2,x, algorithm="maxima")`

[Out] `integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2, x)`

**Fricas** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2,x, algorithm="fricas")`

[Out] Timed out

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^4 + a)^{\frac{3}{2}}}{(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2,x, algorithm="giac")`

```
[Out] integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2, x)
```

$$3.88 \quad \int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2} dx$$

**Optimal.** Leaf size=276

$$\frac{\sqrt[4]{ab}^{3/4} \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{4cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}\sqrt{1 - \frac{bx^4}{a}}(bc - 3ad) \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2d}\sqrt{a-bx^4}}$$

$$- \frac{\sqrt[4]{a}\sqrt{1 - \frac{bx^4}{a}}(bc - 3ad) \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2d}\sqrt{a-bx^4}} + \frac{x\sqrt{a-bx^4}}{4c(c-dx^4)}$$

[Out] (x\*Sqrt[a - b\*x^4])/(4\*c\*(c - d\*x^4)) + (a^(1/4)\*b^(3/4)\*Sqrt[1 - (b\*x^4)/a]\*EllipticF[ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(4\*c\*d\*Sqrt[a - b\*x^4]) - (a^(1/4)\*(b\*c - 3\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d\*Sqrt[a - b\*x^4]) - (a^(1/4)\*(b\*c - 3\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d\*Sqrt[a - b\*x^4])

**Rubi [A]** time = 0.620553, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$

$$\frac{\sqrt[4]{ab}^{3/4} \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{4cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}\sqrt{1 - \frac{bx^4}{a}}(bc - 3ad) \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2d}\sqrt{a-bx^4}}$$

$$- \frac{\sqrt[4]{a}\sqrt{1 - \frac{bx^4}{a}}(bc - 3ad) \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2d}\sqrt{a-bx^4}} + \frac{x\sqrt{a-bx^4}}{4c(c-dx^4)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - b\*x^4]/(c - d\*x^4)^2, x]

[Out] (x\*Sqrt[a - b\*x^4])/(4\*c\*(c - d\*x^4)) + (a^(1/4)\*b^(3/4)\*Sqrt[1 - (b\*x^4)/a]\*EllipticF[ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(4\*c\*d\*Sqrt[a - b\*x^4]) - (a^(1/4)\*(b\*c - 3\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d\*Sqrt[a - b\*x^4]) - (a^(1/4)\*(b\*c - 3\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d\*Sqrt[a - b\*x^4])

**Rubi in Sympy [A]** time = 97.1465, size = 243, normalized size = 0.88

$$\frac{\sqrt[4]{ab}^{3/4} \sqrt{1 - \frac{bx^4}{a}} \left(\operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{4cd\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1 - \frac{bx^4}{a}}(3ad - bc) \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2d}\sqrt{a-bx^4}}$$

$$+ \frac{\sqrt[4]{a}\sqrt{1 - \frac{bx^4}{a}}(3ad - bc) \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2d}\sqrt{a-bx^4}} + \frac{x\sqrt{a-bx^4}}{4c(c-dx^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*4+a)\*\*(1/2)/(-d\*x\*\*4+c)\*\*2, x)

[Out] a\*\*(1/4)\*b\*\*(3/4)\*sqrt(1 - b\*x\*\*4/a)\*elliptic\_f(asin(b\*\*(1/4)\*x/a\*\*(1/4)), -1)/(4\*c\*d\*sqrt(a - b\*x\*\*4)) + a\*\*(1/4)\*sqrt(1 - b\*x\*\*4

/a)\*(3\*a\*d - b\*c)\*elliptic\_pi(-sqrt(a)\*sqrt(d)/(sqrt(b)\*sqrt(c)),  
asin(b\*\*(1/4)\*x/a\*\*(1/4)), -1)/(8\*b\*\*(1/4)\*c\*\*2\*d\*sqrt(a - b\*x\*\*  
4)) + a\*\*(1/4)\*sqrt(1 - b\*x\*\*4/a)\*(3\*a\*d - b\*c)\*elliptic\_pi(sqrt(  
a)\*sqrt(d)/(sqrt(b)\*sqrt(c)), asin(b\*\*(1/4)\*x/a\*\*(1/4)), -1)/(8\*b  
\*\*(1/4)\*c\*\*2\*d\*sqrt(a - b\*x\*\*4)) + x\*sqrt(a - b\*x\*\*4)/(4\*c\*(c - d  
\*x\*\*4))

**Mathematica [C]** time = 0.269328, size = 310, normalized size = 1.12

$$x \left( \frac{75a^2 F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{2x^4 \left(2ad F_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bc F_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right) + 5ac F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{bx^4}{a}, \frac{dx^4}{c}\right)} + \frac{9abx^4 F_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{2x^4 \left(2ad F_1\left(\frac{9}{4}; \frac{1}{2}, 2; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bc F_1\left(\frac{9}{4}; \frac{3}{2}, 1; \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right)} \right) \frac{1}{20\sqrt{a - bx^4}(dx^4 - c)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a - b\*x^4]/(c - d\*x^4)^2, x]

[Out] (x\*((-5\*(a - b\*x^4))/c - (75\*a^2\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c])/(5\*a\*c\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c] + 2\*x^4\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, (b\*x^4)/a, (d\*x^4)/c] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c])) + (9\*a\*b\*x^4\*AppellF1[5/4, 1/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c])/(9\*a\*c\*AppellF1[5/4, 1/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c] + 2\*x^4\*(2\*a\*d\*AppellF1[9/4, 1/2, 2, 13/4, (b\*x^4)/a, (d\*x^4)/c] + b\*c\*AppellF1[9/4, 3/2, 1, 13/4, (b\*x^4)/a, (d\*x^4)/c])))/(20\*sqrt[a - b\*x^4]\*(-c + d\*x^4))

**Maple [C]** time = 0.033, size = 294, normalized size = 1.1

$$-\frac{x}{4c(dx^4 - c)}\sqrt{-bx^4 + a} + \frac{b}{4cd}\sqrt{1 - x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1 + x^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}, i\right)\frac{1}{\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4 + a}} - \frac{1}{32cd^2}\sum_{\alpha=\text{RootOf}(\_Z^4d-c)}\frac{3ad-bc}{\alpha^3}\left(-1\text{Artanh}\left(\frac{-2\alpha^2bx^2+2a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}-2\frac{\alpha^3d}{c\sqrt{-bx^4+a}}\sqrt{1-\frac{\alpha^2bx^2+2a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^4+a)^(1/2)/(-d\*x^4+c)^2, x)

[Out] -1/4/c\*x\*(-b\*x^4+a)^(1/2)/(d\*x^4-c)+1/4\*b/c/d/(1/a^(1/2)\*b^(1/2))^(1/2)\*(1-b^(1/2)\*x^2/a^(1/2))^(1/2)\*(1+b^(1/2)\*x^2/a^(1/2))^(1/2)/(-b\*x^4+a)^(1/2)\*EllipticF(x\*(1/a^(1/2)\*b^(1/2))^(1/2), I)-1/32/c/d^2\*sum((3\*a\*d-b\*c)/\_alpha^3\*(-1/((a\*d-b\*c)/d)^(1/2)\*arctanh(1/2\*(-2\*\_alpha^2\*b\*x^2+2\*a)/((a\*d-b\*c)/d)^(1/2)/(-b\*x^4+a)^(1/2))-2/(1/a^(1/2)\*b^(1/2))^(1/2)\*\_alpha^3\*d/c\*(1-b^(1/2)\*x^2/a^(1/2))^(1/2)\*(1+b^(1/2)\*x^2/a^(1/2))^(1/2)/(-b\*x^4+a)^(1/2)\*EllipticPi(x\*(1/a^(1/2)\*b^(1/2))^(1/2), a^(1/2)/b^(1/2)\*\_alpha^2/c\*d, (-1/a^(1/2)\*b^(1/2))^(1/2)/(1/a^(1/2)\*b^(1/2))^(1/2)), \_alpha=RootOf(\_Z^4\*d-c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^4 + a}}{(dx^4 - c)^2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2,x, algorithm="maxima")`

[Out] `integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2, x)`

---

**Fricas** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2,x, algorithm="fricas")`

[Out] Timed out

---

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)`

[Out] Timed out

---

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^4 + a}}{(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2,x, algorithm="giac")`

[Out] `integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2, x)`

$$3.89 \quad \int \frac{1}{\sqrt{a-bx^4(c-dx^4)^2}} dx$$

**Optimal.** Leaf size=310

$$\begin{aligned} & -\frac{\sqrt[4]{ab}^{3/4} \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{4c\sqrt{a-bx^4}(bc-ad)} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (5bc-3ad) \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2} \sqrt{a-bx^4}(bc-ad)} \\ & + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (5bc-3ad) \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2} \sqrt{a-bx^4}(bc-ad)} - \frac{dx\sqrt{a-bx^4}}{4c(c-dx^4)(bc-ad)} \end{aligned}$$

[Out]  $-(d*x*\text{Sqrt}[a - b*x^4])/(4*c*(b*c - a*d)*(c - d*x^4)) - (a^{(1/4)}*b^{(3/4)}*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticF}[\text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(4*c*(b*c - a*d)*\text{Sqrt}[a - b*x^4]) + (a^{(1/4)}*(5*b*c - 3*a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[-((\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c])), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*(b*c - a*d)*\text{Sqrt}[a - b*x^4]) + (a^{(1/4)}*(5*b*c - 3*a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*(b*c - a*d)*\text{Sqrt}[a - b*x^4])$

**Rubi [A]** time = 0.688348, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$

$$\begin{aligned} & -\frac{\sqrt[4]{ab}^{3/4} \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{4c\sqrt{a-bx^4}(bc-ad)} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (5bc-3ad) \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2} \sqrt{a-bx^4}(bc-ad)} \\ & + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (5bc-3ad) \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2} \sqrt{a-bx^4}(bc-ad)} - \frac{dx\sqrt{a-bx^4}}{4c(c-dx^4)(bc-ad)} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(\text{Sqrt}[a - b*x^4]*(c - d*x^4)^2), x]$

[Out]  $-(d*x*\text{Sqrt}[a - b*x^4])/(4*c*(b*c - a*d)*(c - d*x^4)) - (a^{(1/4)}*b^{(3/4)}*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticF}[\text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(4*c*(b*c - a*d)*\text{Sqrt}[a - b*x^4]) + (a^{(1/4)}*(5*b*c - 3*a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[-((\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c])), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*(b*c - a*d)*\text{Sqrt}[a - b*x^4]) + (a^{(1/4)}*(5*b*c - 3*a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*(b*c - a*d)*\text{Sqrt}[a - b*x^4])$

**Rubi in Sympy [A]** time = 109.539, size = 270, normalized size = 0.87

$$\begin{aligned} & \frac{\sqrt[4]{ab}^{3/4} \sqrt{1 - \frac{bx^4}{a}} F\left(\text{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{4c\sqrt{a-bx^4}(ad-bc)} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (3ad-5bc) \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \text{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2} \sqrt{a-bx^4}(ad-bc)} \\ & + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (3ad-5bc) \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \text{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2} \sqrt{a-bx^4}(ad-bc)} + \frac{dx\sqrt{a-bx^4}}{4c(c-dx^4)(ad-bc)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/(-b*x**4+a)**(1/2)/(-d*x**4+c)**2, x)$

[Out]  $a^{(1/4)}*b^{(3/4)}*\text{sqrt}(1 - b*x^4/a)*\text{elliptic\_f}(\text{asin}(b^{(1/4)}*x/a^{(1/4)}), -1)/(4*c*\text{sqrt}(a - b*x^4)*(a*d - b*c)) + a^{(1/4)}*\text{sqrt}($

$1 - b*x^{**4}/a)^*(3*a*d - 5*b*c)*\text{elliptic\_pi}(-\text{sqrt}(a)*\text{sqrt}(d)/(\text{sqrt}(b)*\text{sqrt}(c)), \text{asin}(b^{**}(1/4)*x/a^{**}(1/4)), -1)/(8*b^{**}(1/4)*c^{**2}*\text{sqrt}(a - b*x^{**4})*(a*d - b*c)) + a^{**}(1/4)*\text{sqrt}(1 - b*x^{**4}/a)^*(3*a*d - 5*b*c)*\text{elliptic\_pi}(\text{sqrt}(a)*\text{sqrt}(d)/(\text{sqrt}(b)*\text{sqrt}(c)), \text{asin}(b^{**}(1/4)*x/a^{**}(1/4)), -1)/(8*b^{**}(1/4)*c^{**2}*\text{sqrt}(a - b*x^{**4})*(a*d - b*c)) + d*x*\text{sqrt}(a - b*x^{**4})/(4*c*(c - d*x^{**4})*(a*d - b*c))$

**Mathematica [C]** time = 0.991384, size = 349, normalized size = 1.13

$$x \left( \frac{9abd x^4 F_1\left(\frac{5}{4}, \frac{1}{2}, 1; \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{(ad-bc)\left(2x^4\left(2adF_1\left(\frac{9}{4}, \frac{1}{2}, 2; \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{9}{4}, \frac{3}{2}, 1; \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right) + 9acF_1\left(\frac{5}{4}, \frac{1}{2}, 1; \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)} + \frac{25a(3ad-4bc)F_1\left(\frac{1}{4}, \frac{1}{2}, 1; \frac{5}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{(bc-ad)\left(2x^4\left(2adF_1\left(\frac{5}{4}, \frac{1}{2}, 2; \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}, \frac{3}{2}, 1; \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right) + 9acF_1\left(\frac{1}{4}, \frac{1}{2}, 1; \frac{5}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)} \right) / (20\sqrt{a-bx^4}(dx^4-c))$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[a - b\*x^4]\*(c - d\*x^4)^2), x]

[Out]  $(x*((5*d*(a - b*x^4))/(c*(b*c - a*d)) + (25*a*(-4*b*c + 3*a*d)*\text{AppellF1}[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/((b*c - a*d)^*(5*a*c*\text{AppellF1}[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*\text{AppellF1}[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*\text{AppellF1}[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))) + (9*a*b*d*x^4*\text{AppellF1}[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/((-b*c) + a*d)^*(9*a*c*\text{AppellF1}[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*\text{AppellF1}[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*\text{AppellF1}[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c]))))/(20*\text{Sqrt}[a - b*x^4]*(-c + d*x^4))$

**Maple [C]** time = 0.035, size = 322, normalized size = 1.

$$\begin{aligned} & -\frac{dx}{(4ad-4bc)c(dx^4-c)}\sqrt{-bx^4+a} \\ & + \frac{b}{(4ad-4bc)c}\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}, i\right)\frac{1}{\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4+a}} \\ & - \frac{1}{32cd}\sum_{\alpha=\text{RootOf}(\_Z^4d-c)}\frac{3ad-5bc}{(ad-bc)\alpha^3}\left(-1\text{Artanh}\left(\frac{-2\alpha^2bx^2+2a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}-2\frac{\alpha}{c\sqrt{-bx^4+a}}\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^4+a)^(1/2)/(-d\*x^4+c)^2, x)

[Out]  $-1/4*d/(a*d-b*c)/c*x*(-b*x^4+a)^(1/2)/(d*x^4-c)+1/4*b/(a*d-b*c)/c/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*\text{EllipticF}(x*(1/a^(1/2)*b^(1/2))^(1/2), I)-1/32/c/d*\text{sum}((3*a*d-5*b*c)/(a*d-b*c)/\alpha^3*(-1/((a*d-b*c)/d)^(1/2)*\text{arctanh}(1/2*(-2*\alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*\alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*\text{EllipticPi}(x*(1/a^(1/2)*b^(1/2))^(1/2), a^(1/2)/b^(1/2)*\alpha^2/c*d, (-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)), \alpha=\text{RootOf}(\_Z^4*d-c))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx^4+a}(dx^4-c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2), x)`

---

**Fricas** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2),x, algorithm="fricas")`

[Out] Timed out

---

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)`

[Out] Timed out

---

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx^4 + a}(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2), x)`

$$3.90 \quad \int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2} dx$$

**Optimal.** Leaf size=362

$$\frac{b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (ad + 2bc) F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{4a^{3/4} c \sqrt{a - bx^4} (bc - ad)^2} - \frac{3\sqrt[4]{ad} \sqrt{1 - \frac{bx^4}{a}} (3bc - ad) \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2} \sqrt{a - bx^4} (bc - ad)^2} - \frac{3\sqrt[4]{ad} \sqrt{1 - \frac{bx^4}{a}} (3bc - ad) \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2} \sqrt{a - bx^4} (bc - ad)^2} + \frac{bx(ad + 2bc)}{4ac \sqrt{a - bx^4} (bc - ad)^2} - \frac{dx}{4c \sqrt{a - bx^4} (c - dx^4) (bc - ad)}$$

[Out] (b\*(2\*b\*c + a\*d)\*x)/(4\*a\*c\*(b\*c - a\*d)^2\*Sqrt[a - b\*x^4]) - (d\*x)/(4\*c\*(b\*c - a\*d)\*Sqrt[a - b\*x^4]\*(c - d\*x^4)) + (b^(3/4)\*(2\*b\*c + a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticF[ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(4\*a^(3/4)\*c\*(b\*c - a\*d)^2\*Sqrt[a - b\*x^4]) - (3\*a^(1/4)\*d\*(3\*b\*c - a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*(b\*c - a\*d)^2\*Sqrt[a - b\*x^4]) - (3\*a^(1/4)\*d\*(3\*b\*c - a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*(b\*c - a\*d)^2\*Sqrt[a - b\*x^4])

**Rubi [A]** time = 1.07374, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$

$$\frac{b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (ad + 2bc) F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{4a^{3/4} c \sqrt{a - bx^4} (bc - ad)^2} - \frac{3\sqrt[4]{ad} \sqrt{1 - \frac{bx^4}{a}} (3bc - ad) \left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2} \sqrt{a - bx^4} (bc - ad)^2} - \frac{3\sqrt[4]{ad} \sqrt{1 - \frac{bx^4}{a}} (3bc - ad) \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2} \sqrt{a - bx^4} (bc - ad)^2} + \frac{bx(ad + 2bc)}{4ac \sqrt{a - bx^4} (bc - ad)^2} - \frac{dx}{4c \sqrt{a - bx^4} (c - dx^4) (bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a - b\*x^4)^(3/2)\*(c - d\*x^4)^2), x]

[Out] (b\*(2\*b\*c + a\*d)\*x)/(4\*a\*c\*(b\*c - a\*d)^2\*Sqrt[a - b\*x^4]) - (d\*x)/(4\*c\*(b\*c - a\*d)\*Sqrt[a - b\*x^4]\*(c - d\*x^4)) + (b^(3/4)\*(2\*b\*c + a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticF[ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(4\*a^(3/4)\*c\*(b\*c - a\*d)^2\*Sqrt[a - b\*x^4]) - (3\*a^(1/4)\*d\*(3\*b\*c - a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*(b\*c - a\*d)^2\*Sqrt[a - b\*x^4]) - (3\*a^(1/4)\*d\*(3\*b\*c - a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*(b\*c - a\*d)^2\*Sqrt[a - b\*x^4])

**Rubi in Sympy [A]** time = 175.193, size = 323, normalized size = 0.89

$$\frac{3\sqrt[4]{ad}\sqrt{1-\frac{bx^4}{a}}(ad-3bc)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle| -1\right)}{8\sqrt[4]{bc^2}\sqrt{a-bx^4}(ad-bc)^2} + \frac{3\sqrt[4]{ad}\sqrt{1-\frac{bx^4}{a}}(ad-3bc)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle| -1\right)}{8\sqrt[4]{bc^2}\sqrt{a-bx^4}(ad-bc)^2} + \frac{dx}{4c\sqrt{a-bx^4}(c-dx^4)(ad-bc)}$$

$$+ \frac{bx(ad+2bc)}{4ac\sqrt{a-bx^4}(ad-bc)^2} + \frac{b^{\frac{3}{4}}\sqrt{1-\frac{bx^4}{a}}(ad+2bc)F\left(\operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\middle| -1\right)}{4a^{\frac{3}{4}}c\sqrt{a-bx^4}(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)`

[Out]  $3*a^{1/4}*d*\sqrt{1-b*x^4/a}*(a*d-3*b*c)*\operatorname{elliptic\_pi}(-\sqrt{a}*\sqrt{d}/(\sqrt{b}*\sqrt{c}), \operatorname{asin}(b^{1/4}*x/a^{1/4}), -1)/(8*b^{1/4}*c^{1/4}) + 3*a^{1/4}*d*\sqrt{1-b*x^4/a}*(a*d-3*b*c)*\operatorname{elliptic\_pi}(\sqrt{a}*\sqrt{d}/(\sqrt{b}*\sqrt{c}), \operatorname{asin}(b^{1/4}*x/a^{1/4}), -1)/(8*b^{1/4}*c^{1/4}) + d*x/(4*c*\sqrt{a-b*x^4}*(c-d*x^4)) + b*x*(a*d+2*b*c)/(4*a*c*\sqrt{a-b*x^4}*(a*d-b*c)^2) + b^{3/4}*\sqrt{1-b*x^4/a}*(a*d+2*b*c)*\operatorname{elliptic\_f}(\operatorname{asin}(b^{1/4}*x/a^{1/4}), -1)/(4*a^{3/4}*c*\sqrt{a-b*x^4}*(a*d-b*c)^2)$

**Mathematica [C]** time = 1.18553, size = 465, normalized size = 1.28

$$x \left( \frac{25(3a^2d^2-8abcd+2b^2c^2)F_1\left(\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{2x^4\left(2adF_1\left(\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right) + 5acF_1\left(\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)} + \frac{9ac(5a^2d^2-6abd^2x^4+2b^2c(5c-6dx^4))F_1\left(\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{ac\left(2x^4\left(2adF_1\left(\frac{5}{4}, \frac{1}{2}, 2, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}, \frac{3}{2}, 1, \frac{13}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right)\right)} \right) / (20\sqrt{a-bx^4}(c-dx^4)(bc-ad)^2)$$

Warning: Unable to verify antiderivative.

[In] `Integrate[1/((a-b*x^4)^(3/2)*(c-d*x^4)^2),x]`

[Out]  $(x*((25*(2*b^2*c^2-8*a*b*c*d+3*a^2*d^2)*\operatorname{AppellF1}[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/(5*a*c*\operatorname{AppellF1}[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*\operatorname{AppellF1}[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*\operatorname{AppellF1}[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])) + (9*a*c*(5*a^2*d^2-6*a*b*d^2*x^4+2*b^2*c*(5*c-6*d*x^4))*\operatorname{AppellF1}[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] - 10*x^4*(-(a^2*d^2)+a*b*d^2*x^4-2*b^2*c*(c-d*x^4))*\operatorname{AppellF1}[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*\operatorname{AppellF1}[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c]))/(20*(b*c-a*d)^2*\sqrt{a-b*x^4}*(c-d*x^4))$

**Maple [C]** time = 0.058, size = 375, normalized size = 1.

$$\begin{aligned}
 & -\frac{d^2x}{4(ad-bc)^2c(dx^4-c)}\sqrt{-bx^4+a} + \frac{b^2x}{2a(ad-bc)^2}\frac{1}{\sqrt{-(x^4-\frac{a}{b})b}} \\
 & + 1\left(\frac{bd}{4(ad-bc)^2c} + \frac{b^2}{2a(ad-bc)^2}\right)\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}, i\right)\frac{1}{\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4+a}} \\
 & -\frac{3}{32c}\sum_{\alpha=\text{RootOf}(\_Z^4d-c)}\frac{ad-3bc}{(ad-bc)^2\alpha^3}\left(-1\text{Artanh}\left(\frac{-2\alpha^2bx^2+2a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}-2\frac{\alpha}{c\sqrt{-bx^4}}\right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^4+a)^(3/2)/(-d\*x^4+c)^2,x)

[Out] 
$$\begin{aligned}
 & -1/4*d^2/(a*d-b*c)^2/c*x*(-b*x^4+a)^{(1/2)}/(d*x^4-c)+1/2*b^2/a*x/( \\
 & a*d-b*c)^2/(-(x^4-a/b)*b)^{(1/2)}+(1/4*d*b/(a*d-b*c)^2/c+1/2*b^2/a/ \\
 & (a*d-b*c)^2)/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-b^{(1/2)}*x^2/a^{(1/2)})^{(1/2)} \\
 & *(1+b^{(1/2)}*x^2/a^{(1/2)})^{(1/2)}/(-b*x^4+a)^{(1/2)}*\text{EllipticF}(x*(1 \\
 & /a^{(1/2)}*b^{(1/2)})^{(1/2)}, I)-3/32/c*\text{sum}((a*d-3*b*c)/(a*d-b*c)^2/_ \\
 & \alpha^3*(-1/((a*d-b*c)/d)^{(1/2)}*\text{arctanh}(1/2*(-2*_ \\
 & \alpha^2*b*x^2+2*a)/( \\
 & (a*d-b*c)/d)^{(1/2)}/(-b*x^4+a)^{(1/2)})-2/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)} \\
 & )*_ \\
 & \alpha^3*d/c*(1-b^{(1/2)}*x^2/a^{(1/2)})^{(1/2)}*(1+b^{(1/2)}*x^2/a^{(1/2)})^{(1/2)}/(-b*x^4+a)^{(1/2)}*\text{EllipticPi}(x*(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}, a^{(1/2)}/b^{(1/2)}*_ \\
 & \alpha^2/c*d, (-1/a^{(1/2)}*b^{(1/2)})^{(1/2)}/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}), \alpha=\text{RootOf}(\_Z^4*d-c))
 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^4+a)^{\frac{3}{2}}(dx^4-c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^4+a)^(3/2)\*(d\*x^4-c)^2),x, algorithm="maxima")

[Out] integrate(1/((-b\*x^4+a)^(3/2)\*(d\*x^4-c)^2), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^4+a)^(3/2)\*(d\*x^4-c)^2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^4 + a)^{\frac{3}{2}}(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2),x, algorithm="giac")`

[Out] `integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2), x)`



$$3.91 \quad \int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2} dx$$

**Optimal.** Leaf size=439

$$\begin{aligned} & \frac{bx(-3a^2d^2 - 17abcd + 5b^2c^2)}{12a^2c\sqrt{a-bx^4}(bc-ad)^3} + \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}(-3a^2d^2 - 17abcd + 5b^2c^2)F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx^4}}{\sqrt[4]{a}}\right)\middle| -1\right)}{12a^{7/4}c\sqrt{a-bx^4}(bc-ad)^3} \\ & + \frac{\sqrt[4]{ad^2}\sqrt{1-\frac{bx^4}{a}}(13bc-3ad)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^4}}{\sqrt[4]{a}}\right)\middle| -1\right)}{8\sqrt[4]{bc^2}\sqrt{a-bx^4}(bc-ad)^3} \\ & + \frac{\sqrt[4]{ad^2}\sqrt{1-\frac{bx^4}{a}}(13bc-3ad)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^4}}{\sqrt[4]{a}}\right)\middle| -1\right)}{8\sqrt[4]{bc^2}\sqrt{a-bx^4}(bc-ad)^3} \\ & - \frac{dx}{4c(a-bx^4)^{3/2}(c-dx^4)(bc-ad)} + \frac{bx(3ad+2bc)}{12ac(a-bx^4)^{3/2}(bc-ad)^2} \end{aligned}$$

[Out] (b\*(2\*b\*c + 3\*a\*d)\*x)/(12\*a\*c\*(b\*c - a\*d)^2\*(a - b\*x^4)^(3/2)) + (b\*(5\*b^2\*c^2 - 17\*a\*b\*c\*d - 3\*a^2\*d^2)\*x)/(12\*a^2\*c\*(b\*c - a\*d)^3\*Sqrt[a - b\*x^4]) - (d\*x)/(4\*c\*(b\*c - a\*d)\*(a - b\*x^4)^(3/2)\*(c - d\*x^4)) + (b^(3/4)\*(5\*b^2\*c^2 - 17\*a\*b\*c\*d - 3\*a^2\*d^2)\*Sqrt[1 - (b\*x^4)/a]\*EllipticF[ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(12\*a^(7/4)\*c\*(b\*c - a\*d)^3\*Sqrt[a - b\*x^4]) + (a^(1/4)\*d^2\*(13\*b\*c - 3\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*(b\*c - a\*d)^3\*Sqrt[a - b\*x^4]) + (a^(1/4)\*d^2\*(13\*b\*c - 3\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*(b\*c - a\*d)^3\*Sqrt[a - b\*x^4])

**Rubi [A]** time = 1.43859, antiderivative size = 439, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$

$$\begin{aligned} & \frac{bx(-3a^2d^2 - 17abcd + 5b^2c^2)}{12a^2c\sqrt{a-bx^4}(bc-ad)^3} + \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}(-3a^2d^2 - 17abcd + 5b^2c^2)F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx^4}}{\sqrt[4]{a}}\right)\middle| -1\right)}{12a^{7/4}c\sqrt{a-bx^4}(bc-ad)^3} \\ & + \frac{\sqrt[4]{ad^2}\sqrt{1-\frac{bx^4}{a}}(13bc-3ad)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^4}}{\sqrt[4]{a}}\right)\middle| -1\right)}{8\sqrt[4]{bc^2}\sqrt{a-bx^4}(bc-ad)^3} \\ & + \frac{\sqrt[4]{ad^2}\sqrt{1-\frac{bx^4}{a}}(13bc-3ad)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^4}}{\sqrt[4]{a}}\right)\middle| -1\right)}{8\sqrt[4]{bc^2}\sqrt{a-bx^4}(bc-ad)^3} \\ & - \frac{dx}{4c(a-bx^4)^{3/2}(c-dx^4)(bc-ad)} + \frac{bx(3ad+2bc)}{12ac(a-bx^4)^{3/2}(bc-ad)^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((a - b\*x^4)^(5/2)\*(c - d\*x^4)^2), x]

[Out] (b\*(2\*b\*c + 3\*a\*d)\*x)/(12\*a\*c\*(b\*c - a\*d)^2\*(a - b\*x^4)^(3/2)) + (b\*(5\*b^2\*c^2 - 17\*a\*b\*c\*d - 3\*a^2\*d^2)\*x)/(12\*a^2\*c\*(b\*c - a\*d)^3\*Sqrt[a - b\*x^4]) - (d\*x)/(4\*c\*(b\*c - a\*d)\*(a - b\*x^4)^(3/2)\*(c - d\*x^4)) + (b^(3/4)\*(5\*b^2\*c^2 - 17\*a\*b\*c\*d - 3\*a^2\*d^2)\*Sqrt[1 - (b\*x^4)/a]\*EllipticF[ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(12\*a^(7/4)\*c\*(b\*c - a\*d)^3\*Sqrt[a - b\*x^4]) + (a^(1/4)\*d^2\*(13\*b\*c - 3\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*(b\*c - a\*d)^3\*Sqrt[a - b\*x^4]) + (a^(1/4)\*d^2\*(13\*b\*c - 3\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*(b\*c - a\*d)^3\*Sqrt[a - b\*x^4])

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)`

[Out] Timed out

**Mathematica [C]** time = 2.18446, size = 617, normalized size = 1.41

$$x \left( \frac{25a(9a^3d^3 - 36a^2bcd^2 + 17ab^2c^2d - 5b^3c^3)F_1\left(\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{2x^4\left(2adF_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right) + 5acF_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)} + \frac{10x^4(3a^4d^3 - 6a^3bd^3x^4 + a^2b^2d(19c^2 - 19cdx^4 + 3d^2x^8) + ab^3c(-19c^2 + 19cdx^4 - 3d^2x^8) + b^4c^3)}{2x^4(2adF_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + 5acF_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right))} \right)$$

Warning: Unable to verify antiderivative.

[In] `Integrate[1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x]`

[Out]  $(x*((25*a*(-5*b^3*c^3 + 17*a*b^2*c^2*d - 36*a^2*b*c*d^2 + 9*a^3*d^3)*\text{AppellF1}[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/(5*a*c*\text{AppellF1}[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*\text{AppellF1}[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*\text{AppellF1}[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])) + (9*a*c*(15*a^4*d^3 - 33*a^3*b*d^3*x^4 + 5*b^4*c^2*x^4*(5*c - 6*d*x^4) + a^2*b^2*d*(95*c^2 - 112*c*d*x^4 + 18*d^2*x^8) + a*b^3*c*(-35*c^2 - 45*c*d*x^4 + 102*d^2*x^8))*\text{AppellF1}[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + 10*x^4*(3*a^4*d^3 - 6*a^3*b*d^3*x^4 + 5*b^4*c^2*x^4*(c - d*x^4) + a^2*b^2*d*(19*c^2 - 19*c*d*x^4 + 3*d^2*x^8) + a*b^3*c*(-7*c^2 - 10*c*d*x^4 + 17*d^2*x^8))* (2*a*d*\text{AppellF1}[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*\text{AppellF1}[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c]))/(c*(a - b*x^4)*(9*a*c*\text{AppellF1}[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*\text{AppellF1}[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*\text{AppellF1}[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c])))))/(60*a^2*(-(b*c) + a*d)^3*\text{Sqrt}[a - b*x^4]*(c - d*x^4))$

**Maple [C]** time = 0.057, size = 484, normalized size = 1.1

$$\begin{aligned} & -\frac{bd^3x}{(4a^2d^2 - 8cabd + 4b^2c^2)(ad - bc)c(bdx^4 - bc)}\sqrt{-bx^4 + a} \\ & + \frac{x}{6(ad - bc)^2a}\sqrt{-bx^4 + a}\left(x^4 - \frac{a}{b}\right)^{-2} + \frac{b^2x(17ad - 5bc)}{12a^2(ad - bc)^3}\frac{1}{\sqrt{-(x^4 - \frac{a}{b})b}} \\ & + 1\left(\frac{d^2b}{(4a^2d^2 - 8cabd + 4b^2c^2)(ad - bc)c} + \frac{b^2(17ad - 5bc)}{12a^2(ad - bc)^3}\right)\sqrt{1 - x^2\sqrt{b}}\frac{1}{\sqrt{a}}\sqrt{1 + x^2\sqrt{b}}\frac{1}{\sqrt{a}}\text{EllipticF}\left(x\sqrt{1\sqrt{b}}\frac{1}{\sqrt{a}}, i\right)\frac{1}{\sqrt{1\sqrt{b}}}} \\ & - \frac{d}{32c}\sum_{\alpha = \text{RootOf}(-Z^4d - c)}\frac{3ad - 13bc}{(ad - bc)^3 - \alpha^3}\left(-1\text{Artanh}\left(\frac{-2\alpha^2bx^2 + 2a}{2}\frac{1}{\sqrt{\frac{ad - bc}{d}}}\frac{1}{\sqrt{-bx^4 + a}}\right)\frac{1}{\sqrt{\frac{ad - bc}{d}}} - 2\frac{\alpha}{c\sqrt{-bx^4}}\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x)`

[Out]  $-1/4*b*d^3/(a^2*d^2 - 2*a*b*c*d + b^2*c^2)/(a*d - b*c)/c*x*(-b*x^4+a)^(1/2)/(b*d*x^4 - b*c) + 1/6/(a*d - b*c)^2/a*x*(-b*x^4+a)^(1/2)/(x^4 - a/b)$

$$\begin{aligned} & ^2+1/12*b^2/a^2*x*(17*a*d-5*b*c)/(a*d-b*c)^3/(-(x^4-a/b)*b)^(1/2) \\ & +(1/4*b*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/c+1/12*b^2/a^2* \\ & (17*a*d-5*b*c)/(a*d-b*c)^3)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)* \\ & x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2) \\ & *EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2), I)-1/32*d/c*sum((3*a*d-13* \\ & b*c)/(a*d-b*c)^3/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2 \\ & *_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a \\ & ^2*(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)* \\ & (1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a \\ & ^2*(1/a^(1/2)*b^(1/2))^(1/2), a^(1/2)/b^(1/2)*_alpha^2/c*d, (-1/a^(1/2)*b^( \\ & 1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))), _alpha=RootOf(_Z^4*d-c)) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^4 + a)^{\frac{5}{2}}(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^4 + a)^(5/2)\*(d\*x^4 - c)^2), x, algorithm="maxima")

[Out] integrate(1/((-b\*x^4 + a)^(5/2)\*(d\*x^4 - c)^2), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^4 + a)^(5/2)\*(d\*x^4 - c)^2), x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x\*\*4+a)\*\*(5/2)/(-d\*x\*\*4+c)\*\*2, x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^4 + a)^{\frac{5}{2}}(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^4 + a)^(5/2)\*(d\*x^4 - c)^2), x, algorithm="giac")

[Out] integrate(1/((-b\*x^4 + a)^(5/2)\*(d\*x^4 - c)^2), x)

$$3.92 \quad \int \frac{\sqrt{a+bx^4}}{ac-bcx^4} dx$$

**Optimal.** Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{bc}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{bc}}$$

[Out] ArcTan[(Sqrt[2]\*a^(1/4)\*b^(1/4)\*x)/Sqrt[a + b\*x^4]]/(2\*Sqrt[2]\*a^(1/4)\*b^(1/4)\*c) + ArcTanh[(Sqrt[2]\*a^(1/4)\*b^(1/4)\*x)/Sqrt[a + b\*x^4]]/(2\*Sqrt[2]\*a^(1/4)\*b^(1/4)\*c)

**Rubi [A]** time = 0.141018, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{bc}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{bc}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x^4]/(a\*c - b\*c\*x^4), x]

[Out] ArcTan[(Sqrt[2]\*a^(1/4)\*b^(1/4)\*x)/Sqrt[a + b\*x^4]]/(2\*Sqrt[2]\*a^(1/4)\*b^(1/4)\*c) + ArcTanh[(Sqrt[2]\*a^(1/4)\*b^(1/4)\*x)/Sqrt[a + b\*x^4]]/(2\*Sqrt[2]\*a^(1/4)\*b^(1/4)\*c)

**Rubi in Sympy [A]** time = 22.7375, size = 94, normalized size = 0.91

$$\frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{4\sqrt[4]{a}\sqrt[4]{bc}} + \frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{4\sqrt[4]{a}\sqrt[4]{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)\*\*(1/2)/(-b\*c\*x\*\*4+a\*c), x)

[Out] sqrt(2)\*atan(sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*x/sqrt(a + b\*x\*\*4))/(4\*a\*\*(1/4)\*b\*\*(1/4)\*c) + sqrt(2)\*atanh(sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*x/sqrt(a + b\*x\*\*4))/(4\*a\*\*(1/4)\*b\*\*(1/4)\*c)

**Mathematica [C]** time = 0.242136, size = 155, normalized size = 1.5

$$\frac{5ax\sqrt{a+bx^4}F_1\left(\frac{1}{4}; -\frac{1}{2}, 1; \frac{5}{4}; -\frac{bx^4}{a}, \frac{bx^4}{a}\right)}{c(a-bx^4)\left(2bx^4\left(2F_1\left(\frac{5}{4}; -\frac{1}{2}, 2; \frac{9}{4}; -\frac{bx^4}{a}, \frac{bx^4}{a}\right) + F_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; -\frac{bx^4}{a}, \frac{bx^4}{a}\right)\right) + 5aF_1\left(\frac{1}{4}; -\frac{1}{2}, 1; \frac{5}{4}; -\frac{bx^4}{a}, \frac{bx^4}{a}\right)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a + b\*x^4]/(a\*c - b\*c\*x^4), x]

[Out] (5\*a\*x\*Sqrt[a + b\*x^4]\*AppellF1[1/4, -1/2, 1, 5/4, -(b\*x^4)/a], (b\*x^4)/a))/(c\*(a - b\*x^4)\*(5\*a\*AppellF1[1/4, -1/2, 1, 5/4, -(b\*x^4)/a], (b\*x^4)/a) + 2\*b\*x^4\*(2\*AppellF1[5/4, -1/2, 2, 9/4, -(b\*x^4)/a], (b\*x^4)/a) + AppellF1[5/4, 1/2, 1, 9/4, -(b\*x^4)/a], (b\*x^4)/a))

**Maple [A]** time = 0.024, size = 103, normalized size = 1.

$$-\frac{\sqrt{2}}{4c} \arctan\left(\frac{\sqrt{2}\sqrt{bx^4+a}}{2x} \frac{1}{\sqrt[4]{ab}}\right) \frac{1}{\sqrt[4]{ab}} + \frac{\sqrt{2}}{8c} \ln\left(1 + \left(\frac{\sqrt{2}\sqrt{bx^4+a} + \sqrt[4]{ab}}{2x}\right) \left(\frac{\sqrt{2}\sqrt{bx^4+a} - \sqrt[4]{ab}}{2x}\right)^{-1}\right) \frac{1}{\sqrt[4]{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(1/2)/(-b\*c\*x^4+a\*c), x)

[Out] -1/4/c\*2^(1/2)/(a\*b)^(1/4)\*arctan(1/2\*(b\*x^4+a)^(1/2)\*2^(1/2)/x/(a\*b)^(1/4))+1/8/c\*2^(1/2)/(a\*b)^(1/4)\*ln((1/2\*(b\*x^4+a)^(1/2)\*2^(1/2)/x+(a\*b)^(1/4))/(1/2\*(b\*x^4+a)^(1/2)\*2^(1/2)/x-(a\*b)^(1/4)))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{bx^4+a}}{bcx^4-ac} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sqrt(b\*x^4 + a)/(b\*c\*x^4 - a\*c), x, algorithm="maxima")

[Out] -integrate(sqrt(b\*x^4 + a)/(b\*c\*x^4 - a\*c), x)

**Fricas [A]** time = 0.592124, size = 450, normalized size = 4.37

$$\begin{aligned} & -\left(\frac{1}{4}\right)^{\frac{1}{4}} \left(\frac{1}{abc^4}\right)^{\frac{1}{4}} \arctan\left(\frac{2\left(2\left(\frac{1}{4}\right)^{\frac{3}{4}} abc^3 x^3 \left(\frac{1}{abc^4}\right)^{\frac{3}{4}} - \left(\frac{1}{4}\right)^{\frac{1}{4}} acx \left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\right)}{\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}} - x^2\right) - \frac{bx^4-a}{\sqrt{b}}}\right) \\ & + \frac{1}{4} \left(\frac{1}{4}\right)^{\frac{1}{4}} \left(\frac{1}{abc^4}\right)^{\frac{1}{4}} \log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}} abc^3 x^3 \left(\frac{1}{abc^4}\right)^{\frac{3}{4}} + 2\left(\frac{1}{4}\right)^{\frac{1}{4}} acx \left(\frac{1}{abc^4}\right)^{\frac{1}{4}} + \sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}} + x^2\right)}{bx^4-a}\right) \\ & - \frac{1}{4} \left(\frac{1}{4}\right)^{\frac{1}{4}} \left(\frac{1}{abc^4}\right)^{\frac{1}{4}} \log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}} abc^3 x^3 \left(\frac{1}{abc^4}\right)^{\frac{3}{4}} + 2\left(\frac{1}{4}\right)^{\frac{1}{4}} acx \left(\frac{1}{abc^4}\right)^{\frac{1}{4}} - \sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}} + x^2\right)}{bx^4-a}\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sqrt(b\*x^4 + a)/(b\*c\*x^4 - a\*c), x, algorithm="fricas")

[Out] -(1/4)^(1/4)\*(1/(a\*b\*c^4))^(1/4)\*arctan(2\*(2\*(1/4)^(3/4)\*a\*b\*c^3\*x^3\*(1/(a\*b\*c^4))^(3/4) - (1/4)^(1/4)\*a\*c\*x\*(1/(a\*b\*c^4))^(1/4))/(sqrt(b\*x^4 + a)\*(a\*c^2\*sqrt(1/(a\*b\*c^4)) - x^2) - (b\*x^4 - a)/sqrt(b))) + 1/4\*(1/4)^(1/4)\*(1/(a\*b\*c^4))^(1/4)\*log((4\*(1/4)^(3/4)\*a\*b\*c^3\*x^3\*(1/(a\*b\*c^4))^(3/4) + 2\*(1/4)^(1/4)\*a\*c\*x\*(1/(a\*b\*c^4))^(1/4) + sqrt(b\*x^4 + a)\*(a\*c^2\*sqrt(1/(a\*b\*c^4)) + x^2))/(b\*x^4 - a) - 1/4\*(1/4)^(1/4)\*(1/(a\*b\*c^4))^(1/4)\*log(-(4\*(1/4)^(3/4)\*a\*b\*c^3\*x^3\*(1/(a\*b\*c^4))^(3/4) + 2\*(1/4)^(1/4)\*a\*c\*x\*(1/(a\*b\*c^4))^(1/4) - sqrt(b\*x^4 + a)\*(a\*c^2\*sqrt(1/(a\*b\*c^4)) + x^2))/(b\*x^4 - a))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^4}}{-a+bx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(1/2)/(-b\*c\*x\*\*4+a\*c), x)

[Out] -Integral(sqrt(a + b\*x\*\*4)/(-a + b\*x\*\*4), x)/c

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{bx^4 + a}}{bcx^4 - ac} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sqrt(b\*x^4 + a)/(b\*c\*x^4 - a\*c), x, algorithm="giac")

[Out] integrate(-sqrt(b\*x^4 + a)/(b\*c\*x^4 - a\*c), x)

$$3.93 \quad \int \frac{\sqrt{a-bx^4}}{ac+bcx^4} dx$$

**Optimal.** Leaf size=116

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}(\sqrt{a+\sqrt{bx^2}})}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{bc}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}(\sqrt{a-\sqrt{bx^2}})}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{bc}}$$

[Out] ArcTan[(b^(1/4)\*x\*(Sqrt[a] + Sqrt[b]\*x^2))/(a^(1/4)\*Sqrt[a - b\*x^4])]/(2\*a^(1/4)\*b^(1/4)\*c) + ArcTanh[(b^(1/4)\*x\*(Sqrt[a] - Sqrt[b]\*x^2))/(a^(1/4)\*Sqrt[a - b\*x^4])]/(2\*a^(1/4)\*b^(1/4)\*c)

**Rubi [A]** time = 0.0790384, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{bx}(\sqrt{a+\sqrt{bx^2}})}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{bc}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bx}(\sqrt{a-\sqrt{bx^2}})}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{bc}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - b\*x^4]/(a\*c + b\*c\*x^4), x]

[Out] ArcTan[(b^(1/4)\*x\*(Sqrt[a] + Sqrt[b]\*x^2))/(a^(1/4)\*Sqrt[a - b\*x^4])]/(2\*a^(1/4)\*b^(1/4)\*c) + ArcTanh[(b^(1/4)\*x\*(Sqrt[a] - Sqrt[b]\*x^2))/(a^(1/4)\*Sqrt[a - b\*x^4])]/(2\*a^(1/4)\*b^(1/4)\*c)

**Rubi in Sympy [A]** time = 12.6542, size = 100, normalized size = 0.86

$$\frac{\operatorname{atan}\left(\frac{\sqrt[4]{bx}(\sqrt{a+\sqrt{bx^2}})}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{bc}} + \frac{\operatorname{atanh}\left(\frac{\sqrt[4]{bx}(\sqrt{a-\sqrt{bx^2}})}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*4+a)\*\*(1/2)/(b\*c\*x\*\*4+a\*c), x)

[Out] atan(b\*\*(1/4)\*x\*(sqrt(a) + sqrt(b)\*x\*\*2)/(a\*\*(1/4)\*sqrt(a - b\*x\*\*4)))/(2\*a\*\*(1/4)\*b\*\*(1/4)\*c) + atanh(b\*\*(1/4)\*x\*(sqrt(a) - sqrt(b)\*x\*\*2)/(a\*\*(1/4)\*sqrt(a - b\*x\*\*4)))/(2\*a\*\*(1/4)\*b\*\*(1/4)\*c)

**Mathematica [C]** time = 0.271918, size = 155, normalized size = 1.34

$$\frac{5ax\sqrt{a-bx^4}F_1\left(\frac{1}{4}; -\frac{1}{2}, 1; \frac{5}{4}, \frac{bx^4}{a}, -\frac{bx^4}{a}\right)}{c(a+bx^4)\left(5aF_1\left(\frac{1}{4}; -\frac{1}{2}, 1; \frac{5}{4}, \frac{bx^4}{a}, -\frac{bx^4}{a}\right) - 2bx^4\left(2F_1\left(\frac{5}{4}; -\frac{1}{2}, 2; \frac{9}{4}, \frac{bx^4}{a}, -\frac{bx^4}{a}\right) + F_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}, \frac{bx^4}{a}, -\frac{bx^4}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a - b\*x^4]/(a\*c + b\*c\*x^4), x]

[Out] (5\*a\*x\*Sqrt[a - b\*x^4]\*AppellF1[1/4, -1/2, 1, 5/4, (b\*x^4)/a, -((b\*x^4)/a)])/(c\*(a + b\*x^4)\*(5\*a\*AppellF1[1/4, -1/2, 1, 5/4, (b\*x^4)/a, -((b\*x^4)/a)]))

4)/a, -((b\*x^4)/a)] - 2\*b\*x^4\*(2\*AppellF1[5/4, -1/2, 2, 9/4, (b\*x^4)/a, -((b\*x^4)/a)] + AppellF1[5/4, 1/2, 1, 9/4, (b\*x^4)/a, -((b\*x^4)/a)]))

**Maple [A]** time = 0.025, size = 158, normalized size = 1.4

$$-\frac{1}{8c} \ln \left( 1 \left( \frac{-bx^4 + a}{2x^2} - \frac{1}{x} \sqrt[4]{ab} \sqrt{-bx^4 + a} + \sqrt{ab} \right) \left( \frac{-bx^4 + a}{2x^2} + \frac{1}{x} \sqrt[4]{ab} \sqrt{-bx^4 + a} + \sqrt{ab} \right)^{-1} \right) \frac{1}{\sqrt[4]{ab}}$$

$$-\frac{1}{4c} \arctan \left( \frac{1}{x} \sqrt{-bx^4 + a} \frac{1}{\sqrt[4]{ab}} + 1 \right) \frac{1}{\sqrt[4]{ab}} + \frac{1}{4c} \arctan \left( -\frac{1}{x} \sqrt{-bx^4 + a} \frac{1}{\sqrt[4]{ab}} + 1 \right) \frac{1}{\sqrt[4]{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^4+a)^(1/2)/(b\*c\*x^4+a\*c), x)

[Out] -1/8/c/(a\*b)^(1/4)\*ln((1/2\*(-b\*x^4+a)/x^2-(a\*b)^(1/4)\*(-b\*x^4+a)^(1/2)/x+(a\*b)^(1/2))/(1/2\*(-b\*x^4+a)/x^2+(a\*b)^(1/4)\*(-b\*x^4+a)^(1/2)/x+(a\*b)^(1/2)))-1/4/c/(a\*b)^(1/4)\*arctan(1/(a\*b)^(1/4)\*(-b\*x^4+a)^(1/2)/x+1)+1/4/c/(a\*b)^(1/4)\*arctan(-1/(a\*b)^(1/4)\*(-b\*x^4+a)^(1/2)/x+1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^4 + a}}{bcx^4 + ac} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-b\*x^4 + a)/(b\*c\*x^4 + a\*c), x, algorithm="maxima")

[Out] integrate(sqrt(-b\*x^4 + a)/(b\*c\*x^4 + a\*c), x)

**Fricas [A]** time = 0.586726, size = 500, normalized size = 4.31

$$\left( \frac{1}{4} \right)^{\frac{1}{4}} \left( -\frac{1}{abc^4} \right)^{\frac{1}{4}} \arctan \left( \frac{2 \left( 2 \left( \frac{1}{4} \right)^{\frac{3}{4}} abc^3 x^3 \left( -\frac{1}{abc^4} \right)^{\frac{3}{4}} + \left( \frac{1}{4} \right)^{\frac{1}{4}} acx \left( -\frac{1}{abc^4} \right)^{\frac{1}{4}} \right)}{\sqrt{-bx^4 + a} ac^2 \sqrt{-\frac{1}{abc^4}} - \sqrt{-bx^4 + ax^2} + (bx^4 + a) \sqrt{-\frac{1}{b}}} \right)$$

$$-\frac{1}{4} \left( \frac{1}{4} \right)^{\frac{1}{4}} \left( -\frac{1}{abc^4} \right)^{\frac{1}{4}} \log \left( \frac{4 \left( \frac{1}{4} \right)^{\frac{3}{4}} abc^3 x^3 \left( -\frac{1}{abc^4} \right)^{\frac{3}{4}} + \sqrt{-bx^4 + a} ac^2 \sqrt{-\frac{1}{abc^4}} - 2 \left( \frac{1}{4} \right)^{\frac{1}{4}} acx \left( -\frac{1}{abc^4} \right)^{\frac{1}{4}} + \sqrt{-bx^4 + ax^2}}{bx^4 + a} \right)$$

$$+\frac{1}{4} \left( \frac{1}{4} \right)^{\frac{1}{4}} \left( -\frac{1}{abc^4} \right)^{\frac{1}{4}} \log \left( \frac{4 \left( \frac{1}{4} \right)^{\frac{3}{4}} abc^3 x^3 \left( -\frac{1}{abc^4} \right)^{\frac{3}{4}} - \sqrt{-bx^4 + a} ac^2 \sqrt{-\frac{1}{abc^4}} - 2 \left( \frac{1}{4} \right)^{\frac{1}{4}} acx \left( -\frac{1}{abc^4} \right)^{\frac{1}{4}} - \sqrt{-bx^4 + ax^2}}{bx^4 + a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-b\*x^4 + a)/(b\*c\*x^4 + a\*c), x, algorithm="fricas")

[Out] (1/4)^(1/4)\*(-1/(a\*b\*c^4))^(1/4)\*arctan(2\*(2\*(1/4)^(3/4)\*a\*b\*c^3\*x^3\*(-1/(a\*b\*c^4))^(3/4)+(1/4)^(1/4)\*a\*c\*x\*(-1/(a\*b\*c^4))^(1/4))/(sqrt(-b\*x^4+a)\*a\*c^2\*sqrt(-1/(a\*b\*c^4))-sqrt(-b\*x^4+a)\*x^2+(b\*x^4+a)\*sqrt(-1/b))-1/4\*(1/4)^(1/4)\*(-1/(a\*b\*c^4))^(1/4)\*log(-4\*(1/4)^(3/4)\*a\*b\*c^3\*x^3\*(-1/(a\*b\*c^4))^(3/4)+sqrt(-b\*x^4+a)\*a\*c^2\*sqrt(-1/(a\*b\*c^4))-2\*(1/4)^(1/4)\*a\*c\*x\*(-1/(a\*b\*c^4))^(1/4)+sqrt(-b\*x^4+ax^2)))+1/4\*(1/4)^(1/4)\*(-1/(a\*b\*c^4))^(1/4)\*log(4\*(1/4)^(3/4)\*a\*b\*c^3\*x^3\*(-1/(a\*b\*c^4))^(3/4)-sqrt(-b\*x^4+a)\*a\*c^2\*sqrt(-1/(a\*b\*c^4))-2\*(1/4)^(1/4)\*a\*c\*x\*(-1/(a\*b\*c^4))^(1/4)-sqrt(-b\*x^4+ax^2)))/bx^4+a



$$b^*c^4)^{(1/4)} + \text{sqrt}(-b^*x^4 + a)^*x^2)/(b^*x^4 + a)) + 1/4*(1/4)^{(1/4)}*(-1/(a^*b^*c^4))^{(1/4)}*\log((4*(1/4)^{(3/4)}*a^*b^*c^3*x^3*(-1/(a^*b^*c^4))^{(3/4)} - \text{sqrt}(-b^*x^4 + a)^*a^*c^2*\text{sqrt}(-1/(a^*b^*c^4)) - 2*(1/4)^{(1/4)}*a^*c*x*(-1/(a^*b^*c^4))^{(1/4)} - \text{sqrt}(-b^*x^4 + a)^*x^2)/(b^*x^4 + a))$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a-bx^4}}{a+bx^4} dx$$

c

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*4+a)\*\*(1/2)/(b\*c\*x\*\*4+a\*c), x)

[Out] Integral(sqrt(a - b\*x\*\*4)/(a + b\*x\*\*4), x)/c

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^4 + a}}{bcx^4 + ac} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-b\*x^4 + a)/(b\*c\*x^4 + a\*c), x, algorithm="giac")

[Out] integrate(sqrt(-b\*x^4 + a)/(b\*c\*x^4 + a\*c), x)

$$3.94 \quad \int \frac{(a+bx^4)^{7/4}}{c+dx^4} dx$$

**Optimal.** Leaf size=211

$$\begin{aligned} & \frac{b^{3/4}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{8d^2} - \frac{b^{3/4}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{8d^2} \\ & + \frac{(bc-ad)^{7/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2} + \frac{(bc-ad)^{7/4}\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2} + \frac{bx(a+bx^4)^{3/4}}{4d} \end{aligned}$$

[Out] (b\*x\*(a+b\*x^4)^(3/4))/(4\*d) - (b^(3/4)\*(4\*b\*c-7\*a\*d)\*ArcTan[(b^(1/4)\*x)/(a+b\*x^4)^(1/4)]/(8\*d^2) + ((b\*c-a\*d)^(7/4)\*ArcTan[((b\*c-a\*d)^(1/4)\*x)/(c^(1/4)\*(a+b\*x^4)^(1/4))]/(2\*c^(3/4)\*d^2) - (b^(3/4)\*(4\*b\*c-7\*a\*d)\*ArcTanh[(b^(1/4)\*x)/(a+b\*x^4)^(1/4)]/(8\*d^2) + ((b\*c-a\*d)^(7/4)\*ArcTanh[((b\*c-a\*d)^(1/4)\*x)/(c^(1/4)\*(a+b\*x^4)^(1/4))]/(2\*c^(3/4)\*d^2)

**Rubi [A]** time = 0.523448, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$\begin{aligned} & \frac{b^{3/4}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{8d^2} - \frac{b^{3/4}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{8d^2} \\ & + \frac{(bc-ad)^{7/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2} + \frac{(bc-ad)^{7/4}\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2} + \frac{bx(a+bx^4)^{3/4}}{4d} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a+b\*x^4)^(7/4)/(c+d\*x^4),x]

[Out] (b\*x\*(a+b\*x^4)^(3/4))/(4\*d) - (b^(3/4)\*(4\*b\*c-7\*a\*d)\*ArcTan[(b^(1/4)\*x)/(a+b\*x^4)^(1/4)]/(8\*d^2) + ((b\*c-a\*d)^(7/4)\*ArcTan[((b\*c-a\*d)^(1/4)\*x)/(c^(1/4)\*(a+b\*x^4)^(1/4))]/(2\*c^(3/4)\*d^2) - (b^(3/4)\*(4\*b\*c-7\*a\*d)\*ArcTanh[(b^(1/4)\*x)/(a+b\*x^4)^(1/4)]/(8\*d^2) + ((b\*c-a\*d)^(7/4)\*ArcTanh[((b\*c-a\*d)^(1/4)\*x)/(c^(1/4)\*(a+b\*x^4)^(1/4))]/(2\*c^(3/4)\*d^2)

**Rubi in Sympy [A]** time = 62.9504, size = 192, normalized size = 0.91

$$\begin{aligned} & \frac{b^{3/4}(7ad-4bc)\operatorname{atan}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{8d^2} + \frac{b^{3/4}(7ad-4bc)\operatorname{atanh}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{8d^2} + \frac{bx(a+bx^4)^{3/4}}{4d} \\ & + \frac{(-ad+bc)^{7/4}\operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2} + \frac{(-ad+bc)^{7/4}\operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)\*\*(7/4)/(d\*x\*\*4+c),x)

[Out] b\*\*(3/4)\*(7\*a\*d-4\*b\*c)\*atan(b\*\*(1/4)\*x/(a+b\*x\*\*4)\*\*(1/4))/(8\*d\*\*2) + b\*\*(3/4)\*(7\*a\*d-4\*b\*c)\*atanh(b\*\*(1/4)\*x/(a+b\*x\*\*4)\*\*(1/4))/(8\*d\*\*2) + b\*x\*(a+b\*x\*\*4)\*\*(3/4)/(4\*d) + (-a\*d+b\*c)\*\*(7/4)\*atan(x\*(-a\*d+b\*c)\*\*(1/4)/(c\*\*(1/4)\*(a+b\*x\*\*4)\*\*(1/4)))/(2\*c\*\*(3/4)\*d\*\*2) + (-a\*d+b\*c)\*\*(7/4)\*atanh(x\*(-a\*d+b\*c)\*\*(1/4)/(c\*\*(1/4)\*(a+b\*x\*\*4)\*\*(1/4)))/(2\*c\*\*(3/4)\*d\*\*2)

**Mathematica [C]** time = 1.80635, size = 396, normalized size = 1.88

$$\frac{1}{80} \left( \frac{5 \left( 4a^2 d \log \left( \frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}} + \sqrt[4]{c} \right) + 4bc^{3/4} x (a+bx^4)^{3/4} \sqrt[4]{bc-ad} + a(bc-4ad) \log \left( \sqrt[4]{c} - \frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}} \right) - abc \log \left( \frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}} \right)}{c^{3/4} d \sqrt[4]{bc-ad}} \right. \\ \left. - \frac{36abcx^5(7ad-4bc)F_1\left(\frac{5}{4}; \frac{1}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{d \sqrt[4]{a+bx^4}(c+dx^4) \left( x^4 \left( 4adF_1\left(\frac{9}{4}; \frac{1}{4}, 2; \frac{13}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + bcF_1\left(\frac{9}{4}; \frac{5}{4}, 1; \frac{13}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right) - 9acF_1\left(\frac{5}{4}; \frac{1}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)} \right.$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^4)^(7/4)/(c + d\*x^4), x]

[Out] ((-36\*a\*b\*c\*(-4\*b\*c + 7\*a\*d)\*x^5\*AppellF1[5/4, 1/4, 1, 9/4, -(b\*x^4)/a, -((d\*x^4)/c)]/(d\*(a + b\*x^4)^(1/4)\*(c + d\*x^4)\*(-9\*a\*c\*AppellF1[5/4, 1/4, 1, 9/4, -(b\*x^4)/a, -((d\*x^4)/c)] + x^4\*(4\*a\*d\*AppellF1[9/4, 1/4, 2, 13/4, -(b\*x^4)/a, -((d\*x^4)/c)] + b\*c\*AppellF1[9/4, 5/4, 1, 13/4, -(b\*x^4)/a, -((d\*x^4)/c)])) + (5\*(4\*b\*c^(3/4)\*(b\*c - a\*d)^(1/4)\*x\*(a + b\*x^4)^(3/4) + 2\*a\*(-(b\*c) + 4\*a\*d)\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(b + a\*x^4)^(1/4))] + a\*(b\*c - 4\*a\*d)\*Log[c^(1/4) - ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)] - a\*b\*c\*Log[c^(1/4) + ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)] + 4\*a^2\*d\*Log[c^(1/4) + ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)]))/(c^(3/4)\*d\*(b\*c - a\*d)^(1/4))/80

**Maple [F]** time = 0.084, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4 + c} (bx^4 + a)^{\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(7/4)/(d\*x^4+c), x)

[Out] int((b\*x^4+a)^(7/4)/(d\*x^4+c), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{7}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(7/4)/(d\*x^4 + c), x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(7/4)/(d\*x^4 + c), x)

**Fricas [A]** time = 2.72271, size = 2776, normalized size = 13.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(7/4)/(d\*x^4 + c),x, algorithm="fricas")

[Out] 
$$\frac{1}{16} \cdot (4 \cdot (b \cdot x^4 + a)^{3/4} \cdot b \cdot x - 16 \cdot d \cdot ((b^7 \cdot c^7 - 7 \cdot a \cdot b^6 \cdot c^6 \cdot d + 21 \cdot a^2 \cdot b^5 \cdot c^5 \cdot d^2 - 35 \cdot a^3 \cdot b^4 \cdot c^4 \cdot d^3 + 35 \cdot a^4 \cdot b^3 \cdot c^3 \cdot d^4 - 21 \cdot a^5 \cdot b^2 \cdot c^2 \cdot d^5 + 7 \cdot a^6 \cdot b \cdot c \cdot d^6 - a^7 \cdot d^7) / (c^3 \cdot d^8))^{1/4} \cdot \arctan(c^2 \cdot d^6 \cdot x \cdot ((b^7 \cdot c^7 - 7 \cdot a \cdot b^6 \cdot c^6 \cdot d + 21 \cdot a^2 \cdot b^5 \cdot c^5 \cdot d^2 - 35 \cdot a^3 \cdot b^4 \cdot c^4 \cdot d^3 + 35 \cdot a^4 \cdot b^3 \cdot c^3 \cdot d^4 - 21 \cdot a^5 \cdot b^2 \cdot c^2 \cdot d^5 + 7 \cdot a^6 \cdot b \cdot c \cdot d^6 - a^7 \cdot d^7) / (c^3 \cdot d^8))^{3/4} / (x \cdot \sqrt{((b^7 \cdot c^8 \cdot d^4 - 7 \cdot a \cdot b^6 \cdot c^7 \cdot d^5 + 21 \cdot a^2 \cdot b^5 \cdot c^6 \cdot d^6 - 35 \cdot a^3 \cdot b^4 \cdot c^5 \cdot d^7 + 35 \cdot a^4 \cdot b^3 \cdot c^4 \cdot d^8 - 21 \cdot a^5 \cdot b^2 \cdot c^3 \cdot d^9 + 7 \cdot a^6 \cdot b \cdot c^2 \cdot d^{10} - a^7 \cdot c \cdot d^{11}) \cdot x^2 \cdot \sqrt{((b^7 \cdot c^7 - 7 \cdot a \cdot b^6 \cdot c^6 \cdot d + 21 \cdot a^2 \cdot b^5 \cdot c^5 \cdot d^2 - 35 \cdot a^3 \cdot b^4 \cdot c^4 \cdot d^3 + 35 \cdot a^4 \cdot b^3 \cdot c^3 \cdot d^4 - 21 \cdot a^5 \cdot b^2 \cdot c^2 \cdot d^5 + 7 \cdot a^6 \cdot b \cdot c \cdot d^6 - a^7 \cdot d^7) / (c^3 \cdot d^8))} + (b^{10} \cdot c^{10} - 10 \cdot a \cdot b^9 \cdot c^9 \cdot d + 45 \cdot a^2 \cdot b^8 \cdot c^8 \cdot d^2 - 120 \cdot a^3 \cdot b^7 \cdot c^7 \cdot d^3 + 210 \cdot a^4 \cdot b^6 \cdot c^6 \cdot d^4 - 252 \cdot a^5 \cdot b^5 \cdot c^5 \cdot d^5 + 210 \cdot a^6 \cdot b^4 \cdot c^4 \cdot d^6 - 120 \cdot a^7 \cdot b^3 \cdot c^3 \cdot d^7 + 45 \cdot a^8 \cdot b^2 \cdot c^2 \cdot d^8 - 10 \cdot a^9 \cdot b \cdot c \cdot d^9 + a^{10} \cdot d^{10}) \cdot \sqrt{(b \cdot x^4 + a)}) / x^2) - (b^5 \cdot c^5 - 5 \cdot a \cdot b^4 \cdot c^4 \cdot d + 10 \cdot a^2 \cdot b^3 \cdot c^3 \cdot d^2 - 10 \cdot a^3 \cdot b^2 \cdot c^2 \cdot d^3 + 5 \cdot a^4 \cdot b \cdot c \cdot d^4 - a^5 \cdot d^5) \cdot (b \cdot x^4 + a)^{1/4}) + 4 \cdot d \cdot ((256 \cdot b^7 \cdot c^4 - 1792 \cdot a \cdot b^6 \cdot c^3 \cdot d + 4704 \cdot a^2 \cdot b^5 \cdot c^2 \cdot d^2 - 5488 \cdot a^3 \cdot b^4 \cdot c \cdot d^3 + 2401 \cdot a^4 \cdot b^3 \cdot d^4) / d^8)^{1/4} \cdot \arctan(d^6 \cdot x \cdot ((256 \cdot b^7 \cdot c^4 - 1792 \cdot a \cdot b^6 \cdot c^3 \cdot d + 4704 \cdot a^2 \cdot b^5 \cdot c^2 \cdot d^2 - 5488 \cdot a^3 \cdot b^4 \cdot c \cdot d^3 + 2401 \cdot a^4 \cdot b^3 \cdot d^4) / d^8)^{3/4} / (x \cdot \sqrt{((256 \cdot b^7 \cdot c^4 \cdot d^4 - 1792 \cdot a \cdot b^6 \cdot c^3 \cdot d^5 + 4704 \cdot a^2 \cdot b^5 \cdot c^2 \cdot d^6 - 5488 \cdot a^3 \cdot b^4 \cdot c \cdot d^7 + 2401 \cdot a^4 \cdot b^3 \cdot d^8) \cdot x^2 \cdot \sqrt{((256 \cdot b^7 \cdot c^4 - 1792 \cdot a \cdot b^6 \cdot c^3 \cdot d + 4704 \cdot a^2 \cdot b^5 \cdot c^2 \cdot d^2 - 5488 \cdot a^3 \cdot b^4 \cdot c \cdot d^3 + 2401 \cdot a^4 \cdot b^3 \cdot d^4) / d^8) + (4096 \cdot b^{10} \cdot c^6 - 43008 \cdot a \cdot b^9 \cdot c^5 \cdot d + 188160 \cdot a^2 \cdot b^8 \cdot c^4 \cdot d^2 - 439040 \cdot a^3 \cdot b^7 \cdot c^3 \cdot d^3 + 576240 \cdot a^4 \cdot b^6 \cdot c^2 \cdot d^4 - 403368 \cdot a^5 \cdot b^5 \cdot c \cdot d^5 + 117649 \cdot a^6 \cdot b^4 \cdot d^6) \cdot \sqrt{(b \cdot x^4 + a)}) / x^2) - (64 \cdot b^5 \cdot c^3 - 336 \cdot a \cdot b^4 \cdot c^2 \cdot d + 588 \cdot a^2 \cdot b^3 \cdot c \cdot d^2 - 343 \cdot a^3 \cdot b^2 \cdot d^3) \cdot (b \cdot x^4 + a)^{1/4}) + 4 \cdot d \cdot ((b^7 \cdot c^7 - 7 \cdot a \cdot b^6 \cdot c^6 \cdot d + 21 \cdot a^2 \cdot b^5 \cdot c^5 \cdot d^2 - 35 \cdot a^3 \cdot b^4 \cdot c^4 \cdot d^3 + 35 \cdot a^4 \cdot b^3 \cdot c^3 \cdot d^4 - 21 \cdot a^5 \cdot b^2 \cdot c^2 \cdot d^5 + 7 \cdot a^6 \cdot b \cdot c \cdot d^6 - a^7 \cdot d^7) / (c^3 \cdot d^8))^{1/4} \cdot \log(-(c^2 \cdot d^6 \cdot x \cdot ((b^7 \cdot c^7 - 7 \cdot a \cdot b^6 \cdot c^6 \cdot d + 21 \cdot a^2 \cdot b^5 \cdot c^5 \cdot d^2 - 35 \cdot a^3 \cdot b^4 \cdot c^4 \cdot d^3 + 35 \cdot a^4 \cdot b^3 \cdot c^3 \cdot d^4 - 21 \cdot a^5 \cdot b^2 \cdot c^2 \cdot d^5 + 7 \cdot a^6 \cdot b \cdot c \cdot d^6 - a^7 \cdot d^7) / (c^3 \cdot d^8))^{3/4} + (b^5 \cdot c^5 - 5 \cdot a \cdot b^4 \cdot c^4 \cdot d + 10 \cdot a^2 \cdot b^3 \cdot c^3 \cdot d^2 - 10 \cdot a^3 \cdot b^2 \cdot c^2 \cdot d^3 + 5 \cdot a^4 \cdot b \cdot c \cdot d^4 - a^5 \cdot d^5) \cdot (b \cdot x^4 + a)^{1/4}) / x) - 4 \cdot d \cdot ((b^7 \cdot c^7 - 7 \cdot a \cdot b^6 \cdot c^6 \cdot d + 21 \cdot a^2 \cdot b^5 \cdot c^5 \cdot d^2 - 35 \cdot a^3 \cdot b^4 \cdot c^4 \cdot d^3 + 35 \cdot a^4 \cdot b^3 \cdot c^3 \cdot d^4 - 21 \cdot a^5 \cdot b^2 \cdot c^2 \cdot d^5 + 7 \cdot a^6 \cdot b \cdot c \cdot d^6 - a^7 \cdot d^7) / (c^3 \cdot d^8))^{1/4} \cdot \log((c^2 \cdot d^6 \cdot x \cdot ((b^7 \cdot c^7 - 7 \cdot a \cdot b^6 \cdot c^6 \cdot d + 21 \cdot a^2 \cdot b^5 \cdot c^5 \cdot d^2 - 35 \cdot a^3 \cdot b^4 \cdot c^4 \cdot d^3 + 35 \cdot a^4 \cdot b^3 \cdot c^3 \cdot d^4 - 21 \cdot a^5 \cdot b^2 \cdot c^2 \cdot d^5 + 7 \cdot a^6 \cdot b \cdot c \cdot d^6 - a^7 \cdot d^7) / (c^3 \cdot d^8))^{3/4} - (b^5 \cdot c^5 - 5 \cdot a \cdot b^4 \cdot c^4 \cdot d + 10 \cdot a^2 \cdot b^3 \cdot c^3 \cdot d^2 - 10 \cdot a^3 \cdot b^2 \cdot c^2 \cdot d^3 + 5 \cdot a^4 \cdot b \cdot c \cdot d^4 - a^5 \cdot d^5) \cdot (b \cdot x^4 + a)^{1/4}) / x) - d \cdot ((256 \cdot b^7 \cdot c^4 - 1792 \cdot a \cdot b^6 \cdot c^3 \cdot d + 4704 \cdot a^2 \cdot b^5 \cdot c^2 \cdot d^2 - 5488 \cdot a^3 \cdot b^4 \cdot c \cdot d^3 + 2401 \cdot a^4 \cdot b^3 \cdot d^4) / d^8)^{1/4} \cdot \log(-(d^6 \cdot x \cdot ((256 \cdot b^7 \cdot c^4 - 1792 \cdot a \cdot b^6 \cdot c^3 \cdot d + 4704 \cdot a^2 \cdot b^5 \cdot c^2 \cdot d^2 - 5488 \cdot a^3 \cdot b^4 \cdot c \cdot d^3 + 2401 \cdot a^4 \cdot b^3 \cdot d^4) / d^8)^{3/4} + (64 \cdot b^5 \cdot c^3 - 336 \cdot a \cdot b^4 \cdot c^2 \cdot d + 588 \cdot a^2 \cdot b^3 \cdot c \cdot d^2 - 343 \cdot a^3 \cdot b^2 \cdot d^3) \cdot (b \cdot x^4 + a)^{1/4}) / x) + d \cdot ((256 \cdot b^7 \cdot c^4 - 1792 \cdot a \cdot b^6 \cdot c^3 \cdot d + 4704 \cdot a^2 \cdot b^5 \cdot c^2 \cdot d^2 - 5488 \cdot a^3 \cdot b^4 \cdot c \cdot d^3 + 2401 \cdot a^4 \cdot b^3 \cdot d^4) / d^8)^{1/4} \cdot \log((d^6 \cdot x \cdot ((256 \cdot b^7 \cdot c^4 - 1792 \cdot a \cdot b^6 \cdot c^3 \cdot d + 4704 \cdot a^2 \cdot b^5 \cdot c^2 \cdot d^2 - 5488 \cdot a^3 \cdot b^4 \cdot c \cdot d^3 + 2401 \cdot a^4 \cdot b^3 \cdot d^4) / d^8)^{3/4} - (64 \cdot b^5 \cdot c^3 - 336 \cdot a \cdot b^4 \cdot c^2 \cdot d + 588 \cdot a^2 \cdot b^3 \cdot c \cdot d^2 - 343 \cdot a^3 \cdot b^2 \cdot d^3) \cdot (b \cdot x^4 + a)^{1/4}) / x)) / d$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(7/4)/(d\*x\*\*4+c),x)

[Out] Timed out

---

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{7}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c), x)
```

$$3.95 \quad \int \frac{(a+bx^4)^{3/4}}{c+dx^4} dx$$

**Optimal.** Leaf size=173

$$\frac{b^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{2d} + \frac{b^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{2d} \\ - \frac{(bc-ad)^{3/4} \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d} - \frac{(bc-ad)^{3/4} \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

[Out]  $(b^{3/4} \text{ArcTan}[(b^{1/4} x)/(a + b x^4)^{1/4}])/(2 d) - ((b^3 c - a^3 d)^{3/4} \text{ArcTan}[(b^3 c - a^3 d)^{1/4} x]/(c^{1/4} (a + b x^4)^{1/4}))/ (2^3 c^{3/4} d) + (b^{3/4} \text{ArcTanh}[(b^{1/4} x)/(a + b x^4)^{1/4}])/(2^3 d) - ((b^3 c - a^3 d)^{3/4} \text{ArcTanh}[(b^3 c - a^3 d)^{1/4} x]/(c^{1/4} (a + b x^4)^{1/4}))/ (2^3 c^{3/4} d)$

**Rubi [A]** time = 0.239638, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$\frac{b^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{2d} + \frac{b^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{2d} \\ - \frac{(bc-ad)^{3/4} \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d} - \frac{(bc-ad)^{3/4} \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(3/4)/(c + d\*x^4), x]

[Out]  $(b^{3/4} \text{ArcTan}[(b^{1/4} x)/(a + b x^4)^{1/4}])/(2 d) - ((b^3 c - a^3 d)^{3/4} \text{ArcTan}[(b^3 c - a^3 d)^{1/4} x]/(c^{1/4} (a + b x^4)^{1/4}))/ (2^3 c^{3/4} d) + (b^{3/4} \text{ArcTanh}[(b^{1/4} x)/(a + b x^4)^{1/4}])/(2^3 d) - ((b^3 c - a^3 d)^{3/4} \text{ArcTanh}[(b^3 c - a^3 d)^{1/4} x]/(c^{1/4} (a + b x^4)^{1/4}))/ (2^3 c^{3/4} d)$

**Rubi in Sympy [A]** time = 37.9971, size = 148, normalized size = 0.86

$$\frac{b^{3/4} \operatorname{atan}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{2d} + \frac{b^{3/4} \operatorname{atanh}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{2d} \\ - \frac{(-ad+bc)^{3/4} \operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d} - \frac{(-ad+bc)^{3/4} \operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)\*\*(3/4)/(d\*x\*\*4+c), x)

[Out]  $b^{3/4} \operatorname{atan}(b^{1/4} x/(a + b x^4)^{1/4})/(2 d) + b^{3/4} \operatorname{atanh}(b^{1/4} x/(a + b x^4)^{1/4})/(2 d) - (-a^3 d + b^3 c)^{3/4} \operatorname{atan}(x(-a^3 d + b^3 c)^{1/4}/(c^{1/4} (a + b x^4)^{1/4}))/ (2^3 c^{3/4} d) - (-a^3 d + b^3 c)^{3/4} \operatorname{atanh}(x(-a^3 d + b^3 c)^{1/4}/(c^{1/4} (a + b x^4)^{1/4}))/ (2^3 c^{3/4} d)$

**Mathematica [C]** time = 0.25475, size = 161, normalized size = 0.93

$$\frac{5acx(a+bx^4)^{3/4}F_1\left(\frac{1}{4};-\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(x^4\left(3bcF_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-4adF_1\left(\frac{5}{4};-\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};-\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^4)^(3/4)/(c + d\*x^4), x]

[Out] (5\*a\*c\*x\*(a + b\*x^4)^(3/4)\*AppellF1[1/4, -3/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)]/((c + d\*x^4)\*(5\*a\*c\*AppellF1[1/4, -3/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + x^4\*(-4\*a\*d\*AppellF1[5/4, -3/4, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 3\*b\*c\*AppellF1[5/4, 1/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)]))

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4 + c} (bx^4 + a)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(3/4)/(d\*x^4+c), x)

[Out] int((b\*x^4+a)^(3/4)/(d\*x^4+c), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(3/4)/(d\*x^4 + c), x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(3/4)/(d\*x^4 + c), x)

**Fricas [A]** time = 0.29378, size = 1006, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(3/4)/(d\*x^4 + c), x, algorithm="fricas")

[Out] -((b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)/(c^3\*d^4))^(1/4)\*arctan(c^2\*d^3\*x\*((b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)/(c^3\*d^4))^(3/4)/(x\*sqrt(((b^3\*c^4\*d^2 - 3\*a\*b^2\*c^3\*d^3 + 3\*a^2\*b\*c^2\*d^4 - a^3\*c\*d^5)\*x^2\*sqrt((b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)/(c^3\*d^4)) + (b^4\*c^4 - 4\*a\*b^3\*c^3\*d + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a^3\*b\*c\*d^3 + a^4\*d^4)\*sqrt(b\*x^4 + a))/x^2) + (b\*x^4 + a)^(1/4)\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)) + (b^3/d^4)^(1/4)\*arctan(d^3\*x\*(b^3/d^4)^(3/4)/((b\*x^4 + a)^(1/4)\*b^2 + x\*sqrt((b^3\*d^2\*x^2\*sqrt(b^3/d^4) + sqrt(b\*x^4 + a)\*b^4)/x^2)) - 1/4\*((b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)/(c^3\*d^4))^(1/4)\*log((c^2\*d^3\*x\*((b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c

$$\begin{aligned} & *d^2 - a^3*d^3)/(c^3*d^4))^{3/4} + (b*x^4 + a)^{1/4}*(b^2*c^2 - 2 \\ & *a*b*c*d + a^2*d^2))/x) + 1/4*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b \\ & *c*d^2 - a^3*d^3)/(c^3*d^4))^{1/4}*\log(-(c^2*d^3*x*(b^3*c^3 - 3* \\ & a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^{3/4} - (b*x^4 \\ & + a)^{1/4}*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*(b^3/d^4)^{1 \\ & /4}*\log((d^3*x*(b^3/d^4)^{3/4} + (b*x^4 + a)^{1/4}*b^2)/x) - 1/4* \\ & (b^3/d^4)^{1/4}*\log(-(d^3*x*(b^3/d^4)^{3/4} - (b*x^4 + a)^{1/4}*b \\ & ^2)/x) \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^4)^{\frac{3}{4}}}{c + dx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(3/4)/(d\*x\*\*4+c), x)

[Out] Integral((a + b\*x\*\*4)\*\*(3/4)/(c + d\*x\*\*4), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(3/4)/(d\*x^4 + c), x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(3/4)/(d\*x^4 + c), x)



$$3.96 \quad \int \frac{1}{\sqrt[4]{a + bx^4}(c+dx^4)} dx$$

**Optimal.** Leaf size=105

$$\frac{\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}} + \frac{\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

[Out] ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))]/(2\*c^(3/4)\*(b\*c - a\*d)^(1/4)) + ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))]/(2\*c^(3/4)\*(b\*c - a\*d)^(1/4))

**Rubi [A]** time = 0.135754, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}} + \frac{\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(1/4)\*(c + d\*x^4)), x]

[Out] ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))]/(2\*c^(3/4)\*(b\*c - a\*d)^(1/4)) + ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))]/(2\*c^(3/4)\*(b\*c - a\*d)^(1/4))

**Rubi in Sympy [A]** time = 23.604, size = 90, normalized size = 0.86

$$\frac{\operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{-ad+bc}} + \frac{\operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{-ad+bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*4+a)\*\*(1/4)/(d\*x\*\*4+c), x)

[Out] atan(x\*(-a\*d + b\*c)\*\*(1/4)/(c\*\*(1/4)\*(a + b\*x\*\*4)\*\*(1/4)))/(2\*c\*\*(3/4)\*(-a\*d + b\*c)\*\*(1/4)) + atanh(x\*(-a\*d + b\*c)\*\*(1/4)/(c\*\*(1/4)\*(a + b\*x\*\*4)\*\*(1/4)))/(2\*c\*\*(3/4)\*(-a\*d + b\*c)\*\*(1/4))

**Mathematica [A]** time = 0.123935, size = 122, normalized size = 1.16

$$\frac{-\log\left(\sqrt[4]{c} - \frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right) + \log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}} + \sqrt[4]{c}\right) + 2 \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)}{4c^{3/4}\sqrt[4]{bc-ad}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(1/4)\*(c + d\*x^4)), x]

[Out] (2\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(b + a\*x^4)^(1/4))]) - Log[c^(1/4) - ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)] + Log[c^(1/4) + ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)]/(4\*c^(3/4)\*(b\*c - a\*d)^(1/4))

---

**Maple [F]** time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4 + c} \frac{1}{\sqrt[4]{bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^4+a)^(1/4)/(d*x^4+c), x)`

[Out] `int(1/(b*x^4+a)^(1/4)/(d*x^4+c), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{1}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x, algorithm="maxima")`

[Out] `integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)`

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a + bx^4}(c + dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c), x)`

[Out] `Integral(1/((a + b*x**4)**(1/4)*(c + d*x**4)), x)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{1}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x, algorithm="giac")`

[Out] `integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)`

$$3.97 \quad \int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)} dx$$

**Optimal.** Leaf size=134

$$-\frac{d \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} - \frac{d \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} + \frac{bx}{a\sqrt[4]{a+bx^4}(bc-ad)}$$

[Out] (b\*x)/(a\*(b\*c - a\*d)\*(a + b\*x^4)^(1/4)) - (d\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(2\*c^(3/4)\*(b\*c - a\*d)^(5/4)) - (d\*ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(2\*c^(3/4)\*(b\*c - a\*d)^(5/4))

**Rubi [A]** time = 0.235548, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$-\frac{d \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} - \frac{d \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} + \frac{bx}{a\sqrt[4]{a+bx^4}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(5/4)\*(c + d\*x^4)), x]

[Out] (b\*x)/(a\*(b\*c - a\*d)\*(a + b\*x^4)^(1/4)) - (d\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(2\*c^(3/4)\*(b\*c - a\*d)^(5/4)) - (d\*ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(2\*c^(3/4)\*(b\*c - a\*d)^(5/4))

**Rubi in Sympy [A]** time = 34.0446, size = 117, normalized size = 0.87

$$-\frac{d \operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(-ad+bc)^{5/4}} - \frac{d \operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(-ad+bc)^{5/4}} - \frac{bx}{a\sqrt[4]{a+bx^4}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*4+a)\*\*(5/4)/(d\*x\*\*4+c), x)

[Out] -d\*atan(x\*(-a\*d + b\*c)\*\*(1/4)/(c\*\*(1/4)\*(a + b\*x\*\*4)\*\*(1/4)))/(2\*c\*\*(3/4)\*(-a\*d + b\*c)\*\*(5/4)) - d\*atanh(x\*(-a\*d + b\*c)\*\*(1/4)/(c\*\*(1/4)\*(a + b\*x\*\*4)\*\*(1/4)))/(2\*c\*\*(3/4)\*(-a\*d + b\*c)\*\*(5/4)) - b\*x/(a\*(a + b\*x\*\*4)\*\*(1/4)\*(a\*d - b\*c))

**Mathematica [A]** time = 0.367768, size = 153, normalized size = 1.14

$$\frac{d\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{c^{3/4}\sqrt[4]{bc-ad}} - \frac{4bx}{a\sqrt[4]{a+bx^4}} \frac{1}{4(ad-bc)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(5/4)\*(c + d\*x^4)), x]

[Out] 
$$\left( \frac{(-4bx)/(a(a+bx^4)^{1/4}) + (d(2\text{ArcTan}[(bc-ad)^{1/4}x]/(c^{1/4}(b+ax^4)^{1/4})) - \text{Log}[c^{1/4} - (bc-ad)^{1/4}x]/(b+ax^4)^{1/4}) + \text{Log}[c^{1/4} + (bc-ad)^{1/4}x]/(b+ax^4)^{1/4})}{c^{3/4}(bc-ad)^{1/4}} \right) / (4(-bc+ad))$$

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4 + c} (bx^4 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^4+a)^(5/4)/(d*x^4+c), x)`

[Out] `int(1/(b*x^4+a)^(5/4)/(d*x^4+c), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{5}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x, algorithm="maxima")`

[Out] `integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^4)^{\frac{5}{4}}(c + dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c), x)`

[Out] `Integral(1/((a + b*x**4)**(5/4)*(c + d*x**4)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{5}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)
```

$$3.98 \quad \int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)} dx$$

**Optimal.** Leaf size=180

$$\frac{bx(4bc - 9ad)}{5a^2\sqrt[4]{a+bx^4}(bc - ad)^2} + \frac{d^2 \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc - ad)^{9/4}} + \frac{d^2 \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc - ad)^{9/4}} + \frac{bx}{5a(a+bx^4)^{5/4}(bc - ad)}$$

[Out] (b\*x)/(5\*a\*(b\*c - a\*d)\*(a + b\*x^4)^(5/4)) + (b\*(4\*b\*c - 9\*a\*d)\*x)/(5\*a^2\*(b\*c - a\*d)^2\*(a + b\*x^4)^(1/4)) + (d^2\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(2\*c^(3/4)\*(b\*c - a\*d)^(9/4)) + (d^2\*ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(2\*c^(3/4)\*(b\*c - a\*d)^(9/4))

**Rubi [A]** time = 0.489959, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{bx(4bc - 9ad)}{5a^2\sqrt[4]{a+bx^4}(bc - ad)^2} + \frac{d^2 \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc - ad)^{9/4}} + \frac{d^2 \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc - ad)^{9/4}} + \frac{bx}{5a(a+bx^4)^{5/4}(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(9/4)\*(c + d\*x^4)), x]

[Out] (b\*x)/(5\*a\*(b\*c - a\*d)\*(a + b\*x^4)^(5/4)) + (b\*(4\*b\*c - 9\*a\*d)\*x)/(5\*a^2\*(b\*c - a\*d)^2\*(a + b\*x^4)^(1/4)) + (d^2\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(2\*c^(3/4)\*(b\*c - a\*d)^(9/4)) + (d^2\*ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(2\*c^(3/4)\*(b\*c - a\*d)^(9/4))

**Rubi in Sympy [A]** time = 80.6157, size = 158, normalized size = 0.88

$$\frac{d^2 \operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(-ad+bc)^{9/4}} + \frac{d^2 \operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(-ad+bc)^{9/4}} - \frac{bx}{5a(a+bx^4)^{5/4}(ad-bc)} - \frac{bx(9ad-4bc)}{5a^2\sqrt[4]{a+bx^4}(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*4+a)\*\*(9/4)/(d\*x\*\*4+c), x)

[Out] d\*\*2\*atan(x\*(-a\*d + b\*c)\*\*(1/4)/(c\*\*(1/4)\*(a + b\*x\*\*4)\*\*(1/4)))/(2\*c\*\*(3/4)\*(-a\*d + b\*c)\*\*(9/4)) + d\*\*2\*atanh(x\*(-a\*d + b\*c)\*\*(1/4)/(c\*\*(1/4)\*(a + b\*x\*\*4)\*\*(1/4)))/(2\*c\*\*(3/4)\*(-a\*d + b\*c)\*\*(9/4)) - b\*x/(5\*a\*(a + b\*x\*\*4)\*\*(5/4)\*(a\*d - b\*c)) - b\*x\*(9\*a\*d - 4\*b\*c)/(5\*a\*\*2\*(a + b\*x\*\*4)\*\*(1/4)\*(a\*d - b\*c)\*\*2)

**Mathematica [A]** time = 0.563055, size = 184, normalized size = 1.02

$$\frac{bx((a+bx^4)(4bc-9ad)+a(bc-ad))}{5a^2(a+bx^4)^{5/4}(bc-ad)^2} + \frac{d^2\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{4c^{3/4}(bc-ad)^{9/4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(9/4)\*(c + d\*x^4)),x]

[Out] (b\*x\*(a\*(b\*c - a\*d) + (4\*b\*c - 9\*a\*d)\*(a + b\*x^4)))/(5\*a^2\*(b\*c - a\*d)^2\*(a + b\*x^4)^(5/4)) + (d^2\*(2\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(b + a\*x^4)^(1/4))]) - Log[c^(1/4) - ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)] + Log[c^(1/4) + ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)])/(4\*c^(3/4)\*(b\*c - a\*d)^(9/4))

**Maple [F]** time = 0.057, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4 + c} (bx^4 + a)^{-\frac{9}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(9/4)/(d\*x^4+c),x)

[Out] int(1/(b\*x^4+a)^(9/4)/(d\*x^4+c),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{9}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)^(9/4)\*(d\*x^4 + c)),x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(9/4)\*(d\*x^4 + c)), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)^(9/4)\*(d\*x^4 + c)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)\*\*(9/4)/(d\*x\*\*4+c),x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{9}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)), x)
```



$$3.99 \quad \int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)} dx$$

**Optimal.** Leaf size=233

$$\frac{bx(8bc - 17ad)}{45a^2(a + bx^4)^{5/4}(bc - ad)^2} + \frac{bx(113a^2d^2 - 100abcd + 32b^2c^2)}{45a^3\sqrt[4]{a + bx^4}(bc - ad)^3}$$

$$- \frac{d^3 \tan^{-1}\left(\frac{x\sqrt[4]{bc - ad}}{\sqrt[4]{c}\sqrt[4]{a + bx^4}}\right)}{2c^{3/4}(bc - ad)^{13/4}} - \frac{d^3 \tanh^{-1}\left(\frac{x\sqrt[4]{bc - ad}}{\sqrt[4]{c}\sqrt[4]{a + bx^4}}\right)}{2c^{3/4}(bc - ad)^{13/4}} + \frac{bx}{9a(a + bx^4)^{9/4}(bc - ad)}$$

[Out] (b\*x)/(9\*a\*(b\*c - a\*d)\*(a + b\*x^4)^(9/4)) + (b\*(8\*b\*c - 17\*a\*d)\*x)/(45\*a^2\*(b\*c - a\*d)^2\*(a + b\*x^4)^(5/4)) + (b\*(32\*b^2\*c^2 - 100\*a\*b\*c\*d + 113\*a^2\*d^2)\*x)/(45\*a^3\*(b\*c - a\*d)^3\*(a + b\*x^4)^(1/4)) - (d^3\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(2\*c^(3/4)\*(b\*c - a\*d)^(13/4)) - (d^3\*ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(2\*c^(3/4)\*(b\*c - a\*d)^(13/4))

**Rubi [A]** time = 0.730128, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{bx(8bc - 17ad)}{45a^2(a + bx^4)^{5/4}(bc - ad)^2} + \frac{bx(113a^2d^2 - 100abcd + 32b^2c^2)}{45a^3\sqrt[4]{a + bx^4}(bc - ad)^3}$$

$$- \frac{d^3 \tan^{-1}\left(\frac{x\sqrt[4]{bc - ad}}{\sqrt[4]{c}\sqrt[4]{a + bx^4}}\right)}{2c^{3/4}(bc - ad)^{13/4}} - \frac{d^3 \tanh^{-1}\left(\frac{x\sqrt[4]{bc - ad}}{\sqrt[4]{c}\sqrt[4]{a + bx^4}}\right)}{2c^{3/4}(bc - ad)^{13/4}} + \frac{bx}{9a(a + bx^4)^{9/4}(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(13/4)\*(c + d\*x^4)), x]

[Out] (b\*x)/(9\*a\*(b\*c - a\*d)\*(a + b\*x^4)^(9/4)) + (b\*(8\*b\*c - 17\*a\*d)\*x)/(45\*a^2\*(b\*c - a\*d)^2\*(a + b\*x^4)^(5/4)) + (b\*(32\*b^2\*c^2 - 100\*a\*b\*c\*d + 113\*a^2\*d^2)\*x)/(45\*a^3\*(b\*c - a\*d)^3\*(a + b\*x^4)^(1/4)) - (d^3\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(2\*c^(3/4)\*(b\*c - a\*d)^(13/4)) - (d^3\*ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(2\*c^(3/4)\*(b\*c - a\*d)^(13/4))

**Rubi in Sympy [A]** time = 133.593, size = 212, normalized size = 0.91

$$- \frac{d^3 \operatorname{atan}\left(\frac{x\sqrt[4]{-ad + bc}}{\sqrt[4]{c}\sqrt[4]{a + bx^4}}\right)}{2c^{3/4}(-ad + bc)^{13/4}} - \frac{d^3 \operatorname{atanh}\left(\frac{x\sqrt[4]{-ad + bc}}{\sqrt[4]{c}\sqrt[4]{a + bx^4}}\right)}{2c^{3/4}(-ad + bc)^{13/4}} - \frac{bx}{9a(a + bx^4)^{9/4}(ad - bc)}$$

$$- \frac{bx(17ad - 8bc)}{45a^2(a + bx^4)^{5/4}(ad - bc)^2} - \frac{bx(113a^2d^2 - 100abcd + 32b^2c^2)}{45a^3\sqrt[4]{a + bx^4}(ad - bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*4+a)\*\*(13/4)/(d\*x\*\*4+c), x)

[Out] -d\*\*3\*atan(x\*(-a\*d + b\*c)\*\*(1/4)/(c\*\*(1/4)\*(a + b\*x\*\*4)\*\*(1/4)))/(2\*c\*\*(3/4)\*(-a\*d + b\*c)\*\*(13/4)) - d\*\*3\*atanh(x\*(-a\*d + b\*c)\*\*(1/4)/(c\*\*(1/4)\*(a + b\*x\*\*4)\*\*(1/4)))/(2\*c\*\*(3/4)\*(-a\*d + b\*c)\*\*(13/4)) - b\*x/(9\*a\*(a + b\*x\*\*4)\*\*(9/4)\*(a\*d - b\*c)) - b\*x\*(17\*a\*d - 8\*b\*c)/(45\*a\*\*2\*(a + b\*x\*\*4)\*\*(5/4)\*(a\*d - b\*c)\*\*2) - b\*x\*(113\*a\*\*2\*d\*\*2 - 100\*a\*b\*c\*d + 32\*b\*\*2\*c\*\*2)/(45\*a\*\*3\*(a + b\*x\*\*4)\*\*(1/4))

) \* (a\*d - b\*c) \*\* 3)

**Mathematica [A]** time = 0.683711, size = 231, normalized size = 0.99

$$\frac{bx \left( (a + bx^4)^2 (113a^2d^2 - 100abcd + 32b^2c^2) + 5a^2(bc - ad)^2 + a(a + bx^4)(ad - bc)(17ad - 8bc) \right)}{45a^3(a + bx^4)^{9/4}(bc - ad)^3} - \frac{d^3 \left( -\log \left( \sqrt[4]{c} - \frac{x\sqrt[4]{bc - ad}}{\sqrt[4]{ax^4 + b}} \right) + \log \left( \frac{x\sqrt[4]{bc - ad}}{\sqrt[4]{ax^4 + b}} + \sqrt[4]{c} \right) + 2 \tan^{-1} \left( \frac{x\sqrt[4]{bc - ad}}{\sqrt[4]{c}\sqrt[4]{ax^4 + b}} \right) \right)}{4c^{3/4}(bc - ad)^{13/4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(13/4)\*(c + d\*x^4)), x]

[Out] (b\*x\*(5\*a^2\*(b\*c - a\*d)^2 + a\*(-(b\*c) + a\*d)\*(-8\*b\*c + 17\*a\*d)\*(a + b\*x^4) + (32\*b^2\*c^2 - 100\*a\*b\*c\*d + 113\*a^2\*d^2)\*(a + b\*x^4)^2)/(45\*a^3\*(b\*c - a\*d)^3\*(a + b\*x^4)^(9/4)) - (d^3\*(2\*ArcTan[(b\*c - a\*d)^(1/4)\*x]/(c^(1/4)\*(b + a\*x^4)^(1/4)))] - Log[c^(1/4) - (b\*c - a\*d)^(1/4)\*x]/(b + a\*x^4)^(1/4)] + Log[c^(1/4) + ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)]]/(4\*c^(3/4)\*(b\*c - a\*d)^(13/4))

**Maple [F]** time = 0.061, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4 + c} (bx^4 + a)^{-\frac{13}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(13/4)/(d\*x^4+c), x)

[Out] int(1/(b\*x^4+a)^(13/4)/(d\*x^4+c), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{13}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)^(13/4)\*(d\*x^4 + c)), x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(13/4)\*(d\*x^4 + c)), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)^(13/4)\*(d\*x^4 + c)), x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)**(13/4)/(d*x**4+c), x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{13}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x, algorithm="giac")`

[Out] `integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x)`

$$3.100 \quad \int \frac{(a+bx^4)^{9/4}}{c+dx^4} dx$$

**Optimal.** Leaf size=316

$$\frac{\sqrt{ab}^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (6bc - 11ad)F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{12d^2 (a + bx^4)^{3/4}} - \frac{bx\sqrt{a + bx^4}(6bc - 11ad)}{12d^2}$$

$$+ \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a + bx^4}(bc - ad)^2 \left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{bx^4 + a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd^2}}$$

$$+ \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a + bx^4}(bc - ad)^2 \left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{bx^4 + a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd^2}} + \frac{bx(a + bx^4)^{5/4}}{6d}$$

[Out]  $-(b*(6*b*c - 11*a*d)*x*(a + b*x^4)^{(1/4)})/(12*d^2) + (b*x*(a + b*x^4)^{(5/4)})/(6*d) + (\text{Sqrt}[a]*b^{(3/2)}*(6*b*c - 11*a*d)*(1 + a/(b*x^4))^{(3/4)}*x^3*\text{EllipticF}[\text{ArcCot}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a]]/2, 2])/(12*d^2*(a + b*x^4)^{(3/4)}) + ((b*c - a*d)^2*\text{Sqrt}[a/(a + b*x^4)]*\text{Sqrt}[a + b*x^4]*\text{EllipticPi}[-(\text{Sqrt}[b*c - a*d]/(\text{Sqrt}[b]*\text{Sqrt}[c]))], \text{ArcSin}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}], -1)]/(2*b^{(1/4)}*c*d^2) + ((b*c - a*d)^2*\text{Sqrt}[a/(a + b*x^4)]*\text{Sqrt}[a + b*x^4]*\text{EllipticPi}[\text{Sqrt}[b*c - a*d]/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}], -1)]/(2*b^{(1/4)}*c*d^2)$

**Rubi [A]** time = 0.921635, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$

$$\frac{\sqrt{ab}^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (6bc - 11ad)F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{12d^2 (a + bx^4)^{3/4}} - \frac{bx\sqrt{a + bx^4}(6bc - 11ad)}{12d^2}$$

$$+ \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a + bx^4}(bc - ad)^2 \left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{bx^4 + a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd^2}}$$

$$+ \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a + bx^4}(bc - ad)^2 \left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{bx^4 + a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd^2}} + \frac{bx(a + bx^4)^{5/4}}{6d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(9/4)/(c + d\*x^4), x]

[Out]  $-(b*(6*b*c - 11*a*d)*x*(a + b*x^4)^{(1/4)})/(12*d^2) + (b*x*(a + b*x^4)^{(5/4)})/(6*d) + (\text{Sqrt}[a]*b^{(3/2)}*(6*b*c - 11*a*d)*(1 + a/(b*x^4))^{(3/4)}*x^3*\text{EllipticF}[\text{ArcCot}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a]]/2, 2])/(12*d^2*(a + b*x^4)^{(3/4)}) + ((b*c - a*d)^2*\text{Sqrt}[a/(a + b*x^4)]*\text{Sqrt}[a + b*x^4]*\text{EllipticPi}[-(\text{Sqrt}[b*c - a*d]/(\text{Sqrt}[b]*\text{Sqrt}[c]))], \text{ArcSin}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}], -1)]/(2*b^{(1/4)}*c*d^2) + ((b*c - a*d)^2*\text{Sqrt}[a/(a + b*x^4)]*\text{Sqrt}[a + b*x^4]*\text{EllipticPi}[\text{Sqrt}[b*c - a*d]/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}], -1)]/(2*b^{(1/4)}*c*d^2)$

**Rubi in Sympy [A]** time = 112.517, size = 282, normalized size = 0.89

$$\frac{\sqrt{ab^{\frac{3}{2}}x^3(11ad-6bc)\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{2}\middle|2\right)}{12d^2(a+bx^4)^{\frac{3}{4}}} + \frac{bx(a+bx^4)^{\frac{5}{4}}}{6d}$$

$$+ \frac{bx\sqrt{a+bx^4}(11ad-6bc)}{12d^2} + \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(ad-bc)^2\left(-\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{bcd^2}}$$

$$+ \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(ad-bc)^2\left(\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{bcd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**4+a)**(9/4)/(d*x**4+c),x)`

[Out] `-sqrt(a)*b**(3/2)*x**3*(11*a*d - 6*b*c)*(a/(b*x**4) + 1)**(3/4)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x**2))/2, 2)/(12*d**2*(a + b*x**4)**(3/4)) + b*x*(a + b*x**4)**(5/4)/(6*d) + b*x*(a + b*x**4)**(1/4)*(11*a*d - 6*b*c)/(12*d**2) + sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*(a*d - b*c)**2*elliptic_pi(-sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(2*b**(1/4)*c*d**2) + sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*(a*d - b*c)**2*elliptic_pi(sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(2*b**(1/4)*c*d**2)`

**Mathematica [C]** time = 1.72467, size = 396, normalized size = 1.25

$$x \left( \frac{9abcx^4(23a^2d^2-30abcd+12b^2c^2)F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(x^4\left(4adF_1\left(\frac{9}{4};\frac{3}{4},2;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{9}{4};\frac{7}{4},1;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-9acF_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}\right) - \frac{25a^2c(12a^2d^2)}{(c+dx^4)\left(x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)} \right) / (60d^2(a+bx^4)^{3/4})$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(a + b*x^4)^(9/4)/(c + d*x^4),x]`

[Out] `(x*(5*b*(a + b*x^4)*(-6*b*c + 13*a*d + 2*b*d*x^4) - (25*a^2*c*(6*b^2*c^2 - 13*a*b*c*d + 12*a^2*d^2)*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)]/(c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])) - (9*a*b*c*(12*b^2*c^2 - 30*a*b*c*d + 23*a^2*d^2)*x^4*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]/(c + d*x^4)*(-9*a*c*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[9/4, 3/4, 2, 13/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[9/4, 7/4, 1, 13/4, -((b*x^4)/a), -((d*x^4)/c)])))/(60*d^2*(a + b*x^4)^(3/4))`

**Maple [F]** time = 0.098, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4 + c} (bx^4 + a)^{\frac{9}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+a)^(9/4)/(d*x^4+c),x)`

[Out] `int((b*x^4+a)^(9/4)/(d*x^4+c),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{9}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(9/4)/(d\*x^4 + c),x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(9/4)/(d\*x^4 + c), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(9/4)/(d\*x^4 + c),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(9/4)/(d\*x\*\*4+c),x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{9}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(9/4)/(d\*x^4 + c),x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(9/4)/(d\*x^4 + c), x)

$$3.101 \quad \int \frac{(a+bx^4)^{5/4}}{c+dx^4} dx$$

**Optimal.** Leaf size=274

$$\begin{aligned} & - \frac{\sqrt{ab^{3/2}}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{2d(a+bx^4)^{3/4}} \\ & - \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4}(bc-ad) \left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd}} \\ & - \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4}(bc-ad) \left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd}} + \frac{bx\sqrt{a+bx^4}}{2d} \end{aligned}$$

[Out] (b\*x\*(a + b\*x^4)^(1/4))/(2\*d) - (Sqrt[a]\*b^(3/2)\*(1 + a/(b\*x^4))^(3/4)\*x^3\*EllipticF[ArcCot[(Sqrt[b]\*x^2)/Sqrt[a]]/2, 2])/(2\*d\*(a + b\*x^4)^(3/4)) - ((b\*c - a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(2\*b^(1/4)\*c\*d) - ((b\*c - a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(2\*b^(1/4)\*c\*d)

**Rubi [A]** time = 0.49314, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$\begin{aligned} & - \frac{\sqrt{ab^{3/2}}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{2d(a+bx^4)^{3/4}} \\ & - \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4}(bc-ad) \left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd}} \\ & - \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4}(bc-ad) \left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{bcd}} + \frac{bx\sqrt{a+bx^4}}{2d} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(5/4)/(c + d\*x^4), x]

[Out] (b\*x\*(a + b\*x^4)^(1/4))/(2\*d) - (Sqrt[a]\*b^(3/2)\*(1 + a/(b\*x^4))^(3/4)\*x^3\*EllipticF[ArcCot[(Sqrt[b]\*x^2)/Sqrt[a]]/2, 2])/(2\*d\*(a + b\*x^4)^(3/4)) - ((b\*c - a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(2\*b^(1/4)\*c\*d) - ((b\*c - a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(2\*b^(1/4)\*c\*d)

**Rubi in Sympy [A]** time = 62.6497, size = 235, normalized size = 0.86

$$\begin{aligned} & -\frac{\sqrt{ab}^{\frac{3}{2}}x^3\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{2}\middle|2\right)}{2d(a+bx^4)^{\frac{3}{4}}}+\frac{bx\sqrt[4]{a+bx^4}}{2d} \\ & +\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(ad-bc)\left(-\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{bcd}} \\ & +\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(ad-bc)\left(\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{bcd}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**4+a)**(5/4)/(d*x**4+c),x)`

[Out] `-sqrt(a)*b**(3/2)*x**3*(a/(b*x**4)+1)**(3/4)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x**2))/2,2)/(2*d*(a+b*x**4)**(3/4))+b*x*(a+b*x**4)**(1/4)/(2*d)+sqrt(a/(a+b*x**4))*sqrt(a+b*x**4)*(a*d-b*c)*elliptic_pi(-sqrt(-a*d+b*c)/(sqrt(b)*sqrt(c)),asin(b**(1/4)*x/(a+b*x**4)**(1/4)),-1)/(2*b**(1/4)*c*d)+sqrt(a/(a+b*x**4))*sqrt(a+b*x**4)*(a*d-b*c)*elliptic_pi(sqrt(-a*d+b*c)/(sqrt(b)*sqrt(c)),asin(b**(1/4)*x/(a+b*x**4)**(1/4)),-1)/(2*b**(1/4)*c*d)`

**Mathematica [C]** time = 0.71372, size = 435, normalized size = 1.59

$$x\left(\frac{b\left(5x^4(a+bx^4)(c+dx^4)\left(4adF_1\left(\frac{9}{4};\frac{3}{4},2;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{9}{4};\frac{7}{4},1;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-9ac(5ac+8adx^4+3bcx^4+5bdx^8)F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{x^4\left(4adF_1\left(\frac{9}{4};\frac{3}{4},2;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{9}{4};\frac{7}{4},1;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-9acF_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}\right)-\frac{70d(a+bx^4)^{3/4}(c+dx^4)}{10d(a+bx^4)^{3/4}(c+dx^4)}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(a+b*x^4)^(5/4)/(c+d*x^4),x]`

[Out] `(x*((-25*a^2*c*(-(b*c)+2*a*d)*AppellF1[1/4,3/4,1,5/4,-((b*x^4)/a),-((d*x^4)/c)]/(-5*a*c*AppellF1[1/4,3/4,1,5/4,-((b*x^4)/a),-((d*x^4)/c)]+x^4*(4*a*d*AppellF1[5/4,3/4,2,9/4,-((b*x^4)/a),-((d*x^4)/c)]+3*b*c*AppellF1[5/4,7/4,1,9/4,-((b*x^4)/a),-((d*x^4)/c)]))+(b*(-9*a*c*(5*a*c+3*b*c*x^4+8*a*d*x^4+5*b*d*x^8)*AppellF1[5/4,3/4,1,9/4,-((b*x^4)/a),-((d*x^4)/c)]+5*x^4*(a+b*x^4)*(c+d*x^4)*(4*a*d*AppellF1[9/4,3/4,2,13/4,-((b*x^4)/a),-((d*x^4)/c)]+3*b*c*AppellF1[9/4,7/4,1,13/4,-((b*x^4)/a),-((d*x^4)/c)])))/(-9*a*c*AppellF1[5/4,3/4,1,9/4,-((b*x^4)/a),-((d*x^4)/c)]+x^4*(4*a*d*AppellF1[9/4,3/4,2,13/4,-((b*x^4)/a),-((d*x^4)/c)]+3*b*c*AppellF1[9/4,7/4,1,13/4,-((b*x^4)/a),-((d*x^4)/c)])))/(10*d*(a+b*x^4)^(3/4)*(c+d*x^4))`

**Maple [F]** time = 0.094, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}(bx^4+a)^{\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+a)^(5/4)/(d*x^4+c),x)`

[Out] `int((b*x^4+a)^(5/4)/(d*x^4+c),x)`



---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{5}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(5/4)/(d\*x^4 + c),x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(5/4)/(d\*x^4 + c), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(5/4)/(d\*x^4 + c),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^4)^{\frac{5}{4}}}{c + dx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(5/4)/(d\*x\*\*4+c),x)

[Out] Integral((a + b\*x\*\*4)\*\*(5/4)/(c + d\*x\*\*4), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{5}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(5/4)/(d\*x^4 + c),x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(5/4)/(d\*x^4 + c), x)

$$3.102 \quad \int \frac{\sqrt[4]{a + bx^4}}{c + dx^4} dx$$

**Optimal.** Leaf size=166

$$\frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \left( -\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1} \left( \frac{\sqrt[4]{bx^4}}{\sqrt[4]{bx^4+a}} \right) \middle| -1 \right)}{2\sqrt[4]{bc}} + \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \left( \frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1} \left( \frac{\sqrt[4]{bx^4}}{\sqrt[4]{bx^4+a}} \right) \middle| -1 \right)}{2\sqrt[4]{bc}}$$

[Out] (Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1)]/(2\*b^(1/4)\*c) + (Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1)]/(2\*b^(1/4)\*c)

**Rubi [A]** time = 0.29281, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \left( -\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1} \left( \frac{\sqrt[4]{bx^4}}{\sqrt[4]{bx^4+a}} \right) \middle| -1 \right)}{2\sqrt[4]{bc}} + \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \left( \frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1} \left( \frac{\sqrt[4]{bx^4}}{\sqrt[4]{bx^4+a}} \right) \middle| -1 \right)}{2\sqrt[4]{bc}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(1/4)/(c + d\*x^4), x]

[Out] (Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1)]/(2\*b^(1/4)\*c) + (Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1)]/(2\*b^(1/4)\*c)

**Rubi in Sympy [A]** time = 41.3291, size = 143, normalized size = 0.86

$$\frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \left( -\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}}; \operatorname{asin} \left( \frac{\sqrt[4]{bx^4}}{\sqrt[4]{a+bx^4}} \right) \middle| -1 \right)}{2\sqrt[4]{bc}} + \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \left( \frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}}; \operatorname{asin} \left( \frac{\sqrt[4]{bx^4}}{\sqrt[4]{a+bx^4}} \right) \middle| -1 \right)}{2\sqrt[4]{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)\*\*(1/4)/(d\*x\*\*4+c), x)

[Out] sqrt(a/(a + b\*x\*\*4))\*sqrt(a + b\*x\*\*4)\*elliptic\_pi(-sqrt(-a\*d + b\*c)/(sqrt(b)\*sqrt(c)), asin(b\*\*(1/4)\*x/(a + b\*x\*\*4)\*\*(1/4)), -1)/(2\*b\*\*(1/4)\*c) + sqrt(a/(a + b\*x\*\*4))\*sqrt(a + b\*x\*\*4)\*elliptic\_pi(sqrt(-a\*d + b\*c)/(sqrt(b)\*sqrt(c)), asin(b\*\*(1/4)\*x/(a + b\*x\*\*4)\*\*(1/4)), -1)/(2\*b\*\*(1/4)\*c)

**Mathematica [C]** time = 0.246518, size = 160, normalized size = 0.96

$$\frac{5acx\sqrt[4]{a+bx^4}F_1\left(\frac{1}{4}; -\frac{1}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{(c+dx^4)\left(x^4\left(bcF_1\left(\frac{5}{4}; \frac{3}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) - 4adF_1\left(\frac{5}{4}; -\frac{1}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) + 5acF_1\left(\frac{1}{4}; -\frac{1}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^4)^(1/4)/(c + d\*x^4), x]

[Out]  $(5*a*c*x*(a + b*x^4)^{(1/4)}*AppellF1[1/4, -1/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)]/((c + d*x^4)*(5*a*c*AppellF1[1/4, -1/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(-4*a*d*AppellF1[5/4, -1/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))$

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4 + c} \sqrt[4]{bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+a)^(1/4)/(d*x^4+c), x)`

[Out] `int((b*x^4+a)^(1/4)/(d*x^4+c), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{1}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x, algorithm="maxima")`

[Out] `integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{a + bx^4}}{c + dx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**4+a)**(1/4)/(d*x**4+c), x)`

[Out] `Integral((a + b*x**4)**(1/4)/(c + d*x**4), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{1}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x)
```

$$3.103 \quad \int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)} dx$$

**Optimal.** Leaf size=259

$$\frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{a}(a+bx^4)^{3/4}(bc-ad)} - \frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4} \left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{a+bx^4}}\right) \middle| -1\right)}{2\sqrt[4]{bc}(bc-ad)} - \frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4} \left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{a+bx^4}}\right) \middle| -1\right)}{2\sqrt[4]{bc}(bc-ad)}$$

[Out]  $-\left(\left(b^{3/2}\right)^{\left(1 + a/\left(b \cdot x^4\right)\right)^{3/4}} \cdot x^3 \cdot \text{EllipticF}\left[\text{ArcCot}\left[\left(\text{Sqrt}[b] \cdot x^2\right)/\text{Sqrt}[a]\right]/2, 2\right]/\left(\text{Sqrt}[a] \cdot \left(b \cdot c - a \cdot d\right) \cdot \left(a + b \cdot x^4\right)^{3/4}\right)\right) - \left(d \cdot \text{Sqrt}\left[a/\left(a + b \cdot x^4\right)\right] \cdot \text{Sqrt}\left[a + b \cdot x^4\right] \cdot \text{EllipticPi}\left[-\left(\text{Sqrt}\left[b \cdot c - a \cdot d\right]/\left(\text{Sqrt}[b] \cdot \text{Sqrt}[c]\right)\right), \text{ArcSin}\left[\left(b^{1/4} \cdot x\right)/\left(a + b \cdot x^4\right)^{1/4}\right], -1\right]/\left(2 \cdot b^{1/4} \cdot c \cdot \left(b \cdot c - a \cdot d\right) - \left(d \cdot \text{Sqrt}\left[a/\left(a + b \cdot x^4\right)\right] \cdot \text{Sqrt}\left[a + b \cdot x^4\right] \cdot \text{EllipticPi}\left[\text{Sqrt}\left[b \cdot c - a \cdot d\right]/\left(\text{Sqrt}[b] \cdot \text{Sqrt}[c]\right), \text{ArcSin}\left[\left(b^{1/4} \cdot x\right)/\left(a + b \cdot x^4\right)^{1/4}\right], -1\right]/\left(2 \cdot b^{1/4} \cdot c \cdot \left(b \cdot c - a \cdot d\right)\right)\right)$

**Rubi [A]** time = 0.471444, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$\frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{a}(a+bx^4)^{3/4}(bc-ad)} - \frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4} \left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{a+bx^4}}\right) \middle| -1\right)}{2\sqrt[4]{bc}(bc-ad)} - \frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4} \left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{a+bx^4}}\right) \middle| -1\right)}{2\sqrt[4]{bc}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(3/4)\*(c + d\*x^4)),x]

[Out]  $-\left(\left(b^{3/2}\right)^{\left(1 + a/\left(b \cdot x^4\right)\right)^{3/4}} \cdot x^3 \cdot \text{EllipticF}\left[\text{ArcCot}\left[\left(\text{Sqrt}[b] \cdot x^2\right)/\text{Sqrt}[a]\right]/2, 2\right]/\left(\text{Sqrt}[a] \cdot \left(b \cdot c - a \cdot d\right) \cdot \left(a + b \cdot x^4\right)^{3/4}\right)\right) - \left(d \cdot \text{Sqrt}\left[a/\left(a + b \cdot x^4\right)\right] \cdot \text{Sqrt}\left[a + b \cdot x^4\right] \cdot \text{EllipticPi}\left[-\left(\text{Sqrt}\left[b \cdot c - a \cdot d\right]/\left(\text{Sqrt}[b] \cdot \text{Sqrt}[c]\right)\right), \text{ArcSin}\left[\left(b^{1/4} \cdot x\right)/\left(a + b \cdot x^4\right)^{1/4}\right], -1\right]/\left(2 \cdot b^{1/4} \cdot c \cdot \left(b \cdot c - a \cdot d\right) - \left(d \cdot \text{Sqrt}\left[a/\left(a + b \cdot x^4\right)\right] \cdot \text{Sqrt}\left[a + b \cdot x^4\right] \cdot \text{EllipticPi}\left[\text{Sqrt}\left[b \cdot c - a \cdot d\right]/\left(\text{Sqrt}[b] \cdot \text{Sqrt}[c]\right), \text{ArcSin}\left[\left(b^{1/4} \cdot x\right)/\left(a + b \cdot x^4\right)^{1/4}\right], -1\right]/\left(2 \cdot b^{1/4} \cdot c \cdot \left(b \cdot c - a \cdot d\right)\right)\right)$

**Rubi in Sympy [A]** time = 62.6977, size = 221, normalized size = 0.85

$$\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4} \left(-\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}}; \text{asin}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{a+bx^4}}\right) \middle| -1\right)}{2\sqrt[4]{bc}(ad-bc)} + \frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4} \left(\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}}; \text{asin}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{a+bx^4}}\right) \middle| -1\right)}{2\sqrt[4]{bc}(ad-bc)} + \frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} F\left(\frac{\text{atan}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{2} \middle| 2\right)}{\sqrt{a}(a+bx^4)^{3/4}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c),x)`

[Out]  $d\sqrt{a/(a + b x^4)}\sqrt{a + b x^4}\operatorname{elliptic\_pi}(-\sqrt{-a d + b c}/(\sqrt{b}\sqrt{c}), \operatorname{asin}(b^{1/4}x/(a + b x^4)^{1/4}), -1)/(2 b^{1/4}c(a d - b c)) + d\sqrt{a/(a + b x^4)}\sqrt{a + b x^4}\operatorname{elliptic\_pi}(\sqrt{-a d + b c}/(\sqrt{b}\sqrt{c}), \operatorname{asin}(b^{1/4}x/(a + b x^4)^{1/4}), -1)/(2 b^{1/4}c(a d - b c)) + b^{3/2}x^3(a/(b x^4) + 1)^{3/4}\operatorname{elliptic\_f}(\operatorname{atan}(\sqrt{a}/(\sqrt{b}x^2))/2, 2)/(\sqrt{a}(a + b x^4)^{3/4}(a d - b c))$

**Mathematica [C]** time = 0.0867231, size = 161, normalized size = 0.62

$$\frac{5acx F_1\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{5}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{(a + bx^4)^{3/4}(c + dx^4)\left(x^4\left(4ad F_1\left(\frac{5}{4}, \frac{3}{4}, 2; \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bc F_1\left(\frac{5}{4}, \frac{7}{4}, 1; \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) - 5ac F_1\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{5}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}\right)$$

Warning: Unable to verify antiderivative.

[In] `Integrate[1/((a + b*x^4)^(3/4)*(c + d*x^4)),x]`

[Out]  $(-5 a^c x \operatorname{AppellF1}[1/4, 3/4, 1, 5/4, -((b x^4)/a), -((d x^4)/c)]) / ((a + b x^4)^{3/4} (c + d x^4) (-5 a^c \operatorname{AppellF1}[1/4, 3/4, 1, 5/4, -((b x^4)/a), -((d x^4)/c)] + x^4 (4 a^d \operatorname{AppellF1}[5/4, 3/4, 2, 9/4, -((b x^4)/a), -((d x^4)/c)] + 3 b^c \operatorname{AppellF1}[5/4, 7/4, 1, 9/4, -((b x^4)/a), -((d x^4)/c)]))$

**Maple [F]** time = 0.052, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4 + c} (bx^4 + a)^{-3/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^4+a)^(3/4)/(d*x^4+c),x)`

[Out] `int(1/(b*x^4+a)^(3/4)/(d*x^4+c),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{3/4}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)),x, algorithm="fricas")`

[Out] Timed out

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^4)^{\frac{3}{4}}(c + dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)\*\*(3/4)/(d\*x\*\*4+c), x)

[Out] Integral(1/((a + b\*x\*\*4)\*\*(3/4)\*(c + d\*x\*\*4)), x)

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{3}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)^(3/4)\*(d\*x^4 + c)), x, algorithm="giac")

[Out] integrate(1/((b\*x^4 + a)^(3/4)\*(d\*x^4 + c)), x)

$$3.104 \quad \int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)} dx$$

**Optimal.** Leaf size=304

$$\begin{aligned} & \frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (2bc - 5ad)F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{3a^{3/2}(a+bx^4)^{3/4}(bc-ad)^2} \\ & + \frac{d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^4+a}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{bc}(bc-ad)^2} \\ & + \frac{d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^4+a}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{bc}(bc-ad)^2} + \frac{bx}{3a(a+bx^4)^{3/4}(bc-ad)} \end{aligned}$$

[Out] (b\*x)/(3\*a\*(b\*c - a\*d)\*(a + b\*x^4)^(3/4)) - (b^(3/2)\*(2\*b\*c - 5\*a\*d)\*(1 + a/(b\*x^4))^(3/4)\*x^3\*EllipticF[ArcCot[(Sqrt[b]\*x^2)/Sqrt[a]]/2, 2])/(3\*a^(3/2)\*(b\*c - a\*d)^2\*(a + b\*x^4)^(3/4)) + (d^2\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(2\*b^(1/4)\*c\*(b\*c - a\*d)^2) + (d^2\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(2\*b^(1/4)\*c\*(b\*c - a\*d)^2)

**Rubi [A]** time = 0.69783, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$\begin{aligned} & \frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (2bc - 5ad)F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{3a^{3/2}(a+bx^4)^{3/4}(bc-ad)^2} \\ & + \frac{d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^4+a}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{bc}(bc-ad)^2} \\ & + \frac{d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^4+a}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{bc}(bc-ad)^2} + \frac{bx}{3a(a+bx^4)^{3/4}(bc-ad)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(7/4)\*(c + d\*x^4)),x]

[Out] (b\*x)/(3\*a\*(b\*c - a\*d)\*(a + b\*x^4)^(3/4)) - (b^(3/2)\*(2\*b\*c - 5\*a\*d)\*(1 + a/(b\*x^4))^(3/4)\*x^3\*EllipticF[ArcCot[(Sqrt[b]\*x^2)/Sqrt[a]]/2, 2])/(3\*a^(3/2)\*(b\*c - a\*d)^2\*(a + b\*x^4)^(3/4)) + (d^2\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(2\*b^(1/4)\*c\*(b\*c - a\*d)^2) + (d^2\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(2\*b^(1/4)\*c\*(b\*c - a\*d)^2)



**Rubi in Sympy [A]** time = 99.6203, size = 265, normalized size = 0.87

$$\frac{d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \left( -\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}}; \operatorname{asin} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}} \right) \Big| -1 \right)}{2\sqrt[4]{bc} (ad-bc)^2} + \frac{d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \left( \frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}}; \operatorname{asin} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}} \right) \Big| -1 \right)}{2\sqrt[4]{bc} (ad-bc)^2} - \frac{bx}{3a(a+bx^4)^{\frac{3}{4}} (ad-bc)} + \frac{b^{\frac{3}{2}} x^3 (5ad-2bc) \left( \frac{a}{bx^4} + 1 \right)^{\frac{3}{4}} F \left( \frac{\operatorname{atan} \left( \frac{\sqrt{a}}{\sqrt{bx^2}} \right)}{2} \Big| 2 \right)}{3a^{\frac{3}{2}} (a+bx^4)^{\frac{3}{4}} (ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c), x)`

[Out] `d**2*sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*elliptic_pi(-sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(2*b**(1/4)*c*(a*d - b*c)**2) + d**2*sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*elliptic_pi(sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(2*b**(1/4)*c*(a*d - b*c)**2) - b*x/(3*a*(a + b*x**4)**(3/4)*(a*d - b*c)) + b**(3/2)*x**3*(5*a*d - 2*b*c)*(a/(b*x**4) + 1)**(3/4)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x**2))/2, 2)/(3*a**(3/2)*(a + b*x**4)**(3/4)*(a*d - b*c)**2)`

**Mathematica [C]** time = 0.536398, size = 343, normalized size = 1.13

$$x \left( \frac{18bcdx^4 F_1 \left( \frac{5}{4}, \frac{3}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right)}{(c+dx^4) \left( x^4 \left( 4adF_1 \left( \frac{9}{4}, \frac{3}{4}, 2, \frac{13}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) + 3bcF_1 \left( \frac{9}{4}, \frac{7}{4}, 1, \frac{13}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) \right) - 9acF_1 \left( \frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) \right)} + \frac{25c(2bc)}{(c+dx^4) \left( x^4 \left( 4adF_1 \left( \frac{5}{4}, \frac{3}{4}, 2, \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) \right) \right)} \right) / (15(a+bx^4)^{3/4} (ad-bc))$$

Warning: Unable to verify antiderivative.

[In] `Integrate[1/((a + b*x^4)^(7/4)*(c + d*x^4)), x]`

[Out] `(x*((-5*b)/a + (25*c*(2*b*c - 3*a*d)*AppellF1[1/4, 3/4, 1, 5/4, -(b*x^4)/a, -((d*x^4)/c)])/((c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -(b*x^4)/a, -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -(b*x^4)/a, -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -(b*x^4)/a, -((d*x^4)/c)])) + (18*b*c*d*x^4*AppellF1[5/4, 3/4, 1, 9/4, -(b*x^4)/a, -((d*x^4)/c)])/((c + d*x^4)*(-9*a*c*AppellF1[5/4, 3/4, 1, 9/4, -(b*x^4)/a, -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[9/4, 3/4, 2, 13/4, -(b*x^4)/a, -((d*x^4)/c)] + 3*b*c*AppellF1[9/4, 7/4, 1, 13/4, -(b*x^4)/a, -((d*x^4)/c)])))/(15*(-(b*c) + a*d)*(a + b*x^4)^(3/4))`

**Maple [F]** time = 0.054, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4 + c} (bx^4 + a)^{-\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^4+a)^(7/4)/(d*x^4+c), x)`

[Out] `int(1/(b*x^4+a)^(7/4)/(d*x^4+c), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{7}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)), x)`

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)),x, algorithm="fricas")`

[Out] Timed out

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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^4)^{\frac{7}{4}}(c + dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c),x)`

[Out] `Integral(1/((a + b*x**4)**(7/4)*(c + d*x**4)), x)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{7}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)), x)`

$$3.105 \quad \int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)} dx$$

**Optimal.** Leaf size=357

$$\frac{bx(6bc - 13ad)}{21a^2(a + bx^4)^{3/4}(bc - ad)^2} - \frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (47a^2d^2 - 38abcd + 12b^2c^2) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{21a^{5/2}(a + bx^4)^{3/4}(bc - ad)^3} - \frac{d^3 \sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} \left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{bc}(bc - ad)^3} - \frac{d^3 \sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} \left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{bc}(bc - ad)^3} + \frac{bx}{7a(a + bx^4)^{7/4}(bc - ad)}$$

[Out] (b\*x)/(7\*a\*(b\*c - a\*d)\*(a + b\*x^4)^(7/4)) + (b\*(6\*b\*c - 13\*a\*d)\*x)/(21\*a^2\*(b\*c - a\*d)^2\*(a + b\*x^4)^(3/4)) - (b^(3/2)\*(12\*b^2\*c^2 - 38\*a\*b\*c\*d + 47\*a^2\*d^2)\*(1 + a/(b\*x^4))^(3/4)\*x^3\*EllipticF[ArcCot[(Sqrt[b]\*x^2)/Sqrt[a]]/2, 2])/(21\*a^(5/2)\*(b\*c - a\*d)^3\*(a + b\*x^4)^(3/4)) - (d^3\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(2\*b^(1/4)\*c\*(b\*c - a\*d)^3) - (d^3\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(2\*b^(1/4)\*c\*(b\*c - a\*d)^3)

**Rubi [A]** time = 1.07988, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$

$$\frac{bx(6bc - 13ad)}{21a^2(a + bx^4)^{3/4}(bc - ad)^2} - \frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (47a^2d^2 - 38abcd + 12b^2c^2) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{21a^{5/2}(a + bx^4)^{3/4}(bc - ad)^3} - \frac{d^3 \sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} \left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{bc}(bc - ad)^3} - \frac{d^3 \sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} \left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{bc}(bc - ad)^3} + \frac{bx}{7a(a + bx^4)^{7/4}(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(11/4)\*(c + d\*x^4)), x]

[Out] (b\*x)/(7\*a\*(b\*c - a\*d)\*(a + b\*x^4)^(7/4)) + (b\*(6\*b\*c - 13\*a\*d)\*x)/(21\*a^2\*(b\*c - a\*d)^2\*(a + b\*x^4)^(3/4)) - (b^(3/2)\*(12\*b^2\*c^2 - 38\*a\*b\*c\*d + 47\*a^2\*d^2)\*(1 + a/(b\*x^4))^(3/4)\*x^3\*EllipticF[ArcCot[(Sqrt[b]\*x^2)/Sqrt[a]]/2, 2])/(21\*a^(5/2)\*(b\*c - a\*d)^3\*(a + b\*x^4)^(3/4)) - (d^3\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(2\*b^(1/4)\*c\*(b\*c - a\*d)^3) - (d^3\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(2\*b^(1/4)\*c\*(b\*c - a\*d)^3)

**Rubi in Sympy [A]** time = 161.555, size = 318, normalized size = 0.89

$$\frac{d^3 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \left( -\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}}; \operatorname{asin} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}} \right) \Big| -1 \right)}{2\sqrt[4]{bc}(ad-bc)^3} + \frac{d^3 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \left( \frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}}; \operatorname{asin} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}} \right) \Big| -1 \right)}{2\sqrt[4]{bc}(ad-bc)^3} - \frac{bx}{7a(a+bx^4)^{\frac{7}{4}}(ad-bc)}$$

$$- \frac{bx(13ad-6bc)}{21a^2(a+bx^4)^{\frac{3}{4}}(ad-bc)^2} + \frac{b^{\frac{3}{2}}x^3 \left( \frac{a}{bx^4} + 1 \right)^{\frac{3}{4}} (47a^2d^2 - 38abcd + 12b^2c^2) F \left( \frac{\operatorname{atan} \left( \frac{\sqrt{a}}{\sqrt{bx^2}} \right)}{2} \Big| 2 \right)}{21a^{\frac{5}{2}}(a+bx^4)^{\frac{3}{4}}(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(b*x**4+a)**(11/4)/(d*x**4+c), x)`

[Out] `d**3*sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*elliptic_pi(-sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(2*b**(1/4)*c*(a*d - b*c)**3) + d**3*sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*elliptic_pi(sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(2*b**(1/4)*c*(a*d - b*c)**3) - b*x/(7*a*(a + b*x**4)**(7/4)*(a*d - b*c)) - b*x*(13*a*d - 6*b*c)/(21*a**2*(a + b*x**4)**(3/4)*(a*d - b*c)**2) + b**(3/2)*x**3*(a/(b*x**4) + 1)**(3/4)*(47*a**2*d**2 - 38*a*b*c*d + 12*b**2*c**2)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x**2))/2, 2)/(21*a**(5/2)*(a + b*x**4)**(3/4)*(a*d - b*c)**3)`

**Mathematica [C]** time = 1.56098, size = 407, normalized size = 1.14

$$x \left( \frac{25ac(21a^2d^2 - 26abcd + 12b^2c^2) F_1 \left( \frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right)}{(c+dx^4) \left( 5ac F_1 \left( \frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) - x^4 \left( 4ad F_1 \left( \frac{5}{4}, \frac{3}{4}, 2, \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) + 3bc F_1 \left( \frac{5}{4}, \frac{7}{4}, 1, \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) \right) \right)} + \frac{5b(-16a^2d+ab(9c-13dx^4)+6b^2cx^4)}{a+bx^4} + \frac{1}{c} \right) \frac{1}{105a^2(a+bx^4)^{3/4}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[1/((a + b*x^4)^(11/4)*(c + d*x^4)), x]`

[Out] `(x*((5*b*(-16*a^2*d + 6*b^2*c*x^4 + a*b*(9*c - 13*d*x^4)))/(a + b*x^4) + (25*a*c*(12*b^2*c^2 - 26*a*b*c*d + 21*a^2*d^2)*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)]/((c + d*x^4)*(5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])) + (18*a*b*c*d*(-6*b*c + 13*a*d)*x^4*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]/((c + d*x^4)*(-9*a*c*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[9/4, 3/4, 2, 13/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[9/4, 7/4, 1, 13/4, -((b*x^4)/a), -((d*x^4)/c)])))/((105*a^2*(b*c - a*d)^2*(a + b*x^4)^(3/4))`

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4 + c} (bx^4 + a)^{-\frac{11}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^4+a)^(11/4)/(d*x^4+c), x)`

[Out] `int(1/(b*x^4+a)^(11/4)/(d*x^4+c),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{11}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)**(11/4)/(d*x**4+c),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{11}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)), x)`

$$3.106 \quad \int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2} dx$$

**Optimal.** Leaf size=280

$$\begin{aligned} & \frac{b^{7/4}(8bc - 11ad) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{8d^3} - \frac{b^{7/4}(8bc - 11ad) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{8d^3} \\ & + \frac{(bc - ad)^{7/4}(3ad + 8bc) \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3} + \frac{(bc - ad)^{7/4}(3ad + 8bc) \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3} \\ & + \frac{bx(a+bx^4)^{3/4}(2bc - ad)}{4cd^2} - \frac{x(a+bx^4)^{7/4}(bc - ad)}{4cd(c+dx^4)} \end{aligned}$$

[Out] (b\*(2\*b\*c - a\*d)\*x\*(a + b\*x^4)^(3/4))/(4\*c\*d^2) - ((b\*c - a\*d)\*x\*(a + b\*x^4)^(7/4))/(4\*c\*d\*(c + d\*x^4)) - (b^(7/4)\*(8\*b\*c - 11\*a\*d)\*ArcTan[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)])/(8\*d^3) + ((b\*c - a\*d)^(7/4)\*(8\*b\*c + 3\*a\*d)\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(8\*c^(7/4)\*d^3) - (b^(7/4)\*(8\*b\*c - 11\*a\*d)\*ArcTanh[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)])/(8\*d^3) + ((b\*c - a\*d)^(7/4)\*(8\*b\*c + 3\*a\*d)\*ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(8\*c^(7/4)\*d^3)

**Rubi [A]** time = 0.893411, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$

$$\begin{aligned} & \frac{b^{7/4}(8bc - 11ad) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{8d^3} - \frac{b^{7/4}(8bc - 11ad) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{8d^3} \\ & + \frac{(bc - ad)^{7/4}(3ad + 8bc) \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3} + \frac{(bc - ad)^{7/4}(3ad + 8bc) \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3} \\ & + \frac{bx(a+bx^4)^{3/4}(2bc - ad)}{4cd^2} - \frac{x(a+bx^4)^{7/4}(bc - ad)}{4cd(c+dx^4)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(11/4)/(c + d\*x^4)^2, x]

[Out] (b\*(2\*b\*c - a\*d)\*x\*(a + b\*x^4)^(3/4))/(4\*c\*d^2) - ((b\*c - a\*d)\*x\*(a + b\*x^4)^(7/4))/(4\*c\*d\*(c + d\*x^4)) - (b^(7/4)\*(8\*b\*c - 11\*a\*d)\*ArcTan[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)])/(8\*d^3) + ((b\*c - a\*d)^(7/4)\*(8\*b\*c + 3\*a\*d)\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(8\*c^(7/4)\*d^3) - (b^(7/4)\*(8\*b\*c - 11\*a\*d)\*ArcTanh[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)])/(8\*d^3) + ((b\*c - a\*d)^(7/4)\*(8\*b\*c + 3\*a\*d)\*ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(8\*c^(7/4)\*d^3)

**Rubi in Sympy [A]** time = 101.332, size = 255, normalized size = 0.91

$$\begin{aligned} & \frac{b^{7/4}(11ad - 8bc) \operatorname{atan}\left(\frac{\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{8d^3} + \frac{b^{7/4}(11ad - 8bc) \operatorname{atanh}\left(\frac{\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{8d^3} \\ & - \frac{bx(a+bx^4)^{3/4}(ad - 2bc)}{4cd^2} + \frac{x(a+bx^4)^{7/4}(ad - bc)}{4cd(c+dx^4)} \\ & + \frac{(-ad + bc)^{7/4}(3ad + 8bc) \operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3} + \frac{(-ad + bc)^{7/4}(3ad + 8bc) \operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**4+a)**(11/4)/(d*x**4+c)**2,x)`

[Out]  $b^{7/4}(11ad - 8b^2c) \operatorname{atan}(b^{1/4}x/(a + b^2x^4)^{1/4})/(8d^3) + b^{7/4}(11ad - 8b^2c) \operatorname{atanh}(b^{1/4}x/(a + b^2x^4)^{1/4})/(8d^3) - b^2x(a + b^2x^4)^{3/4}(ad - 2b^2c)/(4c^2d^2) + x(a + b^2x^4)^{7/4}(ad - b^2c)/(4c^2d(c + dx^4)) + (-ad + b^2c)^{7/4}(3ad + 8b^2c) \operatorname{atan}(x(-ad + b^2c)^{1/4}/(c^{1/4}(a + b^2x^4)^{1/4}))/(8c^{7/4}d^3) + (-ad + b^2c)^{7/4}(3ad + 8b^2c) \operatorname{atanh}(x(-ad + b^2c)^{1/4}/(c^{1/4}(a + b^2x^4)^{1/4}))/(8c^{7/4}d^3)$

**Mathematica [C]** time = 4.04845, size = 735, normalized size = 2.62

$$5 \left( 3a^3d^3x^4 \log \left( \frac{x\sqrt[4]{bc-ad} + \sqrt[4]{c}}{\sqrt[4]{ax^4+b}} \right) + 3a^3cd^2 \log \left( \frac{x\sqrt[4]{bc-ad} + \sqrt[4]{c}}{\sqrt[4]{ax^4+b}} \right) - a(c+dx^4)(3a^2d^2+2abcd-2b^2c^2) \log \left( \sqrt[4]{c} - \frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}} \right) + 2a(c+dx^4)(3a^2d^2+2abcd) \right)$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]`

[Out]  $((-36a^2b^2c^{11/4}(-8b^2c + 11ad)x^5 \operatorname{AppellF1}[5/4, 1/4, 1, 9/4, -((b^2x^4)/a), -((d^2x^4)/c)]) / ((a + b^2x^4)^{1/4}(-9a^2c \operatorname{AppellF1}[5/4, 1/4, 1, 9/4, -((b^2x^4)/a), -((d^2x^4)/c)] + x^4(4a^2d \operatorname{AppellF1}[9/4, 1/4, 2, 13/4, -((b^2x^4)/a), -((d^2x^4)/c)] + b^2c \operatorname{AppellF1}[9/4, 5/4, 1, 13/4, -((b^2x^4)/a), -((d^2x^4)/c)])) + (5(8b^2c^{11/4}(b^2c - ad)^{1/4}x^5(a + b^2x^4)^{3/4} - 8a^2b^2c^{7/4}d(b^2c - ad)^{1/4}x^5(a + b^2x^4)^{3/4} + 4a^2c^{3/4}d^2(b^2c - ad)^{1/4}x^5(a + b^2x^4)^{3/4} + 4b^2c^{7/4}d^2(b^2c - ad)^{1/4}x^5(a + b^2x^4)^{3/4} + 2a^2(-2b^2c^2 + 2ab^2cd + 3a^2d^2)^2(c + d^2x^4) \operatorname{ArcTan}[(b^2c - ad)^{1/4}x/(c^{1/4}(b + a^2x^4)^{1/4})] - a^2(-2b^2c^2 + 2ab^2cd + 3a^2d^2)(c + d^2x^4) \operatorname{Log}[c^{1/4} - ((b^2c - ad)^{1/4}x)/(b + a^2x^4)^{1/4}] - 2a^2b^2c^3 \operatorname{Log}[c^{1/4} + ((b^2c - ad)^{1/4}x)/(b + a^2x^4)^{1/4}] + 2a^2b^2c^2d \operatorname{Log}[c^{1/4} + ((b^2c - ad)^{1/4}x)/(b + a^2x^4)^{1/4}] + 3a^3c^2d^2 \operatorname{Log}[c^{1/4} + ((b^2c - ad)^{1/4}x)/(b + a^2x^4)^{1/4}] - 2a^2b^2c^2d^2x^4 \operatorname{Log}[c^{1/4} + ((b^2c - ad)^{1/4}x)/(b + a^2x^4)^{1/4}] + 2a^2b^2c^2d^2x^4 \operatorname{Log}[c^{1/4} + ((b^2c - ad)^{1/4}x)/(b + a^2x^4)^{1/4}] + 3a^3d^3x^4 \operatorname{Log}[c^{1/4} + ((b^2c - ad)^{1/4}x)/(b + a^2x^4)^{1/4}])) / (b^2c - ad)^{1/4}) / (80c^{7/4}d^2(c + d^2x^4))$

**Maple [F]** time = 0.096, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^4 + c)^2} (bx^4 + a)^{\frac{11}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+a)^(11/4)/(d*x^4+c)^2,x)`

[Out] `int((b*x^4+a)^(11/4)/(d*x^4+c)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{11}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(11/4)/(d\*x^4 + c)^2,x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(11/4)/(d\*x^4 + c)^2, x)

---

**Fricas [A]** time = 19.9538, size = 3868, normalized size = 13.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(11/4)/(d\*x^4 + c)^2,x, algorithm="fricas")

[Out] 
$$-1/16*(4*(c*d^3*x^4 + c^2*d^2)*((4096*b^{11}*c^{11} - 22528*a*b^{10}*c^{10}*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^{10}*b*c*d^{10} - 81*a^{11}*d^{11})/(c^7*d^{12}))^{1/4}*\arctan(c^5*d^9*x*((4096*b^{11}*c^{11} - 22528*a*b^{10}*c^{10}*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^{10}*b*c*d^{10} - 81*a^{11}*d^{11})/(c^7*d^{12}))^{3/4}/(x*\sqrt{((4096*b^{11}*c^{14}*d^6 - 22528*a*b^{10}*c^{13}*d^7 + 46464*a^2*b^9*c^{12}*d^8 - 37664*a^3*b^8*c^{11}*d^9 - 5071*a^4*b^7*c^{10}*d^{10} + 25641*a^5*b^6*c^9*d^{11} - 7931*a^6*b^5*c^8*d^{12} - 6259*a^7*b^4*c^7*d^{13} + 2739*a^8*b^3*c^6*d^{14} + 891*a^9*b^2*c^5*d^{15} - 297*a^{10}*b*c^4*d^{16} - 81*a^{11}*c^3*d^{17})*x^2*\sqrt{((4096*b^{11}*c^{11} - 22528*a*b^{10}*c^{10}*d + 46464*a^2*b^9*c^9*d^2 + 46464*a^2*b^9*c^9*d^2 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^{10}*b*c*d^{10} - 81*a^{11}*d^{11})/(c^7*d^{12}))} + (262144*b^{16}*c^{16} - 2031616*a*b^{15}*c^{15}*d + 6451200*a^2*b^{14}*c^{14}*d^2 - 10168320*a^3*b^{13}*c^{13}*d^3 + 6467520*a^4*b^{12}*c^{12}*d^4 + 3123216*a^5*b^{11}*c^{11}*d^5 - 7258119*a^6*b^{10}*c^{10}*d^6 + 2307030*a^7*b^9*c^9*d^7 + 2428965*a^8*b^8*c^8*d^8 - 1607320*a^9*b^7*c^7*d^9 - 387134*a^{10}*b^6*c^6*d^{10} + 436356*a^{11}*b^5*c^5*d^{11} + 40770*a^{12}*b^4*c^4*d^{12} - 63720*a^{13}*b^3*c^3*d^{13} - 6075*a^{14}*b^2*c^2*d^{14} + 4374*a^{15}*b*c*d^{15} + 729*a^{16}*d^{16})*\sqrt{b*x^4 + a})/x^2) - (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a^2*b^6*c^6*d^2 - 413*a^3*b^5*c^5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c^3*d^5 + 234*a^6*b^2*c^2*d^6 - 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^{1/4})) - 4*(c*d^3*x^4 + c^2*d^2)*((4096*b^{11}*c^4 - 22528*a*b^{10}*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^{12})^{1/4}*\arctan(d^9*x*((4096*b^{11}*c^4 - 22528*a*b^{10}*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^{12})^{3/4}/(x*\sqrt{((4096*b^{11}*c^4*d^6 - 22528*a*b^{10}*c^3*d^7 + 46464*a^2*b^9*c^2*d^8 - 42592*a^3*b^8*c*d^9 + 14641*a^4*b^7*d^{10})*x^2*\sqrt{((4096*b^{11}*c^4 - 22528*a*b^{10}*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^{12})} + (262144*b^{16}*c^6 - 2162688*a*b^{15}*c^5*d + 7434240*a^2*b^{14}*c^4*d^2 - 13629440*a^3*b^{13}*c^3*d^3 + 14055360*a^4*b^{12}*c^2*d^4 - 7730448*a^5*b^{11}*c*d^5 + 1771561*a^6*b^{10}*d^6)*\sqrt{b*x^4 + a})/x^2) - (512*b^8*c^3 - 2112*a*b^7*c^2*d + 2904*a^2*b^6*c*d^2 - 1331*a^3*b^5*d^3)*(b*x^4 + a)^{1/4})) - (c*d^3*x^4 + c^2*d^2)*((4096*b^{11}*c^{11} - 22528*a*b^{10}*c^{10}*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^{10}*b*c*d^{10} - 81*a^{11}*d^{11})/(c^7*d^{12}))^{1/4}*\log(-(c^5*d^9*x*((4096*b^{11}*c^{11} - 22528*a*b^{10}*c^{10}*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^{10}*b*c*d^{10} - 81*a^{11}*d^{11})/(c^7*d^{12}))^{3/4} + (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a^2*b^6*c^6*d^2 - 413*a^3*b^5*c^5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c^3*d^5 + 234*a^6*b^2*c^2*d^6 - 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^{1/4})/x) + (c*d^3*x^4 + c^2*d^2)*((4096*b^{11}*c^{11} - 22528*a*b^{10}*c^{10}*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^{10}*b*c*d^{10} - 81*a^{11}*d^{11}*$$



$$\begin{aligned} & 11)/(c^7 d^{12}))^{1/4} \log((c^5 d^9 x^* ((4096 b^{11} c^{11} - 22528 a^* b \\ & ^{10} c^{10} d + 46464 a^2 b^9 c^9 d^2 - 37664 a^3 b^8 c^8 d^3 - 5071 \\ & * a^4 b^7 c^7 d^4 + 25641 a^5 b^6 c^6 d^5 - 7931 a^6 b^5 c^5 d^6 - \\ & 6259 a^7 b^4 c^4 d^7 + 2739 a^8 b^3 c^3 d^8 + 891 a^9 b^2 c^2 d^9 \\ & 9 - 297 a^{10} b^* c^* d^{10} - 81 a^{11} d^{11})/(c^7 d^{12}))^{3/4} - (512 b^8 \\ & c^8 - 1984 a^* b^7 c^7 d + 2456 a^2 b^6 c^6 d^2 - 413 a^3 b^5 c^5 \\ & * d^3 - 1175 a^4 b^4 c^4 d^4 + 478 a^5 b^3 c^3 d^5 + 234 a^6 b^2 c^2 \\ & ^2 d^6 - 81 a^7 b^* c^* d^7 - 27 a^8 d^8) * (b^* x^4 + a)^{1/4})/x) + (c^* \\ & d^3 x^4 + c^2 d^2) * ((4096 b^{11} c^4 - 22528 a^* b^{10} c^3 d + 46464 a^2 \\ & ^2 b^9 c^2 d^2 - 42592 a^3 b^8 c^* d^3 + 14641 a^4 b^7 d^4)/d^{12})^{1/4} \\ & \log(-(d^9 x^* ((4096 b^{11} c^4 - 22528 a^* b^{10} c^3 d + 46464 a^2 \\ & ^2 b^9 c^2 d^2 - 42592 a^3 b^8 c^* d^3 + 14641 a^4 b^7 d^4)/d^{12})^{3/4} \\ & + (512 b^8 c^3 - 2112 a^* b^7 c^2 d + 2904 a^2 b^6 c^* d^2 - 1331 a^3 \\ & b^5 d^3) * (b^* x^4 + a)^{1/4})/x) - (c^* d^3 x^4 + c^2 d^2) * ((4096 \\ & ^2 b^{11} c^4 - 22528 a^* b^{10} c^3 d + 46464 a^2 b^9 c^2 d^2 - 42592 a^3 \\ & ^3 b^8 c^* d^3 + 14641 a^4 b^7 d^4)/d^{12})^{1/4} \log((d^9 x^* ((4096 b^8 \\ & ^8 c^4 - 22528 a^* b^{10} c^3 d + 46464 a^2 b^9 c^2 d^2 - 42592 a^3 b^8 \\ & ^8 c^* d^3 + 14641 a^4 b^7 d^4)/d^{12})^{3/4} - (512 b^8 c^3 - 2112 a^* \\ & ^2 b^7 c^2 d + 2904 a^2 b^6 c^* d^2 - 1331 a^3 b^5 d^3) * (b^* x^4 + a)^{1/4}) \\ & /x) - 4 * (b^2 c^* d^* x^5 + (2 b^2 c^2 - 2 a^* b^* c^* d + a^2 d^2) * x)^* \\ & (b^* x^4 + a)^{3/4})/(c^* d^3 x^4 + c^2 d^2) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(11/4)/(d\*x\*\*4+c)\*\*2,x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{11}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(11/4)/(d\*x^4 + c)^2,x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(11/4)/(d\*x^4 + c)^2, x)

$$3.107 \quad \int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2} dx$$

**Optimal.** Leaf size=230

$$\frac{b^{7/4} \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{2d^2} + \frac{b^{7/4} \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{2d^2} - \frac{(bc-ad)^{3/4}(3ad+4bc) \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt{a+bx^4}}\right)}{8c^{7/4}d^2}$$

$$- \frac{(bc-ad)^{3/4}(3ad+4bc) \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt{a+bx^4}}\right)}{8c^{7/4}d^2} - \frac{x(a+bx^4)^{3/4}(bc-ad)}{4cd(c+dx^4)}$$

[Out]  $-\left((b^*c - a^*d)*x*(a + b^*x^4)^{(3/4)}\right)/\left(4^*c^*d*(c + d^*x^4)\right) + (b^{(7/4)} * \text{ArcTan}\left[\left(b^{(1/4)}*x\right)/\left(a + b^*x^4\right)^{(1/4)}\right])/ \left(2^*d^2\right) - \left((b^*c - a^*d)^{(3/4)} * (4^*b^*c + 3^*a^*d) * \text{ArcTan}\left[\left((b^*c - a^*d)^{(1/4)}*x\right)/\left(c^{(1/4)}*(a + b^*x^4)^{(1/4)}\right)\right]\right)/ \left(8^*c^{(7/4)}*d^2\right) + (b^{(7/4)} * \text{ArcTanh}\left[\left(b^{(1/4)}*x\right)/\left(a + b^*x^4\right)^{(1/4)}\right])/ \left(2^*d^2\right) - \left((b^*c - a^*d)^{(3/4)} * (4^*b^*c + 3^*a^*d) * \text{ArcTanh}\left[\left((b^*c - a^*d)^{(1/4)}*x\right)/\left(c^{(1/4)}*(a + b^*x^4)^{(1/4)}\right)\right]\right)/ \left(8^*c^{(7/4)}*d^2\right)$

**Rubi [A]** time = 0.438932, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$\frac{b^{7/4} \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{2d^2} + \frac{b^{7/4} \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{2d^2} - \frac{(bc-ad)^{3/4}(3ad+4bc) \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt{a+bx^4}}\right)}{8c^{7/4}d^2}$$

$$- \frac{(bc-ad)^{3/4}(3ad+4bc) \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt{a+bx^4}}\right)}{8c^{7/4}d^2} - \frac{x(a+bx^4)^{3/4}(bc-ad)}{4cd(c+dx^4)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(7/4)/(c + d\*x^4)^2, x]

[Out]  $-\left((b^*c - a^*d)*x*(a + b^*x^4)^{(3/4)}\right)/\left(4^*c^*d*(c + d^*x^4)\right) + (b^{(7/4)} * \text{ArcTan}\left[\left(b^{(1/4)}*x\right)/\left(a + b^*x^4\right)^{(1/4)}\right])/ \left(2^*d^2\right) - \left((b^*c - a^*d)^{(3/4)} * (4^*b^*c + 3^*a^*d) * \text{ArcTan}\left[\left((b^*c - a^*d)^{(1/4)}*x\right)/\left(c^{(1/4)}*(a + b^*x^4)^{(1/4)}\right)\right]\right)/ \left(8^*c^{(7/4)}*d^2\right) + (b^{(7/4)} * \text{ArcTanh}\left[\left(b^{(1/4)}*x\right)/\left(a + b^*x^4\right)^{(1/4)}\right])/ \left(2^*d^2\right) - \left((b^*c - a^*d)^{(3/4)} * (4^*b^*c + 3^*a^*d) * \text{ArcTanh}\left[\left((b^*c - a^*d)^{(1/4)}*x\right)/\left(c^{(1/4)}*(a + b^*x^4)^{(1/4)}\right)\right]\right)/ \left(8^*c^{(7/4)}*d^2\right)$

**Rubi in Sympy [A]** time = 63.5151, size = 206, normalized size = 0.9

$$\frac{b^{7/4} \operatorname{atan}\left(\frac{\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{2d^2} + \frac{b^{7/4} \operatorname{atanh}\left(\frac{\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{2d^2} + \frac{x(a+bx^4)^{3/4}(ad-bc)}{4cd(c+dx^4)}$$

$$- \frac{(-ad+bc)^{3/4}(3ad+4bc) \operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt{a+bx^4}}\right)}{8c^{7/4}d^2} - \frac{(-ad+bc)^{3/4}(3ad+4bc) \operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt{a+bx^4}}\right)}{8c^{7/4}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)\*\*(7/4)/(d\*x\*\*4+c)\*\*2, x)

[Out]  $b^{(7/4)} * \operatorname{atan}\left(b^{(1/4)}*x/\left(a + b^*x^4\right)^{(1/4)}\right)/ \left(2^*d^2\right) + b^{(7/4)} * \operatorname{atanh}\left(b^{(1/4)}*x/\left(a + b^*x^4\right)^{(1/4)}\right)/ \left(2^*d^2\right) + x*(a + b^*x^4)^{(3/4)} * (a*d - b*c)/ \left(4^*c^*d*(c + d^*x^4)\right) - (-a*d + b*c)^{(3/4)} * (3^*a^*d + 4^*b^*c) * \operatorname{atan}\left(x*(-a*d + b*c)^{(1/4)}/\left(c^{(1/4)}*(a + b^*x^4)\right)^{(1/4)}\right) - (-a*d + b*c)^{(3/4)} * (3^*a^*d + 4^*b^*c) * \operatorname{atanh}\left(x*(-a*d + b*c)^{(1/4)}/\left(c^{(1/4)}*(a + b^*x^4)\right)^{(1/4)}\right)$

$1/4)))/(8*c**(7/4)*d**2) - (-a*d + b*c)**(3/4)*(3*a*d + 4*b*c)*at$   
 $anh(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4)))/(8*c**($   
 $7/4)*d**2)$

**Mathematica [C]** time = 0.81934, size = 462, normalized size = 2.01

$$\frac{3a^2 \left( -\log \left( \sqrt[4]{c} - \frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}} \right) + \log \left( \frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}} + \sqrt[4]{c} \right) + 2 \tan^{-1} \left( \frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}} \right) \right)}{16c^{7/4}\sqrt[4]{bc-ad}} - \frac{9ab^2cx^5F_1 \left( \frac{5}{4}; \frac{1}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right)}{5d\sqrt[4]{a+bx^4}(c+dx^4) \left( x^4 \left( 4adF_1 \left( \frac{9}{4}; \frac{1}{4}, 2; \frac{13}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) + bcF_1 \left( \frac{9}{4}; \frac{5}{4}, 1; \frac{13}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) \right) - 9acF_1 \left( \frac{5}{4}; \frac{1}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) \right)} + \frac{ab \left( -\log \left( \sqrt[4]{c} - \frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}} \right) + \log \left( \frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}} + \sqrt[4]{c} \right) + 2 \tan^{-1} \left( \frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}} \right) \right)}{16c^{3/4}d\sqrt[4]{bc-ad}} - \frac{x(a+bx^4)^{3/4}(bc-ad)}{4cd(c+dx^4)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^4)^(7/4)/(c + d\*x^4)^2, x]

[Out]  $-\left((b*c - a*d)*x*(a + b*x^4)^{(3/4)} / (4*c*d*(c + d*x^4)) - (9*a*b^2*c*x^5*AppellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]) / (5*d*(a + b*x^4)^{(1/4)*(c + d*x^4)}*(-9*a*c*AppellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[9/4, 1/4, 2, 13/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[9/4, 5/4, 1, 13/4, -((b*x^4)/a), -((d*x^4)/c)])) + (3*a^2*(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))]) - Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]) / (16*c^(7/4)*(b*c - a*d)^(1/4)) + (a*b*(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))]) - Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]) / (16*c^(3/4)*d*(b*c - a*d)^(1/4))$

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^4 + c)^2} (bx^4 + a)^{\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(7/4)/(d\*x^4+c)^2, x)

[Out] int((b\*x^4+a)^(7/4)/(d\*x^4+c)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{7}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(7/4)/(d\*x^4 + c)^2, x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(7/4)/(d\*x^4 + c)^2, x)

**Fricas** [A] time = 1.42465, size = 1972, normalized size = 8.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(7/4)/(d\*x^4 + c)^2,x, algorithm="fricas")

[Out] 
$$-1/16*(4*(b*x^4 + a)^{3/4}*(b*c - a*d)*x + 4*(c*d^2*x^4 + c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^{1/4}*\arctan(c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^{3/4})/(x*\sqrt{((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^{3/4}} + \sqrt{(b*x^4 + a)})/x^2) + (4096*b^{10}*c^{10} + 2048*a*b^9*c^9*d - 14592*a^2*b^8*c^8*d^2 - 9472*a^3*b^7*c^7*d^3 + 18928*a^4*b^6*c^6*d^4 + 15624*a^5*b^5*c^5*d^5 - 9639*a^6*b^4*c^4*d^6 - 11124*a^7*b^3*c^3*d^7 + 486*a^8*b^2*c^2*d^8 + 2916*a^9*b*c*d^9 + 729*a^{10}*d^{10})*\sqrt{(b*x^4 + a)})/x^2) + (64*b^5*c^5 + 16*a*b^4*c^4*d - 116*a^2*b^3*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b*x^4 + a)^{1/4}) - 16*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^{1/4}*\arctan(d^6*x*(b^7/d^8)^{3/4})/((b*x^4 + a)^{1/4}*b^5 + x*\sqrt{(b^7*d^4*x^2*\sqrt{(b^7/d^8)} + \sqrt{(b*x^4 + a)*b^{10}})/x^2})) + (c*d^2*x^4 + c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^{1/4}*\log((c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^{3/4} + (64*b^5*c^5 + 16*a*b^4*c^4*d - 116*a^2*b^3*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b*x^4 + a)^{1/4})/x) - (c*d^2*x^4 + c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^{1/4}*\log(-(c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^{3/4} - (64*b^5*c^5 + 16*a*b^4*c^4*d - 116*a^2*b^3*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b*x^4 + a)^{1/4})/x) - 4*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^{1/4}*\log((d^6*x*(b^7/d^8)^{3/4} + (b*x^4 + a)^{1/4}*b^5)/x) + 4*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^{1/4}*\log(-(d^6*x*(b^7/d^8)^{3/4} - (b*x^4 + a)^{1/4}*b^5)/x))/(c*d^2*x^4 + c^2*d)$$

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(7/4)/(d\*x\*\*4+c)\*\*2,x)

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{7}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2, x)
```

$$3.108 \quad \int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2} dx$$

**Optimal.** Leaf size=135

$$\frac{3a \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}} + \frac{3a \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}} + \frac{x(a+bx^4)^{3/4}}{4c(c+dx^4)}$$

[Out]  $(x*(a + b*x^4)^{(3/4)})/(4*c*(c + d*x^4)) + (3*a*ArcTan[((b*c - a*d)^{(1/4)*x})/(c^{(1/4)}*(a + b*x^4)^{(1/4)})])/(8*c^{(7/4)}*(b*c - a*d)^{(1/4)}) + (3*a*ArcTanh[((b*c - a*d)^{(1/4)*x})/(c^{(1/4)}*(a + b*x^4)^{(1/4)})])/(8*c^{(7/4)}*(b*c - a*d)^{(1/4)})$

**Rubi [A]** time = 0.203699, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{3a \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}} + \frac{3a \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}} + \frac{x(a+bx^4)^{3/4}}{4c(c+dx^4)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(3/4)/(c + d\*x^4)^2, x]

[Out]  $(x*(a + b*x^4)^{(3/4)})/(4*c*(c + d*x^4)) + (3*a*ArcTan[((b*c - a*d)^{(1/4)*x})/(c^{(1/4)}*(a + b*x^4)^{(1/4)})])/(8*c^{(7/4)}*(b*c - a*d)^{(1/4)}) + (3*a*ArcTanh[((b*c - a*d)^{(1/4)*x})/(c^{(1/4)}*(a + b*x^4)^{(1/4)})])/(8*c^{(7/4)}*(b*c - a*d)^{(1/4)})$

**Rubi in Sympy [A]** time = 31.2364, size = 119, normalized size = 0.88

$$\frac{3a \operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{-ad+bc}} + \frac{3a \operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{-ad+bc}} + \frac{x(a+bx^4)^{3/4}}{4c(c+dx^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)\*\*(3/4)/(d\*x\*\*4+c)\*\*2, x)

[Out]  $3*a*\operatorname{atan}(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4)))/(8*c**(7/4)*(-a*d + b*c)**(1/4)) + 3*a*\operatorname{atanh}(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4)))/(8*c**(7/4)*(-a*d + b*c)**(1/4)) + x*(a + b*x**4)**(3/4)/(4*c*(c + d*x**4))$

**Mathematica [A]** time = 0.350861, size = 152, normalized size = 1.13

$$\frac{3a \left( -\log\left(\sqrt[4]{c} - \frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right) + \log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}} + \sqrt[4]{c}\right) + 2 \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right) \right)}{16c^{7/4}\sqrt[4]{bc-ad}} + \frac{x(a+bx^4)^{3/4}}{4c(c+dx^4)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^4)^(3/4)/(c + d\*x^4)^2, x]

[Out]  $(x*(a + b*x^4)^{(3/4)})/(4*c*(c + d*x^4)) + (3*a*(2*ArcTan[((b*c - a*d)^{(1/4)*x})/(c^{(1/4)}*(b + a*x^4)^{(1/4)})]) - \operatorname{Log}[c^{(1/4)} - ((b*c$

$$- a*d)^{(1/4)*x)/(b + a*x^4)^{(1/4)}] + \text{Log}[c^{(1/4)} + ((b*c - a*d)^{(1/4)*x)/(b + a*x^4)^{(1/4)}])]/(16*c^{(7/4)}*(b*c - a*d)^{(1/4)})$$

**Maple [F]** time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^4 + c)^2} (bx^4 + a)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(3/4)/(d\*x^4+c)^2, x)

[Out] int((b\*x^4+a)^(3/4)/(d\*x^4+c)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(3/4)/(d\*x^4 + c)^2, x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(3/4)/(d\*x^4 + c)^2, x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(3/4)/(d\*x^4 + c)^2, x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(3/4)/(d\*x\*\*4+c)\*\*2, x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)
```



$$3.109 \quad \int \frac{1}{\sqrt[4]{a + bx^4}(c+dx^4)^2} dx$$

**Optimal.** Leaf size=162

$$\frac{(4bc - 3ad) \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}} + \frac{(4bc - 3ad) \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}} - \frac{dx(a+bx^4)^{3/4}}{4c(c+dx^4)(bc-ad)}$$

[Out]  $-(d*x*(a + b*x^4)^{(3/4)})/(4*c*(b*c - a*d)*(c + d*x^4)) + ((4*b*c - 3*a*d)*ArcTan[((b*c - a*d)^{(1/4)*x}/(c^{(1/4)*(a + b*x^4)^{(1/4)})})]/(8*c^{(7/4)*(b*c - a*d)^{(5/4)}) + ((4*b*c - 3*a*d)*ArcTanh[((b*c - a*d)^{(1/4)*x}/(c^{(1/4)*(a + b*x^4)^{(1/4)})})]/(8*c^{(7/4)*(b*c - a*d)^{(5/4)})$

**Rubi [A]** time = 0.300709, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{(4bc - 3ad) \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}} + \frac{(4bc - 3ad) \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}} - \frac{dx(a+bx^4)^{3/4}}{4c(c+dx^4)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(1/4)\*(c + d\*x^4)^2), x]

[Out]  $-(d*x*(a + b*x^4)^{(3/4)})/(4*c*(b*c - a*d)*(c + d*x^4)) + ((4*b*c - 3*a*d)*ArcTan[((b*c - a*d)^{(1/4)*x}/(c^{(1/4)*(a + b*x^4)^{(1/4)})})]/(8*c^{(7/4)*(b*c - a*d)^{(5/4)}) + ((4*b*c - 3*a*d)*ArcTanh[((b*c - a*d)^{(1/4)*x}/(c^{(1/4)*(a + b*x^4)^{(1/4)})})]/(8*c^{(7/4)*(b*c - a*d)^{(5/4)})$

**Rubi in Sympy [A]** time = 36.6484, size = 141, normalized size = 0.87

$$\frac{dx(a+bx^4)^{3/4}}{4c(c+dx^4)(ad-bc)} - \frac{(3ad-4bc) \operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(-ad+bc)^{5/4}} - \frac{(3ad-4bc) \operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(-ad+bc)^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*4+a)\*\*(1/4)/(d\*x\*\*4+c)\*\*2, x)

[Out]  $d*x*(a + b*x**4)**(3/4)/(4*c*(c + d*x**4)*(a*d - b*c)) - (3*a*d - 4*b*c)*\operatorname{atan}(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4)))/(8*c**(7/4)*(-a*d + b*c)**(5/4)) - (3*a*d - 4*b*c)*\operatorname{atanh}(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4)))/(8*c**(7/4)*(-a*d + b*c)**(5/4))$

**Mathematica [A]** time = 0.285242, size = 171, normalized size = 1.06

$$\frac{(4bc - 3ad) \left( -\log\left(\sqrt[4]{c} - \frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right) + \log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}} + \sqrt[4]{c}\right) + 2 \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right) \right)}{16c^{7/4}(bc-ad)^{5/4}} - \frac{dx(a+bx^4)^{3/4}}{4c(c+dx^4)(bc-ad)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(1/4)\*(c + d\*x^4)^2), x]

[Out]  $-(d*x*(a + b*x^4)^{3/4})/(4*c*(b*c - a*d)*(c + d*x^4)) + ((4*b*c - 3*a*d)*(2*\text{ArcTan}[\frac{(b*c - a*d)^{1/4}*x}{c^{1/4}*(b + a*x^4)^{1/4}}]) - \text{Log}[c^{1/4} - \frac{(b*c - a*d)^{1/4}*x}{(b + a*x^4)^{1/4}}] + \text{Log}[c^{1/4} + \frac{(b*c - a*d)^{1/4}*x}{(b + a*x^4)^{1/4}}])/(16*c^{7/4}*(b*c - a*d)^{5/4})$

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^4 + c)^2} \frac{1}{\sqrt[4]{bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(1/4)/(d\*x^4+c)^2, x)

[Out] int(1/(b\*x^4+a)^(1/4)/(d\*x^4+c)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{1/4}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)^(1/4)\*(d\*x^4 + c)^2), x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(1/4)\*(d\*x^4 + c)^2), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)^(1/4)\*(d\*x^4 + c)^2), x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)\*\*(1/4)/(d\*x\*\*4+c)\*\*2, x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{1/4}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2), x)
```

$$3.110 \quad \int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2} dx$$

**Optimal.** Leaf size=205

$$\frac{d(8bc - 3ad) \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}} - \frac{d(8bc - 3ad) \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}} \\ + \frac{bx(ad+4bc)}{4ac\sqrt[4]{a+bx^4}(bc-ad)^2} - \frac{dx}{4c\sqrt[4]{a+bx^4}(c+dx^4)(bc-ad)}$$

[Out]  $(b*(4*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(a + b*x^4)^{(1/4)}) - (d*x)/(4*c*(b*c - a*d)*(a + b*x^4)^{(1/4)*(c + d*x^4)}) - (d*(8*b*c - 3*a*d)*\text{ArcTan}[(b*c - a*d)^{(1/4)*x}/(c^{(1/4)*(a + b*x^4)^{(1/4)})])]/(8*c^{(7/4)*(b*c - a*d)^{(9/4)}) - (d*(8*b*c - 3*a*d)*\text{ArcTanh}[(b*c - a*d)^{(1/4)*x}/(c^{(1/4)*(a + b*x^4)^{(1/4)})])]/(8*c^{(7/4)*(b*c - a*d)^{(9/4)})$

**Rubi [A]** time = 0.495085, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{d(8bc - 3ad) \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}} - \frac{d(8bc - 3ad) \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}} \\ + \frac{bx(ad+4bc)}{4ac\sqrt[4]{a+bx^4}(bc-ad)^2} - \frac{dx}{4c\sqrt[4]{a+bx^4}(c+dx^4)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(5/4)\*(c + d\*x^4)^2), x]

[Out]  $(b*(4*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(a + b*x^4)^{(1/4)}) - (d*x)/(4*c*(b*c - a*d)*(a + b*x^4)^{(1/4)*(c + d*x^4)}) - (d*(8*b*c - 3*a*d)*\text{ArcTan}[(b*c - a*d)^{(1/4)*x}/(c^{(1/4)*(a + b*x^4)^{(1/4)})])]/(8*c^{(7/4)*(b*c - a*d)^{(9/4)}) - (d*(8*b*c - 3*a*d)*\text{ArcTanh}[(b*c - a*d)^{(1/4)*x}/(c^{(1/4)*(a + b*x^4)^{(1/4)})])]/(8*c^{(7/4)*(b*c - a*d)^{(9/4)})$

**Rubi in Sympy [A]** time = 77.0004, size = 180, normalized size = 0.88

$$\frac{dx}{4c\sqrt[4]{a+bx^4}(c+dx^4)(ad-bc)} + \frac{d(3ad-8bc) \operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(-ad+bc)^{9/4}} \\ + \frac{d(3ad-8bc) \operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(-ad+bc)^{9/4}} + \frac{bx(ad+4bc)}{4ac\sqrt[4]{a+bx^4}(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*4+a)\*\*(5/4)/(d\*x\*\*4+c)\*\*2, x)

[Out]  $d*x/(4*c*(a + b*x^4)^{(1/4)*(c + d*x^4)*(a*d - b*c)}) + d*(3*a*d - 8*b*c)*\operatorname{atan}(x*(-a*d + b*c)^{(1/4)}/(c^{(1/4)*(a + b*x^4)^{(1/4)}})))/(8*c^{(7/4)*(-a*d + b*c)^{(9/4)}) + d*(3*a*d - 8*b*c)*\operatorname{atanh}(x*(-a*d + b*c)^{(1/4)}/(c^{(1/4)*(a + b*x^4)^{(1/4)}})))/(8*c^{(7/4)*(-a*d + b*c)^{(9/4)}) + b*x*(a*d + 4*b*c)/(4*a*c*(a + b*x^4)^{(1/4)*(a*d - b*c)^2}$

**Mathematica [A]** time = 0.449289, size = 204, normalized size = 1.

$$\frac{x(a^2d^2 + abd^2x^4 + 4b^2c(c + dx^4))}{4ac\sqrt[4]{a + bx^4}(c + dx^4)(bc - ad)^2} + \frac{d(3ad - 8bc) \left( -\log\left(\sqrt[4]{c} - \frac{x\sqrt[4]{bc - ad}}{\sqrt[4]{ax^4 + b}}\right) + \log\left(\frac{x\sqrt[4]{bc - ad}}{\sqrt[4]{ax^4 + b}} + \sqrt[4]{c}\right) + 2 \tan^{-1}\left(\frac{x\sqrt[4]{bc - ad}}{\sqrt[4]{c}\sqrt[4]{ax^4 + b}}\right) \right)}{16c^{7/4}(bc - ad)^{9/4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(5/4)\*(c + d\*x^4)^2), x]

[Out] (x\*(a^2\*d^2 + a\*b\*d^2\*x^4 + 4\*b^2\*c\*(c + d\*x^4)))/(4\*a\*c\*(b\*c - a\*d)^2\*(a + b\*x^4)^(1/4)\*(c + d\*x^4)) + (d\*(-8\*b\*c + 3\*a\*d)\*(2\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(b + a\*x^4)^(1/4))] - Log[c^(1/4) - ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)] + Log[c^(1/4) + ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)])/(16\*c^(7/4)\*(b\*c - a\*d)^(9/4))

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^4 + c)^2} (bx^4 + a)^{-5/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(5/4)/(d\*x^4+c)^2, x)

[Out] int(1/(b\*x^4+a)^(5/4)/(d\*x^4+c)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{5/4}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)^(5/4)\*(d\*x^4 + c)^2), x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(5/4)\*(d\*x^4 + c)^2), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)^(5/4)\*(d\*x^4 + c)^2), x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{5}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2),x, algorithm="giac")`

[Out] `integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2), x)`

$$3.111 \quad \int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2} dx$$

**Optimal.** Leaf size=266

$$\begin{aligned} & \frac{bx(-5a^2d^2 - 56abcd + 16b^2c^2)}{20a^2c\sqrt[4]{a+bx^4}(bc-ad)^3} + \frac{3d^2(4bc-ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}} \\ & + \frac{3d^2(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}} \\ & - \frac{dx}{4c(a+bx^4)^{5/4}(c+dx^4)(bc-ad)} + \frac{bx(5ad+4bc)}{20ac(a+bx^4)^{5/4}(bc-ad)^2} \end{aligned}$$

[Out] (b\*(4\*b\*c + 5\*a\*d)\*x)/(20\*a\*c\*(b\*c - a\*d)^2\*(a + b\*x^4)^(5/4)) + (b\*(16\*b^2\*c^2 - 56\*a\*b\*c\*d - 5\*a^2\*d^2)\*x)/(20\*a^2\*c\*(b\*c - a\*d)^3\*(a + b\*x^4)^(1/4)) - (d\*x)/(4\*c\*(b\*c - a\*d)\*(a + b\*x^4)^(5/4)\*(c + d\*x^4)) + (3\*d^2\*(4\*b\*c - a\*d)\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(8\*c^(7/4)\*(b\*c - a\*d)^(13/4)) + (3\*d^2\*(4\*b\*c - a\*d)\*ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(8\*c^(7/4)\*(b\*c - a\*d)^(13/4))

**Rubi [A]** time = 0.884767, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned} & \frac{bx(-5a^2d^2 - 56abcd + 16b^2c^2)}{20a^2c\sqrt[4]{a+bx^4}(bc-ad)^3} + \frac{3d^2(4bc-ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}} \\ & + \frac{3d^2(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}} \\ & - \frac{dx}{4c(a+bx^4)^{5/4}(c+dx^4)(bc-ad)} + \frac{bx(5ad+4bc)}{20ac(a+bx^4)^{5/4}(bc-ad)^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(9/4)\*(c + d\*x^4)^2), x]

[Out] (b\*(4\*b\*c + 5\*a\*d)\*x)/(20\*a\*c\*(b\*c - a\*d)^2\*(a + b\*x^4)^(5/4)) + (b\*(16\*b^2\*c^2 - 56\*a\*b\*c\*d - 5\*a^2\*d^2)\*x)/(20\*a^2\*c\*(b\*c - a\*d)^3\*(a + b\*x^4)^(1/4)) - (d\*x)/(4\*c\*(b\*c - a\*d)\*(a + b\*x^4)^(5/4)\*(c + d\*x^4)) + (3\*d^2\*(4\*b\*c - a\*d)\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(8\*c^(7/4)\*(b\*c - a\*d)^(13/4)) + (3\*d^2\*(4\*b\*c - a\*d)\*ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(8\*c^(7/4)\*(b\*c - a\*d)^(13/4))

**Rubi in Sympy [A]** time = 144.937, size = 240, normalized size = 0.9

$$\begin{aligned} & \frac{dx}{4c(a+bx^4)^{5/4}(c+dx^4)(ad-bc)} - \frac{3d^2(ad-4bc)\operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(-ad+bc)^{13/4}} \\ & - \frac{3d^2(ad-4bc)\operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(-ad+bc)^{13/4}} + \frac{bx(5ad+4bc)}{20ac(a+bx^4)^{5/4}(ad-bc)^2} + \frac{bx(5a^2d^2+56abcd-16b^2c^2)}{20a^2c\sqrt[4]{a+bx^4}(ad-bc)^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*4+a)\*\*(9/4)/(d\*x\*\*4+c)\*\*2, x)

[Out]  $d^2 x / (4^2 c^2 (a + b x^4)^{5/4} (c + d x^4) (a d - b^2 c)) - 3 d^2 (a d - 4 b^2 c) \operatorname{atan}(x (-a d + b^2 c)^{1/4} / (c^{1/4} (a + b x^4)^{1/4})) / (8 c^{7/4} (-a d + b^2 c)^{13/4}) - 3 d^2 (a d - 4 b^2 c) a \operatorname{tanh}(x (-a d + b^2 c)^{1/4} / (c^{1/4} (a + b x^4)^{1/4})) / (8 c^{7/4} (-a d + b^2 c)^{13/4}) + b x^5 (5 a d + 4 b^2 c) / (20 a^2 c (a + b x^4)^{5/4} (a d - b^2 c)^2) + b x^5 (5 a^2 d^2 + 56 a b^2 c d - 16 b^2 c^2) / (20 a^2 c^2 (a + b x^4)^{1/4} (a d - b^2 c)^3)$

**Mathematica [A]** time = 0.822363, size = 242, normalized size = 0.91

$$\frac{1}{20} x (a + b x^4)^{3/4} \left( \frac{8 b^2 (7 a d - 2 b c)}{a^2 (a + b x^4) (a d - b c)^3} + \frac{4 b^2}{a (a + b x^4)^2 (b c - a d)^2} - \frac{5 d^3}{c (c + d x^4) (b c - a d)^3} \right) + \frac{3 d^2 (4 b c - a d) \left( -\log \left( \sqrt[4]{c} - \frac{x \sqrt[4]{b c - a d}}{\sqrt[4]{a x^4 + b}} \right) + \log \left( \frac{x \sqrt[4]{b c - a d}}{\sqrt[4]{a x^4 + b}} + \sqrt[4]{c} \right) + 2 \tan^{-1} \left( \frac{x \sqrt[4]{b c - a d}}{\sqrt[4]{c} \sqrt[4]{a x^4 + b}} \right) \right)}{16 c^{7/4} (b c - a d)^{13/4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(9/4)\*(c + d\*x^4)^2), x]

[Out]  $(x (a + b x^4)^{3/4} ((4 b^2) / (a (b^2 c - a^2 d)^2 (a + b x^4)^2) + (8 b^2 (-2 b^2 c + 7 a^2 d) / (a^2 (-b^2 c + a^2 d)^3 (a + b x^4)) - (5 d^3) / (c (b^2 c - a^2 d)^3 (c + d x^4)))) / 20 + (3 d^2 (4 b^2 c - a^2 d) (2 \operatorname{ArcTan}[(b^2 c - a^2 d)^{1/4} x] / (c^{1/4} (b + a x^4)^{1/4})) - \operatorname{Log}[c^{1/4} - ((b^2 c - a^2 d)^{1/4} x) / (b + a x^4)^{1/4}] + \operatorname{Log}[c^{1/4} + ((b^2 c - a^2 d)^{1/4} x) / (b + a x^4)^{1/4}]) / (16 c^{7/4} (b^2 c - a^2 d)^{13/4})$

**Maple [F]** time = 0.064, size = 0, normalized size = 0.

$$\int \frac{1}{(d x^4 + c)^2} (b x^4 + a)^{-\frac{9}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(9/4)/(d\*x^4+c)^2, x)

[Out] int(1/(b\*x^4+a)^(9/4)/(d\*x^4+c)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b x^4 + a)^{\frac{9}{4}} (d x^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^4 + a)^(9/4)\*(d\*x^4 + c)^2), x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(9/4)\*(d\*x^4 + c)^2), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



[In] `integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{9}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2),x, algorithm="giac")`

[Out] `integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2), x)`

$$3.112 \quad \int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2} dx$$

**Optimal.** Leaf size=353

$$\frac{\sqrt{ab^{3/2}}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (3bc - ad)F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{4cd^2 (a + bx^4)^{3/4}} - \frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2}d^2} - \frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2}d^2} + \frac{bx^4\sqrt{a+bx^4}(3bc-ad)}{4cd^2} - \frac{x(a+bx^4)^{5/4}(bc-ad)}{4cd(c+dx^4)}$$

[Out] (b\*(3\*b\*c - a\*d)\*x\*(a + b\*x^4)^(1/4))/(4\*c\*d^2) - ((b\*c - a\*d)\*x\*(a + b\*x^4)^(5/4))/(4\*c\*d\*(c + d\*x^4)) - (Sqrt[a]\*b^(3/2)\*(3\*b\*c - a\*d)\*(1 + a/(b\*x^4))^(3/4)\*x^3\*EllipticF[ArcCot[(Sqrt[b]\*x^2)/Sqrt[a]]/2, 2])/(4\*c\*d^2\*(a + b\*x^4)^(3/4)) - (3\*(b\*c - a\*d)\*(2\*b\*c + a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d^2) - (3\*(b\*c - a\*d)\*(2\*b\*c + a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d^2)

**Rubi [A]** time = 0.987495, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$

$$\frac{\sqrt{ab^{3/2}}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (3bc - ad)F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{4cd^2 (a + bx^4)^{3/4}} - \frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2}d^2} - \frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2}d^2} + \frac{bx^4\sqrt{a+bx^4}(3bc-ad)}{4cd^2} - \frac{x(a+bx^4)^{5/4}(bc-ad)}{4cd(c+dx^4)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(9/4)/(c + d\*x^4)^2, x]

[Out] (b\*(3\*b\*c - a\*d)\*x\*(a + b\*x^4)^(1/4))/(4\*c\*d^2) - ((b\*c - a\*d)\*x\*(a + b\*x^4)^(5/4))/(4\*c\*d\*(c + d\*x^4)) - (Sqrt[a]\*b^(3/2)\*(3\*b\*c - a\*d)\*(1 + a/(b\*x^4))^(3/4)\*x^3\*EllipticF[ArcCot[(Sqrt[b]\*x^2)/Sqrt[a]]/2, 2])/(4\*c\*d^2\*(a + b\*x^4)^(3/4)) - (3\*(b\*c - a\*d)\*(2\*b\*c + a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d^2) - (3\*(b\*c - a\*d)\*(2\*b\*c + a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d^2)

**Rubi in Sympy [A]** time = 112.913, size = 316, normalized size = 0.9

$$\frac{\sqrt{ab}^{\frac{3}{2}} x^3 (ad - 3bc) \left( \frac{a}{bx^4} + 1 \right)^{\frac{3}{4}} F \left( \frac{\operatorname{atan} \left( \frac{\sqrt{a}}{\sqrt{bx^2}} \right)}{2} \middle| 2 \right)}{4cd^2 (a + bx^4)^{\frac{3}{4}}} - \frac{bx^4 \sqrt{a + bx^4} (ad - 3bc)}{4cd^2} + \frac{x (a + bx^4)^{\frac{5}{4}} (ad - bc)}{4cd (c + dx^4)}$$

$$+ \frac{3 \sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} (ad - bc) (ad + 2bc) \left( -\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}}; \operatorname{asin} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a + bx^4}} \right) \middle| -1 \right)}{8 \sqrt[4]{bc^2} d^2}$$

$$+ \frac{3 \sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} (ad - bc) (ad + 2bc) \left( \frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}}; \operatorname{asin} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a + bx^4}} \right) \middle| -1 \right)}{8 \sqrt[4]{bc^2} d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**4+a)**(9/4)/(d*x**4+c)**2, x)`

[Out] `sqrt(a)*b**(3/2)*x**3*(a*d - 3*b*c)*(a/(b*x**4) + 1)**(3/4)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x**2))/2, 2)/(4*c*d**2*(a + b*x**4)**(3/4) - b*x*(a + b*x**4)**(1/4)*(a*d - 3*b*c)/(4*c*d**2) + x*(a + b*x**4)**(5/4)*(a*d - b*c)/(4*c*d*(c + d*x**4)) + 3*sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*(a*d - b*c)*(a*d + 2*b*c)*elliptic_pi(-sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(8*b**(1/4)*c**2*d**2) + 3*sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*(a*d - b*c)*(a*d + 2*b*c)*elliptic_pi(sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(8*b**(1/4)*c**2*d**2)`

**Mathematica [C]** time = 1.29411, size = 506, normalized size = 1.43

$$x \left( \frac{5x^4(a+bx^4)(a^2d^2-2abcd+b^2c(3c+2dx^4)) \left( 4adF_1 \left( \frac{9}{4}, \frac{3}{4}, 2; \frac{13}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) + 3bcF_1 \left( \frac{9}{4}, \frac{7}{4}, 1; \frac{13}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) \right) - 9ac(5a^3d^2+a^2bd(7dx^4-10c))+3ab^2c(5c+2d)}{c \left( x^4 \left( 4adF_1 \left( \frac{9}{4}, \frac{3}{4}, 2; \frac{13}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) + 3bcF_1 \left( \frac{9}{4}, \frac{7}{4}, 1; \frac{13}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) \right) - 9acF_1 \left( \frac{5}{4}, \frac{3}{4}, 1; \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) \right)} \right) 20d^2 (a + bx^4)^3$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(a + b*x^4)^(9/4)/(c + d*x^4)^2, x]`

[Out] `(x*((-25*a^2*(-3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*AppellF1[1/4, 3/4, 1, 5/4, -(b*x^4)/a, -(d*x^4)/c])/(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -(b*x^4)/a, -(d*x^4)/c] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -(b*x^4)/a, -(d*x^4)/c] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -(b*x^4)/a, -(d*x^4)/c])) + (-9*a*c*(5*a^3*d^2 + 3*a*b^2*c*(5*c + 2*d*x^4) + a^2*b*d*(-10*c + 7*d*x^4) + b^3*c*x^4*(9*c + 10*d*x^4))*AppellF1[5/4, 3/4, 1, 9/4, -(b*x^4)/a, -(d*x^4)/c] + 5*x^4*(a + b*x^4)*(-2*a*b*c*d + a^2*d^2 + b^2*c*(3*c + 2*d*x^4))*AppellF1[9/4, 3/4, 2, 13/4, -(b*x^4)/a, -(d*x^4)/c] + 3*b*c*AppellF1[9/4, 7/4, 1, 13/4, -(b*x^4)/a, -(d*x^4)/c]))/(c*(-9*a*c*AppellF1[5/4, 3/4, 1, 9/4, -(b*x^4)/a, -(d*x^4)/c] + x^4*(4*a*d*AppellF1[9/4, 3/4, 2, 13/4, -(b*x^4)/a, -(d*x^4)/c] + 3*b*c*AppellF1[9/4, 7/4, 1, 13/4, -(b*x^4)/a, -(d*x^4)/c])))/(20*d^2*(a + b*x^4)^(3/4)*(c + d*x^4))`

**Maple [F]** time = 0.094, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^4 + c)^2} (bx^4 + a)^{\frac{9}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+a)^(9/4)/(d*x^4+c)^2, x)`

[Out] `int((b*x^4+a)^(9/4)/(d*x^4+c)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{9}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2,x, algorithm="maxima")`

[Out] `integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2, x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2,x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**4+a)**(9/4)/(d*x**4+c)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{9}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2,x, algorithm="giac")`

[Out] `integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2, x)`

$$3.113 \quad \int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2} dx$$

**Optimal.** Leaf size=298

$$\frac{\sqrt{ab}^{3/2} x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{4cd(a+bx^4)^{3/4}} + \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (3ad+2bc) \left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2d}} + \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (3ad+2bc) \left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2d}} - \frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd(c+dx^4)}$$

[Out] -((b\*c - a\*d)\*x\*(a + b\*x^4)^(1/4))/(4\*c\*d\*(c + d\*x^4)) + (Sqrt[a]\*b^(3/2)\*(1 + a/(b\*x^4))^(3/4)\*x^3\*EllipticF[ArcCot[(Sqrt[b]\*x^2)/Sqrt[a]]/2, 2])/(4\*c\*d\*(a + b\*x^4)^(3/4)) + ((2\*b\*c + 3\*a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d) + ((2\*b\*c + 3\*a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d)

**Rubi [A]** time = 0.703559, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$\frac{\sqrt{ab}^{3/2} x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{4cd(a+bx^4)^{3/4}} + \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (3ad+2bc) \left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2d}} + \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (3ad+2bc) \left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2d}} - \frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd(c+dx^4)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(5/4)/(c + d\*x^4)^2, x]

[Out] -((b\*c - a\*d)\*x\*(a + b\*x^4)^(1/4))/(4\*c\*d\*(c + d\*x^4)) + (Sqrt[a]\*b^(3/2)\*(1 + a/(b\*x^4))^(3/4)\*x^3\*EllipticF[ArcCot[(Sqrt[b]\*x^2)/Sqrt[a]]/2, 2])/(4\*c\*d\*(a + b\*x^4)^(3/4)) + ((2\*b\*c + 3\*a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d) + ((2\*b\*c + 3\*a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d)

**Rubi in Sympy [A]** time = 83.37, size = 260, normalized size = 0.87

$$\frac{\sqrt{ab}^{\frac{3}{2}} x^3 \left( \frac{a}{bx^4} + 1 \right)^{\frac{3}{4}} F \left( \frac{\operatorname{atan} \left( \frac{\sqrt{a}}{\sqrt{bx^2}} \right)}{2} \middle| 2 \right)}{4cd (a + bx^4)^{\frac{3}{4}}} + \frac{x \sqrt[4]{a + bx^4} (ad - bc)}{4cd (c + dx^4)}$$

$$+ \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} (3ad + 2bc) \left( -\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}}; \operatorname{asin} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a + bx^4}} \right) \right) \Big| -1}{8\sqrt[4]{bc^2d}}$$

$$+ \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} (3ad + 2bc) \left( \frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}}; \operatorname{asin} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a + bx^4}} \right) \right) \Big| -1}{8\sqrt[4]{bc^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**4+a)**(5/4)/(d*x**4+c)**2, x)`

[Out] `sqrt(a)*b**(3/2)*x**3*(a/(b*x**4) + 1)**(3/4)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x**2))/2, 2)/(4*c*d*(a + b*x**4)**(3/4)) + x*(a + b*x**4)**(1/4)*(a*d - b*c)/(4*c*d*(c + d*x**4)) + sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*(3*a*d + 2*b*c)*elliptic_pi(-sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(8*b**(1/4)*c**2*d) + sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*(3*a*d + 2*b*c)*elliptic_pi(sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(8*b**(1/4)*c**2*d)`

**Mathematica [C]** time = 0.646114, size = 440, normalized size = 1.48

$$x \frac{\left( 9ac(5a^2d+ab(7dx^4-5c)-3b^2cx^4)F_1\left(\frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 5x^4(a+bx^4)(bc-ad)\left(4adF_1\left(\frac{9}{4}, \frac{3}{4}, 2, \frac{13}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bcF_1\left(\frac{9}{4}, \frac{7}{4}, 1, \frac{13}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) \right)}{c\left(9acF_1\left(\frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) - x^4\left(4adF_1\left(\frac{9}{4}, \frac{3}{4}, 2, \frac{13}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bcF_1\left(\frac{9}{4}, \frac{7}{4}, 1, \frac{13}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right)\right)} - \frac{x^4}{20d(a+bx^4)^{3/4}(c+dx^4)}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(a + b*x^4)^(5/4)/(c + d*x^4)^2, x]`

[Out] `(x*((-25*a^2*(b*c + 3*a*d)*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])) + (9*a*c*(5*a^2*d - 3*b^2*c*x^4 + a*b*(-5*c + 7*d*x^4))*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 5*(b*c - a*d)*x^4*(a + b*x^4)*(4*a*d*AppellF1[9/4, 3/4, 2, 13/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[9/4, 7/4, 1, 13/4, -((b*x^4)/a), -((d*x^4)/c)])))/(c*(9*a*c*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] - x^4*(4*a*d*AppellF1[9/4, 3/4, 2, 13/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[9/4, 7/4, 1, 13/4, -((b*x^4)/a), -((d*x^4)/c)])))/(20*d*(a + b*x^4)^(3/4)*(c + d*x^4))`

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^4 + c)^2} (bx^4 + a)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+a)^(5/4)/(d*x^4+c)^2, x)`

[Out] `int((b*x^4+a)^(5/4)/(d*x^4+c)^2, x)`

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**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{5}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(5/4)/(d\*x^4 + c)^2,x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(5/4)/(d\*x^4 + c)^2, x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(5/4)/(d\*x^4 + c)^2,x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(5/4)/(d\*x\*\*4+c)\*\*2,x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{5}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(5/4)/(d\*x^4 + c)^2,x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(5/4)/(d\*x^4 + c)^2, x)

$$3.114 \quad \int \frac{\sqrt[4]{a + bx^4}}{(c + dx^4)^2} dx$$

**Optimal.** Leaf size=308

$$\begin{aligned} & \frac{\sqrt{ab^{3/2}}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{4c(a + bx^4)^{3/4}(bc - ad)} \\ & + \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a + bx^4}(2bc - 3ad) \left(-\frac{\sqrt{bc-ad}}{\sqrt{b\sqrt{c}}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^4 + a}}{\sqrt[4]{bx^4 + a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2}(bc - ad)} \\ & + \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a + bx^4}(2bc - 3ad) \left(\frac{\sqrt{bc-ad}}{\sqrt{b\sqrt{c}}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^4 + a}}{\sqrt[4]{bx^4 + a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2}(bc - ad)} + \frac{x\sqrt[4]{a + bx^4}}{4c(c + dx^4)} \end{aligned}$$

[Out] (x\*(a + b\*x^4)^(1/4))/(4\*c\*(c + d\*x^4)) - (Sqrt[a]\*b^(3/2)\*(1 + a/(b\*x^4))^(3/4)\*x^3\*EllipticF[ArcCot[(Sqrt[b]\*x^2)/Sqrt[a]]/2, 2])/(4\*c\*(b\*c - a\*d)\*(a + b\*x^4)^(3/4)) + ((2\*b\*c - 3\*a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(8\*b^(1/4)\*c^2\*(b\*c - a\*d)) + ((2\*b\*c - 3\*a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(8\*b^(1/4)\*c^2\*(b\*c - a\*d))

**Rubi [A]** time = 0.636605, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$\begin{aligned} & \frac{\sqrt{ab^{3/2}}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{4c(a + bx^4)^{3/4}(bc - ad)} \\ & + \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a + bx^4}(2bc - 3ad) \left(-\frac{\sqrt{bc-ad}}{\sqrt{b\sqrt{c}}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^4 + a}}{\sqrt[4]{bx^4 + a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2}(bc - ad)} \\ & + \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a + bx^4}(2bc - 3ad) \left(\frac{\sqrt{bc-ad}}{\sqrt{b\sqrt{c}}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^4 + a}}{\sqrt[4]{bx^4 + a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2}(bc - ad)} + \frac{x\sqrt[4]{a + bx^4}}{4c(c + dx^4)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(1/4)/(c + d\*x^4)^2, x]

[Out] (x\*(a + b\*x^4)^(1/4))/(4\*c\*(c + d\*x^4)) - (Sqrt[a]\*b^(3/2)\*(1 + a/(b\*x^4))^(3/4)\*x^3\*EllipticF[ArcCot[(Sqrt[b]\*x^2)/Sqrt[a]]/2, 2])/(4\*c\*(b\*c - a\*d)\*(a + b\*x^4)^(3/4)) + ((2\*b\*c - 3\*a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(8\*b^(1/4)\*c^2\*(b\*c - a\*d)) + ((2\*b\*c - 3\*a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(8\*b^(1/4)\*c^2\*(b\*c - a\*d))



**Rubi in Sympy [A]** time = 85.7406, size = 267, normalized size = 0.87

$$\frac{\sqrt{ab}^{\frac{3}{2}} x^3 \left( \frac{a}{bx^4} + 1 \right)^{\frac{3}{4}} F \left( \frac{\operatorname{atan} \left( \frac{\sqrt{a}}{\sqrt{bx^2}} \right)}{2} \middle| 2 \right)}{4c(a+bx^4)^{\frac{3}{4}}(ad-bc)} + \frac{x^4 \sqrt{a+bx^4}}{4c(c+dx^4)}$$

$$+ \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (3ad-2bc) \left( -\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}}; \operatorname{asin} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}} \right) \right) \Big| -1}{8\sqrt[4]{bc^2}(ad-bc)}$$

$$+ \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (3ad-2bc) \left( \frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}}; \operatorname{asin} \left( \frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}} \right) \right) \Big| -1}{8\sqrt[4]{bc^2}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**4+a)**(1/4)/(d*x**4+c)**2,x)`

[Out] `sqrt(a)*b**(3/2)*x**3*(a/(b*x**4)+1)**(3/4)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x**2))/2,2)/(4*c*(a+b*x**4)**(3/4)*(a*d-b*c))+x*(a+b*x**4)**(1/4)/(4*c*(c+d*x**4))+sqrt(a/(a+b*x**4))*sqrt(a+b*x**4)*(3*a*d-2*b*c)*elliptic_pi(-sqrt(-a*d+b*c)/(sqrt(b)*sqrt(c)),asin(b**(1/4)*x/(a+b*x**4)**(1/4)),-1)/(8*b**(1/4)*c**2*(a*d-b*c))+sqrt(a/(a+b*x**4))*sqrt(a+b*x**4)*(3*a*d-2*b*c)*elliptic_pi(sqrt(-a*d+b*c)/(sqrt(b)*sqrt(c)),asin(b**(1/4)*x/(a+b*x**4)**(1/4)),-1)/(8*b**(1/4)*c**2*(a*d-b*c))`

**Mathematica [C]** time = 0.287786, size = 322, normalized size = 1.05

$$x \left( \frac{75a^2 F_1 \left( \frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right)}{x^4 \left( 4ad F_1 \left( \frac{5}{4}, \frac{3}{4}, 2, \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) + 3bc F_1 \left( \frac{5}{4}, \frac{7}{4}, 1, \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) \right) - 5ac F_1 \left( \frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right)}{x^4 \left( 4ad F_1 \left( \frac{9}{4}, \frac{3}{4}, 2, \frac{13}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) + 3bc F_1 \left( \frac{9}{4}, \frac{7}{4}, 1, \frac{13}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) \right)} - \frac{18abx^4 F_1 \left( \frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right)}{20(a+bx^4)^{3/4}(c+dx^4)} \right)$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(a+b*x^4)^(1/4)/(c+d*x^4)^2,x]`

[Out] `(x*((5*(a+b*x^4))/c - (75*a^2*AppellF1[1/4,3/4,1,5/4,-((b*x^4)/a),-((d*x^4)/c)])/(-5*a*c*AppellF1[1/4,3/4,1,5/4,-((b*x^4)/a),-((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4,3/4,2,9/4,-((b*x^4)/a),-((d*x^4)/c)] + 3*b*c*AppellF1[5/4,7/4,1,9/4,-((b*x^4)/a),-((d*x^4)/c)])) - (18*a*b*x^4*AppellF1[5/4,3/4,1,9/4,-((b*x^4)/a),-((d*x^4)/c)])/(-9*a*c*AppellF1[5/4,3/4,1,9/4,-((b*x^4)/a),-((d*x^4)/c)] + x^4*(4*a*d*AppellF1[9/4,3/4,2,13/4,-((b*x^4)/a),-((d*x^4)/c)] + 3*b*c*AppellF1[9/4,7/4,1,13/4,-((b*x^4)/a),-((d*x^4)/c)])))/(20*(a+b*x^4)^(3/4)*(c+d*x^4))`

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^4+c)^2} \sqrt[4]{bx^4+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+a)^(1/4)/(d*x^4+c)^2,x)`

[Out] `int((b*x^4+a)^(1/4)/(d*x^4+c)^2,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{1}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(1/4)/(d\*x^4 + c)^2,x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(1/4)/(d\*x^4 + c)^2, x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(1/4)/(d\*x^4 + c)^2,x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(1/4)/(d\*x\*\*4+c)\*\*2,x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{1}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^(1/4)/(d\*x^4 + c)^2,x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(1/4)/(d\*x^4 + c)^2, x)

$$3.115 \quad \int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2} dx$$

**Optimal.** Leaf size=330

$$\frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (4bc - ad) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{4\sqrt{ac}(a+bx^4)^{3/4}(bc-ad)^2} - \frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2}(bc-ad)^2} - \frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2}(bc-ad)^2} - \frac{dx\sqrt[4]{a+bx^4}}{4c(c+dx^4)(bc-ad)}$$

[Out]  $-(d*x*(a+b*x^4)^{(1/4)})/(4*c*(b*c-a*d)*(c+d*x^4)) - (b^{(3/2)}*(4*b*c-a*d)*(1+a/(b*x^4))^{(3/4)}*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*Sqrt[a]*c*(b*c-a*d)^2*(a+b*x^4)^{(3/4)}) - (3*d*(2*b*c-a*d)*Sqrt[a/(a+b*x^4)]*Sqrt[a+b*x^4]*EllipticPi[-(Sqrt[b*c-a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^{(1/4)}*x)/(a+b*x^4)^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*(b*c-a*d)^2) - (3*d*(2*b*c-a*d)*Sqrt[a/(a+b*x^4)]*Sqrt[a+b*x^4]*EllipticPi[Sqrt[b*c-a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^{(1/4)}*x)/(a+b*x^4)^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*(b*c-a*d)^2)$

**Rubi [A]** time = 0.752525, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$\frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (4bc - ad) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{4\sqrt{ac}(a+bx^4)^{3/4}(bc-ad)^2} - \frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2}(bc-ad)^2} - \frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2}(bc-ad)^2} - \frac{dx\sqrt[4]{a+bx^4}}{4c(c+dx^4)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(3/4)\*(c + d\*x^4)^2), x]

[Out]  $-(d*x*(a+b*x^4)^{(1/4)})/(4*c*(b*c-a*d)*(c+d*x^4)) - (b^{(3/2)}*(4*b*c-a*d)*(1+a/(b*x^4))^{(3/4)}*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*Sqrt[a]*c*(b*c-a*d)^2*(a+b*x^4)^{(3/4)}) - (3*d*(2*b*c-a*d)*Sqrt[a/(a+b*x^4)]*Sqrt[a+b*x^4]*EllipticPi[-(Sqrt[b*c-a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^{(1/4)}*x)/(a+b*x^4)^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*(b*c-a*d)^2) - (3*d*(2*b*c-a*d)*Sqrt[a/(a+b*x^4)]*Sqrt[a+b*x^4]*EllipticPi[Sqrt[b*c-a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^{(1/4)}*x)/(a+b*x^4)^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*(b*c-a*d)^2)$

**Rubi in Sympy [A]** time = 96.9919, size = 292, normalized size = 0.88

$$\frac{dx\sqrt[4]{a+bx^4}}{4c(c+dx^4)(ad-bc)} + \frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(ad-2bc)\left(-\frac{\sqrt{-ad+bc}}{\sqrt{b\sqrt{c}}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)\right|-1)}{8\sqrt[4]{bc^2}(ad-bc)^2}$$

$$+ \frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(ad-2bc)\left(\frac{\sqrt{-ad+bc}}{\sqrt{b\sqrt{c}}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)\right|-1)}{8\sqrt[4]{bc^2}(ad-bc)^2}$$

$$+ \frac{b^{\frac{3}{2}}x^3(ad-4bc)\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{2}\middle|2\right)}{4\sqrt{ac}(a+bx^4)^{\frac{3}{4}}(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c)**2,x)`

[Out] `d*x*(a + b*x**4)**(1/4)/(4*c*(c + d*x**4)*(a*d - b*c)) + 3*d*sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*(a*d - 2*b*c)*elliptic_pi(-sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(8*b**(1/4)*c**2*(a*d - b*c)**2) + 3*d*sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*(a*d - 2*b*c)*elliptic_pi(sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(8*b**(1/4)*c**2*(a*d - b*c)**2) + b**(3/2)*x**3*(a*d - 4*b*c)*(a/(b*x**4) + 1)**(3/4)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x**2))/2, 2)/(4*sqrt(a)*c*(a + b*x**4)**(3/4)*(a*d - b*c)**2)`

**Mathematica [C]** time = 0.890998, size = 341, normalized size = 1.03

$$x \left( \frac{18abd x^4 F_1\left(\frac{5}{4}, \frac{3}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{x^4 \left( 4ad F_1\left(\frac{9}{4}, \frac{3}{4}, 2; \frac{13}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bc F_1\left(\frac{9}{4}, \frac{7}{4}, 1; \frac{13}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right) - 9ac F_1\left(\frac{5}{4}, \frac{3}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)} + \frac{25a(3ad-4bc)F_1\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{x^4 \left( 4ad F_1\left(\frac{5}{4}, \frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bc F_1\left(\frac{5}{4}, \frac{7}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right)} \right) / (20(a+bx^4)^{3/4}(c+dx^4)(bc-ad))$$

Warning: Unable to verify antiderivative.

[In] `Integrate[1/((a + b*x^4)^(3/4)*(c + d*x^4)^2), x]`

[Out] `(x*((-5*d*(a + b*x^4))/c + (25*a*(-4*b*c + 3*a*d)*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)]/(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])) + (18*a*b*d*x^4*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]/(-9*a*c*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[9/4, 3/4, 2, 13/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[9/4, 7/4, 1, 13/4, -((b*x^4)/a), -((d*x^4)/c)])))/(20*(b*c - a*d)*(a + b*x^4)^(3/4)*(c + d*x^4))`

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^4 + c)^2} (bx^4 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x)`

[Out] `int(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{3}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2), x)`

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2),x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c)**2,x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{3}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2),x, algorithm="giac")`

[Out] `integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2), x)`

$$3.116 \quad \int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2} dx$$

**Optimal.** Leaf size=390

$$\begin{aligned} & \frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (3a^2d^2 - 32abcd + 8b^2c^2) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{12a^{3/2}c(a+bx^4)^{3/4}(bc-ad)^3} \\ & + \frac{d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (10bc-3ad) \left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2}(bc-ad)^3} \\ & + \frac{d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (10bc-3ad) \left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2}(bc-ad)^3} \\ & + \frac{bx(3ad+4bc)}{12ac(a+bx^4)^{3/4}(bc-ad)^2} - \frac{dx}{4c(a+bx^4)^{3/4}(c+dx^4)(bc-ad)} \end{aligned}$$

[Out] (b\*(4\*b\*c + 3\*a\*d)\*x)/(12\*a\*c\*(b\*c - a\*d)^2\*(a + b\*x^4)^(3/4)) - (d\*x)/(4\*c\*(b\*c - a\*d)\*(a + b\*x^4)^(3/4)\*(c + d\*x^4)) - (b^(3/2)\*(8\*b^2\*c^2 - 32\*a\*b\*c\*d + 3\*a^2\*d^2)\*(1 + a/(b\*x^4))^(3/4)\*x^3\*EllipticF[ArcCot[(Sqrt[b]\*x^2)/Sqrt[a]]/2, 2])/(12\*a^(3/2)\*c\*(b\*c - a\*d)^3\*(a + b\*x^4)^(3/4)) + (d^2\*(10\*b\*c - 3\*a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1)]/(8\*b^(1/4)\*c^2\*(b\*c - a\*d)^3) + (d^2\*(10\*b\*c - 3\*a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1)]/(8\*b^(1/4)\*c^2\*(b\*c - a\*d)^3)

**Rubi [A]** time = 1.16421, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$

$$\begin{aligned} & \frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (3a^2d^2 - 32abcd + 8b^2c^2) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right) \middle| 2\right)}{12a^{3/2}c(a+bx^4)^{3/4}(bc-ad)^3} \\ & + \frac{d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (10bc-3ad) \left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2}(bc-ad)^3} \\ & + \frac{d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (10bc-3ad) \left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^2}}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{bc^2}(bc-ad)^3} \\ & + \frac{bx(3ad+4bc)}{12ac(a+bx^4)^{3/4}(bc-ad)^2} - \frac{dx}{4c(a+bx^4)^{3/4}(c+dx^4)(bc-ad)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(7/4)\*(c + d\*x^4)^2), x]

[Out] (b\*(4\*b\*c + 3\*a\*d)\*x)/(12\*a\*c\*(b\*c - a\*d)^2\*(a + b\*x^4)^(3/4)) - (d\*x)/(4\*c\*(b\*c - a\*d)\*(a + b\*x^4)^(3/4)\*(c + d\*x^4)) - (b^(3/2)\*(8\*b^2\*c^2 - 32\*a\*b\*c\*d + 3\*a^2\*d^2)\*(1 + a/(b\*x^4))^(3/4)\*x^3\*EllipticF[ArcCot[(Sqrt[b]\*x^2)/Sqrt[a]]/2, 2])/(12\*a^(3/2)\*c\*(b\*c - a\*d)^3\*(a + b\*x^4)^(3/4)) + (d^2\*(10\*b\*c - 3\*a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1)]/(8\*b^(1/4)\*c^2\*(b\*c - a\*d)^3) + (d^2\*(10\*b\*c - 3\*a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1)]/(8\*b^(1/4)\*c^2\*(b\*c - a\*d)^3)

**Rubi in Sympy [A]** time = 157.932, size = 350, normalized size = 0.9

$$\frac{dx}{4c(a+bx^4)^{\frac{3}{4}}(c+dx^4)(ad-bc)} + \frac{d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (3ad-10bc) \left( -\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right) \middle| -1 \right)}{8\sqrt[4]{bc^2}(ad-bc)^3} + \frac{d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (3ad-10bc) \left( \frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}}; \operatorname{asin}\left(\frac{\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right) \middle| -1 \right)}{8\sqrt[4]{bc^2}(ad-bc)^3} + \frac{bx(3ad+4bc)}{12ac(a+bx^4)^{\frac{3}{4}}(ad-bc)^2} + \frac{b^{\frac{3}{2}}x^3 \left(\frac{a}{bx^4} + 1\right)^{\frac{3}{4}} (3a^2d^2 - 32abcd + 8b^2c^2) F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right)}{2} \middle| 2\right)}{12a^{\frac{3}{2}}c(a+bx^4)^{\frac{3}{4}}(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c)**2,x)`

[Out]  $d*x/(4*c*(a+b*x**4)**(3/4)*(c+d*x**4)*(a*d-b*c)) + d**2*sqrt(a/(a+b*x**4))*sqrt(a+b*x**4)*(3*a*d-10*b*c)*elliptic_pi(-sqrt(-a*d+b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a+b*x**4)**(1/4)), -1)/(8*b**(1/4)*c**2*(a*d-b*c)**3) + d**2*sqrt(a/(a+b*x**4))*sqrt(a+b*x**4)*(3*a*d-10*b*c)*elliptic_pi(sqrt(-a*d+b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a+b*x**4)**(1/4)), -1)/(8*b**(1/4)*c**2*(a*d-b*c)**3) + b*x*(3*a*d+4*b*c)/(12*a*c*(a+b*x**4)**(3/4)*(a*d-b*c)**2) + b**(3/2)*x**3*(a/(b*x**4)+1)**(3/4)*(3*a**2*d**2-32*a*b*c*d+8*b**2*c**2)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x**2))/2, 2)/(12*a**(3/2)*c*(a+b*x**4)**(3/4)*(a*d-b*c)**3)$

**Mathematica [C]** time = 1.03826, size = 485, normalized size = 1.24

$$x \frac{\left( 9ac(15a^2d^2+21abd^2x^4+4b^2c(5c+7dx^4))F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-5x^4(3a^2d^2+3abd^2x^4+4b^2c(c+dx^4))\left(4adF_1\left(\frac{9}{4};\frac{3}{4},2;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{9}{4};\frac{7}{4},1;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right) \right)}{ac\left(9acF_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-x^4\left(4adF_1\left(\frac{9}{4};\frac{3}{4},2;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{9}{4};\frac{7}{4},1;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}$$

$60(a+bx^4)^{3/4}(c+dx^4)(bc-ad)$

Warning: Unable to verify antiderivative.

[In] `Integrate[1/((a+b*x^4)^(7/4)*(c+d*x^4)^2),x]`

[Out]  $(x*((-25*(8*b^2*c^2-24*a*b*c*d+9*a^2*d^2)*\operatorname{AppellF1}[1/4, 3/4, 1, 5/4, -(b*x^4)/a, -(d*x^4)/c])/(-5*a*c*\operatorname{AppellF1}[1/4, 3/4, 1, 5/4, -(b*x^4)/a, -(d*x^4)/c]) + x^4*(4*a*d*\operatorname{AppellF1}[5/4, 3/4, 2, 9/4, -(b*x^4)/a, -(d*x^4)/c]) + 3*b*c*\operatorname{AppellF1}[5/4, 7/4, 1, 9/4, -(b*x^4)/a, -(d*x^4)/c])) + (9*a*c*(15*a^2*d^2+21*a*b*d^2*x^4+4*b^2*c*(5*c+7*d*x^4))*\operatorname{AppellF1}[5/4, 3/4, 1, 9/4, -(b*x^4)/a, -(d*x^4)/c] - 5*x^4*(3*a^2*d^2+3*a*b*d^2*x^4+4*b^2*c*(c+d*x^4))*\operatorname{AppellF1}[9/4, 3/4, 2, 13/4, -(b*x^4)/a, -(d*x^4)/c] + 3*b*c*\operatorname{AppellF1}[9/4, 7/4, 1, 13/4, -(b*x^4)/a, -(d*x^4)/c]))/(a*c*(9*a*c*\operatorname{AppellF1}[5/4, 3/4, 1, 9/4, -(b*x^4)/a, -(d*x^4)/c] - x^4*(4*a*d*\operatorname{AppellF1}[9/4, 3/4, 2, 13/4, -(b*x^4)/a, -(d*x^4)/c]) + 3*b*c*\operatorname{AppellF1}[9/4, 7/4, 1, 13/4, -(b*x^4)/a, -(d*x^4)/c])))/(60*(b*c-a*d)^2*(a+b*x^4)^(3/4)*(c+d*x^4))$

**Maple [F]** time = 0.06, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^4+c)^2} (bx^4+a)^{-\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x)`

[Out] `int(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{7}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2), x)`

**Fricas** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2),x, algorithm="fricas")`

[Out] Timed out

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c)**2,x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{7}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2),x, algorithm="giac")`

[Out] `integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2), x)`



$$3.117 \quad \int \frac{1}{\sqrt[4]{1+x^4}(2+x^4)} dx$$

**Optimal.** Leaf size=53

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2^{2^{3/4}}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2^{2^{3/4}}}$$

[Out] ArcTan[x/(2^(1/4)\*(1+x^4)^(1/4))]/(2\*2^(3/4)) + ArcTanh[x/(2^(1/4)\*(1+x^4)^(1/4))]/(2\*2^(3/4))

**Rubi [A]** time = 0.0568344, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2^{2^{3/4}}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2^{2^{3/4}}}$$

Antiderivative was successfully verified.

[In] Int[1/((1+x^4)^(1/4)\*(2+x^4)),x]

[Out] ArcTan[x/(2^(1/4)\*(1+x^4)^(1/4))]/(2\*2^(3/4)) + ArcTanh[x/(2^(1/4)\*(1+x^4)^(1/4))]/(2\*2^(3/4))

**Rubi in Sympy [A]** time = 4.79372, size = 49, normalized size = 0.92

$$\frac{\sqrt[4]{2} \operatorname{atan}\left(\frac{2^{\frac{3}{4}}x}{2\sqrt[4]{x^4+1}}\right)}{4} + \frac{\sqrt[4]{2} \operatorname{atanh}\left(\frac{2^{\frac{3}{4}}x}{2\sqrt[4]{x^4+1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(x\*\*4+1)\*\*(1/4)/(x\*\*4+2),x)

[Out] 2\*\*(1/4)\*atan(2\*\*(3/4)\*x/(2\*(x\*\*4+1)\*\*(1/4)))/4 + 2\*\*(1/4)\*atanh(2\*\*(3/4)\*x/(2\*(x\*\*4+1)\*\*(1/4)))/4

**Mathematica [A]** time = 0.08512, size = 70, normalized size = 1.32

$$\frac{-\log\left(2 - \frac{2^{3/4}x}{\sqrt[4]{x^4+1}}\right) + \log\left(\frac{2^{3/4}x}{\sqrt[4]{x^4+1}} + 2\right) + 2 \tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{4^{2^{3/4}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1+x^4)^(1/4)\*(2+x^4)),x]

[Out] (2\*ArcTan[x/(2^(1/4)\*(1+x^4)^(1/4))]) - Log[2 - (2^(3/4)\*x)/(1+x^4)^(1/4)] + Log[2 + (2^(3/4)\*x)/(1+x^4)^(1/4)]/(4\*2^(3/4))

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^4+2} \frac{1}{\sqrt[4]{x^4+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4+1)^(1/4)/(x^4+2), x)`

[Out] `int(1/(x^4+1)^(1/4)/(x^4+2), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 2)(x^4 + 1)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x, algorithm="maxima")`

[Out] `integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)`

**Fricas [A]** time = 4.31309, size = 273, normalized size = 5.15

$$\frac{1}{64} \cdot 8^{\frac{3}{4}} \left( 4 \arctan \left( \frac{4 \sqrt{x^4 + 1} x^2 - \sqrt{2} (3x^4 + 2)}{2 \cdot 8^{\frac{1}{4}} (x^4 + 1)^{\frac{1}{4}} x^3 - 8^{\frac{3}{4}} (x^4 + 1)^{\frac{3}{4}} x - \sqrt{2} (x^4 + 2)} \right) + \log \left( \frac{2 \left( 2 \cdot 8^{\frac{1}{4}} (x^4 + 1)^{\frac{1}{4}} x^3 + 8^{\frac{3}{4}} (x^4 + 1)^{\frac{3}{4}} x + 4 \sqrt{x^4 + 1} x^2 \right)}{x^4 + 2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x, algorithm="fricas")`

[Out] `1/64*8^(3/4)*(4*arctan(-(4*sqrt(x^4 + 1)*x^2 - sqrt(2)*(3*x^4 + 2))/(2*8^(1/4)*(x^4 + 1)^(1/4)*x^3 - 8^(3/4)*(x^4 + 1)^(3/4)*x - sqrt(2)*(x^4 + 2))) + log(2*(2*8^(1/4)*(x^4 + 1)^(1/4)*x^3 + 8^(3/4)*(x^4 + 1)^(3/4)*x + 4*sqrt(x^4 + 1)*x^2 + sqrt(2)*(3*x^4 + 2)))/(x^4 + 2)) - log(2*(2*8^(1/4)*(x^4 + 1)^(1/4)*x^3 + 8^(3/4)*(x^4 + 1)^(3/4)*x - 4*sqrt(x^4 + 1)*x^2 - sqrt(2)*(3*x^4 + 2)))/(x^4 + 2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{x^4 + 1}(x^4 + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**4+1)**(1/4)/(x**4+2), x)`

[Out] `Integral(1/((x**4 + 1)**(1/4)*(x**4 + 2)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4 + 2)(x^4 + 1)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)
```

$$3.118 \quad \int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}} dx$$

**Optimal.** Leaf size=57

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax}}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{ax}}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

[Out] ArcTan[(a^(1/4)\*x)/(a + b\*x^4)^(1/4)]/(2\*a^(5/4)) + ArcTanh[(a^(1/4)\*x)/(a + b\*x^4)^(1/4)]/(2\*a^(5/4))

**Rubi [A]** time = 0.0694466, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax}}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{ax}}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[1/((a - (a - b)\*x^4)\*(a + b\*x^4)^(1/4)), x]

[Out] ArcTan[(a^(1/4)\*x)/(a + b\*x^4)^(1/4)]/(2\*a^(5/4)) + ArcTanh[(a^(1/4)\*x)/(a + b\*x^4)^(1/4)]/(2\*a^(5/4))

**Rubi in Sympy [A]** time = 9.03974, size = 49, normalized size = 0.86

$$\frac{\operatorname{atan}\left(\frac{\sqrt[4]{ax}}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}} + \frac{\operatorname{atanh}\left(\frac{\sqrt[4]{ax}}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a-(a-b)\*x\*\*4)/(b\*x\*\*4+a)\*\*(1/4), x)

[Out] atan(a\*\*(1/4)\*x/(a + b\*x\*\*4)\*\*(1/4))/(2\*a\*\*(5/4)) + atanh(a\*\*(1/4)\*x/(a + b\*x\*\*4)\*\*(1/4))/(2\*a\*\*(5/4))

**Mathematica [A]** time = 0.109993, size = 76, normalized size = 1.33

$$\frac{-\log\left(1 - \frac{\sqrt[4]{ax}}{\sqrt[4]{ax^4 + b}}\right) + \log\left(\frac{\sqrt[4]{ax}}{\sqrt[4]{ax^4 + b}} + 1\right) + 2 \tan^{-1}\left(\frac{\sqrt[4]{ax}}{\sqrt[4]{ax^4 + b}}\right)}{4a^{5/4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a - (a - b)\*x^4)\*(a + b\*x^4)^(1/4)), x]

[Out] (2\*ArcTan[(a^(1/4)\*x)/(b + a\*x^4)^(1/4)] - Log[1 - (a^(1/4)\*x)/(b + a\*x^4)^(1/4)] + Log[1 + (a^(1/4)\*x)/(b + a\*x^4)^(1/4)])/(4\*a^(5/4))

**Maple [F]** time = 0.076, size = 0, normalized size = 0.

$$\int \frac{1}{a - (a - b)x^4} \frac{1}{\sqrt[4]{bx^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x)`

[Out] `int(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{((a-b)x^4 - a)(bx^4 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/(((a-b)*x^4 - a)*(b*x^4 + a)^(1/4)),x, algorithm="maxima")`

[Out] `-integrate(1/(((a-b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)`

**Fricas** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/(((a-b)*x^4 - a)*(b*x^4 + a)^(1/4)),x, algorithm="fricas")`

[Out] Timed out

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{ax^4\sqrt[4]{a+bx^4} - a\sqrt[4]{a+bx^4} - bx^4\sqrt[4]{a+bx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a-(a-b)*x**4)/(b*x**4+a)**(1/4),x)`

[Out] `-Integral(1/(a*x**4*(a + b*x**4)**(1/4) - a*(a + b*x**4)**(1/4) - b*x**4*(a + b*x**4)**(1/4)), x)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{((a-b)x^4 - a)(bx^4 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/(((a-b)*x^4 - a)*(b*x^4 + a)^(1/4)),x, algorithm="giac")`

[Out] `integrate(-1/(((a-b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)`

### 3.119 $\int (a + bx^4)^p (c + dx^4)^q dx$

**Optimal.** Leaf size=79

$$x (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} (c + dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} F_1\left(\frac{1}{4}; -p, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)$$

[Out]  $(x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(1 + (b*x^4)/a)^p*(1 + (d*x^4)/c)^q$

**Rubi [A]** time = 0.12334, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$x (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} (c + dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} F_1\left(\frac{1}{4}; -p, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^p\*(c + d\*x^4)^q,x]

[Out]  $(x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(1 + (b*x^4)/a)^p*(1 + (d*x^4)/c)^q$

**Rubi in Sympy [A]** time = 21.9447, size = 61, normalized size = 0.77

$$x \left(1 + \frac{bx^4}{a}\right)^{-p} \left(1 + \frac{dx^4}{c}\right)^{-q} (a + bx^4)^p (c + dx^4)^q \text{appellf}_1\left(\frac{1}{4}, -p, -q, \frac{5}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)\*\*p\*(d\*x\*\*4+c)\*\*q,x)

[Out]  $x*(1 + b*x**4/a)**(-p)*(1 + d*x**4/c)**(-q)*(a + b*x**4)**p*(c + d*x**4)**q*\text{appellf}_1(1/4, -p, -q, 5/4, -b*x**4/a, -d*x**4/c)$

**Mathematica [B]** time = 0.358687, size = 172, normalized size = 2.18

$$\frac{5acx (a + bx^4)^p (c + dx^4)^q F_1\left(\frac{1}{4}; -p, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{4x^4 \left( bcpF_1\left(\frac{5}{4}; 1-p, -q; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + adqF_1\left(\frac{5}{4}; -p, 1-q; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right) + 5acF_1\left(\frac{1}{4}; -p, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^4)^p\*(c + d\*x^4)^q,x]

[Out]  $(5*a*c*x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)]/(5*a*c*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 4*x^4*(b*c*p*AppellF1[5/4, 1-p, -q, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + a*d*q*AppellF1[5/4, -p, 1-q, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))$

**Maple [F]** time = 0.115, size = 0, normalized size = 0.

$$\int (bx^4 + a)^p (dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+a)^p*(d*x^4+c)^q,x)`

[Out] `int((b*x^4+a)^p*(d*x^4+c)^q,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^p (dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^p*(d*x^4 + c)^q,x, algorithm="maxima")`

[Out] `integrate((b*x^4 + a)^p*(d*x^4 + c)^q, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4 + a\right)^p \left(dx^4 + c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^p*(d*x^4 + c)^q,x, algorithm="fricas")`

[Out] `integral((b*x^4 + a)^p*(d*x^4 + c)^q, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**4+a)**p*(d*x**4+c)**q,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^p (dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4 + a)^p*(d*x^4 + c)^q,x, algorithm="giac")`

[Out] `integrate((b*x^4 + a)^p*(d*x^4 + c)^q, x)`

### 3.120 $\int (a + bx^4)^2 (c + dx^4)^q dx$

**Optimal.** Leaf size=176

$$\frac{x(c + dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} (a^2 d^2 (16q^2 + 56q + 45) - 2abcd(4q + 9) + 5b^2 c^2) {}_2F_1\left(\frac{1}{4}, -q; \frac{5}{4}; -\frac{dx^4}{c}\right)}{d^2(4q + 5)(4q + 9)} - \frac{bx(c + dx^4)^{q+1} (5bc - ad(4q + 13))}{d^2(4q + 5)(4q + 9)} + \frac{bx(a + bx^4)(c + dx^4)^{q+1}}{d(4q + 9)}$$

[Out]  $-\left(\left(b \cdot (5 \cdot b \cdot c - a \cdot d \cdot (13 + 4 \cdot q))\right) \cdot x \cdot (c + d \cdot x^4)^{(1 + q)}\right) / \left(d^2 \cdot (5 + 4 \cdot q) \cdot (9 + 4 \cdot q)\right) + \left(b \cdot x \cdot (a + b \cdot x^4) \cdot (c + d \cdot x^4)^{(1 + q)}\right) / \left(d \cdot (9 + 4 \cdot q)\right) + \left(\left(5 \cdot b^2 \cdot c^2 - 2 \cdot a \cdot b \cdot c \cdot d \cdot (9 + 4 \cdot q) + a^2 \cdot d^2 \cdot (45 + 56 \cdot q + 16 \cdot q^2)\right) \cdot x \cdot (c + d \cdot x^4)^q \cdot \text{Hypergeometric2F1}\left[\frac{1}{4}, -q, \frac{5}{4}, -\left(\frac{d \cdot x^4}{c}\right)\right]\right) / \left(d^2 \cdot (5 + 4 \cdot q) \cdot (9 + 4 \cdot q) \cdot \left(1 + \frac{d \cdot x^4}{c}\right)^q\right)$

**Rubi [A]** time = 0.316393, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{x(c + dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} (a^2 d^2 (16q^2 + 56q + 45) - 2abcd(4q + 9) + 5b^2 c^2) {}_2F_1\left(\frac{1}{4}, -q; \frac{5}{4}; -\frac{dx^4}{c}\right)}{d^2(4q + 5)(4q + 9)} - \frac{bx(c + dx^4)^{q+1} (5bc - ad(4q + 13))}{d^2(4q + 5)(4q + 9)} + \frac{bx(a + bx^4)(c + dx^4)^{q+1}}{d(4q + 9)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^2\*(c + d\*x^4)^q, x]

[Out]  $-\left(\left(b \cdot (5 \cdot b \cdot c - a \cdot d \cdot (13 + 4 \cdot q))\right) \cdot x \cdot (c + d \cdot x^4)^{(1 + q)}\right) / \left(d^2 \cdot (5 + 4 \cdot q) \cdot (9 + 4 \cdot q)\right) + \left(b \cdot x \cdot (a + b \cdot x^4) \cdot (c + d \cdot x^4)^{(1 + q)}\right) / \left(d \cdot (9 + 4 \cdot q)\right) + \left(\left(5 \cdot b^2 \cdot c^2 - 2 \cdot a \cdot b \cdot c \cdot d \cdot (9 + 4 \cdot q) + a^2 \cdot d^2 \cdot (45 + 56 \cdot q + 16 \cdot q^2)\right) \cdot x \cdot (c + d \cdot x^4)^q \cdot \text{Hypergeometric2F1}\left[\frac{1}{4}, -q, \frac{5}{4}, -\left(\frac{d \cdot x^4}{c}\right)\right]\right) / \left(d^2 \cdot (5 + 4 \cdot q) \cdot (9 + 4 \cdot q) \cdot \left(1 + \frac{d \cdot x^4}{c}\right)^q\right)$

**Rubi in Sympy [A]** time = 32.8228, size = 153, normalized size = 0.87

$$\frac{bx(a + bx^4)(c + dx^4)^{q+1}}{d(4q + 9)} - \frac{bx(c + dx^4)^{q+1}(-ad(4q + 13) + 5bc)}{d^2(4q + 5)(4q + 9)} + \frac{x \left(1 + \frac{dx^4}{c}\right)^{-q} (c + dx^4)^q (-ad(4q + 5)(-ad(4q + 9) + bc) + bc(-ad(4q + 13) + 5bc)) {}_2F_1\left(-q, \frac{1}{4}; \frac{5}{4}; -\frac{dx^4}{c}\right)}{d^2(4q + 5)(4q + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)\*\*2\*(d\*x\*\*4+c)\*\*q, x)

[Out]  $b \cdot x \cdot (a + b \cdot x^4) \cdot (c + d \cdot x^4)^{(q + 1)} / (d \cdot (4 \cdot q + 9)) - b \cdot x \cdot (c + d \cdot x^4)^{(q + 1)} \cdot (-a \cdot d \cdot (4 \cdot q + 13) + 5 \cdot b \cdot c) / (d^2 \cdot (4 \cdot q + 5) \cdot (4 \cdot q + 9)) + x \cdot \left(1 + \frac{d \cdot x^4}{c}\right)^{-q} \cdot (c + d \cdot x^4)^q \cdot (-a \cdot d \cdot (4 \cdot q + 5) \cdot (-a \cdot d \cdot (4 \cdot q + 9) + b \cdot c) + b \cdot c \cdot (-a \cdot d \cdot (4 \cdot q + 13) + 5 \cdot b \cdot c)) \cdot \text{hyper}\left(-q, \frac{1}{4}, \frac{5}{4}, -\frac{d \cdot x^4}{c}\right) / (d^2 \cdot (4 \cdot q + 5) \cdot (4 \cdot q + 9))$

**Mathematica [A]** time = 0.0788624, size = 106, normalized size = 0.6

$$\frac{1}{45} x (c + dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} \left(45a^2 {}_2F_1\left(\frac{1}{4}, -q; \frac{5}{4}; -\frac{dx^4}{c}\right) + bx^4 \left(18a {}_2F_1\left(\frac{5}{4}, -q; \frac{9}{4}; -\frac{dx^4}{c}\right) + 5bx^4 {}_2F_1\left(\frac{9}{4}, -q; \frac{13}{4}; -\frac{dx^4}{c}\right)\right)\right)$$



Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)^2\*(c + d\*x^4)^q,x]

[Out] (x\*(c + d\*x^4)^q\*(45\*a^2\*Hypergeometric2F1[1/4, -q, 5/4, -((d\*x^4)/c)] + b\*x^4\*(18\*a\*Hypergeometric2F1[5/4, -q, 9/4, -((d\*x^4)/c)] + 5\*b\*x^4\*Hypergeometric2F1[9/4, -q, 13/4, -((d\*x^4)/c)])))/(45\*(1 + (d\*x^4)/c)^q)

**Maple [F]** time = 0.057, size = 0, normalized size = 0.

$$\int (bx^4 + a)^2 (dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^2\*(d\*x^4+c)^q,x)

[Out] int((b\*x^4+a)^2\*(d\*x^4+c)^q,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^2 (dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^2\*(d\*x^4 + c)^q,x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^2\*(d\*x^4 + c)^q, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b^2x^8 + 2abx^4 + a^2)(dx^4 + c)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)^2\*(d\*x^4 + c)^q,x, algorithm="fricas")

[Out] integral((b^2\*x^8 + 2\*a\*b\*x^4 + a^2)\*(d\*x^4 + c)^q, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*2\*(d\*x\*\*4+c)\*\*q,x)

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^2 (dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4 + a)^2*(d*x^4 + c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^2*(d*x^4 + c)^q, x)
```

### 3.121 $\int (a + bx^4) (c + dx^4)^q dx$

**Optimal.** Leaf size=93

$$\frac{bx(c+dx^4)^{q+1}}{d(4q+5)} - \frac{x(c+dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} (bc - ad(4q+5)) {}_2F_1\left(\frac{1}{4}, -q; \frac{5}{4}; -\frac{dx^4}{c}\right)}{d(4q+5)}$$

[Out] (b\*x\*(c + d\*x^4)^(1 + q))/(d\*(5 + 4\*q)) - ((b\*c - a\*d\*(5 + 4\*q))\*x\*(c + d\*x^4)^q\*Hypergeometric2F1[1/4, -q, 5/4, -((d\*x^4)/c)])/(d\*(5 + 4\*q)\*(1 + (d\*x^4)/c)^q)

**Rubi [A]** time = 0.111122, antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$x(c+dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} \left(a - \frac{bc}{4dq+5d}\right) {}_2F_1\left(\frac{1}{4}, -q; \frac{5}{4}; -\frac{dx^4}{c}\right) + \frac{bx(c+dx^4)^{q+1}}{d(4q+5)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)\*(c + d\*x^4)^q, x]

[Out] (b\*x\*(c + d\*x^4)^(1 + q))/(d\*(5 + 4\*q)) + ((a - (b\*c)/(5\*d + 4\*d\*q))\*x\*(c + d\*x^4)^q\*Hypergeometric2F1[1/4, -q, 5/4, -((d\*x^4)/c)])/(1 + (d\*x^4)/c)^q

**Rubi in Sympy [A]** time = 11.3714, size = 73, normalized size = 0.78

$$\frac{bx(c+dx^4)^{q+1}}{d(4q+5)} - \frac{x\left(1 + \frac{dx^4}{c}\right)^{-q} (c+dx^4)^q (-ad(4q+5) + bc) {}_2F_1\left(-q, \frac{1}{4}; \frac{5}{4}; -\frac{dx^4}{c}\right)}{d(4q+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*4+a)\*(d\*x\*\*4+c)\*\*q, x)

[Out] b\*x\*(c + d\*x\*\*4)\*\*(q + 1)/(d\*(4\*q + 5)) - x\*(1 + d\*x\*\*4/c)\*\*(-q)\*(c + d\*x\*\*4)\*\*q\*(-a\*d\*(4\*q + 5) + b\*c)\*hyper((-q, 1/4), (5/4, ), -d\*x\*\*4/c)/(d\*(4\*q + 5))

**Mathematica [A]** time = 0.0347268, size = 75, normalized size = 0.81

$$\frac{1}{5}x(c+dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} \left(5a {}_2F_1\left(\frac{1}{4}, -q; \frac{5}{4}; -\frac{dx^4}{c}\right) + bx^4 {}_2F_1\left(\frac{5}{4}, -q; \frac{9}{4}; -\frac{dx^4}{c}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)\*(c + d\*x^4)^q, x]

[Out] (x\*(c + d\*x^4)^q\*(5\*a\*Hypergeometric2F1[1/4, -q, 5/4, -((d\*x^4)/c)] + b\*x^4\*Hypergeometric2F1[5/4, -q, 9/4, -((d\*x^4)/c)]))/(5\*(1 + (d\*x^4)/c)^q)

**Maple [F]** time = 0.049, size = 0, normalized size = 0.

$$\int (bx^4 + a) (dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)\*(d\*x^4+c)^q,x)

[Out] int((b\*x^4+a)\*(d\*x^4+c)^q,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a) (dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)\*(d\*x^4 + c)^q,x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)\*(d\*x^4 + c)^q, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^4 + a)(dx^4 + c)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)\*(d\*x^4 + c)^q,x, algorithm="fricas")

[Out] integral((b\*x^4 + a)\*(d\*x^4 + c)^q, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*(d\*x\*\*4+c)\*\*q,x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a) (dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4 + a)\*(d\*x^4 + c)^q,x, algorithm="giac")

[Out] integrate((b\*x^4 + a)\*(d\*x^4 + c)^q, x)

$$3.122 \quad \int \frac{(c+dx^4)^q}{a+bx^4} dx$$

**Optimal.** Leaf size=57

$$\frac{x (c + dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} F_1\left(\frac{1}{4}; 1, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{a}$$

[Out]  $(x^*(c + d*x^4)^q*AppellF1[1/4, 1, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(a*(1 + (d*x^4)/c)^q)$

**Rubi [A]** time = 0.0838656, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{x (c + dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} F_1\left(\frac{1}{4}; 1, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^q/(a + b\*x^4), x]

[Out]  $(x^*(c + d*x^4)^q*AppellF1[1/4, 1, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(a*(1 + (d*x^4)/c)^q)$

**Rubi in Sympy [A]** time = 21.9034, size = 42, normalized size = 0.74

$$\frac{x \left(1 + \frac{dx^4}{c}\right)^{-q} (c + dx^4)^q \text{appellf1}\left(\frac{1}{4}, 1, -q, \frac{5}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((d\*x\*\*4+c)\*\*q/(b\*x\*\*4+a), x)

[Out]  $x*(1 + d*x**4/c)**(-q)*(c + d*x**4)**q*appellf1(1/4, 1, -q, 5/4, -b*x**4/a, -d*x**4/c)/a$

**Mathematica [B]** time = 0.280766, size = 162, normalized size = 2.84

$$\frac{5acx (c + dx^4)^q F_1\left(\frac{1}{4}; -q, 1; \frac{5}{4}; -\frac{dx^4}{c}, -\frac{bx^4}{a}\right)}{(a + bx^4) \left(4x^4 \left(adqF_1\left(\frac{5}{4}; 1 - q, 1; \frac{9}{4}; -\frac{dx^4}{c}, -\frac{bx^4}{a}\right) - bcF_1\left(\frac{5}{4}; -q, 2; \frac{9}{4}; -\frac{dx^4}{c}, -\frac{bx^4}{a}\right)\right) + 5acF_1\left(\frac{1}{4}; -q, 1; \frac{5}{4}; -\frac{dx^4}{c}, -\frac{bx^4}{a}\right)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^4)^q/(a + b\*x^4), x]

[Out]  $(5*a*c*x*(c + d*x^4)^q*AppellF1[1/4, -q, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)]/((a + b*x^4)*(5*a*c*AppellF1[1/4, -q, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 4*x^4*(a*d*q*AppellF1[5/4, 1 - q, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)] - b*c*AppellF1[5/4, -q, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))$

**Maple [F]** time = 0.072, size = 0, normalized size = 0.

$$\int \frac{(dx^4 + c)^q}{bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^4+c)^q/(b*x^4+a),x)`

[Out] `int((d*x^4+c)^q/(b*x^4+a),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^4 + c)^q}{bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^4 + c)^q/(b*x^4 + a),x, algorithm="maxima")`

[Out] `integrate((d*x^4 + c)^q/(b*x^4 + a), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^4 + c)^q}{bx^4 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^4 + c)^q/(b*x^4 + a),x, algorithm="fricas")`

[Out] `integral((d*x^4 + c)^q/(b*x^4 + a), x)`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**4+c)**q/(b*x**4+a),x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^4 + c)^q}{bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^4 + c)^q/(b*x^4 + a),x, algorithm="giac")`

[Out] `integrate((d*x^4 + c)^q/(b*x^4 + a), x)`

$$3.123 \quad \int \frac{(c+dx^4)^q}{(a+bx^4)^2} dx$$

**Optimal.** Leaf size=57

$$\frac{x (c + dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} F_1\left(\frac{1}{4}; 2, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{a^2}$$

[Out]  $(x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(a^2*(1 + (d*x^4)/c)^q)$

**Rubi [A]** time = 0.0831709, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{x (c + dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} F_1\left(\frac{1}{4}; 2, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^q/(a + b\*x^4)^2, x]

[Out]  $(x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(a^2*(1 + (d*x^4)/c)^q)$

**Rubi in Sympy [A]** time = 20.2223, size = 44, normalized size = 0.77

$$\frac{x \left(1 + \frac{dx^4}{c}\right)^{-q} (c + dx^4)^q \text{appellf1}\left(\frac{1}{4}, 2, -q, \frac{5}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((d\*x\*\*4+c)\*\*q/(b\*x\*\*4+a)\*\*2, x)

[Out]  $x*(1 + d*x**4/c)**(-q)*(c + d*x**4)**q*appellf1(1/4, 2, -q, 5/4, -b*x**4/a, -d*x**4/c)/a**2$

**Mathematica [B]** time = 0.307696, size = 162, normalized size = 2.84

$$\frac{5acx (c + dx^4)^q F_1\left(\frac{1}{4}; 2, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{(a + bx^4)^2 \left(4x^4 \left(adqF_1\left(\frac{5}{4}; 2, 1 - q; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) - 2bcF_1\left(\frac{5}{4}; 3, -q; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) + 5acF_1\left(\frac{1}{4}; 2, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^4)^q/(a + b\*x^4)^2, x]

[Out]  $(5*a*c*x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)]/((a + b*x^4)^2*(5*a*c*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 4*x^4*(a*d*q*AppellF1[5/4, 2, 1 - q, 9/4, -((b*x^4)/a), -((d*x^4)/c)] - 2*b*c*AppellF1[5/4, 3, -q, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))$

**Maple [F]** time = 0.064, size = 0, normalized size = 0.

$$\int \frac{(dx^4 + c)^q}{(bx^4 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^4+c)^q/(b\*x^4+a)^2, x)

[Out] int((d\*x^4+c)^q/(b\*x^4+a)^2, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^4 + c)^q}{(bx^4 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^q/(b\*x^4 + a)^2, x, algorithm="maxima")

[Out] integrate((d\*x^4 + c)^q/(b\*x^4 + a)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^4 + c)^q}{b^2x^8 + 2abx^4 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^q/(b\*x^4 + a)^2, x, algorithm="fricas")

[Out] integral((d\*x^4 + c)^q/(b^2\*x^8 + 2\*a\*b\*x^4 + a^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*4+c)\*\*q/(b\*x\*\*4+a)\*\*2, x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^4 + c)^q}{(bx^4 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4 + c)^q/(b\*x^4 + a)^2, x, algorithm="giac")

[Out] integrate((d\*x^4 + c)^q/(b\*x^4 + a)^2, x)



$$3.124 \quad \int \frac{1}{\sqrt[5]{a + bx^5}(c+dx^5)} dx$$

**Optimal.** Leaf size=545

$$\begin{aligned} & \frac{\log\left(\sqrt[5]{c} - \frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} - \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\sqrt{\frac{1}{5}(5-2\sqrt{5})} - \frac{2\sqrt{\frac{2}{5+\sqrt{5}}x}\sqrt[5]{bc-ad}}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} \\ & + \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{2}{5}(5+\sqrt{5})x}\sqrt[5]{bc-ad}}{\sqrt[5]{c}\sqrt[5]{a+bx^5}} + \sqrt{\frac{1}{5}(5+2\sqrt{5})}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} \\ & + \frac{(1-\sqrt{5}) \log\left(\frac{2c^{2/5}(a+bx^5)^{2/5} - \sqrt{5}\sqrt[5]{cx}\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad} + \sqrt[5]{cx}\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad} + 2x^2(bc-ad)^{2/5}}{(a+bx^5)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}} \\ & + \frac{(1+\sqrt{5}) \log\left(\frac{2c^{2/5}(a+bx^5)^{2/5} + \sqrt{5}\sqrt[5]{cx}\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad} + \sqrt[5]{cx}\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad} + 2x^2(bc-ad)^{2/5}}{(a+bx^5)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}} \end{aligned}$$

[Out]  $-(\text{Sqrt}[(5 + \text{Sqrt}[5])/2]) * \text{ArcTan}[\text{Sqrt}[(5 - 2 * \text{Sqrt}[5])/5] - (2 * \text{Sqrt}[2/(5 + \text{Sqrt}[5])]) * (b * c - a * d)^{(1/5) * x} / (c^{(1/5)} * (a + b * x^5)^{(1/5)})] / (5 * c^{(4/5)} * (b * c - a * d)^{(1/5)}) + (\text{Sqrt}[(5 - \text{Sqrt}[5])/2]) * \text{ArcTan}[\text{Sqrt}[(5 + 2 * \text{Sqrt}[5])/5] + (\text{Sqrt}[(2 * (5 + \text{Sqrt}[5]))/5]) * (b * c - a * d)^{(1/5) * x} / (c^{(1/5)} * (a + b * x^5)^{(1/5)})] / (5 * c^{(4/5)} * (b * c - a * d)^{(1/5)}) - \text{Log}[c^{(1/5)} - ((b * c - a * d)^{(1/5) * x} / (a + b * x^5)^{(1/5)})] / (5 * c^{(4/5)} * (b * c - a * d)^{(1/5)}) + ((1 - \text{Sqrt}[5]) * \text{Log}[(2 * (b * c - a * d)^{(2/5) * x^2} + c^{(1/5)} * (b * c - a * d)^{(1/5) * x} * (a + b * x^5)^{(1/5)} - \text{Sqrt}[5] * c^{(1/5)} * (b * c - a * d)^{(1/5) * x} * (a + b * x^5)^{(1/5)} + 2 * c^{(2/5)} * (a + b * x^5)^{(2/5)}) / (a + b * x^5)^{(2/5)}]) / (20 * c^{(4/5)} * (b * c - a * d)^{(1/5)}) + ((1 + \text{Sqrt}[5]) * \text{Log}[(2 * (b * c - a * d)^{(2/5) * x^2} + c^{(1/5)} * (b * c - a * d)^{(1/5) * x} * (a + b * x^5)^{(1/5)} + \text{Sqrt}[5] * c^{(1/5)} * (b * c - a * d)^{(1/5) * x} * (a + b * x^5)^{(1/5)} + 2 * c^{(2/5)} * (a + b * x^5)^{(2/5)}) / (a + b * x^5)^{(2/5)}]) / (20 * c^{(4/5)} * (b * c - a * d)^{(1/5)})$

**Rubi [A]** time = 2.3109, antiderivative size = 545, normalized size of antiderivative = 1., number of rules used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned} & \frac{\log\left(\sqrt[5]{c} - \frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} - \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\sqrt{\frac{1}{5}(5-2\sqrt{5})} - \frac{2\sqrt{\frac{2}{5+\sqrt{5}}x}\sqrt[5]{bc-ad}}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} \\ & + \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{2}{5}(5+\sqrt{5})x}\sqrt[5]{bc-ad}}{\sqrt[5]{c}\sqrt[5]{a+bx^5}} + \sqrt{\frac{1}{5}(5+2\sqrt{5})}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} \\ & + \frac{(1-\sqrt{5}) \log\left(\frac{2c^{2/5}(a+bx^5)^{2/5} - \sqrt{5}\sqrt[5]{cx}\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad} + \sqrt[5]{cx}\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad} + 2x^2(bc-ad)^{2/5}}{(a+bx^5)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}} \\ & + \frac{(1+\sqrt{5}) \log\left(\frac{2c^{2/5}(a+bx^5)^{2/5} + \sqrt{5}\sqrt[5]{cx}\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad} + \sqrt[5]{cx}\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad} + 2x^2(bc-ad)^{2/5}}{(a+bx^5)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/((a + b * x^5)^{(1/5)} * (c + d * x^5)), x]$

[Out]  $-(\text{Sqrt}[(5 + \text{Sqrt}[5])/2]) * \text{ArcTan}[\text{Sqrt}[(5 - 2 * \text{Sqrt}[5])/5] - (2 * \text{Sqrt}[2/(5 + \text{Sqrt}[5])]) * (b * c - a * d)^{(1/5) * x} / (c^{(1/5)} * (a + b * x^5)^{(1/5)})] / (5 * c^{(4/5)} * (b * c - a * d)^{(1/5)}) + (\text{Sqrt}[(5 - \text{Sqrt}[5])/2]) * \text{ArcTan}[\text{Sqrt}[(5 + 2 * \text{Sqrt}[5])/5] + (\text{Sqrt}[(2 * (5 + \text{Sqrt}[5]))/5]) * (b * c - a * d)^{(1/5) * x} / (c^{(1/5)} * (a + b * x^5)^{(1/5)})] / (5 * c^{(4/5)} * (b * c - a * d)^{(1/5)}) - \text{Log}[c^{(1/5)} - ((b * c - a * d)^{(1/5) * x} / (a + b * x^5)^{(1/5)})] / (5 * c^{(4/5)} * (b * c - a * d)^{(1/5)}) + ((1 - \text{Sqrt}[5]) * \text{Log}[(2 * (b * c - a * d)^{(2/5) * x^2} + c^{(1/5)} * (b * c - a * d)^{(1/5) * x} * (a + b * x^5)^{(1/5)} - \text{Sqrt}[5] * c^{(1/5)} * (b * c - a * d)^{(1/5) * x} * (a + b * x^5)^{(1/5)} + 2 * c^{(2/5)} * (a + b * x^5)^{(2/5)}) / (a + b * x^5)^{(2/5)}]) / (20 * c^{(4/5)} * (b * c - a * d)^{(1/5)}) + ((1 + \text{Sqrt}[5]) * \text{Log}[(2 * (b * c - a * d)^{(2/5) * x^2} + c^{(1/5)} * (b * c - a * d)^{(1/5) * x} * (a + b * x^5)^{(1/5)} + \text{Sqrt}[5] * c^{(1/5)} * (b * c - a * d)^{(1/5) * x} * (a + b * x^5)^{(1/5)} + 2 * c^{(2/5)} * (a + b * x^5)^{(2/5)}) / (a + b * x^5)^{(2/5)}]) / (20 * c^{(4/5)} * (b * c - a * d)^{(1/5)})$

5)) - Log[c^(1/5) - ((b\*c - a\*d)^(1/5)\*x)/(a + b\*x^5)^(1/5)]/(5\*c^(4/5)\*(b\*c - a\*d)^(1/5)) + ((1 - Sqrt[5])\*Log[(2\*(b\*c - a\*d)^(2/5)\*x^2 + c^(1/5)\*(b\*c - a\*d)^(1/5)\*x\*(a + b\*x^5)^(1/5) - Sqrt[5]\*c^(1/5)\*(b\*c - a\*d)^(1/5)\*x\*(a + b\*x^5)^(1/5) + 2\*c^(2/5)\*(a + b\*x^5)^(2/5)]/(a + b\*x^5)^(2/5))]/(20\*c^(4/5)\*(b\*c - a\*d)^(1/5)) + ((1 + Sqrt[5])\*Log[(2\*(b\*c - a\*d)^(2/5)\*x^2 + c^(1/5)\*(b\*c - a\*d)^(1/5)\*x\*(a + b\*x^5)^(1/5) + Sqrt[5]\*c^(1/5)\*(b\*c - a\*d)^(1/5)\*x\*(a + b\*x^5)^(1/5) + 2\*c^(2/5)\*(a + b\*x^5)^(2/5)]/(a + b\*x^5)^(2/5))]/(20\*c^(4/5)\*(b\*c - a\*d)^(1/5))

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*5+a)\*\*(1/5)/(d\*x\*\*5+c), x)

[Out] Timed out

**Mathematica [A]** time = 1.6263, size = 379, normalized size = 0.7

$$-\left(\sqrt{5}-1\right) \log \left(-\frac{\left(\sqrt{5}-1\right) \sqrt[5]{c x} \sqrt[5]{b c-a d}}{2 \sqrt[5]{a x^5+b}}+\frac{x^2(b c-a d)^{2 / 5}}{\left(a x^5+b\right)^{2 / 5}}+c^{2 / 5}\right)+\left(1+\sqrt{5}\right) \log \left(\frac{\left(1+\sqrt{5}\right) \sqrt[5]{c x} \sqrt[5]{b c-a d}}{2 \sqrt[5]{a x^5+b}}+\frac{x^2(b c-a d)^{2 / 5}}{\left(a x^5+b\right)^{2 / 5}}+c^{2 / 5}\right)-$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^5)^(1/5)\*(c + d\*x^5)), x]

[Out] (2\*Sqrt[2\*(5 + Sqrt[5])]\*ArcTan[(2\*Sqrt[2/(5 + Sqrt[5])])\*(-((-1 + Sqrt[5])\*c^(1/5))/4 + ((b\*c - a\*d)^(1/5)\*x)/(b + a\*x^5)^(1/5))]/c^(1/5)] + 2\*Sqrt[10 - 2\*Sqrt[5]]\*ArcTan[(Sqrt[2 + 2/Sqrt[5]])\*((1 + Sqrt[5])\*c^(1/5))/4 + ((b\*c - a\*d)^(1/5)\*x)/(b + a\*x^5)^(1/5)])/c^(1/5)] - 4\*Log[c^(1/5) - ((b\*c - a\*d)^(1/5)\*x)/(b + a\*x^5)^(1/5)] - (-1 + Sqrt[5])\*Log[c^(2/5) + ((b\*c - a\*d)^(2/5)\*x^2)/(b + a\*x^5)^(2/5) - ((-1 + Sqrt[5])\*c^(1/5)\*(b\*c - a\*d)^(1/5)\*x)/(2\*(b + a\*x^5)^(1/5))] + (1 + Sqrt[5])\*Log[c^(2/5) + ((b\*c - a\*d)^(2/5)\*x^2)/(b + a\*x^5)^(2/5) + ((1 + Sqrt[5])\*c^(1/5)\*(b\*c - a\*d)^(1/5)\*x)/(2\*(b + a\*x^5)^(1/5))]/(20\*c^(4/5)\*(b\*c - a\*d)^(1/5))

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{dx^5 + c} \frac{1}{\sqrt[5]{bx^5 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^5+a)^(1/5)/(d\*x^5+c), x)

[Out] int(1/(b\*x^5+a)^(1/5)/(d\*x^5+c), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^5 + a)^{\frac{1}{5}}(dx^5 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)`

---

**Fricas** [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)),x, algorithm="fricas")`

[Out] Exception raised: TypeError

---

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**5+a)**(1/5)/(d*x**5+c),x)`

[Out] Timed out

---

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^5 + a)^{\frac{1}{5}}(dx^5 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)`

$$3.125 \quad \int \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^3 dx$$

**Optimal.** Leaf size=143

$$\frac{d\sqrt{a + \frac{b}{x}} \left(2(-2a^2d^2 + 15abcd + 57b^2c^2) + \frac{bd(2ad+33bc)}{x}\right)}{15b^2} + \frac{c^2(6ad + bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}} + x\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^3 - \frac{7}{5}d\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2$$

[Out]  $(-7*d*\text{Sqrt}[a + b/x]*(c + d/x)^2)/5 - (d*\text{Sqrt}[a + b/x]*(2*(57*b^2*c^2 + 15*a*b*c*d - 2*a^2*d^2) + (b*d*(33*b*c + 2*a*d))/x))/(15*b^2) + \text{Sqrt}[a + b/x]*(c + d/x)^3*x + (c^2*(b*c + 6*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/\text{Sqrt}[a]$

**Rubi [A]** time = 0.417175, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{d\sqrt{a + \frac{b}{x}} \left(2(-2a^2d^2 + 15abcd + 57b^2c^2) + \frac{bd(2ad+33bc)}{x}\right)}{15b^2} + \frac{c^2(6ad + bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}} + x\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^3 - \frac{7}{5}d\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[a + b/x]*(c + d/x)^3, x]$

[Out]  $(-7*d*\text{Sqrt}[a + b/x]*(c + d/x)^2)/5 - (d*\text{Sqrt}[a + b/x]*(2*(57*b^2*c^2 + 15*a*b*c*d - 2*a^2*d^2) + (b*d*(33*b*c + 2*a*d))/x))/(15*b^2) + \text{Sqrt}[a + b/x]*(c + d/x)^3*x + (c^2*(b*c + 6*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/\text{Sqrt}[a]$

**Rubi in Sympy [A]** time = 50.01, size = 131, normalized size = 0.92

$$-\frac{7d\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2}{5} + x\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^3 + \frac{8d\sqrt{a + \frac{b}{x}} \left(\frac{a^2d^2}{2} - \frac{15abcd}{4} - \frac{57b^2c^2}{4} - \frac{bd(2ad+33bc)}{8x}\right)}{15b^2} + \frac{c^2(6ad + bc) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c+d/x)**3*(a+b/x)**(1/2), x)$

[Out]  $-7*d*\text{sqrt}(a + b/x)*(c + d/x)**2/5 + x*\text{sqrt}(a + b/x)*(c + d/x)**3 + 8*d*\text{sqrt}(a + b/x)*(a**2*d**2/2 - 15*a*b*c*d/4 - 57*b**2*c**2/4 - b*d*(2*a*d + 33*b*c)/(8*x))/(15*b**2) + c**2*(6*a*d + b*c)*\text{atanh}(\text{sqrt}(a + b/x)/\text{sqrt}(a))/\text{sqrt}(a)$

**Mathematica [A]** time = 0.20535, size = 131, normalized size = 0.92

$$\sqrt{\frac{ax+b}{x}} \left( -\frac{2d(-2a^2d^2 + 15abcd + 45b^2c^2)}{15b^2} - \frac{2d^2(ad + 15bc)}{15bx} + c^3x - \frac{2d^3}{5x^2} \right) + \frac{c^2(6ad + bc) \log\left(2\sqrt{ax}\sqrt{\frac{ax+b}{x}} + 2ax + b\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x]\*(c + d/x)^3,x]

[Out] Sqrt[(b + a\*x)/x]\*((-2\*d\*(45\*b^2\*c^2 + 15\*a\*b\*c\*d - 2\*a^2\*d^2))/(15\*b^2) - (2\*d^3)/(5\*x^2) - (2\*d^2\*(15\*b\*c + a\*d))/(15\*b\*x) + c^3\*x + (c^2\*(b\*c + 6\*a\*d)\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*x\*Sqrt[(b + a\*x)/x]])/(2\*Sqrt[a])

**Maple [A]** time = 0.02, size = 248, normalized size = 1.7

$$\frac{1}{30x^3b^2} \sqrt{\frac{ax+b}{x}} \left( 90dc^2a \ln\left(\frac{1}{2} \frac{2\sqrt{ax^2+bx}\sqrt{a} + 2ax + b}{\sqrt{a}}\right) b^2x^4 + 180dc^2a^{3/2}\sqrt{ax^2+bx}bx^4 + 30c^3\sqrt{ax^2+bx}\sqrt{ab^2}x^4 + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^3\*(a+b/x)^(1/2),x)

[Out] 1/30\*((a\*x+b)/x)^(1/2)/x^3\*(90\*d\*c^2\*a\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*b^2\*x^4+180\*d\*c^2\*a^(3/2)\*(a\*x^2+b\*x)^(1/2)\*b\*x^4+30\*c^3\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)\*b^2\*x^4+15\*c^3\*b^3\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*x^4-180\*d\*c^2\*(a\*x^2+b\*x)^(3/2)\*a^(1/2)\*b\*x^2+8\*a^(3/2)\*(a\*x^2+b\*x)^(3/2)\*x\*d^3-60\*d^2\*c\*(a\*x^2+b\*x)^(3/2)\*a^(1/2)\*b\*x-12\*a^(1/2)\*(a\*x^2+b\*x)^(3/2)\*b\*d^3)/(x\*(a\*x+b))^(1/2)/a^(1/2)/b^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)\*(c + d/x)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.260376, size = 1, normalized size = 0.01

$$\left[ \frac{15(b^3c^3 + 6ab^2c^2d)x^2 \log\left(2ax\sqrt{\frac{ax+b}{x}} + (2ax+b)\sqrt{a}\right) + 2(15b^2c^3x^3 - 6b^2d^3 - 2(45b^2c^2d + 15abcd^2 - 2a^2d^3)x^2 - 2(15b^2cd^2 + ab^2d^3)x - 2a^2d^3)}{30\sqrt{ab^2}x^2} \right. \\ \left. - \frac{15(b^3c^3 + 6ab^2c^2d)x^2 \arctan\left(\frac{a}{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}\right) - (15b^2c^3x^3 - 6b^2d^3 - 2(45b^2c^2d + 15abcd^2 - 2a^2d^3)x^2 - 2(15b^2cd^2 + ab^2d^3)x - 2a^2d^3)}{15\sqrt{-ab^2}x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)\*(c + d/x)^3,x, algorithm="fricas")

[Out]  $\left[ \frac{1}{30} \cdot (15 \cdot (b^3 \cdot c^3 + 6 \cdot a \cdot b^2 \cdot c^2 \cdot d) \cdot x^2 \cdot \log(2 \cdot a \cdot x \cdot \sqrt{(a \cdot x + b)/x}) + (2 \cdot a \cdot x + b) \cdot \sqrt{a}) + 2 \cdot (15 \cdot b^2 \cdot c^3 \cdot x^3 - 6 \cdot b^2 \cdot d^3 - 2 \cdot (45 \cdot b^2 \cdot c^2 \cdot d + 15 \cdot a \cdot b \cdot c \cdot d^2 - 2 \cdot a^2 \cdot d^3) \cdot x^2 - 2 \cdot (15 \cdot b^2 \cdot c \cdot d^2 + a \cdot b \cdot d^3) \cdot x) \cdot \sqrt{a} \cdot \sqrt{(a \cdot x + b)/x}) / (\sqrt{a} \cdot b^2 \cdot x^2), -1/15 \cdot (15 \cdot (b^3 \cdot c^3 + 6 \cdot a \cdot b^2 \cdot c^2 \cdot d) \cdot x^2 \cdot \arctan(a / (\sqrt{-a} \cdot \sqrt{(a \cdot x + b)/x})) - (15 \cdot b^2 \cdot c^3 \cdot x^3 - 6 \cdot b^2 \cdot d^3 - 2 \cdot (45 \cdot b^2 \cdot c^2 \cdot d + 15 \cdot a \cdot b \cdot c \cdot d^2 - 2 \cdot a^2 \cdot d^3) \cdot x^2 - 2 \cdot (15 \cdot b^2 \cdot c \cdot d^2 + a \cdot b \cdot d^3) \cdot x) \cdot \sqrt{-a} \cdot \sqrt{(a \cdot x + b)/x}) / (\sqrt{-a} \cdot b^2 \cdot x^2) \right]$

**Sympy [A]** time = 15.4931, size = 491, normalized size = 3.43

$$\begin{aligned} & \frac{4a^{\frac{11}{2}}b^{\frac{3}{2}}d^3x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + \frac{2a^{\frac{9}{2}}b^{\frac{5}{2}}d^3x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{8a^{\frac{7}{2}}b^{\frac{7}{2}}d^3x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{6a^{\frac{5}{2}}b^{\frac{9}{2}}d^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} \\ & + 6\sqrt{ac^2d} \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) - \frac{4a^6bd^3x^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{4a^5b^2d^3x^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{6ac^2d\sqrt{x}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}} \\ & + \sqrt{bc^3}\sqrt{x}\sqrt{\frac{ax}{b}+1} - \frac{6\sqrt{bc^2d}}{\sqrt{x}\sqrt{\frac{ax}{b}+1}} + 3cd^2 \left( \begin{cases} -\frac{\sqrt{a}}{x} & \text{for } b = 0 \\ -\frac{2\left(\frac{a+b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases} \right) + \frac{bc^3 \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*\*3\*(a+b/x)\*\*(1/2),x)

[Out]  $4 \cdot a^{11/2} \cdot b^{3/2} \cdot d^3 \cdot x^3 \cdot \sqrt{a \cdot x / b + 1} / (15 \cdot a^{7/2} \cdot b^3 \cdot x^{7/2} + 15 \cdot a^{5/2} \cdot b^4 \cdot x^{5/2}) + 2 \cdot a^{9/2} \cdot b^{5/2} \cdot d^3 \cdot x^2 \cdot \sqrt{a \cdot x / b + 1} / (15 \cdot a^{7/2} \cdot b^3 \cdot x^{7/2} + 15 \cdot a^{5/2} \cdot b^4 \cdot x^{5/2}) - 8 \cdot a^{7/2} \cdot b^{7/2} \cdot d^3 \cdot x \cdot \sqrt{a \cdot x / b + 1} / (15 \cdot a^{7/2} \cdot b^3 \cdot x^{7/2} + 15 \cdot a^{5/2} \cdot b^4 \cdot x^{5/2}) - 6 \cdot a^{5/2} \cdot b^{9/2} \cdot d^3 \cdot \sqrt{a \cdot x / b + 1} / (15 \cdot a^{7/2} \cdot b^3 \cdot x^{7/2} + 15 \cdot a^{5/2} \cdot b^4 \cdot x^{5/2}) + 6 \cdot \sqrt{a} \cdot c^2 \cdot d \cdot \operatorname{asinh}(\sqrt{a} \cdot \sqrt{x} / \sqrt{b}) - 4 \cdot a^6 \cdot b \cdot d^3 \cdot x^{7/2} / (15 \cdot a^{7/2} \cdot b^3 \cdot x^{7/2} + 15 \cdot a^{5/2} \cdot b^4 \cdot x^{5/2}) - 4 \cdot a^5 \cdot b^2 \cdot d^3 \cdot x^{5/2} / (15 \cdot a^{7/2} \cdot b^3 \cdot x^{7/2} + 15 \cdot a^{5/2} \cdot b^4 \cdot x^{5/2}) - 6 \cdot a \cdot c^2 \cdot d \cdot \sqrt{x} / (\sqrt{b} \cdot \sqrt{a \cdot x / b + 1}) + \sqrt{b} \cdot c^3 \cdot \sqrt{x} \cdot \sqrt{a \cdot x / b + 1} - 6 \cdot \sqrt{bc^2d} / (\sqrt{x} \cdot \sqrt{a \cdot x / b + 1}) + 3 \cdot c \cdot d^2 \cdot \operatorname{Piecewise}((- \sqrt{a} / x, \operatorname{Eq}(b, 0)), (-2 \cdot (a + b/x)^{3/2} / (3 \cdot b), \operatorname{True})) + b \cdot c^3 \cdot \operatorname{asinh}(\sqrt{a} \cdot \sqrt{x} / \sqrt{b}) / \sqrt{a}$

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)\*(c + d/x)^3,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.126 \quad \int \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2 dx$$

**Optimal.** Leaf size=99

$$\frac{c^2 x \left(a + \frac{b}{x}\right)^{3/2}}{a} - \frac{c \sqrt{a + \frac{b}{x}} (4ad + bc)}{a} + \frac{c(4ad + bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{3/2}}{3b}$$

[Out]  $-\left(\frac{c^2 (b^2 c + 4 a^2 d) \sqrt{a + b/x}}{a} - \frac{2 d^2 (a + b/x)^{3/2}}{3 b}\right) + \frac{c^2 (a + b/x)^{3/2}}{a} + \frac{c (b^2 c + 4 a^2 d) \operatorname{ArcTanh}\left[\frac{\sqrt{a + b/x}}{\sqrt{a}}\right]}{\sqrt{a}}$

**Rubi [A]** time = 0.218174, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{c^2 x \left(a + \frac{b}{x}\right)^{3/2}}{a} - \frac{c \sqrt{a + \frac{b}{x}} (4ad + bc)}{a} + \frac{c(4ad + bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x]\*(c + d/x)^2, x]

[Out]  $-\left(\frac{c^2 (b^2 c + 4 a^2 d) \sqrt{a + b/x}}{a} - \frac{2 d^2 (a + b/x)^{3/2}}{3 b}\right) + \frac{c^2 (a + b/x)^{3/2}}{a} + \frac{c (b^2 c + 4 a^2 d) \operatorname{ArcTanh}\left[\frac{\sqrt{a + b/x}}{\sqrt{a}}\right]}{\sqrt{a}}$

**Rubi in Sympy [A]** time = 20.3938, size = 82, normalized size = 0.83

$$-\frac{2d^2 \left(a + \frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2 x \left(a + \frac{b}{x}\right)^{3/2}}{a} - \frac{c \sqrt{a + \frac{b}{x}} (4ad + bc)}{a} + \frac{c(4ad + bc) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d/x)\*\*2\*(a+b/x)\*\*(1/2), x)

[Out]  $-2 d^2 (a + b/x)^{3/2} / (3 b) + c^2 x (a + b/x)^{3/2} / a - c \sqrt{a + b/x} (4 a^2 d + b^2 c) / a + c (4 a^2 d + b^2 c) \operatorname{atanh}(\sqrt{a + b/x} / \sqrt{a}) / \sqrt{a}$

**Mathematica [A]** time = 0.133863, size = 85, normalized size = 0.86

$$\sqrt{a + \frac{b}{x}} \left(-\frac{2d^2(ax + b)}{3bx} + c^2 x - 4cd\right) + \frac{c(4ad + bc) \log\left(2\sqrt{ax} \sqrt{a + \frac{b}{x}} + 2ax + b\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x]\*(c + d/x)^2, x]

[Out]  $\sqrt{a + b/x} (-4 c^2 d + c^2 x - (2 d^2 (b + a x)) / (3 b x)) + (c (b^2 c + 4 a^2 d) \operatorname{Log}[b + 2 a x + 2 \sqrt{a} \sqrt{a + b/x} x]) / (2 \sqrt{a})$

a])

---

**Maple [B]** time = 0.017, size = 191, normalized size = 1.9

$$\frac{1}{6bx^2} \sqrt{\frac{ax+b}{x}} \left( 12cda \ln \left( \frac{1}{2} \frac{2\sqrt{ax^2+bx}\sqrt{a} + 2ax+b}{\sqrt{a}} \right) x^3b + 24cda^{3/2}\sqrt{ax^2+bx}x^3 + 6c^2\sqrt{ax^2+bx}\sqrt{ax^3b} + 3c^2b^2 \ln \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^2\*(a+b/x)^(1/2),x)

```
[Out] 1/6*((a*x+b)/x)^(1/2)/x^2*(12*c*d*a*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a
^(1/2)+2*a*x+b)/a^(1/2))*x^3*b+24*c*d*a^(3/2)*(a*x^2+b*x)^(1/2)*x
^3+6*c^2*(a*x^2+b*x)^(1/2)*a^(1/2)*x^3*b+3*c^2*b^2*ln(1/2*(2*(a*x
^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^3-24*c*d*(a*x^2+b*x)^(3
/2)*a^(1/2)*x-4*d^2*(a*x^2+b*x)^(3/2)*a^(1/2))/(x*(a*x+b))^(1/2)/
a^(1/2)/b
```

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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)\*(c + d/x)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

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**Fricas [A]** time = 0.25669, size = 1, normalized size = 0.01

$$\left[ \frac{3(b^2c^2 + 4abcd)x \log\left(2ax\sqrt{\frac{ax+b}{x}} + (2ax+b)\sqrt{a}\right) + 2(3bc^2x^2 - 2bd^2 - 2(6bcd + ad^2)x)\sqrt{a}\sqrt{\frac{ax+b}{x}}}{6\sqrt{abx}}, \right. \\ \left. \frac{3(b^2c^2 + 4abcd)x \arctan\left(\frac{a}{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}\right) - (3bc^2x^2 - 2bd^2 - 2(6bcd + ad^2)x)\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{3\sqrt{-abx}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)\*(c + d/x)^2,x, algorithm="fricas")

```
[Out] [1/6*(3*(b^2*c^2 + 4*a*b*c*d)*x*log(2*a*x*sqrt((a*x + b)/x) + (2*
a*x + b)*sqrt(a)) + 2*(3*b*c^2*x^2 - 2*b*d^2 - 2*(6*b*c*d + a*d^2
)*x)*sqrt(a)*sqrt((a*x + b)/x))/(sqrt(a)*b*x), -1/3*(3*(b^2*c^2 +
4*a*b*c*d)*x*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x))) - (3*b*c^2*x
^2 - 2*b*d^2 - 2*(6*b*c*d + a*d^2)*x)*sqrt(-a)*sqrt((a*x + b)/x))
/(sqrt(-a)*b*x)]
```

---



**Sympy [A]** time = 12.0423, size = 156, normalized size = 1.58

$$4\sqrt{acd} \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) - \frac{4acd\sqrt{x}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}} + \sqrt{bc^2}\sqrt{x}\sqrt{\frac{ax}{b}+1} - \frac{4\sqrt{bcd}}{\sqrt{x}\sqrt{\frac{ax}{b}+1}} + d^2 \left( \begin{cases} -\frac{\sqrt{a}}{x} & \text{for } b = 0 \\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases} \right) + \frac{bc^2 \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*\*2\*(a+b/x)\*\*(1/2),x)

[Out] 4\*sqrt(a)\*c\*d\*asinh(sqrt(a)\*sqrt(x)/sqrt(b)) - 4\*a\*c\*d\*sqrt(x)/(sqrt(b)\*sqrt(a\*x/b + 1)) + sqrt(b)\*c\*\*2\*sqrt(x)\*sqrt(a\*x/b + 1) - 4\*sqrt(b)\*c\*d/(sqrt(x)\*sqrt(a\*x/b + 1)) + d\*\*2\*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2\*(a + b/x)\*\*(3/2)/(3\*b), True)) + b\*c\*\*2\*asinh(sqrt(a)\*sqrt(x)/sqrt(b))/sqrt(a)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)\*(c + d/x)^2,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.127 \quad \int \sqrt{a + \frac{b}{x}} \left( c + \frac{d}{x} \right) dx$$

**Optimal.** Leaf size=74

$$-\frac{\sqrt{a + \frac{b}{x}}(2ad + bc)}{a} + \frac{(2ad + bc) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{\sqrt{a}} + \frac{cx \left( a + \frac{b}{x} \right)^{3/2}}{a}$$

[Out] -(((b\*c + 2\*a\*d)\*Sqrt[a + b/x])/a) + (c\*(a + b/x)^(3/2)\*x)/a + ((b\*c + 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]

**Rubi [A]** time = 0.146402, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$-\frac{\sqrt{a + \frac{b}{x}}(2ad + bc)}{a} + \frac{(2ad + bc) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{\sqrt{a}} + \frac{cx \left( a + \frac{b}{x} \right)^{3/2}}{a}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x]\*(c + d/x), x]

[Out] -(((b\*c + 2\*a\*d)\*Sqrt[a + b/x])/a) + (c\*(a + b/x)^(3/2)\*x)/a + ((b\*c + 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]

**Rubi in Sympy [A]** time = 12.818, size = 63, normalized size = 0.85

$$\frac{cx \left( a + \frac{b}{x} \right)^{3/2}}{a} - \frac{2\sqrt{a + \frac{b}{x}} \left( ad + \frac{bc}{2} \right)}{a} + \frac{2 \left( ad + \frac{bc}{2} \right) \operatorname{atanh} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d/x)\*(a+b/x)\*\*(1/2), x)

[Out] c\*x\*(a + b/x)\*\*(3/2)/a - 2\*sqrt(a + b/x)\*(a\*d + b\*c/2)/a + 2\*(a\*d + b\*c/2)\*atanh(sqrt(a + b/x)/sqrt(a))/sqrt(a)

**Mathematica [A]** time = 0.122252, size = 63, normalized size = 0.85

$$\sqrt{a + \frac{b}{x}}(cx - 2d) + \frac{(2ad + bc) \log \left( 2\sqrt{ax} \sqrt{a + \frac{b}{x}} + 2ax + b \right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x]\*(c + d/x), x]

[Out] Sqrt[a + b/x]\*(-2\*d + c\*x) + ((b\*c + 2\*a\*d)\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]\*x])/(2\*Sqrt[a])

**Maple [B]** time = 0.016, size = 163, normalized size = 2.2

$$\frac{1}{2bx} \sqrt{\frac{ax+b}{x}} \left( 2da \ln \left( \frac{1}{2} \frac{2\sqrt{ax^2+bx}\sqrt{a} + 2ax+b}{\sqrt{a}} \right) x^2b + 4da^{3/2}\sqrt{ax^2+bx}x^2 + 2c\sqrt{ax^2+bx}\sqrt{ax^2b} + cb^2 \ln \left( \frac{1}{2} \left( 2\sqrt{ax^2+bx}\sqrt{a} + 2ax+b \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)\*(a+b/x)^(1/2),x)

[Out] 1/2\*((a\*x+b)/x)^(1/2)/x\*(2\*d\*a\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*x^2\*b+4\*d\*a^(3/2)\*(a\*x^2+b\*x)^(1/2)\*x^2+2\*c\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)\*x^2\*b+c\*b^2\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*x^2-4\*d\*(a\*x^2+b\*x)^(3/2)\*a^(1/2))/(x\*(a\*x+b)^(1/2)/a^(1/2)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)\*(c + d/x),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.259379, size = 1, normalized size = 0.01

$$\left[ \frac{2(cx-2d)\sqrt{a}\sqrt{\frac{ax+b}{x}} + (bc+2ad)\log\left(2ax\sqrt{\frac{ax+b}{x}} + (2ax+b)\sqrt{a}\right)}{2\sqrt{a}}, \frac{(cx-2d)\sqrt{-a}\sqrt{\frac{ax+b}{x}} - (bc+2ad)\arctan\left(\frac{a}{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}\right)}{\sqrt{-a}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)\*(c + d/x),x, algorithm="fricas")

[Out] [1/2\*(2\*(c\*x - 2\*d)\*sqrt(a)\*sqrt((a\*x + b)/x) + (b\*c + 2\*a\*d)\*log(2\*a\*x\*sqrt((a\*x + b)/x) + (2\*a\*x + b)\*sqrt(a)))/sqrt(a), ((c\*x - 2\*d)\*sqrt(-a)\*sqrt((a\*x + b)/x) - (b\*c + 2\*a\*d)\*arctan(a/(sqrt(-a)\*sqrt((a\*x + b)/x))))/sqrt(-a)]

**Sympy [A]** time = 21.0534, size = 121, normalized size = 1.64

$$2\sqrt{ad} \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) - \frac{2ad\sqrt{x}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}} + \sqrt{bc}\sqrt{x}\sqrt{\frac{ax}{b}+1} - \frac{2\sqrt{bd}}{\sqrt{x}\sqrt{\frac{ax}{b}+1}} + \frac{bc \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*(a+b/x)\*\*(1/2),x)

[Out] 2\*sqrt(a)\*d\*asinh(sqrt(a)\*sqrt(x)/sqrt(b)) - 2\*a\*d\*sqrt(x)/(sqrt(b)\*sqrt(a\*x/b + 1)) + sqrt(b)\*c\*sqrt(x)\*sqrt(a\*x/b + 1) - 2\*sqrt(b)\*d/(sqrt(x)\*sqrt(a\*x/b + 1)) + b\*c\*asinh(sqrt(a)\*sqrt(x)/sqrt(b))/sqrt(a)

---

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(a + b/x)*(c + d/x),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.128 \quad \int \sqrt{a + \frac{b}{x}} dx$$

**Optimal.** Leaf size=39

$$x\sqrt{a + \frac{b}{x}} + \frac{b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

[Out] Sqrt[a + b/x]\*x + (b\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]

**Rubi [A]** time = 0.0574037, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$x\sqrt{a + \frac{b}{x}} + \frac{b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x], x]

[Out] Sqrt[a + b/x]\*x + (b\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]

**Rubi in Sympy [A]** time = 5.90255, size = 31, normalized size = 0.79

$$x\sqrt{a + \frac{b}{x}} + \frac{b \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b/x)\*\*(1/2), x)

[Out] x\*sqrt(a + b/x) + b\*atanh(sqrt(a + b/x)/sqrt(a))/sqrt(a)

**Mathematica [A]** time = 0.0350874, size = 50, normalized size = 1.28

$$x\sqrt{a + \frac{b}{x}} + \frac{b \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x], x]

[Out] Sqrt[a + b/x]\*x + (b\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]\*x])/(2\*Sqrt[a])

**Maple [B]** time = 0.006, size = 74, normalized size = 1.9

$$\frac{x}{2}\sqrt{\frac{ax+b}{x}}\left(2\sqrt{ax^2+bx}\sqrt{a}+b\ln\left(\frac{1}{2}\left(2\sqrt{ax^2+bx}\sqrt{a}+2ax+b\right)\frac{1}{\sqrt{a}}\right)\right)\frac{1}{\sqrt{x(ax+b)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x)^(1/2), x)`

[Out]  $\frac{1}{2} \cdot \left( \frac{a \cdot x + b}{x} \right)^{1/2} \cdot x \cdot \left( 2 \cdot (a \cdot x^2 + b \cdot x)^{1/2} \cdot a^{1/2} + b \cdot \ln \left( \frac{1}{2} \cdot \left( 2 \cdot (a \cdot x^2 + b \cdot x)^{1/2} \cdot a^{1/2} + 2 \cdot a \cdot x + b \right) / a^{1/2} \right) \right) / \left( x \cdot (a \cdot x + b)^{1/2} / a^{1/2} \right)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(a + b/x), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.257149, size = 1, normalized size = 0.03

$$\left[ \frac{2 \sqrt{ax} \sqrt{\frac{ax+b}{x}} + b \log \left( 2 ax \sqrt{\frac{ax+b}{x}} + (2 ax + b) \sqrt{a} \right)}{2 \sqrt{a}}, \frac{\sqrt{-ax} \sqrt{\frac{ax+b}{x}} - b \arctan \left( \frac{a}{\sqrt{-a} \sqrt{\frac{ax+b}{x}}} \right)}{\sqrt{-a}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(a + b/x), x, algorithm="fricas")`

[Out]  $\left[ \frac{1}{2} \cdot \left( 2 \cdot \sqrt{a} \cdot x \cdot \sqrt{\frac{a \cdot x + b}{x}} + b \cdot \log \left( 2 \cdot a \cdot x \cdot \sqrt{\frac{a \cdot x + b}{x}} + (2 \cdot a \cdot x + b) \cdot \sqrt{a} \right) \right) / \sqrt{a}, \left( \sqrt{-a} \cdot x \cdot \sqrt{\frac{a \cdot x + b}{x}} - b \cdot \arctan \left( \frac{a}{\sqrt{-a} \cdot \sqrt{\frac{a \cdot x + b}{x}}} \right) \right) / \sqrt{-a} \right]$

**Sympy [A]** time = 6.49426, size = 42, normalized size = 1.08

$$\sqrt{b} \sqrt{x} \sqrt{\frac{ax}{b} + 1} + \frac{b \operatorname{asinh} \left( \frac{\sqrt{a} \sqrt{x}}{\sqrt{b}} \right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x)**(1/2), x)`

[Out]  $\sqrt{b} \cdot \sqrt{x} \cdot \sqrt{a \cdot x / b + 1} + b \cdot \operatorname{asinh}(\sqrt{a} \cdot \sqrt{x} / \sqrt{b}) / \sqrt{a}$

**GIAC/XCAS [A]** time = 0.232805, size = 86, normalized size = 2.21

$$-\frac{b \ln \left( \left| -2 \left( \sqrt{ax} - \sqrt{ax^2 + bx} \right) \sqrt{a} - b \right| \right) \operatorname{sign}(x)}{2 \sqrt{a}} + \frac{b \ln(|b|) \operatorname{sign}(x)}{2 \sqrt{a}} + \sqrt{ax^2 + bx} \operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(a + b/x),x, algorithm="giac")
```

```
[Out] -1/2*b*ln(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))*si  
gn(x)/sqrt(a) + 1/2*b*ln(abs(b))*sign(x)/sqrt(a) + sqrt(a*x^2 + b  
*x)*sign(x)
```

$$3.129 \quad \int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}} dx$$

**Optimal.** Leaf size=104

$$\frac{2\sqrt{d}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{ac^2}} + \frac{x\sqrt{a+\frac{b}{x}}}{c}$$

[Out] (Sqrt[a + b/x]\*x)/c + (2\*Sqrt[d]\*Sqrt[b\*c - a\*d]\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/c^2 + ((b\*c - 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]\*c^2)

**Rubi [A]** time = 0.354658, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{2\sqrt{d}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{ac^2}} + \frac{x\sqrt{a+\frac{b}{x}}}{c}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x]/(c + d/x), x]

[Out] (Sqrt[a + b/x]\*x)/c + (2\*Sqrt[d]\*Sqrt[b\*c - a\*d]\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/c^2 + ((b\*c - 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]\*c^2)

**Rubi in Sympy [A]** time = 41.2538, size = 90, normalized size = 0.87

$$\frac{x\sqrt{a+\frac{b}{x}}}{c} + \frac{2\sqrt{d}\sqrt{ad-bc}\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{c^2} - \frac{2\left(ad-\frac{bc}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{ac^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b/x)\*\*(1/2)/(c+d/x), x)

[Out] x\*sqrt(a + b/x)/c + 2\*sqrt(d)\*sqrt(a\*d - b\*c)\*atanh(sqrt(d)\*sqrt(a + b/x)/sqrt(a\*d - b\*c))/c\*\*2 - 2\*(a\*d - b\*c/2)\*atanh(sqrt(a + b/x)/sqrt(a))/(sqrt(a)\*c\*\*2)

**Mathematica [A]** time = 0.358711, size = 153, normalized size = 1.47

$$\frac{2\sqrt{d}\sqrt{ad-bc}\log(cx+d) + \frac{(bc-2ad)\log\left(2\sqrt{ax}\sqrt{a+\frac{b}{x}}+2ax+b\right)}{\sqrt{a}} - 2\sqrt{d}\sqrt{ad-bc}\log\left(2\sqrt{dx}\sqrt{a+\frac{b}{x}}\sqrt{ad-bc}-2adx+bcx-bd\right)}{2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x]/(c + d/x), x]

[Out] (2\*c\*Sqrt[a + b/x]\*x + 2\*Sqrt[d]\*Sqrt[-(b\*c) + a\*d]\*Log[d + c\*x] + ((b\*c - 2\*a\*d)\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]\*x])/Sqrt



$$[a] - 2 \sqrt{d} \sqrt{-(b^*c) + a^*d} \operatorname{Log}[-(b^*d) + b^*c^*x - 2^*a^*d^*x + 2^*\sqrt{d} \sqrt{-(b^*c) + a^*d} \sqrt{a + b/x}^*x]) / (2^*c^2)$$

**Maple [B]** time = 0.03, size = 286, normalized size = 2.8

$$\frac{x}{2c^3} \sqrt{\frac{ax+b}{x}} \left( 2 \sqrt{x(ax+b)} c^2 \sqrt{a} \sqrt{\frac{(ad-bc)d}{c^2}} - 2 \ln \left( \frac{1}{2} \frac{2 \sqrt{x(ax+b)} \sqrt{a} + 2ax + b}{\sqrt{a}} \right) \sqrt{\frac{(ad-bc)d}{c^2}} acd + \ln \left( \frac{1}{2} \left( 2 \sqrt{x(ax+b)} \sqrt{a} + 2ax + b \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(1/2)/(c+d/x), x)

[Out]  $\frac{1}{2} \left( \frac{(a^*x+b)}{x} \right)^{1/2} x^* \left( 2^* \left( x^* (a^*x+b) \right)^{1/2} c^2 a^{1/2} \left( (a^*d-b^*c)^*d/c^2 \right)^{1/2} - 2^* \ln \left( \frac{1}{2} \left( 2^* \left( x^* (a^*x+b) \right)^{1/2} a^{1/2} + 2^* a^*x+b \right) / a^{1/2} \right) \left( (a^*d-b^*c)^*d/c^2 \right)^{1/2} a^*c^*d + \ln \left( \frac{1}{2} \left( 2^* \left( x^* (a^*x+b) \right)^{1/2} a^{1/2} + 2^* a^*x+b \right) / a^{1/2} \right) \left( (a^*d-b^*c)^*d/c^2 \right)^{1/2} b^*c^2 - 2^* \ln \left( \frac{2^* \left( x^* (a^*x+b) \right)^{1/2} \left( (a^*d-b^*c)^*d/c^2 \right)^{1/2} c - 2^* a^*d^*x + b^*c^*x - b^*d}{(c^*x+d)} \right) a^{3/2} d^2 + 2^* \ln \left( \frac{2^* \left( x^* (a^*x+b) \right)^{1/2} \left( (a^*d-b^*c)^*d/c^2 \right)^{1/2} c - 2^* a^*d^*x + b^*c^*x - b^*d}{(c^*x+d)} \right) a^{1/2} b^*c^*d \right) / (x^* (a^*x+b))^{1/2} / c^3 a^{1/2} / \left( (a^*d-b^*c)^*d/c^2 \right)^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)/(c + d/x), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.288149, size = 1, normalized size = 0.01

$$\frac{2 \sqrt{acx} \sqrt{\frac{ax+b}{x}} - (bc - 2ad) \log \left( -2ax \sqrt{\frac{ax+b}{x}} + (2ax + b) \sqrt{a} \right) + 2 \sqrt{-bcd + ad^2} \sqrt{a} \log \left( \frac{bd - (bc - 2ad)x + 2 \sqrt{-bcd + ad^2} x \sqrt{\frac{ax+b}{x}}}{cx + d} \right)}{2 \sqrt{ac^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)/(c + d/x), x, algorithm="fricas")

[Out]  $\left[ \frac{1}{2} \left( 2^* \sqrt{a} \right)^* c^* x^* \sqrt{\left( \frac{a^*x + b}{x} \right)} - (b^*c - 2^*a^*d)^* \log \left( -2^* a^* x^* \sqrt{\left( \frac{a^*x + b}{x} \right)} + (2^* a^* x^* + b)^* \sqrt{a} \right) + 2^* \sqrt{-b^*c^*d + a^*d^2} \sqrt{a} \log \left( \frac{bd - (bc - 2ad)x + 2 \sqrt{-bcd + ad^2} x \sqrt{\frac{ax+b}{x}}}{cx + d} \right) \right] / \left( \sqrt{a} \right)^* c^2, \frac{1}{2} \left( 2^* \sqrt{a} \right)^* c^* x^* \sqrt{\left( \frac{a^*x + b}{x} \right)} - 4^* \sqrt{b^*c^*d - a^*d^2} \sqrt{a} \arctan \left( \frac{\sqrt{b^*c^*d - a^*d^2}}{d \sqrt{\left( \frac{a^*x + b}{x} \right)}} \right) - (b^*c - 2^*a^*d)^* \log \left( -2^* a^* x^* \sqrt{\left( \frac{a^*x + b}{x} \right)} + (2^* a^* x^* + b)^* \sqrt{a} \right) \right] / \left( \sqrt{a} \right)^* c^2, \left( \sqrt{-a} \right)^* c^* x^* \sqrt{\left( \frac{a^*x + b}{x} \right)} - (b^*c - 2^*a^*d)^* \arctan \left( \frac{a}{\sqrt{-a} \sqrt{\left( \frac{a^*x + b}{x} \right)}} \right) + \sqrt{-b^*c^*d + a^*d^2} \sqrt{-a} \log \left( \frac{bd - (bc - 2ad)x + 2 \sqrt{-bcd + ad^2} x \sqrt{\frac{ax+b}{x}}}{cx + d} \right) \right] / \left( \sqrt{-a} \right)^* c^2, \left( \sqrt{-a} \right)^* c^* x^* \sqrt{\left( \frac{a^*x + b}{x} \right)} - (b^*c - 2^*a^*d)^* \arctan \left( \frac{a}{\sqrt{-a} \sqrt{\left( \frac{a^*x + b}{x} \right)}} \right) - 2^* \sqrt{b^*c^*d - a^*d^2} \sqrt{-a} \arctan \left( \frac{\sqrt{b^*c^*d - a^*d^2}}{d \sqrt{\left( \frac{a^*x + b}{x} \right)}} \right) \right] / \left( \sqrt{-a} \right)^* c^2$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sqrt{a + \frac{b}{x}}}{cx + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(1/2)/(c+d/x), x)

[Out] Integral(x\*sqrt(a + b/x)/(c\*x + d), x)

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)/(c + d/x), x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.130 \quad \int \frac{\sqrt{a + \frac{b}{x}}}{\left(c + \frac{d}{x}\right)^2} dx$$

**Optimal.** Leaf size=147

$$\frac{\sqrt{d}(3bc - 4ad) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^3\sqrt{bc - ad}} + \frac{(bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{ac^3}} + \frac{2d\sqrt{a + \frac{b}{x}}}{c^2\left(c + \frac{d}{x}\right)} + \frac{x\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)}$$

[Out] (2\*d\*Sqrt[a + b/x])/(c^2\*(c + d/x)) + (Sqrt[a + b/x]\*x)/(c\*(c + d/x)) + (Sqrt[d]\*(3\*b\*c - 4\*a\*d)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(c^3\*Sqrt[b\*c - a\*d]) + ((b\*c - 4\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]\*c^3)

**Rubi [A]** time = 0.606933, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{\sqrt{d}(3bc - 4ad) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^3\sqrt{bc - ad}} + \frac{(bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{ac^3}} + \frac{2d\sqrt{a + \frac{b}{x}}}{c^2\left(c + \frac{d}{x}\right)} + \frac{x\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x]/(c + d/x)^2, x]

[Out] (2\*d\*Sqrt[a + b/x])/(c^2\*(c + d/x)) + (Sqrt[a + b/x]\*x)/(c\*(c + d/x)) + (Sqrt[d]\*(3\*b\*c - 4\*a\*d)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(c^3\*Sqrt[b\*c - a\*d]) + ((b\*c - 4\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]\*c^3)

**Rubi in Sympy [A]** time = 67.7229, size = 122, normalized size = 0.83

$$\frac{x\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)} + \frac{2d\sqrt{a + \frac{b}{x}}}{c^2\left(c + \frac{d}{x}\right)} + \frac{\sqrt{d}(4ad - 3bc) \operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{ad - bc}}\right)}{c^3\sqrt{ad - bc}} - \frac{(4ad - bc) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{ac^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b/x)\*\*(1/2)/(c+d/x)\*\*2, x)

[Out] x\*sqrt(a + b/x)/(c\*(c + d/x)) + 2\*d\*sqrt(a + b/x)/(c\*\*2\*(c + d/x)) + sqrt(d)\*(4\*a\*d - 3\*b\*c)\*atanh(sqrt(d)\*sqrt(a + b/x)/sqrt(a\*d - b\*c))/(c\*\*3\*sqrt(a\*d - b\*c)) - (4\*a\*d - b\*c)\*atanh(sqrt(a + b/x)/sqrt(a))/(sqrt(a)\*c\*\*3)

**Mathematica [C]** time = 0.599008, size = 197, normalized size = 1.34

$$\frac{i\sqrt{d}(3bc - 4ad) \log\left(-\frac{2ic^4(-2i\sqrt{dx}\sqrt{a + \frac{b}{x}}\sqrt{bc - ad} - 2adx + bcx - bd)}{d^{3/2}(cx + d)(3bc - 4ad)\sqrt{bc - ad}}\right)}{\sqrt{bc - ad}} + \frac{2cx\sqrt{a + \frac{b}{x}}(cx + 2d)}{cx + d} + \frac{(bc - 4ad) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right)}{\sqrt{a}}$$

$$2c^3$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x]/(c + d/x)^2,x]

[Out] 
$$\frac{((2c\sqrt{a + b/x})x^2(d + cx))/(d + cx) + ((b^2c - 4a^2d)\text{Log}[b + 2ax + 2\sqrt{a}\sqrt{a + b/x}x])/\sqrt{a} + (I\sqrt{d}(3b^2c - 4a^2d)\text{Log}[(-2I)c^4(-(b^2d) + bc^2x - 2a^2dx - (2I)\sqrt{d}\sqrt{b^2c - a^2d}\sqrt{a + b/x}x))]/(d^{3/2})(3b^2c - 4a^2d)\sqrt{b^2c - a^2d}(d + cx))}{\sqrt{b^2c - a^2d}(2c^3)}$$

**Maple [B]** time = 0.028, size = 939, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(1/2)/(c+d/x)^2,x)

[Out] 
$$\frac{1}{2} \left( \frac{(ax+b)}{x} \right)^{1/2} x^2 (-4 \ln(1/2 (2 (x(ax+b))^{1/2} a^{1/2} + 2ax+b)/a^{1/2})) a^2 ((ad-b^2c) d/c^2)^{1/2} x^2 c^2 d^2 - 2 (x(ax+b))^{1/2} a^{3/2} ((ad-b^2c) d/c^2)^{1/2} x^2 c^4 - 4 \ln(1/2 (2 (x(ax+b))^{1/2} a^{1/2} + 2ax+b)/a^{1/2})) a^2 ((ad-b^2c) d/c^2)^{1/2} x^2 c^4 + 5 \ln(1/2 (2 (x(ax+b))^{1/2} a^{1/2} + 2ax+b)/a^{1/2})) a^2 ((ad-b^2c) d/c^2)^{1/2} x^2 b^2 c^3 d + 2 c^4 (x(ax+b))^{3/2} a^{1/2} ((ad-b^2c) d/c^2)^{1/2} + 2 (x(ax+b))^{1/2} a^{3/2} ((ad-b^2c) d/c^2)^{1/2} x^2 c^3 d - 4 (x(ax+b))^{1/2} a^{1/2} ((ad-b^2c) d/c^2)^{1/2} x^2 b^2 c^4 + 5 \ln(1/2 (2 (x(ax+b))^{1/2} a^{1/2} + 2ax+b)/a^{1/2})) a^2 ((ad-b^2c) d/c^2)^{1/2} b^2 c^2 d^2 - \ln(1/2 (2 (x(ax+b))^{1/2} a^{1/2} + 2ax+b)/a^{1/2})) a^{5/2} x^2 c^2 d^3 + 7 \ln((2 (x(ax+b))^{1/2} ((ad-b^2c) d/c^2)^{1/2} c - 2 a^2 dx + b^2 c^2 x - b^2 d)/(cx+d)) a^{5/2} x^2 c^2 d^3 + 7 \ln((2 (x(ax+b))^{1/2} ((ad-b^2c) d/c^2)^{1/2} c - 2 a^2 dx + b^2 c^2 x - b^2 d)/(cx+d)) a^{3/2} x^2 b^2 c^2 d^2 - 3 \ln((2 (x(ax+b))^{1/2} ((ad-b^2c) d/c^2)^{1/2} c - 2 a^2 dx + b^2 c^2 x - b^2 d)/(cx+d)) a^{1/2} x^2 b^2 c^3 d + 4 (x(ax+b))^{1/2} a^{3/2} ((ad-b^2c) d/c^2)^{1/2} b^2 c^3 d - \ln(1/2 (2 (x(ax+b))^{1/2} a^{1/2} + 2ax+b)/a^{1/2})) a^{5/2} x^2 c^2 d^2 - 4 (x(ax+b))^{1/2} a^{1/2} ((ad-b^2c) d/c^2)^{1/2} b^2 c^3 d - 4 \ln((2 (x(ax+b))^{1/2} ((ad-b^2c) d/c^2)^{1/2} c - 2 a^2 dx + b^2 c^2 x - b^2 d)/(cx+d)) a^{5/2} d^4 + 7 \ln((2 (x(ax+b))^{1/2} ((ad-b^2c) d/c^2)^{1/2} c - 2 a^2 dx + b^2 c^2 x - b^2 d)/(cx+d)) a^{3/2} b^2 c^2 d^3 - 3 \ln((2 (x(ax+b))^{1/2} ((ad-b^2c) d/c^2)^{1/2} c - 2 a^2 dx + b^2 c^2 x - b^2 d)/(cx+d)) a^{1/2} b^2 c^2 d^2 / (x(ax+b))^{1/2} / c^4 / a^{1/2} / ((ad-b^2c) d/c^2)^{1/2} / (a^2 d - b^2 c) / (cx+d)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)/(c + d/x)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.300437, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)/(c + d/x)^2,x, algorithm="fricas")

```
[Out] [-1/2*((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(c^2*x^2 + 2*c*d*x)*sqrt(a)*sqrt((a*x + b)/x) + (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*log(-2*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)))/((c^4*x + c^3*d)*sqrt(a)), -1/2*((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(-a)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(c^2*x^2 + 2*c*d*x)*sqrt(-a)*sqrt((a*x + b)/x) + 2*(b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x)))))/((c^4*x + c^3*d)*sqrt(-a)), 1/2*(2*(3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d/(b*c - a*d))/(d*sqrt((a*x + b)/x))) + 2*(c^2*x^2 + 2*c*d*x)*sqrt(a)*sqrt((a*x + b)/x) - (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*log(-2*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)))/((c^4*x + c^3*d)*sqrt(a)), ((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(-a)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d/(b*c - a*d))/(d*sqrt((a*x + b)/x))) + (c^2*x^2 + 2*c*d*x)*sqrt(-a)*sqrt((a*x + b)/x) - (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x)))))/((c^4*x + c^3*d)*sqrt(-a))]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sqrt{a + \frac{b}{x}}}{(cx + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**2,x)
```

```
[Out] Integral(x**2*sqrt(a + b/x)/(c*x + d)**2, x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(a + b/x)/(c + d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.131 \quad \int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3} dx$$

**Optimal.** Leaf size=213

$$\frac{\sqrt{d}(24a^2d^2 - 40abcd + 15b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + (bc - 6ad) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4c^4(bc - ad)^{3/2}} + \frac{d\sqrt{a+\frac{b}{x}}(11bc - 12ad)}{4c^3\left(c+\frac{d}{x}\right)(bc - ad)} + \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}$$

[Out]  $(3*d*\text{Sqrt}[a + b/x])/(2*c^2*(c + d/x)^2) + (d*(11*b*c - 12*a*d)*\text{Sqrt}[a + b/x])/(4*c^3*(b*c - a*d)*(c + d/x)) + (\text{Sqrt}[a + b/x]*x)/(c*(c + d/x)^2) + (\text{Sqrt}[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*\text{ArcTan}[(\text{Sqrt}[d]*\text{Sqrt}[a + b/x])/\text{Sqrt}[b*c - a*d]])/(4*c^4*(b*c - a*d)^{(3/2)}) + ((b*c - 6*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/(\text{Sqrt}[a]*c^4)$

**Rubi [A]** time = 0.968426, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{\sqrt{d}(24a^2d^2 - 40abcd + 15b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + (bc - 6ad) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4c^4(bc - ad)^{3/2}} + \frac{d\sqrt{a+\frac{b}{x}}(11bc - 12ad)}{4c^3\left(c+\frac{d}{x}\right)(bc - ad)} + \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + b/x]/(c + d/x)^3, x]`

[Out]  $(3*d*\text{Sqrt}[a + b/x])/(2*c^2*(c + d/x)^2) + (d*(11*b*c - 12*a*d)*\text{Sqrt}[a + b/x])/(4*c^3*(b*c - a*d)*(c + d/x)) + (\text{Sqrt}[a + b/x]*x)/(c*(c + d/x)^2) + (\text{Sqrt}[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*\text{ArcTan}[(\text{Sqrt}[d]*\text{Sqrt}[a + b/x])/\text{Sqrt}[b*c - a*d]])/(4*c^4*(b*c - a*d)^{(3/2)}) + ((b*c - 6*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/(\text{Sqrt}[a]*c^4)$

**Rubi in Sympy [A]** time = 111.143, size = 182, normalized size = 0.85

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2} + \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d\sqrt{a+\frac{b}{x}}(12ad - 11bc)}{4c^3\left(c+\frac{d}{x}\right)(ad - bc)} + \frac{\sqrt{d}(24a^2d^2 - 40abcd + 15b^2c^2) \operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right) + (6ad - bc) \operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4c^4(ad - bc)^{3/2}} - \frac{1}{\sqrt{ac^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a+b/x)**(1/2)/(c+d/x)**3, x)`

[Out]  $x*\text{sqrt}(a + b/x)/(c*(c + d/x)**2) + 3*d*\text{sqrt}(a + b/x)/(2*c**2*(c + d/x)**2) + d*\text{sqrt}(a + b/x)*(12*a*d - 11*b*c)/(4*c**3*(c + d/x)*c$



$$2) * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(5/2)} * b * c * d^5 - 55 * \ln((2 * (x * (a * x + b))^{(1/2)} * ((a * d - b * c) * d / c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(3/2)} * b^2 * c^2 * d^4 - 24 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * a^3 * ((a * d - b * c) * d / c^2)^{(1/2)} * c * d^5 + 4 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * ((a * d - b * c) * d / c^2)^{(1/2)} * x^2 * b^3 * c^6 + 4 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * ((a * d - b * c) * d / c^2)^{(1/2)} * b^3 * c^4 * d^2 + 15 * \ln((2 * (x * (a * x + b))^{(1/2)} * ((a * d - b * c) * d / c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(1/2)} * b^3 * c^3 * d^3 + 52 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * a^2 * ((a * d - b * c) * d / c^2)^{(1/2)} * x^2 * b * c^4 * d^2 - 24 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * a^3 * ((a * d - b * c) * d / c^2)^{(1/2)} * x^2 * c^3 * d^3 - 32 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * a * ((a * d - b * c) * d / c^2)^{(1/2)} * x^2 * b^2 * c^5 * d - 64 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * a * ((a * d - b * c) * d / c^2)^{(1/2)} * x * b^2 * c^4 * d^2 + 44 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} * ((a * d - b * c) * d / c^2)^{(1/2)} * x * b^2 * c^5 * d - 78 * (x * (a * x + b))^{(1/2)} * a^{(3/2)} * ((a * d - b * c) * d / c^2)^{(1/2)} * x * b * c^4 * d^2 - 18 * (x * (a * x + b))^{(1/2)} * a^{(3/2)} * ((a * d - b * c) * d / c^2)^{(1/2)} * x^2 * b * c^5 * d) / (x * (a * x + b))^{(1/2)} / c^5 / (a * d - b * c)^2 / (c * x + d)^2 / ((a * d - b * c) * d / c^2)^{(1/2)} / a^{(1/2)}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)/(c + d/x)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.402124, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)/(c + d/x)^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/8 * ((15 * b^2 * c^2 * d^2 - 40 * a * b * c * d^3 + 24 * a^2 * d^4 + (15 * b^2 * c^4 - 40 * a * b * c^3 * d + 24 * a^2 * c^2 * d^2) * x^2 + 2 * (15 * b^2 * c^3 * d - 40 * a * b * c^2 * d^2 + 24 * a^2 * c * d^3) * x) * \sqrt{a} * \sqrt{-d / (b * c - a * d)}) * \log(-2 * (b * c - a * d) * x * \sqrt{-d / (b * c - a * d)}) * \sqrt{(a * x + b) / x} - b * d + (b * c - 2 * a * d) * x) / (c * x + d) - 2 * (4 * (b * c^4 - a * c^3 * d) * x^3 + (17 * b * c^3 * d - 18 * a * c^2 * d^2) * x^2 + (11 * b * c^2 * d^2 - 12 * a * c * d^3) * x) * \sqrt{a} * \sqrt{(a * x + b) / x} + 4 * (b^2 * c^2 * d^2 - 7 * a * b * c * d^3 + 6 * a^2 * d^4 + (b^2 * c^4 - 7 * a * b * c^3 * d + 6 * a^2 * c^2 * d^2) * x^2 + 2 * (b^2 * c^3 * d - 7 * a * b * c^2 * d^2 + 6 * a^2 * c * d^3) * x) * \log(-2 * a * x * \sqrt{(a * x + b) / x} + (2 * a * x + b) * \sqrt{a}) / ((b * c^5 * d^2 - a * c^4 * d^3 + (b * c^7 - a * c^6 * d) * x^2 + 2 * (b * c^6 * d - a * c^5 * d^2) * x) * \sqrt{a}), 1/4 * ((15 * b^2 * c^2 * d^2 - 40 * a * b * c * d^3 + 24 * a^2 * d^4 + (15 * b^2 * c^4 - 40 * a * b * c^3 * d + 24 * a^2 * c^2 * d^2) * x^2 + 2 * (15 * b^2 * c^3 * d - 40 * a * b * c^2 * d^2 + 24 * a^2 * c * d^3) * x) * \sqrt{a} * \sqrt{d / (b * c - a * d)}) * \arctan(- (b * c - a * d) * \sqrt{d / (b * c - a * d)}) / (d * \sqrt{d / (b * c - a * d)}) + (4 * (b * c^4 - a * c^3 * d) * x^3 + (17 * b * c^3 * d - 18 * a * c^2 * d^2) * x^2 + (11 * b * c^2 * d^2 - 12 * a * c * d^3) * x) * \sqrt{a} * \sqrt{(a * x + b) / x} - 2 * (b^2 * c^2 * d^2 - 7 * a * b * c * d^3 + 6 * a^2 * d^4 + (b^2 * c^4 - 7 * a * b * c^3 * d + 6 * a^2 * c^2 * d^2) * x^2 + 2 * (b^2 * c^3 * d - 7 * a * b * c^2 * d^2 + 6 * a^2 * c * d^3) * x) * \log(-2 * a * x * \sqrt{(a * x + b) / x} + (2 * a * x + b) * \sqrt{a}) / ((b * c^5 * d^2 - a * c^4 * d^3 + (b * c^7 - a * c^6 * d) * x^2 + 2 * (b * c^6 * d - a * c^5 * d^2) * x) * \sqrt{a}), -1/8 * ((15 * b^2 * c^2 * d^2 - 40 * a * b * c * d^3 + 24 * a^2 * d^4 + (15 * b^2 * c^4 - 40 * a * b * c^3 * d + 24 * a^2 * c^2 * d^2) * x^2 + 2 * (15 * b^2 * c^3 * d - 40 * a * b * c^2 * d^2 + 24 * a^2 * c * d^3) * x) * \sqrt{-a} * \sqrt{-d / (b * c - a * d)}) * \log(-2 * (b * c - a * d) * x * \sqrt{-d / (b * c - a * d)}) * \sqrt{(a * x + b) / x} - b * d + (b * c - 2 * a * d) * x) / (c * x + d) - 2 * (4 * (b * c^4 - a * c^3 * d) * x^3 + (17 * b * c^3 * d - 18 * a * c^2 * d^2) * x^2 + (11 * b * c^2 * d^2 - 12 * a * c * d^3) * x) * \sqrt{-a} * \sqrt{(a * x + b) / x} + 8 * (b^2 * c^2 * d^2 - 7 * a * b * \end{aligned}$$



$$c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x) * \arctan(a/(\sqrt{-a})*\sqrt{(a*x + b)/x}))) / ((b*c^5*d^2 - a*c^4*d^3 + (b*c^7 - a*c^6*d)*x^2 + 2*(b*c^6*d - a*c^5*d^2)*x)*\sqrt{-a}), 1/4*((15*b^2*c^2*d^2 - 40*a*b*c*d^3 + 24*a^2*d^4 + (15*b^2*c^4 - 40*a*b*c^3*d + 24*a^2*c^2*d^2)*x^2 + 2*(15*b^2*c^3*d - 40*a*b*c^2*d^2 + 24*a^2*c*d^3)*x)*\sqrt{-a}*\sqrt{d/(b*c - a*d)}*\arctan(-(b*c - a*d)*\sqrt{d/(b*c - a*d)})/(d*\sqrt{(a*x + b)/x})) + (4*(b*c^4 - a*c^3*d)*x^3 + (17*b*c^3*d - 18*a*c^2*d^2)*x^2 + (11*b*c^2*d^2 - 12*a*c*d^3)*x)*\sqrt{-a}*\sqrt{(a*x + b)/x} - 4*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*\arctan(a/(\sqrt{-a})*\sqrt{(a*x + b)/x}))) / ((b*c^5*d^2 - a*c^4*d^3 + (b*c^7 - a*c^6*d)*x^2 + 2*(b*c^6*d - a*c^5*d^2)*x)*\sqrt{-a})]$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(1/2)/(c+d/x)\*\*3,x)

[Out] Timed out

---

**GIAC/XCAS [A]** time = 0.561121, size = 4, normalized size = 0.02

$sage_0x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)/(c + d/x)^3,x, algorithm="giac")

[Out] sage0\*x

$$3.132 \quad \int \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^3 dx$$

**Optimal.** Leaf size=164

$$\frac{d \left(a + \frac{b}{x}\right)^{3/2} \left(\frac{3bd(2ad+19bc)}{x} + 2(13bc - ad)(2ad + 5bc)\right)}{35b^2} - 3c^2 \sqrt{a + \frac{b}{x}} (2ad + bc) + 3\sqrt{ac^2(2ad + bc)} \operatorname{tanh}^{-1} \left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) + x \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^3 - \frac{9}{7} d \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2$$

[Out]  $-3*c^2*(b*c + 2*a*d)*\operatorname{Sqrt}[a + b/x] - (9*d*(a + b/x)^{(3/2)}*(c + d/x)^2)/7 - (d*(a + b/x)^{(3/2)}*(2*(13*b*c - a*d)*(5*b*c + 2*a*d) + (3*b*d*(19*b*c + 2*a*d))/x))/(35*b^2) + (a + b/x)^{(3/2)}*(c + d/x)^3 + 3*\operatorname{Sqrt}[a]*c^2*(b*c + 2*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]]$

**Rubi [A]** time = 0.44844, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{d \left(a + \frac{b}{x}\right)^{3/2} \left(\frac{3bd(2ad+19bc)}{x} + 2(13bc - ad)(2ad + 5bc)\right)}{35b^2} - 3c^2 \sqrt{a + \frac{b}{x}} (2ad + bc) + 3\sqrt{ac^2(2ad + bc)} \operatorname{tanh}^{-1} \left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) + x \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^3 - \frac{9}{7} d \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b/x)^{(3/2)}*(c + d/x)^3, x]$

[Out]  $-3*c^2*(b*c + 2*a*d)*\operatorname{Sqrt}[a + b/x] - (9*d*(a + b/x)^{(3/2)}*(c + d/x)^2)/7 - (d*(a + b/x)^{(3/2)}*(2*(13*b*c - a*d)*(5*b*c + 2*a*d) + (3*b*d*(19*b*c + 2*a*d))/x))/(35*b^2) + (a + b/x)^{(3/2)}*(c + d/x)^3 + 3*\operatorname{Sqrt}[a]*c^2*(b*c + 2*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]]$

**Rubi in Sympy [A]** time = 48.4643, size = 151, normalized size = 0.92

$$3\sqrt{ac^2(2ad + bc)} \operatorname{atanh} \left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) - 3c^2 \sqrt{a + \frac{b}{x}} (2ad + bc) - \frac{9d \left(a + \frac{b}{x}\right)^{\frac{3}{2}} \left(c + \frac{d}{x}\right)^2}{7} + x \left(a + \frac{b}{x}\right)^{\frac{3}{2}} \left(c + \frac{d}{x}\right)^3 + \frac{8d \left(a + \frac{b}{x}\right)^{\frac{3}{2}} \left(-\frac{9bd(2ad+19bc)}{8x} + \left(\frac{3ad}{4} - \frac{39bc}{4}\right)(2ad + 5bc)\right)}{105b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}((a+b/x)**(3/2)*(c+d/x)**3, x)$

[Out]  $3*\operatorname{sqrt}(a)*c**2*(2*a*d + b*c)*\operatorname{atanh}(\operatorname{sqrt}(a + b/x)/\operatorname{sqrt}(a)) - 3*c**2*\operatorname{sqrt}(a + b/x)*(2*a*d + b*c) - 9*d*(a + b/x)**(3/2)*(c + d/x)**2/7 + x*(a + b/x)**(3/2)*(c + d/x)**3 + 8*d*(a + b/x)**(3/2)*(-9*b*d*(2*a*d + 19*b*c)/(8*x) + (3*a*d/4 - 39*b*c/4)*(2*a*d + 5*b*c))/(105*b**2)$

**Mathematica [A]** time = 0.259178, size = 169, normalized size = 1.03

$$\frac{\sqrt{a + \frac{b}{x}} (4a^3 d^3 x^3 - 2a^2 b d^2 x^2 (21cx + d) + ab^2 x (35c^3 x^3 - 280c^2 dx^2 - 84cd^2 x - 16d^3) - 2b^3 (35c^3 x^3 + 35c^2 dx^2 + 21cd^2 x + 35b^2 x^3) + \frac{3}{2} \sqrt{ac^2} (2ad + bc) \log \left( 2\sqrt{ax} \sqrt{a + \frac{b}{x}} + 2ax + b \right)}{35b^2 x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(3/2) \* (c + d/x)^3, x]

[Out] (Sqrt[a + b/x] \* (4\*a^3\*d^3\*x^3 - 2\*a^2\*b\*d^2\*x^2\*(d + 21\*c\*x) + a\*b^2\*x\*(-16\*d^3 - 84\*c\*d^2\*x - 280\*c^2\*d\*x^2 + 35\*c^3\*x^3) - 2\*b^3\*(5\*d^3 + 21\*c\*d^2\*x + 35\*c^2\*d\*x^2 + 35\*c^3\*x^3)) - 2\*b^3\*(5\*d^3 + 21\*c\*d^2\*x + 35\*c^2\*d\*x^2 + 35\*c^3\*x^3))/ (35\*b^2\*x^3) + (3\*Sqrt[a]\*c^2\*(b\*c + 2\*a\*d)\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]\*x])/2

**Maple [B]** time = 0.02, size = 332, normalized size = 2.

$$\frac{1}{70x^4b^2} \sqrt{\frac{ax+b}{x}} \left( 210c^2a^{3/2} \ln \left( \frac{1}{2} \frac{2\sqrt{ax^2+bx}\sqrt{a}+2ax+b}{\sqrt{a}} \right) db^2x^5 + 105\sqrt{ac^3}b^3 \ln \left( \frac{1}{2} \frac{2\sqrt{ax^2+bx}\sqrt{a}+2ax+b}{\sqrt{a}} \right) x^5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(3/2) \* (c+d/x)^3, x)

[Out] 1/70 \* ((a\*x+b)/x)^(1/2) / x^4 / b^2 \* (210\*c^2\*a^(3/2)\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2)) \* d\*b^2\*x^5 + 105\*a^(1/2)\*c^3\*b^3\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2)) \* x^5 + 420\*c^2\*a^2\*(a\*x^2+b\*x)^(1/2)\*d\*b\*x^5 + 210\*a\*c^3\*(a\*x^2+b\*x)^(1/2)\*b^2\*x^5 - 420\*c^2\*(a\*x^2+b\*x)^(3/2)\*a\*d\*b\*x^3 - 140\*c^3\*(a\*x^2+b\*x)^(3/2)\*b^2\*x^3 + 8\*(a\*x^2+b\*x)^(3/2)\*x^2\*a^2\*d^3 - 84\*(a\*x^2+b\*x)^(3/2)\*x^2\*a\*b\*c\*d^2 - 140\*(a\*x^2+b\*x)^(3/2)\*x^2\*b^2\*c^2\*d - 12\*(a\*x^2+b\*x)^(3/2)\*x\*a\*b\*d^3 - 84\*(a\*x^2+b\*x)^(3/2)\*x\*b^2\*c\*d^2 - 20\*(a\*x^2+b\*x)^(3/2)\*b^2\*d^3) / (x\*(a\*x+b))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(3/2) \* (c + d/x)^3, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.255313, size = 1, normalized size = 0.01

$$\left[ \frac{105(b^3c^3 + 2ab^2c^2d)\sqrt{ax^3} \log \left( 2ax + 2\sqrt{ax} \sqrt{\frac{ax+b}{x}} + b \right) + 2(35ab^2c^3x^4 - 10b^3d^3 - 2(35b^3c^3 + 140ab^2c^2d + 21a^2bcd^2))}{70b^2x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(3/2)\*(c + d/x)^3,x, algorithm="fricas")

[Out] [1/70\*(105\*(b^3\*c^3 + 2\*a\*b^2\*c^2\*d)\*sqrt(a)\*x^3\*log(2\*a\*x + 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + 2\*(35\*a\*b^2\*c^3\*x^4 - 10\*b^3\*d^3 - 2\*(35\*b^3\*c^3 + 140\*a\*b^2\*c^2\*d + 21\*a^2\*b\*c\*d^2 - 2\*a^3\*d^3)\*x^3 - 2\*(35\*b^3\*c^2\*d + 42\*a\*b^2\*c\*d^2 + a^2\*b\*d^3)\*x^2 - 2\*(21\*b^3\*c\*d^2 + 8\*a\*b^2\*d^3)\*x)\*sqrt((a\*x + b)/x))/(b^2\*x^3), 1/35\*(105\*(b^3\*c^3 + 2\*a\*b^2\*c^2\*d)\*sqrt(-a)\*x^3\*arctan(sqrt((a\*x + b)/x)/sqrt(-a)) + (35\*a\*b^2\*c^3\*x^4 - 10\*b^3\*d^3 - 2\*(35\*b^3\*c^3 + 140\*a\*b^2\*c^2\*d + 21\*a^2\*b\*c\*d^2 - 2\*a^3\*d^3)\*x^3 - 2\*(35\*b^3\*c^2\*d + 42\*a\*b^2\*c\*d^2 + a^2\*b\*d^3)\*x^2 - 2\*(21\*b^3\*c\*d^2 + 8\*a\*b^2\*d^3)\*x)\*sqrt((a\*x + b)/x))/(b^2\*x^3)]

**Sympy [A]** time = 41.5849, size = 1862, normalized size = 11.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(3/2)\*(c+d/x)\*\*3,x)

[Out] 
$$\begin{aligned} & -16*a^{(19/2)}*b^{(11/2)}*d^{*3}*x^{*6}*sqrt(a*x/b + 1)/(105*a^{(13/2)}* \\ & b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9} \\ & *x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) - 40*a^{(17/2)}*b^{(13/2)} \\ & *d^{*3}*x^{*5}*sqrt(a*x/b + 1)/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) - 30*a^{(15/2)}*b^{(15/2)}*d^{*3}*x^{*4}*sqrt(a*x/b + 1)/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) - 40*a^{(13/2)}*b^{(17/2)}*d^{*3}*x^{*3}*sqrt(a*x/b + 1)/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) + 4*a^{(13/2)}*b^{(3/2)}*d^{*3}*x^{*3}*sqrt(a*x/b + 1)/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) - 100*a^{(11/2)}*b^{(19/2)}*d^{*3}*x^{*2}*sqrt(a*x/b + 1)/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) + 12*a^{(11/2)}*b^{(5/2)}*c*d^{*2}*x^{*3}*sqrt(a*x/b + 1)/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) + 2*a^{(11/2)}*b^{(5/2)}*d^{*3}*x^{*2}*sqrt(a*x/b + 1)/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) - 96*a^{(9/2)}*b^{(21/2)}*d^{*3}*x*sqrt(a*x/b + 1)/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) + 6*a^{(9/2)}*b^{(7/2)}*c*d^{*2}*x^{*2}*sqrt(a*x/b + 1)/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) - 8*a^{(9/2)}*b^{(7/2)}*d^{*3}*x*sqrt(a*x/b + 1)/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) - 30*a^{(7/2)}*b^{(23/2)}*d^{*3}*sqrt(a*x/b + 1)/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) - 24*a^{(7/2)}*b^{(9/2)}*c*d^{*2}*x*sqrt(a*x/b + 1)/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) - 6*a^{(7/2)}*b^{(9/2)}*d^{*3}*sqrt(a*x/b + 1)/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) + 15*a^{(5/2)}*b^{(11/2)}*c*d^{*2}*sqrt(a*x/b + 1)/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) + 6*a^{(3/2)}*c^{*2}*d*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 3*sqrt(a)*b*c^{*3}*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 16*a^{*10}*b^{*5}*d^{*3}*x^{(13/2)}/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) + 48*a^{*9}*b^{*6}*d^{*3}*x^{(11/2)}/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) + 48*a^{*8}*b^{*7}*d^{*3}*x^{(9/2)}/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) + 16*a^{*7}*b^{*8}*d^{*3}*x^{(7/2)}/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) - 4*a^{*7}*b^{*d^{*3}*x^{(7/2)}/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) - 12*a^{*6}*b^{*2}*c*d^{*2}*x^{(7/2)}/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) - 4*a^{*6}*b^{*2}*d^{*3}*x^{(5/2)}/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) - 12*a^{*5}*b^{*3}*c*d^{*2}*x^{(5/2)}/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) - 6*a^{*2}*c^{*2}*d*sqrt(x)/(sqrt(b)*sqrt(a*x/b + 1)) + \end{aligned}$$

```

a*sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1) - 2*a*sqrt(b)*c**3*sqrt(x)
/sqrt(a*x/b + 1) - 6*a*sqrt(b)*c**2*d/(sqrt(x)*sqrt(a*x/b + 1)) +
3*a*c*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)
)/(3*b), True)) - 2*b**(3/2)*c**3/(sqrt(x)*sqrt(a*x/b + 1)) + 3*b
*c**2*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3
*b), True))

```

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**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(3/2)\*(c + d/x)^3,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.133 \quad \int \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2 dx$$

**Optimal.** Leaf size=126

$$\frac{c^2 x \left(a + \frac{b}{x}\right)^{5/2}}{a} - \frac{c \left(a + \frac{b}{x}\right)^{3/2} (4ad + 3bc)}{3a} - c \sqrt{a + \frac{b}{x}} (4ad + 3bc) + \sqrt{ac} (4ad + 3bc) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) - \frac{2d^2 \left(a + \frac{b}{x}\right)^{5/2}}{5b}$$

[Out]  $-(c*(3*b*c + 4*a*d)*\text{Sqrt}[a + b/x]) - (c*(3*b*c + 4*a*d)*(a + b/x)^{(3/2)})/(3*a) - (2*d^2*(a + b/x)^{(5/2)})/(5*b) + (c^2*(a + b/x)^{(5/2)*x})/a + \text{Sqrt}[a]*c*(3*b*c + 4*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]]$

**Rubi [A]** time = 0.249053, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{c^2 x \left(a + \frac{b}{x}\right)^{5/2}}{a} - \frac{c \left(a + \frac{b}{x}\right)^{3/2} (4ad + 3bc)}{3a} - c \sqrt{a + \frac{b}{x}} (4ad + 3bc) + \sqrt{ac} (4ad + 3bc) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) - \frac{2d^2 \left(a + \frac{b}{x}\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b/x)^{(3/2)}*(c + d/x)^2, x]$

[Out]  $-(c*(3*b*c + 4*a*d)*\text{Sqrt}[a + b/x]) - (c*(3*b*c + 4*a*d)*(a + b/x)^{(3/2)})/(3*a) - (2*d^2*(a + b/x)^{(5/2)})/(5*b) + (c^2*(a + b/x)^{(5/2)*x})/a + \text{Sqrt}[a]*c*(3*b*c + 4*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]]$

**Rubi in Sympy [A]** time = 22.4613, size = 107, normalized size = 0.85

$$\sqrt{ac} (4ad + 3bc) \operatorname{atanh} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) - c \sqrt{a + \frac{b}{x}} (4ad + 3bc) - \frac{2d^2 \left(a + \frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2 x \left(a + \frac{b}{x}\right)^{5/2}}{a} - \frac{c \left(a + \frac{b}{x}\right)^{3/2} (4ad + 3bc)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a+b/x)**(3/2)*(c+d/x)**2, x)$

[Out]  $\text{sqrt}(a)*c*(4*a*d + 3*b*c)*\text{atanh}(\text{sqrt}(a + b/x)/\text{sqrt}(a)) - c*\text{sqrt}(a + b/x)*(4*a*d + 3*b*c) - 2*d**2*(a + b/x)**(5/2)/(5*b) + c**2*x*(a + b/x)**(5/2)/a - c*(a + b/x)**(3/2)*(4*a*d + 3*b*c)/(3*a)$

**Mathematica [A]** time = 0.238831, size = 115, normalized size = 0.91

$$\frac{1}{15} \sqrt{a + \frac{b}{x}} \left( -\frac{6a^2d^2}{b} - \frac{4d(3ad + 5bc)}{x} + 15ac^2x - 80acd - 30bc^2 - \frac{6bd^2}{x^2} \right) + \frac{1}{2} \sqrt{ac}(4ad + 3bc) \log \left( 2\sqrt{ax} \sqrt{a + \frac{b}{x}} + 2ax + b \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(3/2)\*(c + d/x)^2,x]

[Out] (Sqrt[a + b/x]\*(-30\*b\*c^2 - 80\*a\*c\*d - (6\*a^2\*d^2)/b - (6\*b\*d^2)/x^2 - (4\*d\*(5\*b\*c + 3\*a\*d))/x + 15\*a\*c^2\*x))/15 + (Sqrt[a]\*c\*(3\*b\*c + 4\*a\*d)\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]\*x])/2

**Maple [B]** time = 0.019, size = 244, normalized size = 1.9

$$\frac{1}{30bx^3} \sqrt{\frac{ax+b}{x}} \left( 60ca^{3/2} \ln \left( \frac{1}{2} \frac{2\sqrt{ax^2+bx}\sqrt{a} + 2ax+b}{\sqrt{a}} \right) dbx^4 + 45\sqrt{ac^2}b^2 \ln \left( \frac{1}{2} \frac{2\sqrt{ax^2+bx}\sqrt{a} + 2ax+b}{\sqrt{a}} \right) x^4 + 12 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(3/2)\*(c+d/x)^2,x)

[Out] 1/30\*((a\*x+b)/x)^(1/2)/x^3/b\*(60\*c\*a^(3/2)\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*d\*b\*x^4+45\*a^(1/2)\*c^2\*b^2\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*x^4+120\*c\*a^2\*(a\*x^2+b\*x)^(1/2)\*d\*x^4+90\*a\*c^2\*(a\*x^2+b\*x)^(1/2)\*b\*x^4-120\*c\*(a\*x^2+b\*x)^(3/2)\*a\*d\*x^2-60\*(a\*x^2+b\*x)^(3/2)\*c^2\*b\*x^2-12\*(a\*x^2+b\*x)^(3/2)\*x\*a\*d^2-40\*(a\*x^2+b\*x)^(3/2)\*x\*b\*c\*d-12\*(a\*x^2+b\*x)^(3/2)\*b\*d^2)/(x\*(a\*x+b))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(3/2)\*(c + d/x)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.258953, size = 1, normalized size = 0.01

$$\frac{15(3b^2c^2 + 4abcd)\sqrt{ax^2} \log \left( 2ax + 2\sqrt{ax} \sqrt{\frac{ax+b}{x}} + b \right) + 2(15abc^2x^3 - 6b^2d^2 - 2(15b^2c^2 + 40abcd + 3a^2d^2)x^2 - 4(5a^2d^2 + 4abcd)x + 4b^2d^2)}{30bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(3/2)\*(c + d/x)^2,x, algorithm="fricas")

[Out] [1/30\*(15\*(3\*b^2\*c^2 + 4\*a\*b\*c\*d)\*sqrt(a)\*x^2\*log(2\*a\*x + 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + 2\*(15\*a\*b\*c^2\*x^3 - 6\*b^2\*d^2 - 2\*(

$$15*b^2*c^2 + 40*a*b*c*d + 3*a^2*d^2)*x^2 - 4*(5*b^2*c*d + 3*a*b*d^2)*x)*\sqrt{(a*x + b)/x)/(b*x^2)}, 1/15*(15*(3*b^2*c^2 + 4*a*b*c*d)*\sqrt{-a}*x^2*\arctan(\sqrt{(a*x + b)/x)/\sqrt{-a}) + (15*a*b*c^2*x^3 - 6*b^2*d^2 - 2*(15*b^2*c^2 + 40*a*b*c*d + 3*a^2*d^2)*x^2 - 4*(5*b^2*c*d + 3*a*b*d^2)*x)*\sqrt{(a*x + b)/x)/(b*x^2)}}$$

**Sympy [A]** time = 26.8339, size = 576, normalized size = 4.57

$$\begin{aligned} & \frac{4a^{\frac{11}{2}}b^{\frac{5}{2}}d^2x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + \frac{2a^{\frac{9}{2}}b^{\frac{7}{2}}d^2x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{8a^{\frac{7}{2}}b^{\frac{9}{2}}d^2x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} \\ & - \frac{6a^{\frac{5}{2}}b^{\frac{11}{2}}d^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + 4a^{\frac{3}{2}}cd \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) + 3\sqrt{abc^2} \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) \\ & - \frac{4a^6b^2d^2x^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{4a^5b^3d^2x^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} \\ & - \frac{4a^2cd\sqrt{x}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}} + a\sqrt{bc^2}\sqrt{x}\sqrt{\frac{ax}{b}+1} - \frac{2a\sqrt{bc^2}\sqrt{x}}{\sqrt{\frac{ax}{b}+1}} - \frac{4a\sqrt{bcd}}{\sqrt{x}\sqrt{\frac{ax}{b}+1}} \\ & + ad^2 \left( \begin{cases} -\frac{\sqrt{a}}{x} & \text{for } b = 0 \\ \frac{2(a+\frac{b}{x})^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases} \right) - \frac{2b^{\frac{3}{2}}c^2}{\sqrt{x}\sqrt{\frac{ax}{b}+1}} + 2bcd \left( \begin{cases} -\frac{\sqrt{a}}{x} & \text{for } b = 0 \\ -\frac{2(a+\frac{b}{x})^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases} \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(3/2)\*(c+d/x)\*\*2,x)

[Out]  $4*a^{11/2}*b^{5/2}*d^2*x^3*\sqrt{a*x/b + 1}/(15*a^{7/2}*b^3*x^{7/2} + 15*a^{5/2}*b^4*x^{5/2}) + 2*a^{9/2}*b^{7/2}*d^2*x^2*\sqrt{a*x/b + 1}/(15*a^{7/2}*b^3*x^{7/2} + 15*a^{5/2}*b^4*x^{5/2}) - 8*a^{7/2}*b^{9/2}*d^2*x*\sqrt{a*x/b + 1}/(15*a^{7/2}*b^3*x^{7/2} + 15*a^{5/2}*b^4*x^{5/2}) - 6*a^{5/2}*b^{11/2}*d^2*\sqrt{a*x/b + 1}/(15*a^{7/2}*b^3*x^{7/2} + 15*a^{5/2}*b^4*x^{5/2}) + 4*a^{3/2}*c*d*\operatorname{asinh}(\sqrt{a}*\sqrt{x}/\sqrt{b}) + 3*\sqrt{a}*b*c^2*\operatorname{asinh}(\sqrt{a}*\sqrt{x}/\sqrt{b}) - 4*a^6*b^2*d^2*x^{7/2}/(15*a^{7/2}*b^3*x^{7/2} + 15*a^{5/2}*b^4*x^{5/2}) - 4*a^5*b^3*d^2*x^{5/2}/(15*a^{7/2}*b^3*x^{7/2} + 15*a^{5/2}*b^4*x^{5/2}) - 4*a^2*c*d*\sqrt{x}/(\sqrt{b}*\sqrt{a*x/b + 1}) + a*\sqrt{b}*c^2*\sqrt{x}*\sqrt{a*x/b + 1} - 2*a*\sqrt{b}*c^2*\sqrt{x}/\sqrt{a*x/b + 1} - 4*a*\sqrt{b}*c*d/(\sqrt{x}*\sqrt{a*x/b + 1}) + a*d^2*\operatorname{Piecewise}((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) - 2*b^{3/2}*c^2/(\sqrt{x}*\sqrt{a*x/b + 1}) + 2*b*c*d*\operatorname{Piecewise}((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))$

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(3/2)\*(c + d/x)^2,x, algorithm="giac")

[Out] Exception raised: TypeError



$$3.134 \quad \int \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right) dx$$

**Optimal.** Leaf size=100

$$-\frac{\left(a + \frac{b}{x}\right)^{3/2} (2ad + 3bc)}{3a} - \sqrt{a + \frac{b}{x}} (2ad + 3bc) + \sqrt{a} (2ad + 3bc) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) + \frac{cx \left(a + \frac{b}{x}\right)^{5/2}}{a}$$

[Out]  $-\left((3*b*c + 2*a*d)*\text{Sqrt}[a + b/x]\right) - \left((3*b*c + 2*a*d)*(a + b/x)^{(3/2)}\right)/(3*a) + (c*(a + b/x)^{(5/2)*x})/a + \text{Sqrt}[a]*(3*b*c + 2*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]]$

**Rubi [A]** time = 0.181951, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$-\frac{\left(a + \frac{b}{x}\right)^{3/2} (2ad + 3bc)}{3a} - \sqrt{a + \frac{b}{x}} (2ad + 3bc) + \sqrt{a} (2ad + 3bc) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) + \frac{cx \left(a + \frac{b}{x}\right)^{5/2}}{a}$$

Antiderivative was successfully verified.

[In]  $\text{Int}\left[\left(a + \frac{b}{x}\right)^{(3/2)} * \left(c + \frac{d}{x}\right), x\right]$

[Out]  $-\left((3*b*c + 2*a*d)*\text{Sqrt}[a + b/x]\right) - \left((3*b*c + 2*a*d)*(a + b/x)^{(3/2)}\right)/(3*a) + (c*(a + b/x)^{(5/2)*x})/a + \text{Sqrt}[a]*(3*b*c + 2*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]]$

**Rubi in Sympy [A]** time = 14.7568, size = 87, normalized size = 0.87

$$2\sqrt{a} \left(ad + \frac{3bc}{2}\right) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) - \sqrt{a + \frac{b}{x}} (2ad + 3bc) + \frac{cx \left(a + \frac{b}{x}\right)^{5/2}}{a} - \frac{2 \left(a + \frac{b}{x}\right)^{3/2} \left(ad + \frac{3bc}{2}\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}\left(\left(a + \frac{b}{x}\right)^{(3/2)} * \left(c + \frac{d}{x}\right), x\right)$

[Out]  $2*\text{sqrt}(a)*(a*d + 3*b*c/2)*\text{atanh}(\text{sqrt}(a + b/x)/\text{sqrt}(a)) - \text{sqrt}(a + b/x)*(2*a*d + 3*b*c) + c*x*(a + b/x)^{(5/2)}/a - 2*(a + b/x)^{(3/2)}*(a*d + 3*b*c/2)/(3*a)$

**Mathematica [A]** time = 0.162674, size = 84, normalized size = 0.84

$$\frac{\sqrt{a + \frac{b}{x}}(ax(3cx - 8d) - 2b(3cx + d))}{3x} + \frac{1}{2}\sqrt{a}(2ad + 3bc) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}\left[\left(a + \frac{b}{x}\right)^{(3/2)} * \left(c + \frac{d}{x}\right), x\right]$

[Out]  $(\text{Sqrt}[a + b/x]*(a*x*(-8*d + 3*c*x) - 2*b*(d + 3*c*x)))/(3*x) + (\text{Sqrt}[a]*(3*b*c + 2*a*d)*\text{Log}[b + 2*a*x + 2*\text{Sqrt}[a]*\text{Sqrt}[a + b/x]*x])/2$

**Maple [B]** time = 0.016, size = 194, normalized size = 1.9

$$\frac{1}{6bx^2} \sqrt{\frac{ax+b}{x}} \left( 6a^{3/2} \ln \left( \frac{1}{2} \frac{2\sqrt{ax^2+bx}\sqrt{a} + 2ax + b}{\sqrt{a}} \right) dx^3b + 9\sqrt{a} \ln \left( \frac{1}{2} \frac{2\sqrt{ax^2+bx}\sqrt{a} + 2ax + b}{\sqrt{a}} \right) b^2cx^3 + 12a^2\sqrt{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(3/2)\*(c+d/x),x)

[Out] 1/6\*((a\*x+b)/x)^(1/2)/x^2\*(6\*a^(3/2)\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*d\*x^3\*b+9\*a^(1/2)\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*b^2\*c\*x^3+12\*a^2\*(a\*x^2+b\*x)^(1/2)\*d\*x^3+18\*a\*(a\*x^2+b\*x)^(1/2)\*c\*x^3\*b-12\*(a\*x^2+b\*x)^(3/2)\*a\*d\*x-12\*(a\*x^2+b\*x)^(3/2)\*c\*x\*b-4\*d\*(a\*x^2+b\*x)^(3/2)\*b/(x\*(a\*x+b))^(1/2)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(3/2)\*(c + d/x),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.25885, size = 1, normalized size = 0.01

$$\left[ \frac{3(3bc + 2ad)\sqrt{ax} \log \left( 2ax + 2\sqrt{ax}\sqrt{\frac{ax+b}{x}} + b \right) + 2(3acx^2 - 2bd - 2(3bc + 4ad)x)\sqrt{\frac{ax+b}{x}}}{6x}, \frac{3(3bc + 2ad)\sqrt{-ax} \arctan \left( \sqrt{\frac{ax+b}{x}} \right)}{6x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(3/2)\*(c + d/x),x, algorithm="fricas")

[Out] [1/6\*(3\*(3\*b\*c + 2\*a\*d)\*sqrt(a)\*x\*log(2\*a\*x + 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + 2\*(3\*a\*c\*x^2 - 2\*b\*d - 2\*(3\*b\*c + 4\*a\*d)\*x)\*sqrt((a\*x + b)/x)/x, 1/3\*(3\*(3\*b\*c + 2\*a\*d)\*sqrt(-a)\*x\*arctan(sqrt((a\*x + b)/x)/sqrt(-a)) + (3\*a\*c\*x^2 - 2\*b\*d - 2\*(3\*b\*c + 4\*a\*d)\*x)\*sqrt((a\*x + b)/x)/x]

**Sympy [A]** time = 32.5682, size = 204, normalized size = 2.04

$$2a^{\frac{3}{2}}d \operatorname{asinh} \left( \frac{\sqrt{a}\sqrt{x}}{\sqrt{b}} \right) + 3\sqrt{abc} \operatorname{asinh} \left( \frac{\sqrt{a}\sqrt{x}}{\sqrt{b}} \right) - \frac{2a^2d\sqrt{x}}{\sqrt{b}\sqrt{\frac{ax}{b} + 1}} + a\sqrt{bc}\sqrt{x}\sqrt{\frac{ax}{b} + 1} - \frac{2a\sqrt{bc}\sqrt{x}}{\sqrt{\frac{ax}{b} + 1}} - \frac{2a\sqrt{bd}}{\sqrt{x}\sqrt{\frac{ax}{b} + 1}} - \frac{2b^{\frac{3}{2}}c}{\sqrt{x}\sqrt{\frac{ax}{b} + 1}} + bd \left( \begin{array}{ll} \left( -\frac{\sqrt{a}}{x} \right) & \text{for } b = 0 \\ \left( -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} \right) & \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*(c+d/x),x)
```

```
[Out] 2*a**(3/2)*d*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 3*sqrt(a)*b*c*asinh
(sqrt(a)*sqrt(x)/sqrt(b)) - 2*a**2*d*sqrt(x)/(sqrt(b)*sqrt(a*x/b
+ 1)) + a*sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1) - 2*a*sqrt(b)*c*sqrt(
x)/sqrt(a*x/b + 1) - 2*a*sqrt(b)*d/(sqrt(x)*sqrt(a*x/b + 1)) - 2*
b**(3/2)*c/(sqrt(x)*sqrt(a*x/b + 1)) + b*d*Piecewise((-sqrt(a)/x,
Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a + b/x)^(3/2)*(c + d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.135 \quad \int \left(a + \frac{b}{x}\right)^{3/2} dx$$

**Optimal.** Leaf size=54

$$x \left(a + \frac{b}{x}\right)^{3/2} - 3b\sqrt{a + \frac{b}{x}} + 3\sqrt{ab} \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)$$

[Out]  $-3*b*\text{Sqrt}[a + b/x] + (a + b/x)^{(3/2)}*x + 3*\text{Sqrt}[a]*b*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]]$

**Rubi [A]** time = 0.0767684, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$

$$x \left(a + \frac{b}{x}\right)^{3/2} - 3b\sqrt{a + \frac{b}{x}} + 3\sqrt{ab} \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b/x)^{(3/2)}, x]$

[Out]  $-3*b*\text{Sqrt}[a + b/x] + (a + b/x)^{(3/2)}*x + 3*\text{Sqrt}[a]*b*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]]$

**Rubi in Sympy [A]** time = 7.71145, size = 44, normalized size = 0.81

$$3\sqrt{ab} \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) - 3b\sqrt{a + \frac{b}{x}} + x \left(a + \frac{b}{x}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a+b/x)**(3/2), x)$

[Out]  $3*\text{sqrt}(a)*b*\text{atanh}(\text{sqrt}(a + b/x)/\text{sqrt}(a)) - 3*b*\text{sqrt}(a + b/x) + x*(a + b/x)**(3/2)$

**Mathematica [A]** time = 0.0458004, size = 56, normalized size = 1.04

$$\sqrt{a + \frac{b}{x}}(ax - 2b) + \frac{3}{2}\sqrt{ab} \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b/x)^{(3/2)}, x]$

[Out]  $\text{Sqrt}[a + b/x]*(-2*b + a*x) + (3*\text{Sqrt}[a]*b*\text{Log}[b + 2*a*x + 2*\text{Sqrt}[a]*\text{Sqrt}[a + b/x]*x])/2$

**Maple [B]** time = 0.006, size = 94, normalized size = 1.7

$$\frac{1}{2x}\sqrt{\frac{ax+b}{x}}\left(3\sqrt{ab}\ln\left(\frac{1}{2}\frac{2\sqrt{ax^2+bx}\sqrt{a}+2ax+b}{\sqrt{a}}\right)x^2+6a\sqrt{ax^2+bx}x^2-4(ax^2+bx)^{3/2}\right)\frac{1}{\sqrt{x(ax+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x)^(3/2), x)`

[Out]  $\frac{1}{2} \cdot \left( \frac{a^2 x + b}{x} \right)^{1/2} / x \cdot \left( 3 \cdot a^{1/2} \cdot b \cdot \ln \left( \frac{1}{2} \cdot \left( 2 \cdot (a^2 x + b) \right)^{1/2} \cdot a^{1/2} + 2 \cdot a^2 x + b \right) / a^{1/2} \right) \cdot x^2 + 6 \cdot a \cdot (a^2 x + b)^{1/2} \cdot x^2 - 4 \cdot (a^2 x + b)^{3/2} \right) / \left( x \cdot (a^2 x + b)^{1/2} \right)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a + b/x)^(3/2), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.263802, size = 1, normalized size = 0.02

$$\left[ \frac{3}{2} \sqrt{ab} \log \left( 2ax + 2\sqrt{ax}\sqrt{\frac{ax+b}{x}} + b \right) + (ax - 2b)\sqrt{\frac{ax+b}{x}}, 3\sqrt{-ab} \arctan \left( \frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}} \right) + (ax - 2b)\sqrt{\frac{ax+b}{x}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a + b/x)^(3/2), x, algorithm="fricas")`

[Out]  $\left[ \frac{3}{2} \sqrt{a} \cdot b \cdot \log(2 \cdot a \cdot x + 2 \cdot \sqrt{a} \cdot x \cdot \sqrt{(a \cdot x + b)/x} + b) + (a \cdot x - 2 \cdot b) \cdot \sqrt{(a \cdot x + b)/x}, 3 \cdot \sqrt{-a} \cdot b \cdot \arctan(\sqrt{(a \cdot x + b)/x} / \sqrt{-a}) + (a \cdot x - 2 \cdot b) \cdot \sqrt{(a \cdot x + b)/x} \right]$

**Sympy [A]** time = 9.18015, size = 92, normalized size = 1.7

$$3\sqrt{ab} \operatorname{asinh} \left( \frac{\sqrt{a}\sqrt{x}}{\sqrt{b}} \right) + \frac{a^2 x^{3/2}}{\sqrt{b}\sqrt{\frac{ax}{b} + 1}} - \frac{a\sqrt{b}\sqrt{x}}{\sqrt{\frac{ax}{b} + 1}} - \frac{2b^{3/2}}{\sqrt{x}\sqrt{\frac{ax}{b} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x)**(3/2), x)`

[Out]  $3 \cdot \sqrt{a} \cdot b \cdot \operatorname{asinh}(\sqrt{a} \cdot \sqrt{x} / \sqrt{b}) + a^{3/2} \cdot x^{3/2} / (\sqrt{b} \cdot \sqrt{a \cdot x / b + 1}) - a \cdot \sqrt{b} \cdot \sqrt{x} / \sqrt{a \cdot x / b + 1} - 2 \cdot b^{3/2} / (\sqrt{x} \cdot \sqrt{a \cdot x / b + 1})$

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a + b/x)^(3/2), x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.136 \quad \int \frac{\left(a + \frac{b}{x}\right)^{3/2}}{c + \frac{d}{x}} dx$$

**Optimal.** Leaf size=106

$$-\frac{2(bc - ad)^{3/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^2\sqrt{d}} + \frac{\sqrt{a}(3bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^2} + \frac{ax\sqrt{a + \frac{b}{x}}}{c}$$

[Out] (a\*Sqrt[a + b/x]\*x)/c - (2\*(b\*c - a\*d)^(3/2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(c^2\*Sqrt[d]) + (Sqrt[a]\*(3\*b\*c - 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^2

**Rubi [A]** time = 0.384434, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{2(bc - ad)^{3/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^2\sqrt{d}} + \frac{\sqrt{a}(3bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^2} + \frac{ax\sqrt{a + \frac{b}{x}}}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(3/2)/(c + d/x), x]

[Out] (a\*Sqrt[a + b/x]\*x)/c - (2\*(b\*c - a\*d)^(3/2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(c^2\*Sqrt[d]) + (Sqrt[a]\*(3\*b\*c - 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^2

**Rubi in Sympy [A]** time = 37.9565, size = 92, normalized size = 0.87

$$-\frac{\sqrt{a}(2ad - 3bc) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^2} + \frac{ax\sqrt{a + \frac{b}{x}}}{c} + \frac{2(ad - bc)^{3/2} \operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{ad - bc}}\right)}{c^2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b/x)\*\*(3/2)/(c+d/x), x)

[Out] -sqrt(a)\*(2\*a\*d - 3\*b\*c)\*atanh(sqrt(a + b/x)/sqrt(a))/c\*\*2 + a\*x\*sqrt(a + b/x)/c + 2\*(a\*d - b\*c)\*\*(3/2)\*atanh(sqrt(d)\*sqrt(a + b/x)/sqrt(a\*d - b\*c))/(c\*\*2\*sqrt(d))

**Mathematica [A]** time = 0.49446, size = 156, normalized size = 1.47

$$\frac{2(ad - bc)^{3/2} \log(cx + d)}{\sqrt{d}} - \frac{2(ad - bc)^{3/2} \log\left(2\sqrt{d}x\sqrt{a + \frac{b}{x}}\sqrt{ad - bc} - 2adx + bcx - bd\right)}{\sqrt{d}} - \frac{\sqrt{a}(2ad - 3bc) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right) + 2acx\sqrt{a + \frac{b}{x}}}{2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(3/2)/(c + d/x), x]

[Out] (2\*a\*c\*Sqrt[a + b/x]\*x + (2\*(-b\*c) + a\*d)^(3/2)\*Log[d + c\*x])/Sqrt[d] - Sqrt[a]\*(-3\*b\*c + 2\*a\*d)\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]]/c^2

+ b/x]\*x] - (2\*(-(b\*c) + a\*d)^(3/2)\*Log[-(b\*d) + b\*c\*x - 2\*a\*d\*x + 2\*Sqrt[d]\*Sqrt[-(b\*c) + a\*d]\*Sqrt[a + b/x]\*x])/Sqrt[d])/(2\*c^2)

**Maple [B]** time = 0.02, size = 528, normalized size = 5.

$$\frac{x}{2dc^3} \sqrt{\frac{ax+b}{x}} \left( -2d^2 \ln \left( \frac{1}{2} \frac{2\sqrt{x(ax+b)}\sqrt{a} + 2ax + b}{\sqrt{a}} \right) a^2 c \sqrt{\frac{(ad-bc)d}{c^2}} + 3 \ln \left( \frac{1}{2} \frac{2\sqrt{x(ax+b)}\sqrt{a} + 2ax + b}{\sqrt{a}} \right) abdc \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(3/2)/(c+d/x),x)

[Out] 1/2\*((a\*x+b)/x)^(1/2)\*x\*(-2\*d^2\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^2\*c\*((a\*d-b\*c)\*d/c^2)^(1/2)+3\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a\*b\*d\*c^2\*((a\*d-b\*c)\*d/c^2)^(1/2)+2\*(x\*(a\*x+b))^(1/2)\*a^(3/2)\*d\*c^2\*((a\*d-b\*c)\*d/c^2)^(1/2)-2\*(x\*(a\*x+b))^(1/2)\*b\*a^(1/2)\*c^3\*((a\*d-b\*c)\*d/c^2)^(1/2)-2\*d^3\*ln((2\*(x\*(a\*x+b))^(1/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*c-2\*a\*d\*x+b\*c\*x-b\*d)/(c\*x+d))\*a^(5/2)+4\*d^2\*ln((2\*(x\*(a\*x+b))^(1/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*c-2\*a\*d\*x+b\*c\*x-b\*d)/(c\*x+d))\*a^(3/2)\*b\*c-2\*ln((2\*(x\*(a\*x+b))^(1/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*c-2\*a\*d\*x+b\*c\*x-b\*d)/(c\*x+d))\*b^2\*d\*a^(1/2)\*c^2+2\*b\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)\*c^3\*((a\*d-b\*c)\*d/c^2)^(1/2)+b^2\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*c^3\*((a\*d-b\*c)\*d/c^2)^(1/2)-ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*b^2\*c^3\*((a\*d-b\*c)\*d/c^2)^(1/2))/(x\*(a\*x+b))^(1/2)/d/a^(1/2)/c^3/((a\*d-b\*c)\*d/c^2)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(3/2)/(c + d/x),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.296247, size = 1, normalized size = 0.01

$$\frac{2acx\sqrt{\frac{ax+b}{x}} - (3bc - 2ad)\sqrt{a}\log\left(2ax - 2\sqrt{ax}\sqrt{\frac{ax+b}{x}} + b\right) - 2(bc - ad)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}} + bd - (bc - 2ad)x}{cx+d}\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(3/2)/(c + d/x),x, algorithm="fricas")

[Out] [1/2\*(2\*a\*c\*x\*sqrt((a\*x + b)/x) - (3\*b\*c - 2\*a\*d)\*sqrt(a)\*log(2\*a\*x - 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) - 2\*(b\*c - a\*d)\*sqrt(-(b\*c - a\*d)/d)\*log((2\*d\*x\*sqrt(-(b\*c - a\*d)/d)\*sqrt((a\*x + b)/x) + b\*d - (b\*c - 2\*a\*d)\*x)/(c\*x + d))/c^2, (a\*c\*x\*sqrt((a\*x + b)/x) + (3\*b\*c - 2\*a\*d)\*sqrt(-a)\*arctan(sqrt((a\*x + b)/x)/sqrt(-a)) - (b\*c - a\*d)\*sqrt(-(b\*c - a\*d)/d)\*log((2\*d\*x\*sqrt(-(b\*c - a\*d)/d)\*sqrt((a\*x + b)/x) + b\*d - (b\*c - 2\*a\*d)\*x)/(c\*x + d))/c^2, 1/2\*(2\*a\*c\*x\*sqrt((a\*x + b)/x) - 4\*(b\*c - a\*d)\*sqrt((b\*c - a\*d)/d)\*arctan(sqrt((a\*x + b)/x)/sqrt((b\*c - a\*d)/d)) - (3\*b\*c - 2\*a\*d)\*sqrt(a

```
) * log(2 * a * x - 2 * sqrt(a) * x * sqrt((a * x + b) / x) + b) / c^2, (a * c * x * sqrt((a * x + b) / x) + (3 * b * c - 2 * a * d) * sqrt(-a) * arctan(sqrt((a * x + b) / x) / sqrt(-a)) - 2 * (b * c - a * d) * sqrt((b * c - a * d) / d) * arctan(sqrt((a * x + b) / x) / sqrt((b * c - a * d) / d))) / c^2]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x \left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{cx + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/(c+d/x), x)
```

```
[Out] Integral(x*(a + b/x)**(3/2)/(c*x + d), x)
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a + b/x)^(3/2)/(c + d/x), x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```



$$3.137 \quad \int \frac{\left(a + \frac{b}{x}\right)^{3/2}}{\left(c + \frac{d}{x}\right)^2} dx$$

**Optimal.** Leaf size=156

$$\frac{(bc - 4ad)\sqrt{bc - ad} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^3\sqrt{d}} + \frac{\sqrt{a}(3bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^3} - \frac{\sqrt{a + \frac{b}{x}}(bc - 2ad)}{c^2\left(c + \frac{d}{x}\right)} + \frac{ax\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)}$$

[Out] -(((b\*c - 2\*a\*d)\*Sqrt[a + b/x])/(c^2\*(c + d/x))) + (a\*Sqrt[a + b/x]\*x)/(c\*(c + d/x)) - ((b\*c - 4\*a\*d)\*Sqrt[b\*c - a\*d]\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(c^3\*Sqrt[d]) + (Sqrt[a]\*(3\*b\*c - 4\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^3

**Rubi [A]** time = 0.670228, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{(bc - 4ad)\sqrt{bc - ad} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^3\sqrt{d}} + \frac{\sqrt{a}(3bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^3} - \frac{\sqrt{a + \frac{b}{x}}(bc - 2ad)}{c^2\left(c + \frac{d}{x}\right)} + \frac{ax\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(3/2)/(c + d/x)^2, x]

[Out] -(((b\*c - 2\*a\*d)\*Sqrt[a + b/x])/(c^2\*(c + d/x))) + (a\*Sqrt[a + b/x]\*x)/(c\*(c + d/x)) - ((b\*c - 4\*a\*d)\*Sqrt[b\*c - a\*d]\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(c^3\*Sqrt[d]) + (Sqrt[a]\*(3\*b\*c - 4\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^3

**Rubi in Sympy [A]** time = 68.3839, size = 129, normalized size = 0.83

$$\frac{\sqrt{a}(4ad - 3bc) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^3} + \frac{ax\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}(2ad - bc)}{c^2\left(c + \frac{d}{x}\right)} + \frac{\sqrt{ad - bc}(4ad - bc) \operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{ad - bc}}\right)}{c^3\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b/x)\*\*(3/2)/(c+d/x)\*\*2, x)

[Out] -sqrt(a)\*(4\*a\*d - 3\*b\*c)\*atanh(sqrt(a + b/x)/sqrt(a))/c\*\*3 + a\*x\*sqrt(a + b/x)/(c\*(c + d/x)) + sqrt(a + b/x)\*(2\*a\*d - b\*c)/(c\*\*2\*(c + d/x)) + sqrt(a\*d - b\*c)\*(4\*a\*d - b\*c)\*atanh(sqrt(d)\*sqrt(a + b/x)/sqrt(a\*d - b\*c))/(c\*\*3\*sqrt(d))

**Mathematica [C]** time = 0.478662, size = 231, normalized size = 1.48

$$\frac{i(4a^2d^2 - 5abcd + b^2c^2) \log\left(\frac{2c^4\left(2\sqrt{d}x\sqrt{a + \frac{b}{x}}\sqrt{bc - ad} - 2iadx - ib(d - cx)\right)}{\sqrt{d}(cx + d)\sqrt{bc - ad}(4a^2d^2 - 5abcd + b^2c^2)}\right)}{\sqrt{d}\sqrt{bc - ad}} - \frac{2cx\sqrt{a + \frac{b}{x}}(acx + 2ad - bc)}{cx + d} + \sqrt{a}(4ad - 3bc) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + \dots\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(3/2)/(c + d/x)^2, x]

[Out]  $-\frac{(-2c\sqrt{a + b/x})x^2(-bc + 2ad + acx)}{(d + cx) + \sqrt{t[a](-3b^2c + 4a^2d)\text{Log}[b + 2ax + 2\sqrt{a}\sqrt{a + b/x}x]} + \frac{(I(b^2c^2 - 5ab^2cd + 4a^2d^2)\text{Log}[(2c^4((-2I)ad^2x + 2\sqrt{d}\sqrt{bc - ad})\sqrt{a + b/x}x - I^2b(d - cx))])}{(\sqrt{t[d]\sqrt{bc - ad}}(b^2c^2 - 5ab^2cd + 4a^2d^2)(d + cx))} + \frac{1}{(\sqrt{d}\sqrt{bc - ad})}(2c^3)$

**Maple [B]** time = 0.02, size = 1202, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(3/2)/(c+d/x)^2, x)

[Out]  $\frac{1}{2} \left( \frac{(ax+b)}{x} \right)^{1/2} x^2 \left( -4a^{5/2} \ln\left(\frac{1}{2} \left( 2 \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} a^{1/2} + 2a \sqrt{ax+b} \right) / a^{1/2} \right) \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} c^2 d^4 + 2c^4 \left( x \left( \frac{ax+b}{x} \right) \right)^{3/2} a \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} d + 2 \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} c - 2a^2 dx + b^2 c^2 x - b^2 d \right) / (cx+d) \right) x^2 a^3 c^2 d^4 + \ln\left(\frac{1}{2} \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} c - 2a^2 dx + b^2 c^2 x - b^2 d \right) / (cx+d) \right) x^2 b^3 c^4 d + 4 \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} a^2 c^2 d^3 + 2 \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} b^2 c^4 d + 9 \ln\left(\frac{1}{2} \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} c - 2a^2 dx + b^2 c^2 x - b^2 d \right) / (cx+d) \right) a^2 b^2 c^2 d^4 - 6 \ln\left(\frac{1}{2} \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} c - 2a^2 dx + b^2 c^2 x - b^2 d \right) / (cx+d) \right) a^2 b^2 c^2 d^3 - 4a^{5/2} \ln\left(\frac{1}{2} \left( 2 \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} a^{1/2} + 2a \sqrt{ax+b} \right) / a^{1/2} \right) \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} x^2 c^2 d^3 - 2 \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} x^2 a^2 b^2 c^5 + 7a^{3/2} \ln\left(\frac{1}{2} \left( 2 \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} a^{1/2} + 2a \sqrt{ax+b} \right) / a^{1/2} \right) \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} b^2 c^2 d^3 + 2 \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} x^2 a^2 c^3 d^2 - 3a^{1/2} \ln\left(\frac{1}{2} \left( 2 \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} a^{1/2} + 2a \sqrt{ax+b} \right) / a^{1/2} \right) \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} b^2 c^3 d^2 + 9 \ln\left(\frac{1}{2} \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} c - 2a^2 dx + b^2 c^2 x - b^2 d \right) / (cx+d) \right) x^2 a^2 b^2 c^2 d^3 - 6 \ln\left(\frac{1}{2} \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} c - 2a^2 dx + b^2 c^2 x - b^2 d \right) / (cx+d) \right) x^2 a^2 b^2 c^3 d^2 - 6 \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} a^2 b^2 c^3 d^2 - 4 \ln\left(\frac{1}{2} \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} c - 2a^2 dx + b^2 c^2 x - b^2 d \right) / (cx+d) \right) a^3 d^5 + 7a^{3/2} \ln\left(\frac{1}{2} \left( 2 \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} a^{1/2} + 2a \sqrt{ax+b} \right) / a^{1/2} \right) \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} x^2 b^2 c^3 d^2 - 3a^{1/2} \ln\left(\frac{1}{2} \left( 2 \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} a^{1/2} + 2a \sqrt{ax+b} \right) / a^{1/2} \right) \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} x^2 b^2 c^3 d^2 - 4 \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} x^2 a^2 b^2 c^4 d - 2c^5 \left( x \left( \frac{ax+b}{x} \right) \right)^{3/2} b^2 \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} + \ln\left(\frac{1}{2} \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} c - 2a^2 dx + b^2 c^2 x - b^2 d \right) / (cx+d) \right) b^3 c^3 d^2 / \left( x \left( \frac{ax+b}{x} \right) \right)^{1/2} / c^4 / \left( \frac{ad-bc}{c} \right)^{1/2} \left( \frac{d}{c^2} \right)^{1/2} / \left( \frac{ad-bc}{c} \right) / (cx+d) / d$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(3/2)/(c + d/x)^2, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.292952, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a + b/x)^(3/2)/(c + d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*
a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (b*c*d - 4*a*d^2 + (b*
c^2 - 4*a*c*d)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*
d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(
a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqrt((a*x + b)/x)/(c^4*x + c^3*
d), -1/2*(2*(b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt((b*c - a*
*d)/d)*arctan(sqrt((a*x + b)/x)/sqrt((b*c - a*d)/d)) + (3*b*c*d -
4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x
*sqrt((a*x + b)/x) + b) - 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqr
t((a*x + b)/x)/(c^4*x + c^3*d), 1/2*(2*(3*b*c*d - 4*a*d^2 + (3*b*
c^2 - 4*a*c*d)*x)*sqrt(-a)*arctan(sqrt((a*x + b)/x)/sqrt(-a)) -
(b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(-(b*c - a*d)/d)*log(
(2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*
d)*x)/(c*x + d)) + 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqrt((a*x
+ b)/x)/(c^4*x + c^3*d), ((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*
d)*x)*sqrt(-a)*arctan(sqrt((a*x + b)/x)/sqrt(-a)) - (b*c*d - 4*a*
d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt((b*c - a*d)/d)*arctan(sqrt((a*x +
b)/x)/sqrt((b*c - a*d)/d)) + (a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*s
qrt((a*x + b)/x)/(c^4*x + c^3*d)]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{(cx + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/(c+d/x)**2,x)
```

```
[Out] Integral(x**2*(a + b/x)**(3/2)/(c*x + d)**2, x)
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a + b/x)^(3/2)/(c + d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.138 \quad \int \frac{\left(a + \frac{b}{x}\right)^{3/2}}{\left(c + \frac{d}{x}\right)^3} dx$$

Optimal. Leaf size=209

$$\frac{3(8a^2d^2 - 8abcd + b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + 3\sqrt{a}(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}} + \frac{3\sqrt{a+\frac{b}{x}}(bc - 4ad)}{4c^3\left(c + \frac{d}{x}\right)} - \frac{\sqrt{a+\frac{b}{x}}(bc - 3ad)}{2c^2\left(c + \frac{d}{x}\right)^2} + \frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c + \frac{d}{x}\right)^2}$$

[Out]  $-\left((b*c - 3*a*d)*\text{Sqrt}[a + b/x]\right)/(2*c^2*(c + d/x)^2) - (3*(b*c - 4*a*d)*\text{Sqrt}[a + b/x])/(4*c^3*(c + d/x)) + (a*\text{Sqrt}[a + b/x]*x)/(c*(c + d/x)^2) - (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*\text{ArcTan}[(\text{Sqrt}[d]*\text{Sqrt}[a + b/x])/\text{Sqrt}[b*c - a*d]])/(4*c^4*\text{Sqrt}[d]*\text{Sqrt}[b*c - a*d]) + (3*\text{Sqrt}[a]*(b*c - 2*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/c^4$

Rubi [A] time = 0.983858, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{3(8a^2d^2 - 8abcd + b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + 3\sqrt{a}(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}} + \frac{3\sqrt{a+\frac{b}{x}}(bc - 4ad)}{4c^3\left(c + \frac{d}{x}\right)} - \frac{\sqrt{a+\frac{b}{x}}(bc - 3ad)}{2c^2\left(c + \frac{d}{x}\right)^2} + \frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c + \frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(3/2)/(c + d/x)^3, x]

[Out]  $-\left((b*c - 3*a*d)*\text{Sqrt}[a + b/x]\right)/(2*c^2*(c + d/x)^2) - (3*(b*c - 4*a*d)*\text{Sqrt}[a + b/x])/(4*c^3*(c + d/x)) + (a*\text{Sqrt}[a + b/x]*x)/(c*(c + d/x)^2) - (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*\text{ArcTan}[(\text{Sqrt}[d]*\text{Sqrt}[a + b/x])/\text{Sqrt}[b*c - a*d]])/(4*c^4*\text{Sqrt}[d]*\text{Sqrt}[b*c - a*d]) + (3*\text{Sqrt}[a]*(b*c - 2*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/c^4$

Rubi in Sympy [A] time = 106.166, size = 182, normalized size = 0.87

$$\frac{6\sqrt{a}\left(ad - \frac{bc}{2}\right) \operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4} + \frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c + \frac{d}{x}\right)^2} + \frac{\sqrt{a+\frac{b}{x}}(3ad - bc)}{2c^2\left(c + \frac{d}{x}\right)^2} + \frac{3\sqrt{a+\frac{b}{x}}(4ad - bc)}{4c^3\left(c + \frac{d}{x}\right)} + \frac{3(8a^2d^2 - 8abcd + b^2c^2) \operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{4c^4\sqrt{d}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b/x)\*\*(3/2)/(c+d/x)\*\*3, x)

[Out]  $-6*\text{sqrt}(a)*(a*d - b*c/2)*\operatorname{atanh}(\text{sqrt}(a + b/x)/\text{sqrt}(a))/c**4 + a*x*\text{sqrt}(a + b/x)/(c*(c + d/x)**2) + \text{sqrt}(a + b/x)*(3*a*d - b*c)/(2*c$

$$3 \sqrt{c + d/x} \sqrt{a + b/x} (4a^2d - b^2c) / (4c^3 \sqrt{c + d/x}) + 3(8a^2d^2 - 8ab^2cd + b^2c^2) \operatorname{atanh}(\sqrt{d} \sqrt{a + b/x} / \sqrt{ad - b^2c}) / (4c^4 \sqrt{d} \sqrt{ad - b^2c})$$

**Mathematica [C]** time = 0.512074, size = 256, normalized size = 1.22

$$\frac{3i(8a^2d^2 - 8abcd + b^2c^2) \log\left(\frac{8c^5 \left(2\sqrt{d}x\sqrt{a + \frac{b}{x}}\sqrt{bc - ad} + 2iadx + ib(d - cx)\right)}{3\sqrt{d}(cx + d)\sqrt{bc - ad}(8a^2d^2 - 8abcd + b^2c^2)}\right)}{\sqrt{d}\sqrt{bc - ad}} + \frac{2cx\sqrt{a + \frac{b}{x}}(2a(2c^2x^2 + 9cdx + 6d^2) - bc(5cx + 3d))}{(cx + d)^2} - 12\sqrt{a}(2ad - bc) \log\left(2\sqrt{a}\right)}{8c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(3/2)/(c + d/x)^3, x]

[Out] 
$$\frac{((2c \sqrt{a + b/x})^3 x^3 - (b^2c^3 d + 5c^4 x)) + 2a^2(6d^2 + 9c^2 d x + 2c^2 x^2)}{(d + cx)^2} - 12 \sqrt{a} (-b^2c + 2ad) \operatorname{Log}[b + 2ax + 2\sqrt{a} \sqrt{a + b/x} x] + ((3I)(b^2c^2 - 8ab^2cd + 8a^2d^2) \operatorname{Log}[(8c^5((2I)ad^2x + 2\sqrt{d} \sqrt{bc - ad} \sqrt{a + b/x} x + I b(d - cx)))/(3\sqrt{d} \sqrt{bc - ad} (b^2c^2 - 8ab^2cd + 8a^2d^2)(d + cx))]) / (\sqrt{d} \sqrt{bc - ad})}{(8c^4)}$$

**Maple [B]** time = 0.021, size = 2373, normalized size = 11.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(3/2)/(c+d/x)^3, x)

[Out] 
$$\frac{1}{8} \left( \frac{(ax+b)^{1/2}}{x} \right)^2 x^3 (-24 \ln((2(x(ax+b))^{1/2})^2 ((ad-b^2c)d/c^2)^{1/2})^2 c - 2a^2 d^2 x + b^2 c^2 x - b^2 d) / (c^2 x + d)) + 60 a^{5/2} \ln(1/2 (2(x(ax+b))^{1/2})^2 a^{1/2} + 2ax + b) / a^{1/2})^2 ((ad-b^2c)d/c^2)^{1/2} x^2 b^2 c^4 d^3 + 120 a^{5/2} \ln(1/2 (2(x(ax+b))^{1/2})^2 a^{1/2} + 2ax + b) / a^{1/2})^2 ((ad-b^2c)d/c^2)^{1/2} x^2 b^2 c^3 d^4 - 48 a^{3/2} \ln(1/2 (2(x(ax+b))^{1/2})^2 a^{1/2} + 2ax + b) / a^{1/2})^2 ((ad-b^2c)d/c^2)^{1/2} x^2 b^2 c^5 d^2 + 18 (x(ax+b))^{1/2} ((ad-b^2c)d/c^2)^{1/2} x^3 a^2 b^2 c^6 d - 96 a^{3/2} \ln(1/2 (2(x(ax+b))^{1/2})^2 a^{1/2} + 2ax + b) / a^{1/2})^2 ((ad-b^2c)d/c^2)^{1/2} x^2 b^2 c^4 d^3 + 12 a^{1/2} \ln(1/2 (2(x(ax+b))^{1/2})^2 a^{1/2} + 2ax + b) / a^{1/2})^2 ((ad-b^2c)d/c^2)^{1/2} x^2 b^3 c^6 d - 18 (x(ax+b))^{3/2} ((ad-b^2c)d/c^2)^{1/2} x^2 a^2 b^2 c^6 d - 18 (x(ax+b))^{1/2} ((ad-b^2c)d/c^2)^{1/2} x^2 a^2 b^2 c^5 d^2 + 24 (x(ax+b))^{1/2} ((ad-b^2c)d/c^2)^{1/2} x^2 a^2 b^2 c^6 d + 24 a^{1/2} \ln(1/2 (2(x(ax+b))^{1/2})^2 a^{1/2} + 2ax + b) / a^{1/2})^2 ((ad-b^2c)d/c^2)^{1/2} x^2 b^3 c^5 d^2 - 90 (x(ax+b))^{1/2} ((ad-b^2c)d/c^2)^{1/2} x^2 a^2 b^2 c^4 d^3 + 66 (x(ax+b))^{1/2} ((ad-b^2c)d/c^2)^{1/2} x^2 a^2 b^2 c^5 d^2 - 48 a^{3/2} \ln(1/2 (2(x(ax+b))^{1/2})^2 a^{1/2} + 2ax + b) / a^{1/2})^2 ((ad-b^2c)d/c^2)^{1/2} b^2 c^3 d^4 - 3 \ln((2(x(ax+b))^{1/2})^2 ((ad-b^2c)d/c^2)^{1/2})^2 c - 2a^2 d^2 x + b^2 c^2 x - b^2 d) / (c^2 x + d)) + b^4 c^4 d^3 - 24 a^{7/2} \ln(1/2 (2(x(ax+b))^{1/2})^2 a^{1/2} + 2ax + b) / a^{1/2})^2 ((ad-b^2c)d/c^2)^{1/2} c^2 d^6 + 6 (x(ax+b))^{3/2} ((ad-b^2c)d/c^2)^{1/2} x^2 b^2 c^7 - 6 (x(ax+b))^{1/2} ((ad-b^2c)d/c^2)^{1/2} x^2 b^3 c^7 - 24 \ln((2(x(ax+b))^{1/2})^2 ((ad-b^2c)d/c^2)^{1/2})^2 c - 2a^2 d^2 x + b^2 c^2 x - b^2 d) / (c^2 x + d)) + x^2 a^4 c^2 d^5 - 3 \ln((2(x(ax+b))^{1/2})^2 ((ad-b^2c)d/c^2)^{1/2})^2 c - 2a^2 d^2 x + b^2 c^2 x - b^2 d) / (c^2 x + d)) + x^2 b^4 c^6 d + 8 (x(ax+b))^{3/2} ((ad-b^2c)d/c^2)^{1/2} a^2 c^4 d^3 + 2 (x(ax+b))^{3/2} ((ad-b^2c)d/c^2)^{1/2} b^2 c^6 d - 48 \ln((2(x(ax+b))^{1/2})^2 ((ad-b^2c)d/c^2)^{1/2})^2 c - 2a^2 d^2 x + b^2 c^2 x - b^2 d) / (c^2 x + d)) + x^2 a^4 c^2 d^6 - 6 \ln((2(x(ax+b))^{1/2})^2 ((ad-b^2c)d/c^2)^{1/2})^2 c - 2a^2 d^2 x + b^2 c^2 x - b^2 d) / (c^2 x + d)) + x^2 b^4 c^5 d^2 + 24 (x(ax+b))^{1/2} ((ad-b^2c)d/c^2)^{1/2} a^3 c^2 d^5 - 6 (x(ax+b))^{1/2} ((ad-b^2c)d/c^2)^{1/2} b^3 c^5 d^2 + 72 \ln((2(x(ax+b))^{1/2})^2 ((ad-b^2c)d/c^2)^{1/2})$$

$$2) * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^3 * b * c * d^6 - 75 * \ln((2 * (x * (a * x + b))^{1/2} * ((a * d - b * c) * d / c^2)^{1/2} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^2 * b^2 * c^2 * d^5 + 30 * \ln((2 * (x * (a * x + b))^{1/2} * ((a * d - b * c) * d / c^2)^{1/2} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a * b^3 * c^3 * d^4 + 12 * (x * (a * x + b))^{3/2} * ((a * d - b * c) * d / c^2)^{1/2} * x * a^2 * c^5 * d^2 + 36 * (x * (a * x + b))^{1/2} * ((a * d - b * c) * d / c^2)^{1/2} * x * a^3 * c^3 * d^4 - 12 * (x * (a * x + b))^{1/2} * ((a * d - b * c) * d / c^2)^{1/2} * x * b^3 * c^6 * d + 12 * a^{1/2} * \ln(1/2 * (2 * (x * (a * x + b))^{1/2} * a^{1/2} + 2 * a * x + b) / a^{1/2}) * ((a * d - b * c) * d / c^2)^{1/2} * b^3 * c^4 * d^3 + 144 * \ln((2 * (x * (a * x + b))^{1/2} * ((a * d - b * c) * d / c^2)^{1/2} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * x * a^3 * b * c^2 * d^5 - 150 * \ln((2 * (x * (a * x + b))^{1/2} * ((a * d - b * c) * d / c^2)^{1/2} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * x * a^2 * b^2 * c^3 * d^4 + 60 * \ln((2 * (x * (a * x + b))^{1/2} * ((a * d - b * c) * d / c^2)^{1/2} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * x * a * b^3 * c^4 * d^3 - 54 * (x * (a * x + b))^{1/2} * ((a * d - b * c) * d / c^2)^{1/2} * a^2 * b * c^3 * d^4 + 36 * (x * (a * x + b))^{1/2} * ((a * d - b * c) * d / c^2)^{1/2} * a * b^2 * c^4 * d^3 - 6 * (x * (a * x + b))^{1/2} * ((a * d - b * c) * d / c^2)^{1/2} * x^3 * a * b^2 * c^7 + 60 * a^{5/2} * \ln(1/2 * (2 * (x * (a * x + b))^{1/2} * a^{1/2} + 2 * a * x + b) / a^{1/2}) * ((a * d - b * c) * d / c^2)^{1/2} * b * c^2 * d^5 - 48 * a^{7/2} * \ln(1/2 * (2 * (x * (a * x + b))^{1/2} * a^{1/2} + 2 * a * x + b) / a^{1/2}) * ((a * d - b * c) * d / c^2)^{1/2} * x * c^2 * d^5 - 12 * (x * (a * x + b))^{1/2} * ((a * d - b * c) * d / c^2)^{1/2} * x^3 * a^3 * c^5 * d^2 - 24 * a^{7/2} * \ln(1/2 * (2 * (x * (a * x + b))^{1/2} * a^{1/2} + 2 * a * x + b) / a^{1/2}) * ((a * d - b * c) * d / c^2)^{1/2} * x^2 * c^3 * d^4 + 72 * \ln((2 * (x * (a * x + b))^{1/2} * ((a * d - b * c) * d / c^2)^{1/2} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * x^2 * a^3 * b * c^3 * d^4 - 75 * \ln((2 * (x * (a * x + b))^{1/2} * ((a * d - b * c) * d / c^2)^{1/2} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * x^2 * a^2 * b^2 * c^4 * d^3 + 30 * \ln((2 * (x * (a * x + b))^{1/2} * ((a * d - b * c) * d / c^2)^{1/2} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * x^2 * a * b^3 * c^5 * d^2 - 10 * (x * (a * x + b))^{3/2} * ((a * d - b * c) * d / c^2)^{1/2} * a * b * c^5 * d^2) / (x * (a * x + b))^{1/2} / c^5 / (a * d - b * c)^2 / ((a * d - b * c) * d / c^2)^{1/2} / (c * x + d)^2 / d$$


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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(3/2)/(c + d/x)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

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**Fricas [A]** time = 0.328166, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(3/2)/(c + d/x)^3,x, algorithm="fricas")

[Out] 
$$[-1/8 * (12 * (b * c * d^2 - 2 * a * d^3 + (b * c^3 - 2 * a * c^2 * d) * x^2 + 2 * (b * c^2 * d - 2 * a * c * d^2) * x) * \sqrt{-b * c * d + a * d^2}) * \sqrt{a} * \log(2 * a * x - 2 * \sqrt{t(a) * x * \sqrt{(a * x + b) / x} + b} - 2 * (4 * a * c^3 * x^3 - (5 * b * c^3 - 18 * a * c^2 * d) * x^2 - 3 * (b * c^2 * d - 4 * a * c * d^2) * x) * \sqrt{-b * c * d + a * d^2}) * \sqrt{(a * x + b) / x} - 3 * (b^2 * c^2 * d^2 - 8 * a * b * c * d^3 + 8 * a^2 * d^4 + (b^2 * c^4 - 8 * a * b * c^3 * d + 8 * a^2 * c^2 * d^2) * x^2 + 2 * (b^2 * c^3 * d - 8 * a * b * c^2 * d^2 + 8 * a^2 * c * d^3) * x) * \log(-(2 * (b * c * d - a * d^2) * x * \sqrt{(a * x + b) / x} - \sqrt{-b * c * d + a * d^2}) * (b * d - (b * c - 2 * a * d) * x)) / (c * x + d)) / ((c^6 * x^2 + 2 * c^5 * d * x + c^4 * d^2) * \sqrt{-b * c * d + a * d^2}), 1/8 * (24 * (b * c * d^2 - 2 * a * d^3 + (b * c^3 - 2 * a * c^2 * d) * x^2 + 2 * (b * c^2 * d - 2 * a * c * d^2) * x) * \sqrt{-b * c * d + a * d^2}) * \sqrt{-a} * \arctan(\sqrt{(a * x + b) / x} / \sqrt{-a}) + 2 * (4 * a * c^3 * x^3 - (5 * b * c^3 - 18 * a * c^2 * d) * x^2 - 3 * (b * c^2 * d - 4 * a * c * d^2) * x) * \sqrt{-b * c * d + a * d^2}) * \sqrt{(a * x + b) / x} + 3 * (b^2 * c^2 * d^2 - 8 * a * b * c * d^3 + 8 * a^2 * d^4 + (b^2 * c^4 - 8 * a * b * c^3 * d + 8 * a^2 * c^2 * d^2) * x^2 + 2 * (b^2 * c^3 * d - 8 * a * b * c^2 * d^2 + 8 * a^2 * c * d^3) * x) * \log(-(2 * (b * c * d - a * d^2) * x * \sqrt{(a * x + b) / x} - \sqrt{-b * c * d + a * d^2}) * (b * d - (b * c - 2 * a * d) * x)) / (c * x + d)) / ((c^6 * x^2 + 2 * c^5 * d * x + c^4 * d^2) * \sqrt{-b * c * d + a * d^2}), -1/4 * (6 * (b * c * d^2 - 2 * a * d^3 + (b * c^3 - 2$$

```
*a*c^2*d)*x^2 + 2*(b*c^2*d - 2*a*c*d^2)*x)*sqrt(b*c*d - a*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (4*a*c^3*x^3 - (5*b*c^3 - 18*a*c^2*d)*x^2 - 3*(b*c^2*d - 4*a*c*d^2)*x)*sqrt(b*c*d - a*d^2)*sqrt((a*x + b)/x) + 3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*arctan(-(b*c - a*d)/(sqrt(b*c*d - a*d^2)*sqrt((a*x + b)/x)))/((c^6*x^2 + 2*c^5*d*x + c^4*d^2)*sqrt(b*c*d - a*d^2)), 1/4*(12*(b*c*d^2 - 2*a*d^3 + (b*c^3 - 2*a*c^2*d)*x^2 + 2*(b*c^2*d - 2*a*c*d^2)*x)*sqrt(b*c*d - a*d^2)*sqrt(-a)*arctan(sqrt((a*x + b)/x)/sqrt(-a)) + (4*a*c^3*x^3 - (5*b*c^3 - 18*a*c^2*d)*x^2 - 3*(b*c^2*d - 4*a*c*d^2)*x)*sqrt(b*c*d - a*d^2)*sqrt((a*x + b)/x) - 3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*arctan(-(b*c - a*d)/(sqrt(b*c*d - a*d^2)*sqrt((a*x + b)/x)))/((c^6*x^2 + 2*c^5*d*x + c^4*d^2)*sqrt(b*c*d - a*d^2))]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{(cx + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(3/2)/(c+d/x)\*\*3,x)

[Out] Integral(x\*\*3\*(a + b/x)\*\*(3/2)/(c\*x + d)\*\*3, x)

**GIAC/XCAS [A]** time = 0.550936, size = 4, normalized size = 0.02

*sage0*x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(3/2)/(c + d/x)^3,x, algorithm="giac")

[Out] sage0\*x

$$3.139 \quad \int \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^3 dx$$

**Optimal.** Leaf size=198

$$a^{3/2}c^2(6ad + 5bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) - \frac{d\left(a + \frac{b}{x}\right)^{5/2} \left(2(-10a^2d^2 + 135abcd + 469b^2c^2) + \frac{5bd(10ad+89bc)}{x}\right)}{315b^2}$$

$$- \frac{1}{3}c^2\left(a + \frac{b}{x}\right)^{3/2} (6ad + 5bc) - ac^2\sqrt{a + \frac{b}{x}}(6ad + 5bc) + x\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^3 - \frac{11}{9}d\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2$$

[Out]  $-(a^2c^2(5b^2c + 6a^2d)\sqrt{a + b/x}) - (c^2(5b^2c + 6a^2d)(a + b/x)^{3/2})/3 - (11d^2(a + b/x)^{5/2}(c + d/x)^2)/9 - (d^2(a + b/x)^{5/2}(2(469b^2c^2 + 135ab^2cd - 10a^2d^2) + (5b^2d(89b^2c + 10a^2d))/x))/(315b^2) + (a + b/x)^{5/2}(c + d/x)^3x + a^{3/2}c^2(5b^2c + 6a^2d)\text{ArcTanh}[\sqrt{a + b/x}/\sqrt{a}]$

**Rubi [A]** time = 0.499922, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$a^{3/2}c^2(6ad + 5bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) - \frac{d\left(a + \frac{b}{x}\right)^{5/2} \left(2(-10a^2d^2 + 135abcd + 469b^2c^2) + \frac{5bd(10ad+89bc)}{x}\right)}{315b^2}$$

$$- \frac{1}{3}c^2\left(a + \frac{b}{x}\right)^{3/2} (6ad + 5bc) - ac^2\sqrt{a + \frac{b}{x}}(6ad + 5bc) + x\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^3 - \frac{11}{9}d\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(5/2)\*(c + d/x)^3, x]

[Out]  $-(a^2c^2(5b^2c + 6a^2d)\sqrt{a + b/x}) - (c^2(5b^2c + 6a^2d)(a + b/x)^{3/2})/3 - (11d^2(a + b/x)^{5/2}(c + d/x)^2)/9 - (d^2(a + b/x)^{5/2}(2(469b^2c^2 + 135ab^2cd - 10a^2d^2) + (5b^2d(89b^2c + 10a^2d))/x))/(315b^2) + (a + b/x)^{5/2}(c + d/x)^3x + a^{3/2}c^2(5b^2c + 6a^2d)\text{ArcTanh}[\sqrt{a + b/x}/\sqrt{a}]$

**Rubi in Sympy [A]** time = 54.8412, size = 184, normalized size = 0.93

$$a^{3/2}c^2(6ad + 5bc) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) - ac^2\sqrt{a + \frac{b}{x}}(6ad + 5bc)$$

$$- \frac{c^2\left(a + \frac{b}{x}\right)^{3/2} (6ad + 5bc)}{3} - \frac{11d\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2}{9} + x\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^3$$

$$+ \frac{8d\left(a + \frac{b}{x}\right)^{5/2} \left(\frac{5a^2d^2}{2} - \frac{135abcd}{4} - \frac{469b^2c^2}{4} - \frac{5bd(10ad+89bc)}{8x}\right)}{315b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b/x)\*\*(5/2)\*(c+d/x)\*\*3, x)

[Out]  $a^{3/2}c^2(6a^2d + 5b^2c)\operatorname{atanh}(\sqrt{a + b/x}/\sqrt{a}) - a^2c^2\sqrt{a + b/x}(6a^2d + 5b^2c) - c^2(a + b/x)^{3/2}(6a^2d + 5b^2c)/3 - 11d^2(a + b/x)^{5/2}(c + d/x)^2/9 + x(a + b/x)^{5/2}(c + d/x)^3 + 8d^2(a + b/x)^{5/2}(5a^2d^2/2 - 135abd^2/4 - 469b^2c^2/4 - 5bd(10ad + 89b^2c)/(8x))/(315b^2)$



**Mathematica [A]** time = 0.354917, size = 212, normalized size = 1.07

$$\frac{\frac{1}{2}a^{3/2}c^2(6ad + 5bc) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right) + \sqrt{a + \frac{b}{x}}(20a^4d^3x^4 - 10a^3bd^2x^3(27cx + d) - 3a^2b^2x^2(-105c^3x^3 + 966c^2dx^2 + 270cd^2x + 50d^3) - 2ab^3x(735c^3x^3 + 693c^2d^2x + 210cd^3) + 210ab^3d^3) - 2ab^3x(735c^3x^3 + 693c^2d^2x + 210cd^3) + 210ab^3d^3}{315b^2x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(5/2)\*(c + d/x)^3,x]

[Out] (Sqrt[a + b/x]\*(20\*a^4\*d^3\*x^4 - 10\*a^3\*b\*d^2\*x^3\*(d + 27\*c\*x) - 3\*a^2\*b^2\*x^2\*(50\*d^3 + 270\*c\*d^2\*x + 966\*c^2\*d\*x^2 - 105\*c^3\*x^3) - 2\*b^4\*(35\*d^3 + 135\*c\*d^2\*x + 189\*c^2\*d\*x^2 + 105\*c^3\*x^3) - 2\*a\*b^3\*x\*(95\*d^3 + 405\*c\*d^2\*x + 693\*c^2\*d\*x^2 + 735\*c^3\*x^3)))/(315\*b^2\*x^4) + (a^(3/2)\*c^2\*(5\*b\*c + 6\*a\*d)\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]\*x])/2

**Maple [B]** time = 0.022, size = 434, normalized size = 2.2

$$\frac{1}{630x^5b^2}\sqrt{\frac{ax+b}{x}}\left(1890a^{5/2}c^2\ln\left(\frac{1}{2}\frac{2\sqrt{ax^2+bx}\sqrt{a}+2ax+b}{\sqrt{a}}\right)dx^6b^2+1575a^{3/2}c^3b^3\ln\left(\frac{1}{2}\frac{2\sqrt{ax^2+bx}\sqrt{a}+2ax+b}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(5/2)\*(c+d/x)^3,x)

[Out] 1/630\*((a\*x+b)/x)^(1/2)/x^5/b^2\*(1890\*a^(5/2)\*c^2\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*d\*x^6\*b^2+1575\*a^(3/2)\*c^3\*b^3\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*x^6+3780\*a^3\*c^2\*(a\*x^2+b\*x)^(1/2)\*d\*x^6\*b+3150\*a^2\*c^3\*(a\*x^2+b\*x)^(1/2)\*x^6\*b^2-3780\*a^2\*c^2\*(a\*x^2+b\*x)^(3/2)\*d\*x^4\*b-2520\*a\*c^3\*(a\*x^2+b\*x)^(3/2)\*x^4\*b^2+40\*(a\*x^2+b\*x)^(3/2)\*x^3\*a^3\*d^3-540\*(a\*x^2+b\*x)^(3/2)\*x^3\*a^2\*b\*c\*d^2-2016\*(a\*x^2+b\*x)^(3/2)\*x^3\*a\*b^2\*c^2\*d-420\*(a\*x^2+b\*x)^(3/2)\*x^3\*b^3\*c^3-60\*(a\*x^2+b\*x)^(3/2)\*x^2\*a^2\*b\*d^3-1080\*(a\*x^2+b\*x)^(3/2)\*x^2\*a\*b^2\*c\*d^2-756\*(a\*x^2+b\*x)^(3/2)\*x^2\*b^3\*c^2\*d-240\*(a\*x^2+b\*x)^(3/2)\*x\*a\*b^2\*d^3-540\*(a\*x^2+b\*x)^(3/2)\*x\*b^3\*c\*d^2-140\*(a\*x^2+b\*x)^(3/2)\*b^3\*d^3)/(x\*(a\*x+b))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(5/2)\*(c + d/x)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.261345, size = 1, normalized size = 0.01

$$\frac{315(5ab^3c^3 + 6a^2b^2c^2d)\sqrt{ax^4}\log\left(2ax + 2\sqrt{ax}\sqrt{\frac{ax+b}{x}} + b\right) + 2(315a^2b^2c^3x^5 - 70b^4d^3 - 2(735ab^3c^3 + 1449a^2b^2c^2d + 210ab^3d^3))}{315b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(5/2)\*(c + d/x)^3,x, algorithm="fricas")

[Out] [1/630\*(315\*(5\*a\*b^3\*c^3 + 6\*a^2\*b^2\*c^2\*d)\*sqrt(a)\*x^4\*log(2\*a\*x + 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + 2\*(315\*a^2\*b^2\*c^3\*x^5 - 70\*b^4\*d^3 - 2\*(735\*a\*b^3\*c^3 + 1449\*a^2\*b^2\*c^2\*d + 135\*a^3\*b\*c\*d^2 - 10\*a^4\*d^3)\*x^4 - 2\*(105\*b^4\*c^3 + 693\*a\*b^3\*c^2\*d + 405\*a^2\*b^2\*c\*d^2 + 5\*a^3\*b\*d^3)\*x^3 - 6\*(63\*b^4\*c^2\*d + 135\*a\*b^3\*c\*d^2 + 25\*a^2\*b^2\*d^3)\*x^2 - 10\*(27\*b^4\*c\*d^2 + 19\*a\*b^3\*d^3)\*x)\*sqrt((a\*x + b)/x))/(b^2\*x^4), 1/315\*(315\*(5\*a\*b^3\*c^3 + 6\*a^2\*b^2\*c^2\*d)\*sqrt(-a)\*x^4\*arctan(sqrt((a\*x + b)/x)/sqrt(-a)) + (315\*a^2\*b^2\*c^3\*x^5 - 70\*b^4\*d^3 - 2\*(735\*a\*b^3\*c^3 + 1449\*a^2\*b^2\*c^2\*d + 135\*a^3\*b\*c\*d^2 - 10\*a^4\*d^3)\*x^4 - 2\*(105\*b^4\*c^3 + 693\*a\*b^3\*c^2\*d + 405\*a^2\*b^2\*c\*d^2 + 5\*a^3\*b\*d^3)\*x^3 - 6\*(63\*b^4\*c^2\*d + 135\*a\*b^3\*c\*d^2 + 25\*a^2\*b^2\*d^3)\*x^2 - 10\*(27\*b^4\*c\*d^2 + 19\*a\*b^3\*d^3)\*x)\*sqrt((a\*x + b)/x))/(b^2\*x^4)]

**Sympy [A]** time = 82.497, size = 5557, normalized size = 28.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(5/2)\*(c+d/x)\*\*3,x)

[Out] 32\*a\*\*(29/2)\*b\*\*(27/2)\*d\*\*3\*x\*\*10\*sqrt(a\*x/b + 1)/(315\*a\*\*(21/2)\*b\*\*15\*x\*\*(21/2) + 1890\*a\*\*(19/2)\*b\*\*16\*x\*\*(19/2) + 4725\*a\*\*(17/2)\*b\*\*17\*x\*\*(17/2) + 6300\*a\*\*(15/2)\*b\*\*18\*x\*\*(15/2) + 4725\*a\*\*(13/2)\*b\*\*19\*x\*\*(13/2) + 1890\*a\*\*(11/2)\*b\*\*20\*x\*\*(11/2) + 315\*a\*\*(9/2)\*b\*\*21\*x\*\*(9/2)) + 176\*a\*\*(27/2)\*b\*\*(29/2)\*d\*\*3\*x\*\*9\*sqrt(a\*x/b + 1)/(315\*a\*\*(21/2)\*b\*\*15\*x\*\*(21/2) + 1890\*a\*\*(19/2)\*b\*\*16\*x\*\*(19/2) + 4725\*a\*\*(17/2)\*b\*\*17\*x\*\*(17/2) + 6300\*a\*\*(15/2)\*b\*\*18\*x\*\*(15/2) + 4725\*a\*\*(13/2)\*b\*\*19\*x\*\*(13/2) + 1890\*a\*\*(11/2)\*b\*\*20\*x\*\*(11/2) + 315\*a\*\*(9/2)\*b\*\*21\*x\*\*(9/2)) + 396\*a\*\*(25/2)\*b\*\*(31/2)\*d\*\*3\*x\*\*8\*sqrt(a\*x/b + 1)/(315\*a\*\*(21/2)\*b\*\*15\*x\*\*(21/2) + 1890\*a\*\*(19/2)\*b\*\*16\*x\*\*(19/2) + 4725\*a\*\*(17/2)\*b\*\*17\*x\*\*(17/2) + 6300\*a\*\*(15/2)\*b\*\*18\*x\*\*(15/2) + 4725\*a\*\*(13/2)\*b\*\*19\*x\*\*(13/2) + 1890\*a\*\*(11/2)\*b\*\*20\*x\*\*(11/2) + 315\*a\*\*(9/2)\*b\*\*21\*x\*\*(9/2)) + 462\*a\*\*(23/2)\*b\*\*(33/2)\*d\*\*3\*x\*\*7\*sqrt(a\*x/b + 1)/(315\*a\*\*(21/2)\*b\*\*15\*x\*\*(21/2) + 1890\*a\*\*(19/2)\*b\*\*16\*x\*\*(19/2) + 4725\*a\*\*(17/2)\*b\*\*17\*x\*\*(17/2) + 6300\*a\*\*(15/2)\*b\*\*18\*x\*\*(15/2) + 4725\*a\*\*(13/2)\*b\*\*19\*x\*\*(13/2) + 1890\*a\*\*(11/2)\*b\*\*20\*x\*\*(11/2) + 315\*a\*\*(9/2)\*b\*\*21\*x\*\*(9/2)) + 210\*a\*\*(21/2)\*b\*\*(35/2)\*d\*\*3\*x\*\*6\*sqrt(a\*x/b + 1)/(315\*a\*\*(21/2)\*b\*\*15\*x\*\*(21/2) + 1890\*a\*\*(19/2)\*b\*\*16\*x\*\*(19/2) + 4725\*a\*\*(17/2)\*b\*\*17\*x\*\*(17/2) + 6300\*a\*\*(15/2)\*b\*\*18\*x\*\*(15/2) + 4725\*a\*\*(13/2)\*b\*\*19\*x\*\*(13/2) + 1890\*a\*\*(11/2)\*b\*\*20\*x\*\*(11/2) + 315\*a\*\*(9/2)\*b\*\*21\*x\*\*(9/2)) - 32\*a\*\*(21/2)\*b\*\*(11/2)\*d\*\*3\*x\*\*6\*sqrt(a\*x/b + 1)/(105\*a\*\*(13/2)\*b\*\*7\*x\*\*(13/2) + 315\*a\*\*(11/2)\*b\*\*8\*x\*\*(11/2) + 315\*a\*\*(9/2)\*b\*\*9\*x\*\*(9/2) + 105\*a\*\*(7/2)\*b\*\*10\*x\*\*(7/2)) - 378\*a\*\*(19/2)\*b\*\*(37/2)\*d\*\*3\*x\*\*5\*sqrt(a\*x/b + 1)/(315\*a\*\*(21/2)\*b\*\*15\*x\*\*(21/2) + 1890\*a\*\*(19/2)\*b\*\*16\*x\*\*(19/2) + 4725\*a\*\*(17/2)\*b\*\*17\*x\*\*(17/2) + 6300\*a\*\*(15/2)\*b\*\*18\*x\*\*(15/2) + 4725\*a\*\*(13/2)\*b\*\*19\*x\*\*(13/2) + 1890\*a\*\*(11/2)\*b\*\*20\*x\*\*(11/2) + 315\*a\*\*(9/2)\*b\*\*21\*x\*\*(9/2)) - 48\*a\*\*(19/2)\*b\*\*(13/2)\*c\*d\*\*2\*x\*\*6\*sqrt(a\*x/b + 1)/(105\*a\*\*(13/2)\*b\*\*7\*x\*\*(13/2) + 315\*a\*\*(11/2)\*b\*\*8\*x\*\*(11/2) + 315\*a\*\*(9/2)\*b\*\*9\*x\*\*(9/2) + 105\*a\*\*(7/2)\*b\*\*10\*x\*\*(7/2)) - 80\*a\*\*(19/2)\*b\*\*(13/2)\*d\*\*3\*x\*\*5\*sqrt(a\*x/b + 1)/(105\*a\*\*(13/2)\*b\*\*7\*x\*\*(13/2) + 315\*a\*\*(11/2)\*b\*\*8\*x\*\*(11/2) + 315\*a\*\*(9/2)\*b\*\*9\*x\*\*(9/2) + 105\*a\*\*(7/2)\*b\*\*10\*x\*\*(7/2)) - 1134\*a\*\*(17/2)\*b\*\*(39/2)\*d\*\*3\*x\*\*4\*sqrt(a\*x/b + 1)/(315\*a\*\*(21/2)\*b\*\*15\*x\*\*(21/2) + 1890\*a\*\*(19/2)\*b\*\*16\*x\*\*(19/2) + 4725\*a\*\*(17/2)\*b\*\*17\*x\*\*(17/2) + 6300\*a\*\*(15/2)\*b\*\*18\*x\*\*(15/2) + 4725\*a\*\*(13/2)\*b\*\*19\*x\*\*(13/2) + 1890\*a\*\*(11/2)\*b\*\*20\*x\*\*(11/2) + 315\*a\*\*(9/2)\*b\*\*21\*x\*\*(9/2)) - 120\*a\*\*(17/2)\*b\*\*(15/2)\*c\*d\*\*2\*x\*\*5\*sqrt(a\*x/b + 1)/(105\*a\*\*(13/2)\*b\*\*7\*x\*\*(13/2) + 315\*a\*\*(11/2)\*b\*\*8\*x\*\*(11/2) + 315\*a\*\*(9/2)\*b\*\*9\*x\*\*(9/2) + 105\*a\*\*(7/2)\*b\*\*10\*x\*\*(7/2)) - 60\*a\*\*(17/2)\*b\*\*(15/2)\*d\*\*3\*x\*\*4\*sqrt(a\*x/b + 1)/(105\*a\*\*(13/2)\*b\*\*7\*x\*\*(13/2) + 315\*a\*\*(11/2)\*b\*\*8\*x\*\*(11/2) + 315\*a\*\*(9/2)\*b\*\*9\*x\*\*(9/2) + 105\*a\*\*(7/2)\*b\*\*10\*x\*\*(7/2)) + 105\*a\*\*(11/2)\*b\*\*8\*x\*\*(11/2) + 315\*a\*\*(9/2)\*b\*\*9\*x\*\*(9/2) + 105\*a\*\*(7/2)\*b\*\*10\*x\*\*(7/2) + 105\*a\*\*(5/2)\*b\*\*8\*x\*\*(5/2) + 105\*a\*\*(3/2)\*b\*\*8\*x\*\*(3/2) + 105\*a\*\*(1/2)\*b\*\*8\*x\*\*(1/2)



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- 640*a**12*b**16*d**3*x**(15/2)/(315*a**(21/2)*b**15*x**(21/2)
+ 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2)
+ 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2)
) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2))
- 480*a**11*b**17*d**3*x**(13/2)/(315*a**(21/2)*b**15*x**(21/2)
+ 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2)
+ 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2)
) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2))
+ 32*a**11*b**5*d**3*x**(13/2)/(105*a**(13/2)*b**7*x**(13/2) + 3
15*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a*
*(7/2)*b**10*x**(7/2)) - 192*a**10*b**18*d**3*x**(11/2)/(315*a**(
21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**
(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a*
*(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a*
*(9/2)*b**21*x**(9/2)) + 48*a**10*b**6*c*d**2*x**(13/2)/(105*a**
(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)
)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 96*a**10*b**6*d*
**3*x**(11/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x
**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/
2)) - 32*a**9*b**19*d**3*x**(9/2)/(315*a**(21/2)*b**15*x**(21/2)
+ 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2)
+ 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2)
) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2))
+ 144*a**9*b**7*c*d**2*x**(11/2)/(105*a**(13/2)*b**7*x**(13/2) +
315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*
a**(7/2)*b**10*x**(7/2)) + 96*a**9*b**7*d**3*x**(9/2)/(105*a**
(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*
b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 144*a**8*b**8*c*d*
**2*x**(9/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x*
*(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)
)) + 32*a**8*b**8*d**3*x**(7/2)/(105*a**(13/2)*b**7*x**(13/2) + 3
15*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a*
*(7/2)*b**10*x**(7/2)) - 4*a**8*b*d**3*x**(7/2)/(15*a**(7/2)*b**3
*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + 48*a**7*b**9*c*d**2*x**
(7/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2)
+ 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 24
*a**7*b**2*c*d**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/
2)*b**4*x**(5/2)) - 4*a**7*b**2*d**3*x**(5/2)/(15*a**(7/2)*b**3*x
**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**6*b**3*c**2*d*x**(7/
2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 24*a
**6*b**3*c*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)
)*b**4*x**(5/2)) - 12*a**5*b**4*c**2*d*x**(5/2)/(15*a**(7/2)*b**3*
x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6*a**3*c**2*d*sqrt(x)/(sq
rt(b)*sqrt(a*x/b + 1)) + a**2*sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1)
) - 4*a**2*sqrt(b)*c**3*sqrt(x)/sqrt(a*x/b + 1) - 6*a**2*sqrt(b)*
c**2*d/(sqrt(x)*sqrt(a*x/b + 1)) + 3*a**2*c*d**2*Piecewise((-sqrt
(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) - 4*a*b**(3/
2)*c**3/(sqrt(x)*sqrt(a*x/b + 1)) + 6*a*b*c**2*d*Piecewise((-sqrt
(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c**3*
Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), Tru
e))

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**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(5/2)\*(c + d/x)^3,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.140 \quad \int \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2 dx$$

**Optimal.** Leaf size=152

$$a^{3/2}c(4ad + 5bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) + \frac{c^2x\left(a + \frac{b}{x}\right)^{7/2}}{a} - \frac{c\left(a + \frac{b}{x}\right)^{5/2}(4ad + 5bc)}{5a}$$

$$- \frac{1}{3}c\left(a + \frac{b}{x}\right)^{3/2}(4ad + 5bc) - ac\sqrt{a + \frac{b}{x}}(4ad + 5bc) - \frac{2d^2\left(a + \frac{b}{x}\right)^{7/2}}{7b}$$

[Out]  $-(a*c*(5*b*c + 4*a*d)*\text{Sqrt}[a + b/x]) - (c*(5*b*c + 4*a*d)*(a + b/x)^{(3/2)})/3 - (c*(5*b*c + 4*a*d)*(a + b/x)^{(5/2)})/(5*a) - (2*d^2*(a + b/x)^{(7/2)})/(7*b) + (c^2*x*(a + b/x)^{(7/2)})/a + a^{(3/2)*c*(5*b*c + 4*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]]$

**Rubi [A]** time = 0.299007, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$a^{3/2}c(4ad + 5bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) + \frac{c^2x\left(a + \frac{b}{x}\right)^{7/2}}{a} - \frac{c\left(a + \frac{b}{x}\right)^{5/2}(4ad + 5bc)}{5a}$$

$$- \frac{1}{3}c\left(a + \frac{b}{x}\right)^{3/2}(4ad + 5bc) - ac\sqrt{a + \frac{b}{x}}(4ad + 5bc) - \frac{2d^2\left(a + \frac{b}{x}\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(5/2)\*(c + d/x)^2, x]

[Out]  $-(a*c*(5*b*c + 4*a*d)*\text{Sqrt}[a + b/x]) - (c*(5*b*c + 4*a*d)*(a + b/x)^{(3/2)})/3 - (c*(5*b*c + 4*a*d)*(a + b/x)^{(5/2)})/(5*a) - (2*d^2*(a + b/x)^{(7/2)})/(7*b) + (c^2*x*(a + b/x)^{(7/2)})/a + a^{(3/2)*c*(5*b*c + 4*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]]$

**Rubi in Sympy [A]** time = 26.7544, size = 131, normalized size = 0.86

$$a^{3/2}c(4ad + 5bc) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) - ac\sqrt{a + \frac{b}{x}}(4ad + 5bc) - \frac{c\left(a + \frac{b}{x}\right)^{3/2}(4ad + 5bc)}{3}$$

$$- \frac{2d^2\left(a + \frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2x\left(a + \frac{b}{x}\right)^{7/2}}{a} - \frac{c\left(a + \frac{b}{x}\right)^{5/2}(4ad + 5bc)}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b/x)\*\*(5/2)\*(c+d/x)\*\*2, x)

[Out]  $a^{(3/2)*c*(4*a*d + 5*b*c)*\operatorname{atanh}(\text{sqrt}(a + b/x)/\text{sqrt}(a)) - a*c*\text{sqrt}(a + b/x)*(4*a*d + 5*b*c) - c*(a + b/x)^{(3/2)*(4*a*d + 5*b*c)}/3 - 2*d^2*(a + b/x)^{(7/2)}/(7*b) + c^2*x*(a + b/x)^{(7/2)}/a - c*(a + b/x)^{(5/2)*(4*a*d + 5*b*c)}/(5*a)$

**Mathematica [A]** time = 0.249973, size = 156, normalized size = 1.03

$$\frac{\frac{1}{2}a^{3/2}c(4ad + 5bc) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right) + \sqrt{a + \frac{b}{x}}(-30a^3d^2x^3 + a^2bx^2(105c^2x^2 - 644cdx - 90d^2) - 2ab^2x(245c^2x^2 + 154cdx + 45d^2) - 2b^3(35c^2x^2 + 42cdx + 15d^2))}{105bx^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(5/2)\*(c + d/x)^2,x]

[Out] (Sqrt[a + b/x]\*(-30\*a^3\*d^2\*x^3 - 2\*b^3\*(15\*d^2 + 42\*c\*d\*x + 35\*c^2\*x^2) + a^2\*b\*x^2\*(-90\*d^2 - 644\*c\*d\*x + 105\*c^2\*x^2) - 2\*a\*b^2\*x\*(45\*d^2 + 154\*c\*d\*x + 245\*c^2\*x^2)))/(105\*b\*x^3) + (a^(3/2)\*c\*(5\*b\*c + 4\*a\*d)\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]\*x])/2

**Maple [B]** time = 0.02, size = 318, normalized size = 2.1

$$\frac{1}{210bx^4}\sqrt{\frac{ax+b}{x}}\left(420a^{5/2}c\ln\left(\frac{1}{2}\frac{2\sqrt{ax^2+bx}\sqrt{a}+2ax+b}{\sqrt{a}}\right)dbx^5+525a^{3/2}c^2b^2\ln\left(\frac{1}{2}\frac{2\sqrt{ax^2+bx}\sqrt{a}+2ax+b}{\sqrt{a}}\right)x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(5/2)\*(c+d/x)^2,x)

[Out] 1/210\*((a\*x+b)/x)^(1/2)/x^4/b\*(420\*a^(5/2)\*c\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*d\*b\*x^5+525\*a^(3/2)\*c^2\*b^2\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*x^5+840\*a^3\*c\*(a\*x^2+b\*x)^(1/2)\*d\*x^5+1050\*a^2\*c^2\*(a\*x^2+b\*x)^(1/2)\*b\*x^5-840\*a^2\*c\*(a\*x^2+b\*x)^(3/2)\*d\*x^3-840\*a\*c^2\*(a\*x^2+b\*x)^(3/2)\*b\*x^3-60\*(a\*x^2+b\*x)^(3/2)\*x^2\*a^2\*d^2-448\*(a\*x^2+b\*x)^(3/2)\*x^2\*a\*b\*c\*d-140\*(a\*x^2+b\*x)^(3/2)\*x^2\*b^2\*c^2-120\*(a\*x^2+b\*x)^(3/2)\*x\*a\*b\*d^2-168\*(a\*x^2+b\*x)^(3/2)\*x\*b^2\*c\*d-60\*(a\*x^2+b\*x)^(3/2)\*b^2\*d^2)/(x\*(a\*x+b))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(5/2)\*(c + d/x)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.252331, size = 1, normalized size = 0.01

$$\frac{105(5ab^2c^2 + 4a^2bcd)\sqrt{ax^3}\log\left(2ax + 2\sqrt{ax}\sqrt{\frac{ax+b}{x}} + b\right) + 2(105a^2bc^2x^4 - 30b^3d^2 - 2(245ab^2c^2 + 322a^2bcd + 15a^3d^2))}{210bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(5/2)\*(c + d/x)^2,x, algorithm="fricas")

```
[Out] [1/210*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(a)*x^3*log(2*a*x + 2
*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(105*a^2*b*c^2*x^4 - 30*b^3
*d^2 - 2*(245*a*b^2*c^2 + 322*a^2*b*c*d + 15*a^3*d^2)*x^3 - 2*(35
*b^3*c^2 + 154*a*b^2*c*d + 45*a^2*b*d^2)*x^2 - 6*(14*b^3*c*d + 15
*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b*x^3), 1/105*(105*(5*a*b^2*c^
2 + 4*a^2*b*c*d)*sqrt(-a)*x^3*arctan(sqrt((a*x + b)/x)/sqrt(-a))
+ (105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(245*a*b^2*c^2 + 322*a^2*b*
c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*c*d + 45*a^2*b*
d^2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b
*x^3)]
```

---

**Sympy [A]** time = 48.817, size = 1884, normalized size = 12.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*(c+d/x)**2,x)
```

```
[Out] -16*a**(19/2)*b**(13/2)*d**2*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*
b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9
*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(15/2)
*d**2*x**5*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a*
*(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)
)*b**10*x**(7/2)) - 30*a**(15/2)*b**(17/2)*d**2*x**4*sqrt(a*x/b +
1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2)
+ 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*
a**(13/2)*b**(19/2)*d**2*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7
*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**
(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 8*a**(13/2)*b**(5/2)*d**2*
x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**
4*x**(5/2)) - 100*a**(11/2)*b**(21/2)*d**2*x**2*sqrt(a*x/b + 1)/(
105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315
*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 8*a**(11
/2)*b**(7/2)*c*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2)
+ 15*a**(5/2)*b**4*x**(5/2)) + 4*a**(11/2)*b**(7/2)*d**2*x**2*sqr
t(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/
2)) - 96*a**(9/2)*b**(23/2)*d**2*x*sqrt(a*x/b + 1)/(105*a**(13/2)
*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**
9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 4*a**(9/2)*b**(9/2)*c
*d*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) - 16*a**(9/2)*b**(9/2)*d**2*x*sqrt(a*x/b + 1)/(15*
a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 30*a**(7/2)
*b**(25/2)*d**2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 3
15*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a*
*(7/2)*b**10*x**(7/2)) - 16*a**(7/2)*b**(11/2)*c*d*x*sqrt(a*x/b +
1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*
a**(7/2)*b**(11/2)*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2)
) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**(5/2)*b**(13/2)*c*d*sqrt(a
*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2))
+ 4*a**(5/2)*c*d*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 5*a**(3/2)*b*c
**2*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 16*a**10*b**6*d**2*x**(13/2)
/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 3
15*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**
9*b**7*d**2*x**(11/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/
2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**
10*x**(7/2)) + 48*a**8*b**8*d**2*x**(9/2)/(105*a**(13/2)*b**7*x**
(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2)
) + 105*a**(7/2)*b**10*x**(7/2)) + 16*a**7*b**9*d**2*x**(7/2)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a
**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 8*a**7*b**
2*d**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**
(5/2)) - 8*a**6*b**3*c*d*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15
*a**(5/2)*b**4*x**(5/2)) - 8*a**6*b**3*d**2*x**(5/2)/(15*a**(7/2)
*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**5*b**4*c*d*x**
(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4
*a**3*c*d*sqrt(x)/(sqrt(b)*sqrt(a*x/b + 1)) + a**2*sqrt(b)*c**2*s
qrt(x)*sqrt(a*x/b + 1) - 4*a**2*sqrt(b)*c**2*sqrt(x)/sqrt(a*x/b +
1) - 4*a**2*sqrt(b)*c*d/(sqrt(x)*sqrt(a*x/b + 1)) + a**2*d**2*Pi
ecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)
```

```
) - 4*a*b**(3/2)*c**2/(sqrt(x)*sqrt(a*x/b + 1)) + 4*a*b*c*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a + b/x)^(5/2)*(c + d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```



$$3.141 \quad \int \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right) dx$$

**Optimal.** Leaf size=125

$$a^{3/2}(2ad + 5bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) - \frac{\left(a + \frac{b}{x}\right)^{5/2} (2ad + 5bc)}{5a}$$

$$- \frac{1}{3} \left(a + \frac{b}{x}\right)^{3/2} (2ad + 5bc) - a\sqrt{a + \frac{b}{x}}(2ad + 5bc) + \frac{cx \left(a + \frac{b}{x}\right)^{7/2}}{a}$$

[Out] -(a\*(5\*b\*c + 2\*a\*d)\*Sqrt[a + b/x]) - ((5\*b\*c + 2\*a\*d)\*(a + b/x)^(3/2))/3 - ((5\*b\*c + 2\*a\*d)\*(a + b/x)^(5/2))/(5\*a) + (c\*(a + b/x)^(7/2)\*x)/a + a^(3/2)\*(5\*b\*c + 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]

**Rubi [A]** time = 0.208579, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$a^{3/2}(2ad + 5bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) - \frac{\left(a + \frac{b}{x}\right)^{5/2} (2ad + 5bc)}{5a}$$

$$- \frac{1}{3} \left(a + \frac{b}{x}\right)^{3/2} (2ad + 5bc) - a\sqrt{a + \frac{b}{x}}(2ad + 5bc) + \frac{cx \left(a + \frac{b}{x}\right)^{7/2}}{a}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(5/2)\*(c + d/x), x]

[Out] -(a\*(5\*b\*c + 2\*a\*d)\*Sqrt[a + b/x]) - ((5\*b\*c + 2\*a\*d)\*(a + b/x)^(3/2))/3 - ((5\*b\*c + 2\*a\*d)\*(a + b/x)^(5/2))/(5\*a) + (c\*(a + b/x)^(7/2)\*x)/a + a^(3/2)\*(5\*b\*c + 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]

**Rubi in Sympy [A]** time = 17.6048, size = 110, normalized size = 0.88

$$2a^{\frac{3}{2}} \left(ad + \frac{5bc}{2}\right) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) - a\sqrt{a + \frac{b}{x}}(2ad + 5bc)$$

$$- \left(a + \frac{b}{x}\right)^{\frac{3}{2}} \left(\frac{2ad}{3} + \frac{5bc}{3}\right) + \frac{cx \left(a + \frac{b}{x}\right)^{\frac{7}{2}}}{a} - \frac{2 \left(a + \frac{b}{x}\right)^{\frac{5}{2}} \left(ad + \frac{5bc}{2}\right)}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b/x)\*\*(5/2)\*(c+d/x), x)

[Out] 2\*a\*\*(3/2)\*(a\*d + 5\*b\*c/2)\*atanh(sqrt(a + b/x)/sqrt(a)) - a\*sqrt(a + b/x)\*(2\*a\*d + 5\*b\*c) - (a + b/x)\*\*(3/2)\*(2\*a\*d/3 + 5\*b\*c/3) + c\*x\*(a + b/x)\*\*(7/2)/a - 2\*(a + b/x)\*\*(5/2)\*(a\*d + 5\*b\*c/2)/(5\*a)

**Mathematica [A]** time = 0.252027, size = 105, normalized size = 0.84

$$\frac{1}{2}a^{3/2}(2ad + 5bc)\log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right) + \frac{\sqrt{a + \frac{b}{x}}(a^2x^2(15cx - 46d) - 2abx(35cx + 11d) - 2b^2(5cx + 3d))}{15x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(5/2)\*(c + d/x), x]

[Out] (Sqrt[a + b/x]\*(-2\*b^2\*(3\*d + 5\*c\*x) + a^2\*x^2\*(-46\*d + 15\*c\*x) - 2\*a\*b\*x\*(11\*d + 35\*c\*x))/(15\*x^2) + (a^(3/2)\*(5\*b\*c + 2\*a\*d)\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]\*x])/2

**Maple [B]** time = 0.016, size = 240, normalized size = 1.9

$$\frac{1}{30bx^3}\sqrt{\frac{ax+b}{x}}\left(30a^{5/2}\ln\left(\frac{1}{2}\frac{2\sqrt{ax^2+bx}\sqrt{a}+2ax+b}{\sqrt{a}}\right)dbx^4+75a^{3/2}\ln\left(\frac{1}{2}\frac{2\sqrt{ax^2+bx}\sqrt{a}+2ax+b}{\sqrt{a}}\right)b^2cx^4+60\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(5/2)\*(c+d/x), x)

[Out] 1/30\*((a\*x+b)/x)^(1/2)/x^3/b\*(30\*a^(5/2)\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*d\*b\*x^4+75\*a^(3/2)\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*b^2\*c\*x^4+60\*a^3\*(a\*x^2+b\*x)^(1/2)\*d\*x^4+150\*a^2\*(a\*x^2+b\*x)^(1/2)\*c\*b\*x^4-60\*(a\*x^2+b\*x)^(3/2)\*a^2\*d\*x^2-120\*a\*(a\*x^2+b\*x)^(3/2)\*c\*b\*x^2-32\*(a\*x^2+b\*x)^(3/2)\*x\*a\*b\*d-20\*(a\*x^2+b\*x)^(3/2)\*x\*b^2\*c-12\*(a\*x^2+b\*x)^(3/2)\*b^2\*d)/(x\*(a\*x+b))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(5/2)\*(c + d/x), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.262862, size = 1, normalized size = 0.01

$$\left[ \frac{15(5abc + 2a^2d)\sqrt{ax^2}\log\left(2ax + 2\sqrt{ax}\sqrt{\frac{ax+b}{x}} + b\right) + 2(15a^2cx^3 - 6b^2d - 2(35abc + 23a^2d)x^2 - 2(5b^2c + 11abd)x)}{30x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(5/2)\*(c + d/x), x, algorithm="fricas")

[Out] [1/30\*(15\*(5\*a\*b\*c + 2\*a^2\*d)\*sqrt(a)\*x^2\*log(2\*a\*x + 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + 2\*(15\*a^2\*c\*x^3 - 6\*b^2\*d - 2\*(35\*a\*b\*c

$$+ 23*a^2*d)*x^2 - 2*(5*b^2*c + 11*a*b*d)*x)*\sqrt{(a*x + b)/x)}/x^2, 1/15*(15*(5*a*b*c + 2*a^2*d)*\sqrt{-a}*x^2*\arctan(\sqrt{(a*x + b)/x)/\sqrt{-a})) + (15*a^2*c*x^3 - 6*b^2*d - 2*(35*a*b*c + 23*a^2*d)*x^2 - 2*(5*b^2*c + 11*a*b*d)*x)*\sqrt{(a*x + b)/x)}/x^2]$$

**Sympy [A]** time = 47.6987, size = 561, normalized size = 4.49

$$\begin{aligned} & \frac{4a^{\frac{11}{2}}b^{\frac{7}{2}}dx^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + \frac{2a^{\frac{9}{2}}b^{\frac{9}{2}}dx^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{8a^{\frac{7}{2}}b^{\frac{11}{2}}dx\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} \\ & - \frac{6a^{\frac{5}{2}}b^{\frac{13}{2}}d\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + 2a^{\frac{5}{2}}d\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) + 5a^{\frac{3}{2}}bc\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) - \frac{4a^6b^3dx^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} \\ & - \frac{4a^5b^4dx^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{2a^3d\sqrt{x}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}} + a^2\sqrt{bc}\sqrt{x}\sqrt{\frac{ax}{b}+1} - \frac{4a^2\sqrt{bc}\sqrt{x}}{\sqrt{\frac{ax}{b}+1}} - \frac{2a^2\sqrt{bd}}{\sqrt{x}\sqrt{\frac{ax}{b}+1}} \\ & - \frac{4ab^{\frac{3}{2}}c}{\sqrt{x}\sqrt{\frac{ax}{b}+1}} + 2abd\left(\begin{cases} -\frac{\sqrt{a}}{x} & \text{for } b = 0 \\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}\right) + b^2c\left(\begin{cases} -\frac{\sqrt{a}}{x} & \text{for } b = 0 \\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(5/2)\*(c+d/x),x)

[Out]  $4*a^{11/2}*b^{7/2}*d*x^3*\sqrt{a*x/b + 1}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) + 2*a^{9/2}*b^{9/2}*d*x^2*\sqrt{a*x/b + 1}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) - 8*a^{7/2}*b^{11/2}*d*x*\sqrt{a*x/b + 1}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) - 6*a^{5/2}*b^{13/2}*d*\sqrt{a*x/b + 1}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) + 2*a^{5/2}*d*\operatorname{asinh}(\sqrt{a}*\sqrt{x}/\sqrt{b}) + 5*a^{3/2}*b*c*\operatorname{asinh}(\sqrt{a}*\sqrt{x}/\sqrt{b}) - 4*a^6*b^3*d*x^{7/2}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) - 4*a^5*b^4*d*x^{5/2}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) - 2*a^3*d*\sqrt{x}/(\sqrt{b}*\sqrt{a*x/b + 1}) + a^2*\sqrt{bc}*\sqrt{x}\sqrt{a*x/b + 1} - 4*a^2*\sqrt{bc}\sqrt{x}/\sqrt{a*x/b + 1} - 2*a^2*\sqrt{bd}/(\sqrt{x}\sqrt{a*x/b + 1}) - 4*a*b^{3/2}*c/(\sqrt{x}\sqrt{a*x/b + 1}) + 2*a*b*d*\operatorname{Piecewise}((-sqrt(a)/x, \operatorname{Eq}(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), \operatorname{True})) + b^2*c*\operatorname{Piecewise}((-sqrt(a)/x, \operatorname{Eq}(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), \operatorname{True}))$

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(5/2)\*(c + d/x),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.142 \quad \int \left(a + \frac{b}{x}\right)^{5/2} dx$$

**Optimal.** Leaf size=71

$$5a^{3/2}b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) + x\left(a + \frac{b}{x}\right)^{5/2} - \frac{5}{3}b\left(a + \frac{b}{x}\right)^{3/2} - 5ab\sqrt{a + \frac{b}{x}}$$

[Out]  $-5*a*b*\text{Sqrt}[a + b/x] - (5*b*(a + b/x)^{(3/2)})/3 + (a + b/x)^{(5/2)}*x + 5*a^{(3/2)}*b*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]]$

**Rubi [A]** time = 0.101272, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$

$$5a^{3/2}b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) + x\left(a + \frac{b}{x}\right)^{5/2} - \frac{5}{3}b\left(a + \frac{b}{x}\right)^{3/2} - 5ab\sqrt{a + \frac{b}{x}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b/x)^{(5/2)}, x]$

[Out]  $-5*a*b*\text{Sqrt}[a + b/x] - (5*b*(a + b/x)^{(3/2)})/3 + (a + b/x)^{(5/2)}*x + 5*a^{(3/2)}*b*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]]$

**Rubi in Sympy [A]** time = 9.5882, size = 60, normalized size = 0.85

$$5a^{3/2}b \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) - 5ab\sqrt{a + \frac{b}{x}} - \frac{5b\left(a + \frac{b}{x}\right)^{3/2}}{3} + x\left(a + \frac{b}{x}\right)^{5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a+b/x)**(5/2), x)$

[Out]  $5*a^{(3/2)}*b*\operatorname{atanh}(\text{sqrt}(a + b/x)/\text{sqrt}(a)) - 5*a*b*\text{sqrt}(a + b/x) - 5*b*(a + b/x)^{(3/2)}/3 + x*(a + b/x)^{(5/2)}$

**Mathematica [A]** time = 0.102443, size = 71, normalized size = 1.

$$\frac{5}{2}a^{3/2}b \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right) + \sqrt{a + \frac{b}{x}}\left(a^2x - \frac{14ab}{3} - \frac{2b^2}{3x}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b/x)^{(5/2)}, x]$

[Out]  $\text{Sqrt}[a + b/x]*((-14*a*b)/3 - (2*b^2)/(3*x) + a^2*x) + (5*a^{(3/2)}*b*\text{Log}[b + 2*a*x + 2*\text{Sqrt}[a]*\text{Sqrt}[a + b/x]*x])/2$

**Maple [A]** time = 0.005, size = 112, normalized size = 1.6

$$\frac{1}{6x^2} \sqrt{\frac{ax+b}{x}} \left( 15a^{3/2} \ln \left( \frac{1}{2} \frac{2\sqrt{ax^2+bx}\sqrt{a} + 2ax + b}{\sqrt{a}} \right) x^3b + 30a^2\sqrt{ax^2+bx}x^3 - 24a(ax^2+bx)^{3/2}x - 4(ax^2+bx)^{3/2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(5/2), x)

[Out] 1/6\*((a\*x+b)/x)^(1/2)/x^2\*(15\*a^(3/2)\*ln(1/2\*(2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*x^3\*b+30\*a^2\*(a\*x^2+b\*x)^(1/2)\*x^3-24\*a\*(a\*x^2+b\*x)^(3/2)\*x-4\*(a\*x^2+b\*x)^(3/2)/(x\*(a\*x+b))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(5/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.256117, size = 1, normalized size = 0.01

$$\left[ \frac{15a^{\frac{3}{2}}bx \log\left(2ax + 2\sqrt{ax}\sqrt{\frac{ax+b}{x}} + b\right) + 2(3a^2x^2 - 14abx - 2b^2)\sqrt{\frac{ax+b}{x}}}{6x}, \frac{15\sqrt{-a}bx \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right) + (3a^2x^2 - 14abx)}{3x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(5/2), x, algorithm="fricas")

[Out] [1/6\*(15\*a^(3/2)\*b\*x\*log(2\*a\*x + 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + 2\*(3\*a^2\*x^2 - 14\*a\*b\*x - 2\*b^2)\*sqrt((a\*x + b)/x))/x, 1/3\*(15\*sqrt(-a)\*a\*b\*x\*arctan(sqrt((a\*x + b)/x)/sqrt(-a)) + (3\*a^2\*x^2 - 14\*a\*b\*x - 2\*b^2)\*sqrt((a\*x + b)/x))/x]

**Sympy [A]** time = 13.3606, size = 99, normalized size = 1.39

$$a^{\frac{5}{2}}x\sqrt{1+\frac{b}{ax}} - \frac{14a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax}}}{3} - \frac{5a^{\frac{3}{2}}b\log\left(\frac{b}{ax}\right)}{2} + 5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{b}{ax}}+1\right) - \frac{2\sqrt{ab^2}\sqrt{1+\frac{b}{ax}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(5/2), x)

[Out] a\*\*(5/2)\*x\*sqrt(1 + b/(a\*x)) - 14\*a\*\*(3/2)\*b\*sqrt(1 + b/(a\*x))/3 - 5\*a\*\*(3/2)\*b\*log(b/(a\*x))/2 + 5\*a\*\*(3/2)\*b\*log(sqrt(1 + b/(a\*x)) + 1) - 2\*sqrt(a)\*b\*\*2\*sqrt(1 + b/(a\*x))/(3\*x)

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a + b/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.143 \quad \int \frac{\left(a + \frac{b}{x}\right)^{5/2}}{c + \frac{d}{x}} dx$$

**Optimal.** Leaf size=134

$$\frac{a^{3/2}(5bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^2} + \frac{2(bc - ad)^{5/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^2 d^{3/2}} - \frac{b\sqrt{a + \frac{b}{x}}(ad + 2bc)}{cd} + \frac{ax\left(a + \frac{b}{x}\right)^{3/2}}{c}$$

[Out]  $-\left(\frac{b(2bc + ad)\sqrt{a + b/x}}{c^2 d}\right) + \frac{a(a + b/x)^{3/2}x}{c} + \frac{(2(bc - ad)^{5/2} \operatorname{ArcTan}[\frac{\sqrt{d}\sqrt{a + b/x}}{\sqrt{bc - ad}}])}{c^2 d^{3/2}} + \frac{a^{3/2}(5bc - 2ad) \operatorname{ArcTanh}[\frac{\sqrt{a + b/x}}{\sqrt{a}}]}{c^2}$

**Rubi [A]** time = 0.639039, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{a^{3/2}(5bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^2} + \frac{2(bc - ad)^{5/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^2 d^{3/2}} - \frac{b\sqrt{a + \frac{b}{x}}(ad + 2bc)}{cd} + \frac{ax\left(a + \frac{b}{x}\right)^{3/2}}{c}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\left(a + \frac{b}{x}\right)^{5/2} / \left(c + \frac{d}{x}\right), x\right]$

[Out]  $-\left(\frac{b(2bc + ad)\sqrt{a + b/x}}{c^2 d}\right) + \frac{a(a + b/x)^{3/2}x}{c} + \frac{(2(bc - ad)^{5/2} \operatorname{ArcTan}[\frac{\sqrt{d}\sqrt{a + b/x}}{\sqrt{bc - ad}}])}{c^2 d^{3/2}} + \frac{a^{3/2}(5bc - 2ad) \operatorname{ArcTanh}[\frac{\sqrt{a + b/x}}{\sqrt{a}}]}{c^2}$

**Rubi in Sympy [A]** time = 62.0592, size = 114, normalized size = 0.85

$$-\frac{a^{3/2}(2ad - 5bc) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^2} + \frac{ax\left(a + \frac{b}{x}\right)^{3/2}}{c} - \frac{b\sqrt{a + \frac{b}{x}}(ad + 2bc)}{cd} + \frac{2(ad - bc)^{5/2} \operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{ad - bc}}\right)}{c^2 d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}\left(\left(a + \frac{b}{x}\right)^{5/2} / \left(c + \frac{d}{x}\right), x\right)$

[Out]  $-a^{3/2}(2ad - 5bc) \operatorname{atanh}\left(\frac{\sqrt{a + b/x}}{\sqrt{a}}\right) / c^2 + a x \left(a + \frac{b}{x}\right)^{3/2} / c - b \sqrt{a + \frac{b}{x}} (ad + 2bc) / (c d) + 2 (ad - bc)^{5/2} \operatorname{atanh}\left(\frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{ad - bc}}\right) / (c^2 d^{3/2})$

**Mathematica [A]** time = 0.228124, size = 172, normalized size = 1.28

$$-\frac{a^{3/2}(2ad - 5bc) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right)}{2c^2} + \sqrt{a + \frac{b}{x}} \left(\frac{a^2 x}{c} - \frac{2b^2}{d}\right) + \frac{(ad - bc)^{5/2} \log(cx + d)}{c^2 d^{3/2}} - \frac{(ad - bc)^{5/2} \log\left(2\sqrt{dx}\sqrt{a + \frac{b}{x}}\sqrt{ad - bc} - 2adx + bcx - bd\right)}{c^2 d^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(5/2)/(c + d/x), x]

[Out] Sqrt[a + b/x]\*((-2\*b^2)/d + (a^2\*x)/c) + ((-(b\*c) + a\*d)^(5/2)\*Log[d + c\*x])/(c^2\*d^(3/2)) - (a^(3/2)\*(-5\*b\*c + 2\*a\*d)\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]\*x])/(2\*c^2) - ((-(b\*c) + a\*d)^(5/2)\*Log[-(b\*d) + b\*c\*x - 2\*a\*d\*x + 2\*Sqrt[d]\*Sqrt[-(b\*c) + a\*d]\*Sqrt[a + b/x]\*x])/(c^2\*d^(3/2))

**Maple [B]** time = 0.023, size = 859, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(5/2)/(c+d/x), x)

[Out]  $\frac{1}{2} \left( \frac{(a^2 x + b)}{x} \right)^{1/2} / x^2 \left( -2 d^3 \ln \left( \frac{1}{2} \left( 2 \left( x \left( a^2 x + b \right) \right)^{1/2} \right) a^{1/2} + 2 a^2 x + b \right) / a^{1/2} \right) a^3 x^2 c \left( \left( a^2 d - b^2 c \right) d / c^2 \right)^{1/2} + 5 \ln \left( \frac{1}{2} \left( 2 \left( x \left( a^2 x + b \right) \right)^{1/2} a^{1/2} + 2 a^2 x + b \right) / a^{1/2} \right) a^2 b^2 x^2 d^2 c^2 \left( \left( a^2 d - b^2 c \right) d / c^2 \right)^{1/2} + 4 b^2 a^2 \ln \left( \frac{1}{2} \left( 2 \left( a^2 x^2 + b^2 x \right)^{1/2} a^{1/2} + 2 a^2 x + b \right) / a^{1/2} \right) x^2 d^2 c^3 \left( \left( a^2 d - b^2 c \right) d / c^2 \right)^{1/2} - 4 \ln \left( \frac{1}{2} \left( 2 \left( x \left( a^2 x + b \right) \right)^{1/2} a^{1/2} + 2 a^2 x + b \right) / a^{1/2} \right) a^2 b^2 x^2 d^2 c^3 \left( \left( a^2 d - b^2 c \right) d / c^2 \right)^{1/2} + 2 \left( x \left( a^2 x + b \right) \right)^{1/2} a^{5/2} x^2 d^2 c^2 \left( \left( a^2 d - b^2 c \right) d / c^2 \right)^{1/2} - 4 \left( x \left( a^2 x + b \right) \right)^{1/2} a^{3/2} b^2 x^2 d^2 c^3 \left( \left( a^2 d - b^2 c \right) d / c^2 \right)^{1/2} + 2 c^4 \left( x \left( a^2 x + b \right) \right)^{1/2} b^2 x^2 a^{1/2} \left( \left( a^2 d - b^2 c \right) d / c^2 \right)^{1/2} - 2 d^4 \ln \left( \frac{1}{2} \left( 2 \left( x \left( a^2 x + b \right) \right)^{1/2} \left( \left( a^2 d - b^2 c \right) d / c^2 \right)^{1/2} \right) c - 2 a^2 d^2 x + b^2 c x - b^2 d \right) / \left( c^2 x + d \right) a^{7/2} x^2 + 6 d^3 \ln \left( \frac{1}{2} \left( 2 \left( x \left( a^2 x + b \right) \right)^{1/2} \left( \left( a^2 d - b^2 c \right) d / c^2 \right)^{1/2} \right) c - 2 a^2 d^2 x + b^2 c x - b^2 d \right) / \left( c^2 x + d \right) a^{5/2} b^2 x^2 c - 6 \ln \left( \frac{1}{2} \left( 2 \left( x \left( a^2 x + b \right) \right)^{1/2} \left( \left( a^2 d - b^2 c \right) d / c^2 \right)^{1/2} \right) c - 2 a^2 d^2 x + b^2 c x - b^2 d \right) / \left( c^2 x + d \right) a^{3/2} b^2 x^2 d^2 c^2 + 2 \ln \left( \frac{1}{2} \left( 2 \left( x \left( a^2 x + b \right) \right)^{1/2} \left( \left( a^2 d - b^2 c \right) d / c^2 \right)^{1/2} \right) c - 2 a^2 d^2 x + b^2 c x - b^2 d \right) / \left( c^2 x + d \right) b^3 x^2 d^2 c^3 a^{1/2} + 8 b^2 a^{3/2} \left( a^2 x^2 + b^2 x \right)^{1/2} x^2 d^2 c^3 \left( \left( a^2 d - b^2 c \right) d / c^2 \right)^{1/2} - 2 b^2 \left( a^2 x^2 + b^2 x \right)^{1/2} c^4 x^2 a^{1/2} \left( \left( a^2 d - b^2 c \right) d / c^2 \right)^{1/2} - b^3 \ln \left( \frac{1}{2} \left( 2 \left( a^2 x^2 + b^2 x \right)^{1/2} a^{1/2} + 2 a^2 x + b \right) / a^{1/2} \right) c^4 x^2 \left( \left( a^2 d - b^2 c \right) d / c^2 \right)^{1/2} + c^4 \ln \left( \frac{1}{2} \left( 2 \left( x \left( a^2 x + b \right) \right)^{1/2} a^{1/2} + 2 a^2 x + b \right) / a^{1/2} \right) b^3 x^2 \left( \left( a^2 d - b^2 c \right) d / c^2 \right)^{1/2} - 4 b^2 \left( a^2 x^2 + b^2 x \right)^{3/2} d^2 c^3 a^{1/2} \left( \left( a^2 d - b^2 c \right) d / c^2 \right)^{1/2} \right) / \left( x \left( a^2 x + b \right) \right)^{1/2} / d^2 / c^3 / a^{1/2} / \left( \left( a^2 d - b^2 c \right) d / c^2 \right)^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(5/2)/(c + d/x), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.435569, size = 1, normalized size = 0.01

$$\frac{\left( 5abcd - 2a^2d^2 \right) \sqrt{a} \log \left( 2ax - 2\sqrt{ax} \sqrt{\frac{ax+b}{x}} + b \right) - 2 \left( b^2c^2 - 2abcd + a^2d^2 \right) \sqrt{-\frac{bc-ad}{d}} \log \left( \frac{2dx \sqrt{-\frac{bc-ad}{d}} \sqrt{\frac{ax+b}{x}} + bd - (bc-2cd)}{cx+d} \right)}{2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(5/2)/(c + d/x), x, algorithm="fricas")



```
[Out] [-1/2*((5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d), ((5*a*b*c*d - 2*a^2*d^2)*sqrt(-a)*arctan(sqrt((a*x + b)/x)/sqrt(-a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) + (a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d), 1/2*(4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/d)*arctan(sqrt((a*x + b)/x)/sqrt((b*c - a*d)/d)) - (5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d), ((5*a*b*c*d - 2*a^2*d^2)*sqrt(-a)*arctan(sqrt((a*x + b)/x)/sqrt(-a)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/d)*arctan(sqrt((a*x + b)/x)/sqrt((b*c - a*d)/d)) + (a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d)]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x \left(a + \frac{b}{x}\right)^{\frac{5}{2}}}{cx + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/(c+d/x), x)
```

```
[Out] Integral(x*(a + b/x)**(5/2)/(c*x + d), x)
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a + b/x)^(5/2)/(c + d/x), x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.144 \quad \int \frac{\left(a + \frac{b}{x}\right)^{5/2}}{\left(c + \frac{d}{x}\right)^2} dx$$

Optimal. Leaf size=166

$$\frac{a^{3/2}(5bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^3} - \frac{(bc - ad)^{3/2}(4ad + bc) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^3 d^{3/2}} + \frac{\sqrt{a + \frac{b}{x}}(bc - 2ad)(bc - ad)}{c^2 d \left(c + \frac{d}{x}\right)} + \frac{ax \left(a + \frac{b}{x}\right)^{3/2}}{c \left(c + \frac{d}{x}\right)}$$

[Out]  $((b*c - 2*a*d)*(b*c - a*d)*\text{Sqrt}[a + b/x])/(c^2*d*(c + d/x)) + (a*(a + b/x)^{(3/2)*x})/(c*(c + d/x)) - ((b*c - a*d)^{(3/2)*(b*c + 4*a*d)}*\text{ArcTan}[(\text{Sqrt}[d]*\text{Sqrt}[a + b/x])/\text{Sqrt}[b*c - a*d]])/(c^3*d^{(3/2)}) + (a^{(3/2)}*(5*b*c - 4*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/c^3$

Rubi [A] time = 0.668661, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{a^{3/2}(5bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^3} - \frac{(bc - ad)^{3/2}(4ad + bc) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^3 d^{3/2}} + \frac{\sqrt{a + \frac{b}{x}}(bc - 2ad)(bc - ad)}{c^2 d \left(c + \frac{d}{x}\right)} + \frac{ax \left(a + \frac{b}{x}\right)^{3/2}}{c \left(c + \frac{d}{x}\right)}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(5/2)/(c + d/x)^2, x]

[Out]  $((b*c - 2*a*d)*(b*c - a*d)*\text{Sqrt}[a + b/x])/(c^2*d*(c + d/x)) + (a*(a + b/x)^{(3/2)*x})/(c*(c + d/x)) - ((b*c - a*d)^{(3/2)*(b*c + 4*a*d)}*\text{ArcTan}[(\text{Sqrt}[d]*\text{Sqrt}[a + b/x])/\text{Sqrt}[b*c - a*d]])/(c^3*d^{(3/2)}) + (a^{(3/2)}*(5*b*c - 4*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/c^3$

Rubi in Sympy [A] time = 63.2027, size = 138, normalized size = 0.83

$$-\frac{a^{3/2}(4ad - 5bc) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^3} + \frac{ax \left(a + \frac{b}{x}\right)^{3/2}}{c \left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}(ad - bc)(2ad - bc)}{c^2 d \left(c + \frac{d}{x}\right)} + \frac{(ad - bc)^{3/2}(4ad + bc) \operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{ad - bc}}\right)}{c^3 d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b/x)\*\*(5/2)/(c+d/x)\*\*2, x)

[Out]  $-a^{(3/2)}*(4*a*d - 5*b*c)*\operatorname{atanh}(\text{sqrt}(a + b/x)/\text{sqrt}(a))/c^{**3} + a*x*(a + b/x)^{(3/2)}/(c*(c + d/x)) + \text{sqrt}(a + b/x)*(a*d - b*c)*(2*a*d - b*c)/(c^{**2}*d*(c + d/x)) + (a*d - b*c)^{(3/2)}*(4*a*d + b*c)*\operatorname{atanh}(\text{sqrt}(d)*\text{sqrt}(a + b/x)/\text{sqrt}(a*d - b*c))/(c^{**3}*d^{(3/2)})$

**Mathematica [C]** time = 0.604074, size = 219, normalized size = 1.32

$$\frac{a^{3/2}(4ad - 5bc) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right) - \frac{2cx\sqrt{a + \frac{b}{x}}(a^2d(cx+2d) - 2abcd + b^2c^2)}{d(cx+d)} + \frac{i(bc-ad)^{3/2}(4ad+bc) \log\left(\frac{2c^4\left(2dx\sqrt{a + \frac{b}{x}}\sqrt{bc-ad} - 2ic\right)}{(cx+d)(bc-ad)^{5/2}}\right)}{d^{3/2}}}{2c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(5/2)/(c + d/x)^2, x]

[Out]  $-\left(-2^*c*\text{Sqrt}[a + b/x]*x*(b^2*c^2 - 2*a*b*c*d + a^2*d*(2*d + c*x))\right)/(d*(d + c*x)) + a^{(3/2)}*(-5*b*c + 4*a*d)*\text{Log}[b + 2*a*x + 2*\text{Sqrt}[a]*\text{Sqrt}[a + b/x]*x] + (I*(b*c - a*d)^{(3/2)}*(b*c + 4*a*d)*\text{Log}[(2*c^4*((-2*I)*a*d^{(3/2)}*x + 2*d*\text{Sqrt}[b*c - a*d]*\text{Sqrt}[a + b/x]*x - I*b*\text{Sqrt}[d]*(d - c*x)))/(b*c - a*d)^{(5/2)}*(b*c + 4*a*d)*(d + c*x)])/d^{(3/2)}/(2*c^3)$

**Maple [B]** time = 0.024, size = 2014, normalized size = 12.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(5/2)/(c+d/x)^2, x)

[Out]  $\frac{1}{2}*(a*x+b)/x)^{(1/2)}*x*(-4*a^{(9/2)}*\ln((2*(x*(a*x+b))^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))^d+2*c^4*(x*(a*x+b))^{(3/2)}*a^{(5/2)}*d^2*((a*d-b*c)*d/c^2)^{(1/2)}-4*a^{(9/2)}*\ln((2*(x*(a*x+b))^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))^x*c*d^5+4*(x*(a*x+b))^{(1/2)}*a^{(7/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c^2*d^4+11*a^{(7/2)}*\ln((2*(x*(a*x+b))^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))^b*c*d^5-9*a^{(5/2)}*\ln((2*(x*(a*x+b))^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))^b^2*c^2*d^4+a^{(3/2)}*\ln((2*(x*(a*x+b))^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))^b^3*c^3*d^3-4*a^4*\ln(1/2*(2*(x*(a*x+b))^{(1/2)}*a^{(1/2)+2*a*x+b}/a^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c^2*d^5+2*c^6*(x*(a*x+b))^{(3/2)}*b^2*a^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}+1*\ln(1/2*(2*(x*(a*x+b))^{(1/2)}*a^{(1/2)+2*a*x+b}/a^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*x*b^4*c^6-\ln(1/2*(2*(a*x^2+b*x)^{(1/2)}*a^{(1/2)+2*a*x+b}/a^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*x*b^4*c^6+a^{(1/2)}*\ln((2*(x*(a*x+b))^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))^b^4*c^4*d^2+\ln(1/2*(2*(x*(a*x+b))^{(1/2)}*a^{(1/2)+2*a*x+b}/a^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*b^4*c^5*d-\ln(1/2*(2*(a*x^2+b*x)^{(1/2)}*a^{(1/2)+2*a*x+b}/a^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*b^4*c^5*d-5*a^2*\ln(1/2*(2*(x*(a*x+b))^{(1/2)}*a^{(1/2)+2*a*x+b}/a^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*d/c^2)^{(1/2)}*x*b^2*c^4*d^2-5*a^2*\ln(1/2*(2*(x*(a*x+b))^{(1/2)}*a^{(1/2)+2*a*x+b}/a^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*b^2*c^3*d^3+a^{(1/2)}*\ln((2*(x*(a*x+b))^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))^x*b^4*c^5*d-2*a^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*(a*x^2+b*x)^{(1/2)}*x*b^3*c^6-2*a^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*(a*x^2+b*x)^{(1/2)}*b^3*c^5*d-\ln(1/2*(2*(x*(a*x+b))^{(1/2)}*a^{(1/2)+2*a*x+b}/a^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*a*b^3*c^4*d^2+\ln(1/2*(a*x^2+b*x)^{(1/2)}*a^{(1/2)+2*a*x+b}/a^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*a*b^3*c^4*d^2+11*a^{(7/2)}*\ln((2*(x*(a*x+b))^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))^x*b*c^2*d^4-9*a^{(5/2)}*\ln((2*(x*(a*x+b))^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))^x*b^2*c^3*d^3-2*(x*(a*x+b))^{(1/2)}*a^{(7/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*x^2*c^4*d^2-2*(x*(a*x+b))^{(1/2)}*a^{(3/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*x^2*b^2*c^6-4*c^5*(x*(a*x+b))^{(3/2)}*a^{(3/2)}*b*d*((a*d-b*c)*d/c^2)^{(1/2)}+2*(x*(a*x+b))^{(1/2)}*a^{(7/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*x^3*c^3*d^3-8*(x*(a*x+b))^{(1/2)}*a^{(5/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*b*c^3*d^3+4*(x*(a*x+b))^{(1/2)}*a^{(3/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*b^2*c^4*d^2+2*a^{(3/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*(a*x^2+b*x)^{(1/2)}*b^2*c^4*d^2-4*a^4*\ln(1/2*(2*(x*(a*x+b))^{(1/2)}*a^{(1/2)+2*a*x+b}/a^{(1/2)}*((a$

```

*d-b*c)*d/c^2)^(1/2)*x*c^2*d^4+9*a^3*ln(1/2*(2*(x*(a*x+b))^(1/2)*
a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*b*c^2*d^4-ln(1/
2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2
)^(1/2)*x*a*b^3*c^5*d+ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x*a*b^3*c^5*d-4*(x*(a*x+b))^(1
/2)*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^4*d^2+4*(x*(a*x+b))^(1/
2)*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*x^2*b*c^5*d+2*a^(3/2)*((a*d-b*
c)*d/c^2)^(1/2)*(a*x^2+b*x)^(1/2)*x*b^2*c^5*d+2*(x*(a*x+b))^(1/2)
*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b^2*c^5*d+9*a^3*ln(1/2*(2*(x*(
a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x
*b*c^3*d^3+a^(3/2)*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2
))*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b^3*c^4*d^2)/(x*(a*x+b))^(1/2)/
c^4/d^2/a^(1/2)/(a*d-b*c)/((a*d-b*c)*d/c^2)^(1/2)/(c*x+d)

```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(5/2)/(c + d/x)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.373037, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(5/2)/(c + d/x)^2,x, algorithm="fricas")

[Out] [-1/2\*((5\*a\*b\*c\*d^2 - 4\*a^2\*d^3 + (5\*a\*b\*c^2\*d - 4\*a^2\*c\*d^2)\*x)\*sqrt(a)\*log(2\*a\*x - 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + (b^2\*c^2\*d + 3\*a\*b\*c\*d^2 - 4\*a^2\*d^3 + (b^2\*c^3 + 3\*a\*b\*c^2\*d - 4\*a^2\*c\*d^2)\*x)\*sqrt(-(b\*c - a\*d)/d)\*log((2\*d\*x\*sqrt(-(b\*c - a\*d)/d)\*sqrt((a\*x + b)/x) + b\*d - (b\*c - 2\*a\*d)\*x)/(c\*x + d)) - 2\*(a^2\*c^2\*d\*x^2 + (b^2\*c^3 - 2\*a\*b\*c^2\*d + 2\*a^2\*c\*d^2)\*x)\*sqrt((a\*x + b)/x))/(c^4\*d\*x + c^3\*d^2), 1/2\*(2\*(5\*a\*b\*c\*d^2 - 4\*a^2\*d^3 + (5\*a\*b\*c^2\*d - 4\*a^2\*c\*d^2)\*x)\*sqrt(-a)\*arctan(sqrt((a\*x + b)/x)/sqrt(-a)) - (b^2\*c^2\*d + 3\*a\*b\*c\*d^2 - 4\*a^2\*d^3 + (b^2\*c^3 + 3\*a\*b\*c^2\*d - 4\*a^2\*c\*d^2)\*x)\*sqrt(-(b\*c - a\*d)/d)\*log((2\*d\*x\*sqrt(-(b\*c - a\*d)/d)\*sqrt((a\*x + b)/x) + b\*d - (b\*c - 2\*a\*d)\*x)/(c\*x + d)) + 2\*(a^2\*c^2\*d\*x^2 + (b^2\*c^3 - 2\*a\*b\*c^2\*d + 2\*a^2\*c\*d^2)\*x)\*sqrt((a\*x + b)/x))/(c^4\*d\*x + c^3\*d^2), -1/2\*(2\*(b^2\*c^2\*d + 3\*a\*b\*c\*d^2 - 4\*a^2\*d^3 + (b^2\*c^3 + 3\*a\*b\*c^2\*d - 4\*a^2\*c\*d^2)\*x)\*sqrt((b\*c - a\*d)/d)\*arctan(sqrt((a\*x + b)/x)/sqrt((b\*c - a\*d)/d)) + (5\*a\*b\*c\*d^2 - 4\*a^2\*d^3 + (5\*a\*b\*c^2\*d - 4\*a^2\*c\*d^2)\*x)\*sqrt(a)\*log(2\*a\*x - 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) - 2\*(a^2\*c^2\*d\*x^2 + (b^2\*c^3 - 2\*a\*b\*c^2\*d + 2\*a^2\*c\*d^2)\*x)\*sqrt((a\*x + b)/x))/(c^4\*d\*x + c^3\*d^2), ((5\*a\*b\*c\*d^2 - 4\*a^2\*d^3 + (5\*a\*b\*c^2\*d - 4\*a^2\*c\*d^2)\*x)\*sqrt(-a)\*arctan(sqrt((a\*x + b)/x)/sqrt(-a)) - (b^2\*c^2\*d + 3\*a\*b\*c\*d^2 - 4\*a^2\*d^3 + (b^2\*c^3 + 3\*a\*b\*c^2\*d - 4\*a^2\*c\*d^2)\*x)\*sqrt((b\*c - a\*d)/d)\*arctan(sqrt((a\*x + b)/x)/sqrt((b\*c - a\*d)/d)) + (a^2\*c^2\*d\*x^2 + (b^2\*c^3 - 2\*a\*b\*c^2\*d + 2\*a^2\*c\*d^2)\*x)\*sqrt((a\*x + b)/x))/(c^4\*d\*x + c^3\*d^2)]

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(a + \frac{b}{x}\right)^{\frac{5}{2}}}{(cx + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/(c+d/x)**2,x)
```

```
[Out] Integral(x**2*(a + b/x)**(5/2)/(c*x + d)**2, x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a + b/x)^(5/2)/(c + d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.145 \quad \int \frac{\left(a + \frac{b}{x}\right)^{5/2}}{\left(c + \frac{d}{x}\right)^3} dx$$

**Optimal.** Leaf size=237

$$\frac{a^{3/2}(5bc - 6ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^4} - \frac{\sqrt{bc - ad}(-24a^2d^2 + 8abcd + b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{4c^4d^{3/2}}$$

$$- \frac{\sqrt{a + \frac{b}{x}}(-12a^2d^2 + 7abcd + b^2c^2)}{4c^3d\left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}(bc - 3ad)(bc - ad)}{2c^2d\left(c + \frac{d}{x}\right)^2} + \frac{ax\left(a + \frac{b}{x}\right)^{3/2}}{c\left(c + \frac{d}{x}\right)^2}$$

[Out]  $((b*c - 3*a*d)*(b*c - a*d)*\text{Sqrt}[a + b/x])/(2*c^2*d*(c + d/x)^2) - ((b^2*c^2 + 7*a*b*c*d - 12*a^2*d^2)*\text{Sqrt}[a + b/x])/(4*c^3*d*(c + d/x)) + (a*(a + b/x)^(3/2)*x)/(c*(c + d/x)^2) - (\text{Sqrt}[b*c - a*d]*(b^2*c^2 + 8*a*b*c*d - 24*a^2*d^2)*\text{ArcTan}[(\text{Sqrt}[d]*\text{Sqrt}[a + b/x])/(\text{Sqrt}[b*c - a*d])])/(4*c^4*d^(3/2)) + (a^(3/2)*(5*b*c - 6*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/c^4$

**Rubi [A]** time = 1.0225, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$\frac{a^{3/2}(5bc - 6ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^4} - \frac{\sqrt{bc - ad}(-24a^2d^2 + 8abcd + b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{4c^4d^{3/2}}$$

$$- \frac{\sqrt{a + \frac{b}{x}}(-12a^2d^2 + 7abcd + b^2c^2)}{4c^3d\left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}(bc - 3ad)(bc - ad)}{2c^2d\left(c + \frac{d}{x}\right)^2} + \frac{ax\left(a + \frac{b}{x}\right)^{3/2}}{c\left(c + \frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}\left[\left(a + \frac{b}{x}\right)^{5/2}/\left(c + \frac{d}{x}\right)^3, x\right]$

[Out]  $((b*c - 3*a*d)*(b*c - a*d)*\text{Sqrt}[a + b/x])/(2*c^2*d*(c + d/x)^2) - ((b^2*c^2 + 7*a*b*c*d - 12*a^2*d^2)*\text{Sqrt}[a + b/x])/(4*c^3*d*(c + d/x)) + (a*(a + b/x)^(3/2)*x)/(c*(c + d/x)^2) - (\text{Sqrt}[b*c - a*d]*(b^2*c^2 + 8*a*b*c*d - 24*a^2*d^2)*\text{ArcTan}[(\text{Sqrt}[d]*\text{Sqrt}[a + b/x])/(\text{Sqrt}[b*c - a*d])])/(4*c^4*d^(3/2)) + (a^(3/2)*(5*b*c - 6*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/c^4$

**Rubi in Sympy [A]** time = 106.304, size = 204, normalized size = 0.86

$$-\frac{a^{3/2}(6ad - 5bc) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^4} + \frac{ax\left(a + \frac{b}{x}\right)^{3/2}}{c\left(c + \frac{d}{x}\right)^2} + \frac{\sqrt{a + \frac{b}{x}}(ad - bc)(3ad - bc)}{2c^2d\left(c + \frac{d}{x}\right)^2}$$

$$+ \frac{\sqrt{a + \frac{b}{x}}(12a^2d^2 - 7abcd - b^2c^2)}{4c^3d\left(c + \frac{d}{x}\right)} + \frac{\sqrt{ad - bc}(24a^2d^2 - 8abcd - b^2c^2) \operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{ad - bc}}\right)}{4c^4d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}\left(\left(a + \frac{b}{x}\right)^{5/2}/\left(c + \frac{d}{x}\right)^3, x\right)$

[Out]  $-a^{3/2}(6ad - 5b^2c) \operatorname{atanh}(\sqrt{a + b/x}/\sqrt{a})/c^4 + a^2x^2(a + b/x)^{3/2}/(c^2(c + d/x)^2) + \sqrt{a + b/x}(ad - b^2c)(3ad - b^2c)/(2c^2d^2(c + d/x)^2) + \sqrt{a + b/x}(12a^2d^2 - 7ab^2cd - b^2c^2)/(4c^3d(c + d/x)) + \sqrt{ad - b^2c}(24a^2d^2 - 8ab^2cd - b^2c^2) \operatorname{atanh}(\sqrt{d})\sqrt{a + b/x}/\sqrt{ad - b^2c}/(4c^4d^{3/2})$

**Mathematica [C]** time = 0.651414, size = 304, normalized size = 1.28

$$\frac{-4a^{3/2}(6ad - 5bc) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right) + \frac{2cx\sqrt{a + \frac{b}{x}}(2a^2d(2c^2x^2 + 9cdx + 6d^2) - abcd(11cx + 7d) + b^2c^2(cx - d))}{d(cx + d)^2} - \frac{i(24a^3d^3 - 32a^2bcd^2)}{8c^4}}{8c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(5/2)/(c + d/x)^3, x]

[Out]  $((2c\sqrt{a + b/x}x^2(b^2c^2(-d + cx) - ab^2cd(7d + 11cx) + 2a^2d(6d^2 + 9cdx + 2c^2x^2)))/(d(d + cx)^2) - 4a^{3/2}(-5b^2c + 6ad)\operatorname{Log}[b + 2ax + 2\sqrt{a}\sqrt{a + b/x}x] - (I(b^3c^3 + 7ab^2c^2d - 32a^2b^2cd^2 + 24a^3d^3)\operatorname{Log}[(8c^5((-2I)ad^{3/2}x + 2d\sqrt{bc - ad})\sqrt{a + b/x}x - I\sqrt{d}(d - cx))]/(\sqrt{bc - ad}(b^3c^3 + 7ab^2c^2d - 32a^2b^2cd^2 + 24a^3d^3)(d + cx)))]/(d^{3/2}\sqrt{bc - ad}))/8c^4$

**Maple [B]** time = 0.022, size = 2807, normalized size = 11.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(5/2)/(c+d/x)^3, x)

[Out]  $1/8*((a*x+b)/x)^{1/2}x^{3/2}(-22*(x*(a*x+b))^{3/2}*((a*d-b*c)*d/c^2)^{1/2}(1/2)*x^2a^2b^2c^6d^2+8*(x*(a*x+b))^{3/2}*((a*d-b*c)*d/c^2)^{1/2}x^2a^2b^2c^6d^2-\ln((2*(x*(a*x+b))^{1/2}*((a*d-b*c)*d/c^2)^{1/2}c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^5c^5d^3+20*a^{3/2}\ln(1/2*(2*(x*(a*x+b))^{1/2}a^{1/2}+2*a*x+b)/a^{1/2})*((a*d-b*c)*d/c^2)^{1/2}x^2b^3c^6d^2+22*(x*(a*x+b))^{1/2}*((a*d-b*c)*d/c^2)^{1/2}x^3a^3b^2c^6d^2-128*a^{5/2}\ln(1/2*(2*(x*(a*x+b))^{1/2}a^{1/2}+2*a*x+b)/a^{1/2})*((a*d-b*c)*d/c^2)^{1/2}x^2b^2c^4d^4-24*a^{9/2}\ln(1/2*(2*(x*(a*x+b))^{1/2}a^{1/2}+2*a*x+b)/a^{1/2})*((a*d-b*c)*d/c^2)^{1/2}c^2d^7+2*(x*(a*x+b))^{3/2}*((a*d-b*c)*d/c^2)^{1/2}x^2b^3c^8-2*(x*(a*x+b))^{1/2}*((a*d-b*c)*d/c^2)^{1/2}x^2b^4c^8-24*\ln((2*(x*(a*x+b))^{1/2}*((a*d-b*c)*d/c^2)^{1/2}c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2a^5c^2d^6-\ln((2*(x*(a*x+b))^{1/2}*((a*d-b*c)*d/c^2)^{1/2}c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2b^5c^7d+8*(x*(a*x+b))^{3/2}*((a*d-b*c)*d/c^2)^{1/2}a^3c^4d^4+6*(x*(a*x+b))^{3/2}*((a*d-b*c)*d/c^2)^{1/2}b^3c^7d-48*\ln((2*(x*(a*x+b))^{1/2}*((a*d-b*c)*d/c^2)^{1/2}c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2a^5c^2d^7-2*\ln((2*(x*(a*x+b))^{1/2}*((a*d-b*c)*d/c^2)^{1/2}c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2b^5c^6d^2+24*(x*(a*x+b))^{1/2}*((a*d-b*c)*d/c^2)^{1/2}a^4c^2d^6-2*(x*(a*x+b))^{1/2}*((a*d-b*c)*d/c^2)^{1/2}b^4c^6d^2+80*\ln((2*(x*(a*x+b))^{1/2}*((a*d-b*c)*d/c^2)^{1/2}c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^4b^2c^7-95*\ln((2*(x*(a*x+b))^{1/2}*((a*d-b*c)*d/c^2)^{1/2}c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^3b^2c^2d^6+45*\ln((2*(x*(a*x+b))^{1/2}*((a*d-b*c)*d/c^2)^{1/2}c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^2b^3c^3d^5-5*\ln((2*(x*(a*x+b))^{1/2}*((a*d-b*c)*d/c^2)^{1/2}c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^4b^4c^4d^4+68*a^{7/2}\ln(1/2*(2*(x*(a*x+b))^{1/2}a^{1/2}+2*a*x+b)/a^{1/2})*((a*d-b*c)*d/c^2)^{1/2}x^2b^2c^4d^4+136*a^{7/2}\ln(1/2*(2*(x*(a*x+b))^{1/2}a^{1/2}+2*a*x+b)/a^{1/2})*((a*d-b*c)*d/c^2)^{1/2}$

$$\begin{aligned}
& x^*b^*c^3*d^5-64^*a^{(5/2)}*\ln(1/2*(2*(x*(a^*x+b))^{(1/2)}*a^{(1/2)}+2^*a^*x+ \\
& b)/a^{(1/2)})*((a^*d-b^*c)^*d/c^2)^{(1/2)}*x^2*b^2*c^5*d^3-102^*(x*(a^*x+b) \\
& ))^{(1/2)}*((a^*d-b^*c)^*d/c^2)^{(1/2)}*x^*a^3*b^2*c^4*d^4+92^*(x*(a^*x+b))^{(1/2)} \\
& *((a^*d-b^*c)^*d/c^2)^{(1/2)}*x^*a^2*b^2*c^5*d^3-22^*(x*(a^*x+b))^{(1/2)} \\
& *((a^*d-b^*c)^*d/c^2)^{(1/2)}*x^*a^*b^3*c^6*d^2-8^*(x*(a^*x+b))^{(1/2)}*(( \\
& a^*d-b^*c)^*d/c^2)^{(1/2)}*x^3*a^2*b^2*c^7*d+40^*a^{(3/2)}*\ln(1/2*(2*(x*( \\
& a^*x+b))^{(1/2)}*a^{(1/2)}+2^*a^*x+b)/a^{(1/2)})*((a^*d-b^*c)^*d/c^2)^{(1/2)}*x \\
& *b^3*c^5*d^3-190^*\ln((2*(x*(a^*x+b))^{(1/2)}*((a^*d-b^*c)^*d/c^2)^{(1/2)} \\
& c-2^*a^*d^*x+b^*c^*x-b^*d)/(c^*x+d))*x^*a^3*b^2*c^3*d^5+90^*\ln((2*(x*(a^*x+ \\
& b))^{(1/2)}*((a^*d-b^*c)^*d/c^2)^{(1/2)}*c-2^*a^*d^*x+b^*c^*x-b^*d)/(c^*x+d))*x \\
& *a^2*b^3*c^4*d^4-10^*\ln((2*(x*(a^*x+b))^{(1/2)}*((a^*d-b^*c)^*d/c^2)^{(1/2)} \\
& c-2^*a^*d^*x+b^*c^*x-b^*d)/(c^*x+d))*x^*a^*b^4*c^5*d^3-62^*(x*(a^*x+b))^{(1/2)} \\
& *((a^*d-b^*c)^*d/c^2)^{(1/2)}*a^3*b^2*c^3*d^5+50^*(x*(a^*x+b))^{(1/2)}*(( \\
& a^*d-b^*c)^*d/c^2)^{(1/2)}*a^2*b^2*c^4*d^4-10^*(x*(a^*x+b))^{(1/2)}*((a^*d \\
& -b^*c)^*d/c^2)^{(1/2)}*a^*b^3*c^5*d^3-24^*a^{(9/2)}*\ln(1/2*(2*(x*(a^*x+b)) \\
& )^{(1/2)}*a^{(1/2)}+2^*a^*x+b)/a^{(1/2)})*((a^*d-b^*c)^*d/c^2)^{(1/2)}*x^2*c^3* \\
& d^5-24^*\ln((2*(x*(a^*x+b))^{(1/2)}*((a^*d-b^*c)^*d/c^2)^{(1/2)}*c-2^*a^*d^*x+ \\
& b^*c^*x-b^*d)/(c^*x+d))*a^5*d^8-48^*a^{(9/2)}*\ln(1/2*(2*(x*(a^*x+b))^{(1/2)} \\
& )^*a^{(1/2)}+2^*a^*x+b)/a^{(1/2)})*((a^*d-b^*c)^*d/c^2)^{(1/2)}*x^*c^2*d^6+68^* \\
& a^{(7/2)}*\ln(1/2*(2*(x*(a^*x+b))^{(1/2)}*a^{(1/2)}+2^*a^*x+b)/a^{(1/2)})*((a \\
& *d-b^*c)^*d/c^2)^{(1/2)}*b^*c^2*d^6-12^*(x*(a^*x+b))^{(1/2)}*((a^*d-b^*c)^*d/ \\
& c^2)^{(1/2)}*x^3*a^4*c^5*d^3-2^*(x*(a^*x+b))^{(1/2)}*((a^*d-b^*c)^*d/c^2)^{(1/2)} \\
& *x^3*a^*b^3*c^8-64^*a^{(5/2)}*\ln(1/2*(2*(x*(a^*x+b))^{(1/2)}*a^{(1/2)} \\
& )+2^*a^*x+b)/a^{(1/2)})*((a^*d-b^*c)^*d/c^2)^{(1/2)}*b^2*c^3*d^5+12^*(x^*(a^* \\
& x+b))^{(3/2)}*((a^*d-b^*c)^*d/c^2)^{(1/2)}*x^*a^3*c^5*d^3+20^*a^{(3/2)}*\ln(1 \\
& /2*(2*(x*(a^*x+b))^{(1/2)}*a^{(1/2)}+2^*a^*x+b)/a^{(1/2)})*((a^*d-b^*c)^*d/c^2 \\
& )^{(1/2)}*b^3*c^4*d^4+80^*\ln((2*(x*(a^*x+b))^{(1/2)}*((a^*d-b^*c)^*d/c^2) \\
& )^{(1/2)}*c-2^*a^*d^*x+b^*c^*x-b^*d)/(c^*x+d))*x^2*a^4*b^*c^3*d^5-95^*\ln((2^*( \\
& x*(a^*x+b))^{(1/2)}*((a^*d-b^*c)^*d/c^2)^{(1/2)}*c-2^*a^*d^*x+b^*c^*x-b^*d)/(c^* \\
& x+d))*x^2*a^3*b^2*c^4*d^4+45^*\ln((2*(x*(a^*x+b))^{(1/2)}*((a^*d-b^*c)^*d \\
& /c^2)^{(1/2)}*c-2^*a^*d^*x+b^*c^*x-b^*d)/(c^*x+d))*x^2*a^2*b^3*c^5*d^3-5^*l \\
& n((2*(x*(a^*x+b))^{(1/2)}*((a^*d-b^*c)^*d/c^2)^{(1/2)}*c-2^*a^*d^*x+b^*c^*x-b^* \\
& d)/(c^*x+d))*x^2*a^*b^4*c^6*d^2-10^*(x*(a^*x+b))^{(3/2)}*((a^*d-b^*c)^*d/c \\
& ^2)^{(1/2)}*a^2*b^*c^5*d^3-4^*(x*(a^*x+b))^{(3/2)}*((a^*d-b^*c)^*d/c^2)^{(1/2)} \\
& )^*a^*b^2*c^6*d^2+36^*(x*(a^*x+b))^{(1/2)}*((a^*d-b^*c)^*d/c^2)^{(1/2)}*x^*a \\
& ^4*c^3*d^5-4^*(x*(a^*x+b))^{(1/2)}*((a^*d-b^*c)^*d/c^2)^{(1/2)}*x^*b^4*c^7 \\
& d+160^*\ln((2*(x*(a^*x+b))^{(1/2)}*((a^*d-b^*c)^*d/c^2)^{(1/2)}*c-2^*a^*d^*x+b \\
& ^*c^*x-b^*d)/(c^*x+d))*x^*a^4*b^*c^2*d^6-18^*(x*(a^*x+b))^{(1/2)}*((a^*d-b^*c \\
& )^*d/c^2)^{(1/2)}*x^2*a^3*b^*c^5*d^3-14^*(x*(a^*x+b))^{(1/2)}*((a^*d-b^*c)^* \\
& d/c^2)^{(1/2)}*x^2*a^*b^3*c^7*d+34^*(x*(a^*x+b))^{(1/2)}*((a^*d-b^*c)^*d/c^2 \\
& )^{(1/2)}*x^2*a^2*b^2*c^6*d^2)/(x*(a^*x+b))^{(1/2)}/c^5/(a^*d-b^*c)^2/d \\
& ^2/((a^*d-b^*c)^*d/c^2)^{(1/2)}/(c^*x+d)^2
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(5/2)/(c + d/x)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.342362, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(5/2)/(c + d/x)^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned}
& [-1/8*(4*(5*a^*b^*c^*d^3 - 6*a^2*d^4 + (5*a^*b^*c^3*d - 6*a^2*c^2*d^2) \\
& *x^2 + 2*(5*a^*b^*c^2*d^2 - 6*a^2*c^*d^3)*x)*\sqrt{a}*\log(2^*a^*x - 2^*s \\
& \sqrt{a}*x*\sqrt{(a^*x + b)/x} + b) + (b^2*c^2*d^2 + 8^*a^*b^*c^*d^3 - 24 \\
& *a^2*d^4 + (b^2*c^4 + 8^*a^*b^*c^3*d - 24^*a^2*c^2*d^2)*x^2 + 2^*(b^2^*
\end{aligned}$$



$$c^3d + 8ab^2c^2d^2 - 24a^2c^3d^3)x) \sqrt{-(b^2c - a^2d)/d} \log \left( \frac{(2dx \sqrt{-(b^2c - a^2d)/d} \sqrt{(ax + b)/x} + b^2d - (b^2c - 2a^2d)x)/(cx + d) - 2(4a^2c^3d^2x^3 + (b^2c^4 - 11a^2b^2c^3d + 18a^2c^2d^2)x^2 - (b^2c^3d + 7a^2b^2c^2d^2 - 12a^2c^2d^3)x) \sqrt{(ax + b)/x})/(c^6d^2x^2 + 2c^5d^2x + c^4d^3)}{1/4((b^2c^2d^2 + 8a^2b^2c^2d^3 - 24a^2d^4 + (b^2c^4 + 8a^2b^2c^3d - 24a^2c^2d^2)x^2 + 2(b^2c^3d + 8a^2b^2c^2d^2 - 24a^2c^2d^3)x) \sqrt{(b^2c - a^2d)/d} \arctan(\sqrt{(ax + b)/x}/\sqrt{(b^2c - a^2d)/d}) + 2(5a^2b^2c^2d^3 - 6a^2d^4 + (5a^2b^2c^3d - 6a^2c^2d^2)x^2 + 2(5a^2b^2c^2d^2 - 6a^2c^2d^3)x) \sqrt{a} \log(2ax - 2\sqrt{a}x \sqrt{(ax + b)/x} + b) - (4a^2c^3d^2x^3 + (b^2c^4 - 11a^2b^2c^3d + 18a^2c^2d^2)x^2 - (b^2c^3d + 7a^2b^2c^2d^2 - 12a^2c^2d^3)x) \sqrt{(ax + b)/x})/(c^6d^2x^2 + 2c^5d^2x + c^4d^3)}{1/8(8(5a^2b^2c^2d^3 - 6a^2d^4 + (5a^2b^2c^3d - 6a^2c^2d^2)x^2 + 2(5a^2b^2c^2d^2 - 6a^2c^2d^3)x) \sqrt{-a} \arctan(\sqrt{(ax + b)/x}/\sqrt{-a}) - (b^2c^2d^2 + 8a^2b^2c^2d^3 - 24a^2d^4 + (b^2c^4 + 8a^2b^2c^3d - 24a^2c^2d^2)x^2 + 2(b^2c^3d + 8a^2b^2c^2d^2 - 24a^2c^2d^3)x) \sqrt{-(b^2c - a^2d)/d} \log \left( \frac{(2dx \sqrt{-(b^2c - a^2d)/d} \sqrt{(ax + b)/x} + b^2d - (b^2c - 2a^2d)x)/(cx + d) + 2(4a^2c^3d^2x^3 + (b^2c^4 - 11a^2b^2c^3d + 18a^2c^2d^2)x^2 - (b^2c^3d + 7a^2b^2c^2d^2 - 12a^2c^2d^3)x) \sqrt{(ax + b)/x})/(c^6d^2x^2 + 2c^5d^2x + c^4d^3)}{1/4(4(5a^2b^2c^2d^3 - 6a^2d^4 + (5a^2b^2c^3d - 6a^2c^2d^2)x^2 + 2(5a^2b^2c^2d^2 - 6a^2c^2d^3)x) \sqrt{-a} \arctan(\sqrt{(ax + b)/x}/\sqrt{-a}) - (b^2c^2d^2 + 8a^2b^2c^2d^3 - 24a^2d^4 + (b^2c^4 + 8a^2b^2c^3d - 24a^2c^2d^2)x^2 + 2(b^2c^3d + 8a^2b^2c^2d^2 - 24a^2c^2d^3)x) \sqrt{(b^2c - a^2d)/d} \arctan(\sqrt{(ax + b)/x}/\sqrt{(b^2c - a^2d)/d}) + (4a^2c^3d^2x^3 + (b^2c^4 - 11a^2b^2c^3d + 18a^2c^2d^2)x^2 - (b^2c^3d + 7a^2b^2c^2d^2 - 12a^2c^2d^3)x) \sqrt{(ax + b)/x})/(c^6d^2x^2 + 2c^5d^2x + c^4d^3)} \right]$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(a + \frac{b}{x}\right)^{\frac{5}{2}}}{(cx + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(5/2)/(c+d/x)\*\*3,x)

[Out] Integral(x\*\*3\*(a + b/x)\*\*(5/2)/(c\*x + d)\*\*3, x)

**GIAC/XCAS [A]** time = 0.549039, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(5/2)/(c + d/x)^3,x, algorithm="giac")

[Out] sage0\*x

$$3.146 \quad \int \frac{\left(c + \frac{d}{x}\right)^3}{\sqrt{a + \frac{b}{x}}} dx$$

**Optimal.** Leaf size=126

$$\frac{c^2(bc - 6ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{d\sqrt{a + \frac{b}{x}} \left(2(-2a^2d^2 + 9abcd + 3b^2c^2) + \frac{bd(2ad+3bc)}{x}\right)}{3ab^2} + \frac{cx\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2}{a}$$

[Out]  $-(d*\text{Sqrt}[a + b/x]*(2*(3*b^2*c^2 + 9*a*b*c*d - 2*a^2*d^2) + (b*d*(3*b*c + 2*a*d))/x))/(3*a*b^2) + (c*\text{Sqrt}[a + b/x]*(c + d/x)^2*x)/a - (c^2*(b*c - 6*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/a^{3/2}$

**Rubi [A]** time = 0.283658, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{c^2(bc - 6ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{d\sqrt{a + \frac{b}{x}} \left(2(-2a^2d^2 + 9abcd + 3b^2c^2) + \frac{bd(2ad+3bc)}{x}\right)}{3ab^2} + \frac{cx\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2}{a}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d/x)^3/\text{Sqrt}[a + b/x], x]$

[Out]  $-(d*\text{Sqrt}[a + b/x]*(2*(3*b^2*c^2 + 9*a*b*c*d - 2*a^2*d^2) + (b*d*(3*b*c + 2*a*d))/x))/(3*a*b^2) + (c*\text{Sqrt}[a + b/x]*(c + d/x)^2*x)/a - (c^2*(b*c - 6*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/a^{3/2}$

**Rubi in Sympy [A]** time = 25.5374, size = 114, normalized size = 0.9

$$\frac{cx\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2}{a} + \frac{4d\sqrt{a + \frac{b}{x}} \left(a^2d^2 - \frac{9abcd}{2} - \frac{3b^2c^2}{2} - \frac{bd(2ad+3bc)}{4x}\right)}{3ab^2} + \frac{c^2(6ad - bc) \text{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c+d/x)**3/(a+b/x)**(1/2), x)$

[Out]  $c*x*\text{sqrt}(a + b/x)*(c + d/x)**2/a + 4*d*\text{sqrt}(a + b/x)*(a**2*d**2 - 9*a*b*c*d/2 - 3*b**2*c**2/2 - b*d*(2*a*d + 3*b*c)/(4*x))/(3*a*b**2) + c**2*(6*a*d - b*c)*\text{atanh}(\text{sqrt}(a + b/x)/\text{sqrt}(a))/a**(3/2)$

**Mathematica [A]** time = 0.288788, size = 99, normalized size = 0.79

$$\frac{c^2(6ad - bc) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right)}{2a^{3/2}} + \sqrt{a + \frac{b}{x}} \left(\frac{4ad^3}{3b^2} + \frac{c^3x}{a} - \frac{2d^2(9cx + d)}{3bx}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(c + d/x)^3/\text{Sqrt}[a + b/x], x]$

[Out]  $\text{Sqrt}[a + b/x]*((4*a*d^3)/(3*b^2) + (c^3*x)/a - (2*d^2*(d + 9*c*x))/(3*b*x)) + (c^2*(-(b*c) + 6*a*d)*\text{Log}[b + 2*a*x + 2*\text{Sqrt}[a]*\text{Sqrt}$

$[a + b/x]^*x)/(2^*a^{(3/2)})$

---

**Maple [B]** time = 0.021, size = 540, normalized size = 4.3

$$\frac{1}{6b^3x^2}\sqrt{\frac{ax+b}{x}}\left(3a^4\ln\left(\frac{1}{2}\frac{2\sqrt{x(ax+b)}\sqrt{a}+2ax+b}{\sqrt{a}}\right)d^3bx^3-3d^3\ln\left(\frac{1}{2}\frac{2\sqrt{ax^2+bx}\sqrt{a}+2ax+b}{\sqrt{a}}\right)a^4bx^3-9a^3\ln\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^3/(a+b/x)^(1/2), x)

[Out]  $\frac{1}{6}((a*x+b)/x)^{(1/2)}/x^2*(3*a^4*\ln(1/2*(2*(x*(a*x+b)))^{(1/2)}*a^{(1/2)+2*a*x+b)/a^{(1/2)})^3*d^3*b*x^3-3*d^3*\ln(1/2*(2*(a*x^2+b*x))^{(1/2)}*a^{(1/2)+2*a*x+b)/a^{(1/2)})^3*a^4*b*x^3-9*a^3*\ln(1/2*(2*(x*(a*x+b)))^{(1/2)}*a^{(1/2)+2*a*x+b)/a^{(1/2)})^3*c*d^2*b^2*x^3+9*d^2*\ln(1/2*(2*(a*x^2+b*x))^{(1/2)}*a^{(1/2)+2*a*x+b)/a^{(1/2)})^3*a^3*c*b^2*x^3+9*\ln(1/2*(2*(x*(a*x+b))^{(1/2)}*a^{(1/2)+2*a*x+b)/a^{(1/2)})^3*c^2*d*b^3*x^3*a^{(2)+9*d*\ln(1/2*(2*(a*x^2+b*x))^{(1/2)}*a^{(1/2)+2*a*x+b)/a^{(1/2)})^3*c^2*b^3*x^3*a^{(2)+9*d*\ln(1/2*(2*(a*x^2+b*x))^{(1/2)}*a^{(1/2)+2*a*x+b)/a^{(1/2)})^3*c^2*b^3*x^3*a^{(2)+6*d^3*(a*x^2+b*x)^{(1/2)}*a^{(9/2)}*x^3+18*d^2*(a*x^2+b*x)^{(1/2)}*c*a^{(7/2)}*b*x^3+18*d*(a*x^2+b*x)^{(1/2)}*c^2*b^2*x^3*a^{(5/2)}-6*a^{(9/2)}*(x*(a*x+b))^{(1/2)}*d^3*x^3+18*a^{(7/2)}*(x*(a*x+b))^{(1/2)}*c*d^2*b*x^3-18*(x*(a*x+b))^{(1/2)}*c^2*d*b^2*x^3*a^{(5/2)}+6*(x*(a*x+b))^{(1/2)}*c^3*b^3*x^3*a^{(3/2)}-3*b^4*\ln(1/2*(2*(x*(a*x+b))^{(1/2)}*a^{(1/2)+2*a*x+b)/a^{(1/2)})^3*c^3*x^3*a^{(3/2)}+12*d^3*(a*x^2+b*x)^{(3/2)}*a^{(7/2)}*x-36*d^2*(a*x^2+b*x)^{(3/2)}*c*b*x*a^{(5/2)}-4*d^3*(a*x^2+b*x)^{(3/2)}*b*a^{(5/2)})/(x*(a*x+b))^{(1/2)}/b^3/a^{(5/2)}$

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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)^3/sqrt(a + b/x), x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.253138, size = 1, normalized size = 0.01

$$\left[ \frac{3(b^3c^3 - 6ab^2c^2d)x \log\left(2ax\sqrt{\frac{ax+b}{x}} + (2ax+b)\sqrt{a}\right) - 2(3b^2c^3x^2 - 2abd^3 - 2(9abcd^2 - 2a^2d^3)x)\sqrt{a}\sqrt{\frac{ax+b}{x}}}{6a^{\frac{3}{2}}b^2x}, \frac{3(b^3c^3 - 6ab^2c^2d)x \arctan\left(\frac{\sqrt{a}\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}\right) + (3b^2c^3x^2 - 2abd^3 - 2(9abcd^2 - 2a^2d^3)x)\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{(\sqrt{-a})^2b^2x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)^3/sqrt(a + b/x), x, algorithm="fricas")

[Out]  $[-1/6*(3*(b^3*c^3 - 6*a*b^2*c^2*d)*x*\log(2*a*x*\sqrt{(a*x + b)/x}) + (2*a*x + b)*\sqrt{a}) - 2*(3*b^2*c^3*x^2 - 2*a*b*d^3 - 2*(9*a*b*c*d^2 - 2*a^2*d^3)*x)*\sqrt{a}*\sqrt{(a*x + b)/x})/(a^{(3/2)}*b^2*x), 1/3*(3*(b^3*c^3 - 6*a*b^2*c^2*d)*x*\arctan(a/(\sqrt{-a}*\sqrt{(a*x + b)/x})) + (3*b^2*c^3*x^2 - 2*a*b*d^3 - 2*(9*a*b*c*d^2 - 2*a^2*d^3)*x)*\sqrt{-a}*\sqrt{(a*x + b)/x})/(\sqrt{-a}*a*b^2*x)]$

---

**Sympy [A]** time = 15.5631, size = 377, normalized size = 2.99

$$\frac{4a^{\frac{7}{2}}b^{\frac{3}{2}}d^3x^2\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}} + \frac{2a^{\frac{5}{2}}b^{\frac{5}{2}}d^3x\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}} - \frac{2a^{\frac{3}{2}}b^{\frac{7}{2}}d^3\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}} - \frac{4a^4bd^3x^{\frac{5}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}} - \frac{4a^3b^2d^3x^{\frac{3}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}} + 3cd^2 \left( \begin{cases} -\frac{1}{\sqrt{ax}} & \text{for } b = 0 \\ -\frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{otherwise} \end{cases} \right) + \frac{\sqrt{bc^3}\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a} + \frac{6c^2d \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}} - \frac{bc^3 \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*\*3/(a+b/x)\*\*(1/2),x)

[Out]  $4*a^{(7/2)}*b^{(3/2)}*d^{*3}*x^{*2}*\sqrt{a*x/b + 1}/(3*a^{(5/2)}*b^{*3}*x^{*(5/2)} + 3*a^{(3/2)}*b^{*4}*x^{*(3/2)}) + 2*a^{(5/2)}*b^{(5/2)}*d^{*3}*x*s\sqrt{a*x/b + 1}/(3*a^{(5/2)}*b^{*3}*x^{*(5/2)} + 3*a^{(3/2)}*b^{*4}*x^{*(3/2)}) - 2*a^{(3/2)}*b^{(7/2)}*d^{*3}*\sqrt{a*x/b + 1}/(3*a^{(5/2)}*b^{*3}*x^{*(5/2)} + 3*a^{(3/2)}*b^{*4}*x^{*(3/2)}) - 4*a^{*4}*b*d^{*3}*x^{*(5/2)}/(3*a^{(5/2)}*b^{*3}*x^{*(5/2)} + 3*a^{(3/2)}*b^{*4}*x^{*(3/2)}) - 4*a^{*3}*b^{*2}*d^{*3}*x^{*(3/2)}/(3*a^{(5/2)}*b^{*3}*x^{*(5/2)} + 3*a^{(3/2)}*b^{*4}*x^{*(3/2)}) + 3*c*d^{*2}*Piecewise((-1/(sqrt(a)*x), Eq(b, 0)), (-2*sqrt(a + b/x)/b, True)) + sqrt(b)*c^{*3}*sqrt(x)*sqrt(a*x/b + 1)/a + 6*c^{*2}*d*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a) - b*c^{*3}*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a^{(3/2)}$

**GIAC/XCAS [A]** time = 0.256988, size = 207, normalized size = 1.64

$$-\frac{1}{3} \left( \frac{3c^3\sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right)a} - \frac{3(bc^3 - 6ac^2d) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-aab}} + \frac{2\left(9b^7cd^2\sqrt{\frac{ax+b}{x}} - 3ab^6d^3\sqrt{\frac{ax+b}{x}} + \frac{(ax+b)b^6d^3\sqrt{\frac{ax+b}{x}}}{x}\right)}{b^9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)^3/sqrt(a + b/x),x, algorithm="giac")

[Out]  $-1/3*(3*c^3*\sqrt{((a*x + b)/x)/((a - (a*x + b)/x)*a)} - 3*(b*c^3 - 6*a*c^2*d)*\arctan(\sqrt{((a*x + b)/x)/\sqrt{-a}})/(\sqrt{-a}*a*b) + 2*(9*b^7*c*d^2*\sqrt{((a*x + b)/x)} - 3*a*b^6*d^3*\sqrt{((a*x + b)/x)} + (a*x + b)*b^6*d^3*\sqrt{((a*x + b)/x)}/x)/b^9*b$

$$3.147 \quad \int \frac{\left(c + \frac{d}{x}\right)^2}{\sqrt{a + \frac{b}{x}}} dx$$

**Optimal.** Leaf size=73

$$-\frac{c(bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{c^2 x \sqrt{a + \frac{b}{x}}}{a} - \frac{2d^2 \sqrt{a + \frac{b}{x}}}{b}$$

[Out]  $(-2*d^2*\text{Sqrt}[a + b/x])/b + (c^2*\text{Sqrt}[a + b/x]*x)/a - (c*(b*c - 4*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/a^{(3/2)}$

**Rubi [A]** time = 0.180524, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$-\frac{c(bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{c^2 x \sqrt{a + \frac{b}{x}}}{a} - \frac{2d^2 \sqrt{a + \frac{b}{x}}}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d/x)^2/Sqrt[a + b/x], x]

[Out]  $(-2*d^2*\text{Sqrt}[a + b/x])/b + (c^2*\text{Sqrt}[a + b/x]*x)/a - (c*(b*c - 4*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/a^{(3/2)}$

**Rubi in Sympy [A]** time = 16.5146, size = 60, normalized size = 0.82

$$-\frac{2d^2 \sqrt{a + \frac{b}{x}}}{b} + \frac{c^2 x \sqrt{a + \frac{b}{x}}}{a} + \frac{c(4ad - bc) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d/x)\*\*2/(a+b/x)\*\*(1/2), x)

[Out]  $-2*d**2*\text{sqrt}(a + b/x)/b + c**2*x*\text{sqrt}(a + b/x)/a + c*(4*a*d - b*c)*\text{atanh}(\text{sqrt}(a + b/x)/\text{sqrt}(a))/a**(3/2)$

**Mathematica [A]** time = 0.161446, size = 75, normalized size = 1.03

$$\frac{c(4ad - bc) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right)}{2a^{3/2}} + \sqrt{a + \frac{b}{x}} \left(\frac{c^2 x}{a} - \frac{2d^2}{b}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d/x)^2/Sqrt[a + b/x], x]

[Out]  $\text{Sqrt}[a + b/x]*((-2*d^2)/b + (c^2*x)/a) + (c*(-(b*c) + 4*a*d)*\text{Log}[b + 2*a*x + 2*\text{Sqrt}[a]*\text{Sqrt}[a + b/x]*x])/(2*a^{(3/2)})$

**Maple [B]** time = 0.02, size = 354, normalized size = 4.9

$$\frac{1}{2xb^2} \sqrt{\frac{ax+b}{x}} \left( d^2 a^3 \ln \left( \frac{1}{2} \left( 2 \sqrt{ax^2+bx}\sqrt{a} + 2ax + b \right) \frac{1}{\sqrt{a}} \right) x^2 b - a^3 \ln \left( \frac{1}{2} \left( 2 \sqrt{x(ax+b)}\sqrt{a} + 2ax + b \right) \frac{1}{\sqrt{a}} \right) d^2 x^2 b + 2a \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^2/(a+b/x)^(1/2), x)

[Out]  $\frac{1}{2} \left( \frac{(a^2 x + b)}{x} \right)^{1/2} / x \left( d^2 a^3 \ln \left( \frac{1}{2} \left( 2 \sqrt{ax^2+bx}\sqrt{a} + 2ax + b \right) \frac{1}{\sqrt{a}} \right) x^2 b - a^3 \ln \left( \frac{1}{2} \left( 2 \sqrt{x(ax+b)}\sqrt{a} + 2ax + b \right) \frac{1}{\sqrt{a}} \right) d^2 x^2 b + 2a \right)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)^2/sqrt(a + b/x), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.248367, size = 1, normalized size = 0.01

$$\left[ \frac{2(bc^2x - 2ad^2)\sqrt{a}\sqrt{\frac{ax+b}{x}} - (b^2c^2 - 4abcd) \log\left(2ax\sqrt{\frac{ax+b}{x}} + (2ax+b)\sqrt{a}\right)}{2a^{\frac{3}{2}}b}, \frac{(bc^2x - 2ad^2)\sqrt{-a}\sqrt{\frac{ax+b}{x}} + (b^2c^2 - 4abcd) \log\left(2ax\sqrt{\frac{ax+b}{x}} + (2ax+b)\sqrt{a}\right)}{\sqrt{-aab}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)^2/sqrt(a + b/x), x, algorithm="fricas")

[Out]  $\left[ \frac{1}{2} \left( 2 \left( b^2 c^2 x - 2 a^2 d^2 \right) \sqrt{a} \sqrt{\frac{a x + b}{x}} - \left( b^2 c^2 - 4 a b c d \right) \log \left( 2 a x \sqrt{\frac{a x + b}{x}} + \left( 2 a x + b \right) \sqrt{a} \right) \right) / \left( a^{\frac{3}{2}} b \right), \left( \left( b^2 c^2 x - 2 a^2 d^2 \right) \sqrt{-a} \sqrt{\frac{a x + b}{x}} + \left( b^2 c^2 - 4 a b c d \right) \arctan \left( a / \left( \sqrt{-a} \sqrt{\frac{a x + b}{x}} \right) \right) \right) / \left( \sqrt{-a} a^{\frac{3}{2}} b \right) \right]$

**Sympy [A]** time = 12.382, size = 105, normalized size = 1.44

$$d^2 \left( \begin{cases} -\frac{1}{\sqrt{ax}} & \text{for } b = 0 \\ \frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{otherwise} \end{cases} \right) + \frac{\sqrt{bc^2}\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a} + \frac{4cd \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}} - \frac{bc^2 \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*\*2/(a+b/x)\*\*(1/2), x)

```
[Out] d**2*Piecewise((-1/(sqrt(a)*x), Eq(b, 0)), (-2*sqrt(a + b/x)/b, True)) + sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1)/a + 4*c*d*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a) - b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/2)
```

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**GIAC/XCAS [A]** time = 0.255677, size = 131, normalized size = 1.79

$$-b \left( \frac{c^2 \sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right)a} + \frac{2d^2 \sqrt{\frac{ax+b}{x}}}{b^2} - \frac{(bc^2 - 4acd) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-aab}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c + d/x)^2/sqrt(a + b/x),x, algorithm="giac")
```

```
[Out] -b*(c^2*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a) + 2*d^2*sqrt((a*x + b)/x)/b^2 - (b*c^2 - 4*a*c*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/sqrt(-a)*a*b)
```

$$3.148 \quad \int \frac{c + \frac{d}{x}}{\sqrt{a + \frac{b}{x}}} dx$$

**Optimal.** Leaf size=51

$$\frac{cx\sqrt{a + \frac{b}{x}}}{a} - \frac{(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

[Out] (c\*Sqrt[a + b/x]\*x)/a - ((b\*c - 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)

**Rubi [A]** time = 0.109843, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{cx\sqrt{a + \frac{b}{x}}}{a} - \frac{(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c + d/x)/Sqrt[a + b/x], x]

[Out] (c\*Sqrt[a + b/x]\*x)/a - ((b\*c - 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)

**Rubi in Sympy [A]** time = 9.44179, size = 42, normalized size = 0.82

$$\frac{cx\sqrt{a + \frac{b}{x}}}{a} + \frac{2\left(ad - \frac{bc}{2}\right) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d/x)/(a+b/x)\*\*(1/2), x)

[Out] c\*x\*sqrt(a + b/x)/a + 2\*(a\*d - b\*c/2)\*atanh(sqrt(a + b/x)/sqrt(a))/a\*\*(3/2)

**Mathematica [A]** time = 0.0753147, size = 62, normalized size = 1.22

$$\frac{(2ad - bc) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right)}{2a^{3/2}} + \frac{cx\sqrt{a + \frac{b}{x}}}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d/x)/Sqrt[a + b/x], x]

[Out] (c\*Sqrt[a + b/x]\*x)/a + ((-(b\*c) + 2\*a\*d)\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]\*x])/(2\*a^(3/2))



**Maple [B]** time = 0.017, size = 179, normalized size = 3.5

$$-\frac{x}{2b}\sqrt{\frac{ax+b}{x}}\left(2\sqrt{x(ax+b)}da^{5/2}-2\sqrt{x(ax+b)}cba^{3/2}-2d\sqrt{ax^2+bx}a^{5/2}-d\ln\left(\frac{1}{2}\left(2\sqrt{ax^2+bx}\sqrt{a}+2ax+b\right)\frac{1}{\sqrt{a}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)/(a+b/x)^(1/2),x)

[Out] 
$$-1/2*((a*x+b)/x)^(1/2)*x*(2*(x*(a*x+b))^(1/2)*d*a^(5/2)-2*(x*(a*x+b))^(1/2)*c*b*a^(3/2)-2*d*(a*x^2+b*x)^(1/2)*a^(5/2)-d*\ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b*a^2-\ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*d*b*a^2+b^2*\ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*c*a)/(x*(a*x+b))^(1/2)/b/a^(5/2)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)/sqrt(a + b/x),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.249634, size = 1, normalized size = 0.02

$$\left[ \frac{2\sqrt{acx}\sqrt{\frac{ax+b}{x}} - (bc - 2ad)\log\left(2ax\sqrt{\frac{ax+b}{x}} + (2ax+b)\sqrt{a}\right)}{2a^{\frac{3}{2}}}, \frac{\sqrt{-acx}\sqrt{\frac{ax+b}{x}} + (bc - 2ad)\arctan\left(\frac{a}{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}\right)}{\sqrt{-aa}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)/sqrt(a + b/x),x, algorithm="fricas")

[Out] 
$$[1/2*(2*\sqrt{a}*c*x*\sqrt{(a*x+b)/x} - (b*c - 2*a*d)*\log(2*a*x*\sqrt{(a*x+b)/x} + (2*a*x+b)*\sqrt{a}))/a^{3/2}, (\sqrt{-a}*c*x*\sqrt{(a*x+b)/x} + (b*c - 2*a*d)*\arctan(a/(\sqrt{-a}*\sqrt{(a*x+b)/x}))) / (\sqrt{-a}*a)]$$

**Sympy [A]** time = 25.0676, size = 73, normalized size = 1.43

$$\frac{\sqrt{bc}\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a} + \frac{2d\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}} - \frac{bc\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)/(a+b/x)\*\*(1/2),x)

[Out] 
$$\sqrt{b}*c*\sqrt{x}*\sqrt{a*x/b+1}/a + 2*d*\operatorname{asinh}(\sqrt{a}*\sqrt{x}/\sqrt{b})/\sqrt{a} - b*c*\operatorname{asinh}(\sqrt{a}*\sqrt{x}/\sqrt{b})/a^{3/2}$$

---

**GIAC/XCAS [A]** time = 0.250741, size = 99, normalized size = 1.94

$$-b \left( \frac{c \sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right)a} - \frac{(bc - 2ad) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)/sqrt(a + b/x),x, algorithm="giac")

[Out] -b\*(c\*sqrt((a\*x + b)/x)/((a - (a\*x + b)/x)\*a) - (b\*c - 2\*a\*d)\*arc  
tan(sqrt((a\*x + b)/x)/sqrt(-a))/(sqrt(-a)\*a\*b)

$$3.149 \quad \int \frac{1}{\sqrt{a + \frac{b}{x}}} dx$$

**Optimal.** Leaf size=43

$$\frac{x\sqrt{a + \frac{b}{x}}}{a} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

[Out] (Sqrt[a + b/x]\*x)/a - (b\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)

**Rubi [A]** time = 0.059137, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\frac{x\sqrt{a + \frac{b}{x}}}{a} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b/x], x]

[Out] (Sqrt[a + b/x]\*x)/a - (b\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)

**Rubi in Sympy [A]** time = 5.35715, size = 32, normalized size = 0.74

$$\frac{x\sqrt{a + \frac{b}{x}}}{a} - \frac{b \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a+b/x)\*\*(1/2), x)

[Out] x\*sqrt(a + b/x)/a - b\*atanh(sqrt(a + b/x)/sqrt(a))/a\*\*(3/2)

**Mathematica [A]** time = 0.0467834, size = 53, normalized size = 1.23

$$\frac{x\sqrt{a + \frac{b}{x}}}{a} - \frac{b \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b/x], x]

[Out] (Sqrt[a + b/x]\*x)/a - (b\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]\*x])/(2\*a^(3/2))

**Maple [A]** time = 0.005, size = 70, normalized size = 1.6

$$-\frac{x}{2}\sqrt{\frac{ax+b}{x}}\left(b\ln\left(\frac{1}{2}\left(2\sqrt{x(ax+b)}\sqrt{a}+2ax+b\right)\frac{1}{\sqrt{a}}\right)-2\sqrt{x(ax+b)}\sqrt{a}\right)\frac{1}{\sqrt{x(ax+b)}}a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b/x)^(1/2), x)`

[Out]  $-1/2 * ((a * x + b) / x)^{(1/2)} * x * (b * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) - 2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)}) / (x * (a * x + b))^{(1/2)} / a^{(3/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(a + b/x), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.247426, size = 1, normalized size = 0.02

$$\left[ \frac{2\sqrt{ax}\sqrt{\frac{ax+b}{x}} + b \log\left(-2ax\sqrt{\frac{ax+b}{x}} + (2ax+b)\sqrt{a}\right)}{2a^{\frac{3}{2}}}, \frac{\sqrt{-ax}\sqrt{\frac{ax+b}{x}} + b \arctan\left(\frac{a}{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}\right)}{\sqrt{-aa}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(a + b/x), x, algorithm="fricas")`

[Out]  $[1/2 * (2 * \sqrt{a} * x * \sqrt{(a * x + b) / x} + b * \log(-2 * a * x * \sqrt{(a * x + b) / x} + (2 * a * x + b) * \sqrt{a})) / a^{(3/2)}, (\sqrt{-a} * x * \sqrt{(a * x + b) / x} + b * \arctan(a / (\sqrt{-a} * \sqrt{(a * x + b) / x}))) / (\sqrt{-a} * a)]$

**Sympy [A]** time = 7.83873, size = 44, normalized size = 1.02

$$\frac{\sqrt{b}\sqrt{x}\sqrt{\frac{ax}{b} + 1}}{a} - \frac{b \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b/x)**(1/2), x)`

[Out]  $\sqrt{b} * \sqrt{x} * \sqrt{a * x / b + 1} / a - b * \operatorname{asinh}(\sqrt{a} * \sqrt{x} / \sqrt{b}) / a^{(3/2)}$

**GIAC/XCAS [A]** time = 0.237001, size = 96, normalized size = 2.23

$$-\frac{b \ln(|b|) \operatorname{sign}(x)}{2a^{\frac{3}{2}}} + \frac{b \ln\left(\left|-2\left(\sqrt{ax} - \sqrt{ax^2 + bx}\right)\sqrt{a-b}\right|\right)}{2a^{\frac{3}{2}} \operatorname{sign}(x)} + \frac{\sqrt{ax^2 + bx}}{a \operatorname{sign}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(a + b/x),x, algorithm="giac")
```

```
[Out] -1/2*b*ln(abs(b))*sign(x)/a^(3/2) + 1/2*b*ln(abs(-2*(sqrt(a)*x -  
sqrt(a*x^2 + b*x))*sqrt(a) - b))/(a^(3/2)*sign(x)) + sqrt(a*x^2 +  
b*x)/(a*sign(x))
```

$$3.150 \quad \int \frac{1}{\sqrt{a + \frac{b}{x}} \left( c + \frac{d}{x} \right)} dx$$

**Optimal.** Leaf size=108

$$\frac{(2ad + bc) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{a^{3/2} c^2} - \frac{2d^{3/2} \tan^{-1} \left( \frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}} \right)}{c^2 \sqrt{bc - ad}} + \frac{x \sqrt{a + \frac{b}{x}}}{ac}$$

[Out] (Sqrt[a + b/x]\*x)/(a\*c) - (2\*d^(3/2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(c^2\*Sqrt[b\*c - a\*d]) - ((b\*c + 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(3/2)\*c^2)

**Rubi [A]** time = 0.316674, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{(2ad + bc) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{a^{3/2} c^2} - \frac{2d^{3/2} \tan^{-1} \left( \frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}} \right)}{c^2 \sqrt{bc - ad}} + \frac{x \sqrt{a + \frac{b}{x}}}{ac}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b/x]\*(c + d/x)),x]

[Out] (Sqrt[a + b/x]\*x)/(a\*c) - (2\*d^(3/2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(c^2\*Sqrt[b\*c - a\*d]) - ((b\*c + 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(3/2)\*c^2)

**Rubi in Sympy [A]** time = 40.7588, size = 92, normalized size = 0.85

$$\frac{2d^{3/2} \operatorname{atanh} \left( \frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{ad - bc}} \right)}{c^2 \sqrt{ad - bc}} + \frac{x \sqrt{a + \frac{b}{x}}}{ac} - \frac{2 \left( ad + \frac{bc}{2} \right) \operatorname{atanh} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{a^{3/2} c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c+d/x)/(a+b/x)\*\*(1/2),x)

[Out] 2\*d\*\*(3/2)\*atanh(sqrt(d)\*sqrt(a + b/x)/sqrt(a\*d - b\*c))/(c\*\*2\*sqrt(a\*d - b\*c)) + x\*sqrt(a + b/x)/(a\*c) - 2\*(a\*d + b\*c/2)\*atanh(sqrt(a + b/x)/sqrt(a))/(a\*\*(3/2)\*c\*\*2)

**Mathematica [A]** time = 0.467142, size = 157, normalized size = 1.45

$$\frac{(2ad+bc) \log \left( 2\sqrt{ax} \sqrt{a + \frac{b}{x}} + 2ax + b \right)}{a^{3/2}} + \frac{2d^{3/2} \log(cx+d)}{\sqrt{ad-bc}} - \frac{2d^{3/2} \log \left( 2\sqrt{dx} \sqrt{a + \frac{b}{x}} \sqrt{ad-bc} - 2adx + bcx - bd \right)}{\sqrt{ad-bc}} + \frac{2cx \sqrt{a + \frac{b}{x}}}{a}$$

$$2c^2$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b/x]\*(c + d/x)),x]

[Out] ((2\*c\*Sqrt[a + b/x]\*x)/a + (2\*d^(3/2)\*Log[d + c\*x])/Sqrt[-(b\*c) + a\*d]) - ((b\*c + 2\*a\*d)\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]\*x])

$\left. \right) / a^{3/2} - (2^2 d^{3/2}) \cdot \text{Log}[-(b^2 d) + b^2 c^2 x - 2^2 a^2 d^2 x + 2^2 \text{Sqrt}[d] \cdot \text{Sqrt}[-(b^2 c) + a^2 d] \cdot \text{Sqrt}[a + b/x] \cdot x) / \text{Sqrt}[-(b^2 c) + a^2 d] / (2^2 c^2)$

**Maple [B]** time = 0.019, size = 389, normalized size = 3.6

$$\frac{x}{2c^3(ad-bc)} \sqrt{\frac{ax+b}{x}} \left( -2d^2 \ln \left( \frac{1}{2} \frac{2\sqrt{x(ax+b)}\sqrt{a} + 2ax + b}{\sqrt{a}} \right) a^3 c \sqrt{\frac{(ad-bc)d}{c^2}} + 2d\sqrt{x(ax+b)}c^2 a^{5/2} \sqrt{\frac{(ad-bc)d}{c^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+d/x)/(a+b/x)^(1/2), x)

[Out]  $\frac{1}{2} \left( (a^2 x + b) / x \right)^{1/2} x^2 \left( -2^2 d^2 \ln \left( \frac{1}{2} \left( 2^2 (x^2 (a^2 x + b))^{1/2} a^{1/2} + 2^2 a^2 x + b \right) / a^{1/2} \right) a^3 c^2 \left( (a^2 d - b^2 c) d / c^2 \right)^{1/2} + 2^2 d^2 (x^2 (a^2 x + b))^{1/2} c^2 a^{5/2} \left( (a^2 d - b^2 c) d / c^2 \right)^{1/2} - 2^2 b^2 (x^2 (a^2 x + b))^{1/2} c^2 a^{3/2} a^{3/2} \left( (a^2 d - b^2 c) d / c^2 \right)^{1/2} - 2^2 d^2 \ln \left( \frac{1}{2} \left( 2^2 (x^2 (a^2 x + b))^{1/2} a^{1/2} + 2^2 a^2 x + b \right) / a^{1/2} \right) \left( (a^2 d - b^2 c) d / c^2 \right)^{1/2} c - 2^2 a^2 d^2 x + b^2 c^2 x - b^2 d \right) / (c^2 x + d) a^{7/2} + 2^2 d^2 \ln \left( \frac{1}{2} \left( 2^2 (x^2 (a^2 x + b))^{1/2} a^{1/2} + 2^2 a^2 x + b \right) / a^{1/2} \right) \left( (a^2 d - b^2 c) d / c^2 \right)^{1/2} c - 2^2 a^2 d^2 x + b^2 c^2 x - b^2 d \right) / (c^2 x + d) b^2 c^2 a^{5/2} + d^2 \ln \left( \frac{1}{2} \left( 2^2 (x^2 (a^2 x + b))^{1/2} a^{1/2} + 2^2 a^2 x + b \right) / a^{1/2} \right) b^2 c^2 a^{5/2} \left( (a^2 d - b^2 c) d / c^2 \right)^{1/2} + b^2 a^2 \ln \left( \frac{1}{2} \left( 2^2 (x^2 (a^2 x + b))^{1/2} a^{1/2} + 2^2 a^2 x + b \right) / a^{1/2} \right) c^2 a^{3/2} \left( (a^2 d - b^2 c) d / c^2 \right)^{1/2} a \right) / (x^2 (a^2 x + b))^{1/2} / c^3 / (a^2 d - b^2 c) / a^{5/2} / \left( (a^2 d - b^2 c) d / c^2 \right)^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(a + b/x)\*(c + d/x)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.277234, size = 1, normalized size = 0.01

$$\left[ \frac{2 a^{\frac{3}{2}} d \sqrt{-\frac{d}{bc-ad}} \log \left( -\frac{2(bc-ad)x \sqrt{-\frac{d}{bc-ad}} \sqrt{\frac{ax+b}{x}} - bd + (bc-2ad)x}{cx+d} \right) + 2 \sqrt{acx} \sqrt{\frac{ax+b}{x}} + (bc+2ad) \log \left( -2ax \sqrt{\frac{ax+b}{x}} + (2ax+b) \sqrt{a} \right)}{2 a^{\frac{3}{2}} c^2}, \right. \\ \left. \frac{4 a^{\frac{3}{2}} d \sqrt{\frac{d}{bc-ad}} \arctan \left( -\frac{(bc-ad) \sqrt{\frac{d}{bc-ad}}}{d \sqrt{\frac{ax+b}{x}}} \right) - 2 \sqrt{acx} \sqrt{\frac{ax+b}{x}} - (bc+2ad) \log \left( -2ax \sqrt{\frac{ax+b}{x}} + (2ax+b) \sqrt{a} \right)}{2 a^{\frac{3}{2}} c^2}, \right. \\ \left. \frac{2 \sqrt{-aad} \sqrt{\frac{d}{bc-ad}} \arctan \left( -\frac{(bc-ad) \sqrt{\frac{d}{bc-ad}}}{d \sqrt{\frac{ax+b}{x}}} \right) - \sqrt{-acx} \sqrt{\frac{ax+b}{x}} - (bc+2ad) \arctan \left( \frac{a}{\sqrt{-a} \sqrt{\frac{ax+b}{x}}} \right)}{\sqrt{-aac^2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(a + b/x)\*(c + d/x)), x, algorithm="fricas")

[Out]  $\left[ \frac{1}{2} \left( 2^2 a^{3/2} \right) d^2 \text{sqrt}(-d/(b^2 c - a^2 d)) \cdot \log(-2^2 (b^2 c - a^2 d) x^2 \text{sqrt}(-d/(b^2 c - a^2 d)) \cdot \text{sqrt}((a^2 x + b)/x) - b^2 d + (b^2 c - 2^2 a^2 d) x) / (c^2 x$

+ d)) + 2\*sqrt(a)\*c\*x\*sqrt((a\*x + b)/x) + (b\*c + 2\*a\*d)\*log(-2\*a\*x\*sqrt((a\*x + b)/x) + (2\*a\*x + b)\*sqrt(a))/(a^(3/2)\*c^2), (sqrt(-a)\*a\*d\*sqrt(-d/(b\*c - a\*d))\*log(-(2\*(b\*c - a\*d)\*x\*sqrt(-d/(b\*c - a\*d))\*sqrt((a\*x + b)/x) - b\*d + (b\*c - 2\*a\*d)\*x)/(c\*x + d)) + sqrt(-a)\*c\*x\*sqrt((a\*x + b)/x) + (b\*c + 2\*a\*d)\*arctan(a/(sqrt(-a)\*sqrt((a\*x + b)/x)))/(sqrt(-a)\*a\*c^2), -1/2\*(4\*a^(3/2)\*d\*sqrt(d/(b\*c - a\*d))\*arctan(-(b\*c - a\*d)\*sqrt(d/(b\*c - a\*d))/(d\*sqrt((a\*x + b)/x))) - 2\*sqrt(a)\*c\*x\*sqrt((a\*x + b)/x) - (b\*c + 2\*a\*d)\*log(-2\*a\*x\*sqrt((a\*x + b)/x) + (2\*a\*x + b)\*sqrt(a))/(a^(3/2)\*c^2), -(2\*sqrt(-a)\*a\*d\*sqrt(d/(b\*c - a\*d))\*arctan(-(b\*c - a\*d)\*sqrt(d/(b\*c - a\*d))/(d\*sqrt((a\*x + b)/x))) - sqrt(-a)\*c\*x\*sqrt((a\*x + b)/x) - (b\*c + 2\*a\*d)\*arctan(a/(sqrt(-a)\*sqrt((a\*x + b)/x)))/(sqrt(-a)\*a\*c^2)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + \frac{b}{x}}(cx + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d/x)/(a+b/x)\*\*(1/2),x)

[Out] Integral(x/(sqrt(a + b/x)\*(c\*x + d)), x)

**GIAC/XCAS [A]** time = 0.252986, size = 174, normalized size = 1.61

$$-b \left( \frac{2d^2 \arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}bc^2} + \frac{\sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right)ac} - \frac{(bc + 2ad) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}bc^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(a + b/x)\*(c + d/x)),x, algorithm="giac")

[Out] -b\*(2\*d^2\*arctan(d\*sqrt((a\*x + b)/x)/sqrt(b\*c\*d - a\*d^2))/(sqrt(b\*c\*d - a\*d^2)\*b\*c^2) + sqrt((a\*x + b)/x)/((a - (a\*x + b)/x)\*a\*c) - (b\*c + 2\*a\*d)\*arctan(sqrt((a\*x + b)/x)/sqrt(-a))/(sqrt(-a)\*a\*b\*c^2)



$$3.151 \quad \int \frac{1}{\sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)^2} dx$$

**Optimal.** Leaf size=172

$$-\frac{(4ad+bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3} - \frac{d^{3/2}(5bc-4ad) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}} + \frac{d\sqrt{a+\frac{b}{x}}(bc-2ad)}{ac^2\left(c+\frac{d}{x}\right)(bc-ad)} + \frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

[Out] (d\*(b\*c - 2\*a\*d)\*Sqrt[a + b/x])/(a\*c^2\*(b\*c - a\*d)\*(c + d/x)) + (Sqrt[a + b/x]\*x)/(a\*c\*(c + d/x)) - (d^(3/2)\*(5\*b\*c - 4\*a\*d)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(c^3\*(b\*c - a\*d)^(3/2)) - ((b\*c + 4\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(3/2)\*c^3)

**Rubi [A]** time = 0.650209, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{(4ad+bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3} - \frac{d^{3/2}(5bc-4ad) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}} + \frac{d\sqrt{a+\frac{b}{x}}(bc-2ad)}{ac^2\left(c+\frac{d}{x}\right)(bc-ad)} + \frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b/x]\*(c + d/x)^2), x]

[Out] (d\*(b\*c - 2\*a\*d)\*Sqrt[a + b/x])/(a\*c^2\*(b\*c - a\*d)\*(c + d/x)) + (Sqrt[a + b/x]\*x)/(a\*c\*(c + d/x)) - (d^(3/2)\*(5\*b\*c - 4\*a\*d)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(c^3\*(b\*c - a\*d)^(3/2)) - ((b\*c + 4\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(3/2)\*c^3)

**Rubi in Sympy [A]** time = 68.0127, size = 141, normalized size = 0.82

$$-\frac{dx\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)(ad-bc)} + \frac{d^{3/2}(4ad-5bc) \operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{c^3(ad-bc)^{3/2}} + \frac{x\sqrt{a+\frac{b}{x}}(2ad-bc)}{ac^2(ad-bc)} - \frac{(4ad+bc) \operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c+d/x)\*\*2/(a+b/x)\*\*(1/2), x)

[Out] -d\*x\*sqrt(a + b/x)/(c\*(c + d/x)\*(a\*d - b\*c)) + d\*\*(3/2)\*(4\*a\*d - 5\*b\*c)\*atanh(sqrt(d)\*sqrt(a + b/x)/sqrt(a\*d - b\*c))/(c\*\*3\*(a\*d - b\*c)\*\*(3/2)) + x\*sqrt(a + b/x)\*(2\*a\*d - b\*c)/(a\*c\*\*2\*(a\*d - b\*c)) - (4\*a\*d + b\*c)\*atanh(sqrt(a + b/x)/sqrt(a))/(a\*\*(3/2)\*c\*\*3)

**Mathematica [C]** time = 0.84081, size = 224, normalized size = 1.3

$$-\frac{(4ad+bc) \log\left(2\sqrt{ax}\sqrt{a+\frac{b}{x}}+2ax+b\right)}{a^{3/2}} + \frac{id^{3/2}(5bc-4ad) \log\left(\frac{2c^4\sqrt{bc-ad}\left(2\sqrt{d}x\sqrt{a+\frac{b}{x}}\sqrt{bc-ad}-2iadx-ib(d-cx)\right)}{d^{5/2}(cx+d)(5bc-4ad)}\right)}{(bc-ad)^{3/2}} + \frac{2cx\sqrt{a+\frac{b}{x}}(bc(cx+d)-ad(cx+2d))}{a(cx+d)(ad-bc)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b/x]\*(c + d/x)^2),x]

[Out] 
$$-\left(\frac{2c\sqrt{a + b/x}x(bcd + c^2x) - ad(2d + cx)}{(a(-bc) + ad)(d + cx)} + \frac{(bc + 4ad)\text{Log}[b + 2ax + 2\sqrt{a}\sqrt{a + b/x}x]}{a^{3/2}} + \frac{(I d^{3/2}(5bc - 4ad)\text{Log}[(2c^4\sqrt{bc - ad})^{(-2I)adx + 2\sqrt{d}\sqrt{bc - ad}\sqrt{a + b/x}x - I b(d - cx)])]}{d^{5/2}(5bc - 4ad)(d + cx)}\right) / (bc - ad)^{3/2} / (2c^3)$$

**Maple [B]** time = 0.029, size = 1137, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+d/x)^2/(a+b/x)^(1/2),x)

[Out] 
$$\frac{1}{2} \left( \frac{(ax+b)^{1/2}}{x} x^{-4} \ln\left(\frac{1}{2} (2(x(ax+b))^{1/2} a^{1/2} + 2ax+b)/a^{1/2}\right) \left(\frac{ad-bc}{c^2}\right)^{1/2} a^4 x^2 c^2 d^3 - 2(x(ax+b))^{1/2} \left(\frac{ad-bc}{c^2}\right)^{1/2} a^{7/2} x^2 c^4 d - 4 \ln\left(\frac{1}{2} (2(x(ax+b))^{1/2} a^{1/2} + 2ax+b)/a^{1/2}\right) \left(\frac{ad-bc}{c^2}\right)^{1/2} a^4 c^2 d^4 + 7 \ln\left(\frac{1}{2} (2(x(ax+b))^{1/2} a^{1/2} + 2ax+b)/a^{1/2}\right) \left(\frac{ad-bc}{c^2}\right)^{1/2} a^3 x^2 b^2 c^3 d^2 + 2d^2 c^4 (x(ax+b))^{3/2} a^{5/2} \left(\frac{ad-bc}{c^2}\right)^{1/2} + 2(x(ax+b))^{1/2} \left(\frac{ad-bc}{c^2}\right)^{1/2} a^{7/2} x^2 c^3 d^2 - 6(x(ax+b))^{1/2} \left(\frac{ad-bc}{c^2}\right)^{1/2} a^{5/2} x^2 b^2 c^4 d + 2(x(ax+b))^{1/2} \left(\frac{ad-bc}{c^2}\right)^{1/2} a^{3/2} x^2 b^2 c^5 + 7 \ln\left(\frac{1}{2} (2(x(ax+b))^{1/2} a^{1/2} + 2ax+b)/a^{1/2}\right) \left(\frac{ad-bc}{c^2}\right)^{1/2} a^3 b^2 c^2 d^3 - 2 \ln\left(\frac{1}{2} (2(x(ax+b))^{1/2} a^{1/2} + 2ax+b)/a^{1/2}\right) \left(\frac{ad-bc}{c^2}\right)^{1/2} a^2 x^2 b^2 c^4 d - 4 \ln\left(\frac{1}{2} (2(x(ax+b))^{1/2} \left(\frac{ad-bc}{c^2}\right)^{1/2} c - 2ad^2x + b^2cx - b^2d)/(cx+d)\right) a^{9/2} x^2 c^2 d^4 + 9 \ln\left(\frac{1}{2} (2(x(ax+b))^{1/2} \left(\frac{ad-bc}{c^2}\right)^{1/2} c - 2ad^2x + b^2cx - b^2d)/(cx+d)\right) a^{7/2} x^2 b^2 c^2 d^3 - 5 \ln\left(\frac{1}{2} (2(x(ax+b))^{1/2} \left(\frac{ad-bc}{c^2}\right)^{1/2} c - 2ad^2x + b^2cx - b^2d)/(cx+d)\right) a^{5/2} x^2 b^2 c^3 d^2 + 4(x(ax+b))^{1/2} \left(\frac{ad-bc}{c^2}\right)^{1/2} a^{7/2} c^2 d^3 - 6(x(ax+b))^{1/2} \left(\frac{ad-bc}{c^2}\right)^{1/2} a^{5/2} b^2 c^3 d^2 + 2(x(ax+b))^{1/2} \left(\frac{ad-bc}{c^2}\right)^{1/2} a^{3/2} b^2 c^4 d - 2 \ln\left(\frac{1}{2} (2(x(ax+b))^{1/2} a^{1/2} + 2ax+b)/a^{1/2}\right) \left(\frac{ad-bc}{c^2}\right)^{1/2} a^2 b^2 c^3 d^2 - \ln\left(\frac{1}{2} (2(x(ax+b))^{1/2} a^{1/2} + 2ax+b)/a^{1/2}\right) \left(\frac{ad-bc}{c^2}\right)^{1/2} x^2 a^3 c^5 - 4 \ln\left(\frac{1}{2} (2(x(ax+b))^{1/2} \left(\frac{ad-bc}{c^2}\right)^{1/2} c - 2ad^2x + b^2cx - b^2d)/(cx+d)\right) a^{9/2} d^5 + 9 \ln\left(\frac{1}{2} (2(x(ax+b))^{1/2} \left(\frac{ad-bc}{c^2}\right)^{1/2} c - 2ad^2x + b^2cx - b^2d)/(cx+d)\right) a^{7/2} b^2 c^2 d^4 - 5 \ln\left(\frac{1}{2} (2(x(ax+b))^{1/2} \left(\frac{ad-bc}{c^2}\right)^{1/2} c - 2ad^2x + b^2cx - b^2d)/(cx+d)\right) a^{5/2} b^2 c^2 d^3 - \ln\left(\frac{1}{2} (2(x(ax+b))^{1/2} a^{1/2} + 2ax+b)/a^{1/2}\right) \left(\frac{ad-bc}{c^2}\right)^{1/2} a^2 b^3 c^4 d / (x(ax+b))^{1/2} / a^{5/2} / (ad-bc)^{1/2} / c^4 / \left(\frac{ad-bc}{c^2}\right)^{1/2} / (cx+d)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(a + b/x)\*(c + d/x)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.359771, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(a + b/x)\*(c + d/x)^2),x, algorithm="fricas")

[Out] 
$$\left[ \frac{1}{2} \left( (5ab^2cd^2 - 4a^2d^3 + (5ab^2c^2d - 4a^2c^2d^2)x) \sqrt{a} \sqrt{-d/(b^2c - a^2d)} \log\left(-2(b^2c - a^2d)x \sqrt{-d/(b^2c - a^2d)} \sqrt{(ax+b)/x} - b^2d + (b^2c - 2a^2d)x/(cx+d)\right) + 2 \left( (b^2c^3 - a^2c^2d)x^2 + (b^2c^2d - 2a^2c^2d^2)x \right) \sqrt{a} \sqrt{(ax+b)/x} + (b^2c^2d + 3a^2b^2c^2d^2 - 4a^2d^3 + (b^2c^3 + 3a^2b^2c^2d - 4a^2c^2d^2)x) \log\left(-2ax \sqrt{(ax+b)/x} + (2ax+b) \sqrt{a}\right) \right) / \left( (a^2b^2c^4d - a^2c^3d^2 + (a^2b^2c^5 - a^2c^4d)x) \sqrt{a} \right), -\frac{1}{2} \left( 2(5ab^2cd^2 - 4a^2d^3 + (5ab^2c^2d - 4a^2c^2d^2)x) \sqrt{a} \sqrt{d/(b^2c - a^2d)} \arctan\left(-\frac{b^2c - a^2d}{\sqrt{d/(b^2c - a^2d)}} \sqrt{(ax+b)/x}\right) - 2 \left( (b^2c^3 - a^2c^2d)x^2 + (b^2c^2d - 2a^2c^2d^2)x \right) \sqrt{a} \sqrt{(ax+b)/x} - (b^2c^2d + 3a^2b^2c^2d^2 - 4a^2d^3 + (b^2c^3 + 3a^2b^2c^2d - 4a^2c^2d^2)x) \log\left(-2ax \sqrt{(ax+b)/x} + (2ax+b) \sqrt{a}\right) \right) / \left( (a^2b^2c^4d - a^2c^3d^2 + (a^2b^2c^5 - a^2c^4d)x) \sqrt{a} \right), \frac{1}{2} \left( (5ab^2cd^2 - 4a^2d^3 + (5ab^2c^2d - 4a^2c^2d^2)x) \sqrt{-a} \sqrt{-d/(b^2c - a^2d)} \log\left(-2(b^2c - a^2d)x \sqrt{-d/(b^2c - a^2d)} \sqrt{(ax+b)/x} - b^2d + (b^2c - 2a^2d)x/(cx+d)\right) + 2 \left( (b^2c^3 - a^2c^2d)x^2 + (b^2c^2d - 2a^2c^2d^2)x \right) \sqrt{-a} \sqrt{(ax+b)/x} + 2 \left( b^2c^2d + 3a^2b^2c^2d^2 - 4a^2d^3 + (b^2c^3 + 3a^2b^2c^2d - 4a^2c^2d^2)x \right) \arctan\left(\frac{a}{\sqrt{-a} \sqrt{(ax+b)/x}}\right) \right) / \left( (a^2b^2c^4d - a^2c^3d^2 + (a^2b^2c^5 - a^2c^4d)x) \sqrt{-a} \right), - \left( (5ab^2cd^2 - 4a^2d^3 + (5ab^2c^2d - 4a^2c^2d^2)x) \sqrt{-a} \sqrt{d/(b^2c - a^2d)} \arctan\left(-\frac{b^2c - a^2d}{\sqrt{d/(b^2c - a^2d)}} \sqrt{(ax+b)/x}\right) - \left( (b^2c^3 - a^2c^2d)x^2 + (b^2c^2d - 2a^2c^2d^2)x \right) \sqrt{-a} \sqrt{(ax+b)/x} - (b^2c^2d + 3a^2b^2c^2d^2 - 4a^2d^3 + (b^2c^3 + 3a^2b^2c^2d - 4a^2c^2d^2)x) \arctan\left(\frac{a}{\sqrt{-a} \sqrt{(ax+b)/x}}\right) \right) / \left( (a^2b^2c^4d - a^2c^3d^2 + (a^2b^2c^5 - a^2c^4d)x) \sqrt{-a} \right) \right]$$

**Sympy** [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d/x)\*\*2/(a+b/x)\*\*(1/2),x)

[Out] Exception raised: ValueError

**GIAC/XCAS** [A] time = 0.25707, size = 393, normalized size = 2.28

$$-b \left( \frac{(5bcd^2 - 4ad^3) \arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{(b^2c^4 - abc^3d)\sqrt{bcd-ad^2}} + \frac{b^2c^2\sqrt{\frac{ax+b}{x}} - 2abcd\sqrt{\frac{ax+b}{x}} + 2a^2d^2\sqrt{\frac{ax+b}{x}} + \frac{(ax+b)bcd\sqrt{\frac{ax+b}{x}}}{x} - \frac{2(ax+b)ad^2\sqrt{\frac{ax+b}{x}}}{x}}{(abc^3 - a^2c^2d)\left(abc - a^2d - \frac{(ax+b)bc}{x} + \frac{2(ax+b)ad}{x} - \frac{(ax+b)^2d}{x^2}\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(a + b/x)\*(c + d/x)^2),x, algorithm="giac")

[Out] 
$$-b \left( (5b^2cd^2 - 4a^2d^3) \arctan\left(\frac{d\sqrt{(ax+b)/x}}{\sqrt{bcd-ad^2}}\right) / \sqrt{b^2c^2d - a^2d^2} + (b^2c^2d + 3a^2b^2c^2d^2 - 4a^2d^3 + (b^2c^3 + 3a^2b^2c^2d - 4a^2c^2d^2)x) \sqrt{a} \sqrt{(ax+b)/x} - 2a^2b^2c^2d^2 \sqrt{(ax+b)/x} + 2a^2d^2 \sqrt{(ax+b)/x} + (a^2b^2c^5 - a^2c^4d)x \sqrt{a} \sqrt{(ax+b)/x} + (a^2b^2c^4d - a^2c^3d^2) \sqrt{b^2c^2d - a^2d^2} \right) / \left( (a^2b^2c^4d - a^2c^3d^2 + (a^2b^2c^5 - a^2c^4d)x) \sqrt{a} \right) - \left( (b^2c^3 - a^2c^2d)x^2 + (b^2c^2d - 2a^2c^2d^2)x \right) \sqrt{-a} \sqrt{(ax+b)/x} - (b^2c^2d + 3a^2b^2c^2d^2 - 4a^2d^3 + (b^2c^3 + 3a^2b^2c^2d - 4a^2c^2d^2)x) \arctan\left(\frac{a}{\sqrt{-a} \sqrt{(ax+b)/x}}\right) \right) / \left( (a^2b^2c^4d - a^2c^3d^2 + (a^2b^2c^5 - a^2c^4d)x) \sqrt{-a} \right)$$

$$3.152 \quad \int \frac{1}{\sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)^3} dx$$

**Optimal.** Leaf size=250

$$\frac{(6ad+bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^4} - \frac{d^{3/2}(24a^2d^2-56abcd+35b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{5/2}} \\ + \frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{4ac^3\left(c+\frac{d}{x}\right)(bc-ad)^2} + \frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{2ac^2\left(c+\frac{d}{x}\right)^2(bc-ad)} + \frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

[Out]  $(d*(2*b*c - 3*a*d)*\text{Sqrt}[a + b/x])/(2*a*c^2*(b*c - a*d)*(c + d/x)^2) + (d*(b*c - 4*a*d)*(4*b*c - 3*a*d)*\text{Sqrt}[a + b/x])/(4*a*c^3*(b*c - a*d)^2*(c + d/x)) + (\text{Sqrt}[a + b/x]*x)/(a*c*(c + d/x)^2) - (d^{3/2}*(35*b^2*c^2 - 56*a*b*c*d + 24*a^2*d^2)*\text{ArcTan}[(\text{Sqrt}[d]*\text{Sqrt}[a + b/x])/\text{Sqrt}[b*c - a*d]])/(4*c^4*(b*c - a*d)^{(5/2)}) - ((b*c + 6*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/(a^{(3/2)}*c^4)$

**Rubi [A]** time = 1.08312, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{(6ad+bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^4} - \frac{d^{3/2}(24a^2d^2-56abcd+35b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{5/2}} \\ + \frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{4ac^3\left(c+\frac{d}{x}\right)(bc-ad)^2} + \frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{2ac^2\left(c+\frac{d}{x}\right)^2(bc-ad)} + \frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b/x]\*(c + d/x)^3), x]

[Out]  $(d*(2*b*c - 3*a*d)*\text{Sqrt}[a + b/x])/(2*a*c^2*(b*c - a*d)*(c + d/x)^2) + (d*(b*c - 4*a*d)*(4*b*c - 3*a*d)*\text{Sqrt}[a + b/x])/(4*a*c^3*(b*c - a*d)^2*(c + d/x)) + (\text{Sqrt}[a + b/x]*x)/(a*c*(c + d/x)^2) - (d^{3/2}*(35*b^2*c^2 - 56*a*b*c*d + 24*a^2*d^2)*\text{ArcTan}[(\text{Sqrt}[d]*\text{Sqrt}[a + b/x])/\text{Sqrt}[b*c - a*d]])/(4*c^4*(b*c - a*d)^{(5/2)}) - ((b*c + 6*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/(a^{(3/2)}*c^4)$

**Rubi in Sympy [A]** time = 117.385, size = 218, normalized size = 0.87

$$\frac{dx\sqrt{a+\frac{b}{x}}}{2c\left(c+\frac{d}{x}\right)^2(ad-bc)} - \frac{3dx\sqrt{a+\frac{b}{x}}(2ad-3bc)}{4c^2\left(c+\frac{d}{x}\right)(ad-bc)^2} + \frac{d^{3/2}(24a^2d^2-56abcd+35b^2c^2) \operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{4c^4(ad-bc)^{5/2}} \\ + \frac{x\sqrt{a+\frac{b}{x}}(3ad-4bc)(4ad-bc)}{4ac^3(ad-bc)^2} - \frac{(6ad+bc) \operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c+d/x)\*\*3/(a+b/x)\*\*(1/2), x)

[Out]  $-d*x*\text{sqrt}(a + b/x)/(2*c*(c + d/x)**2*(a*d - b*c)) - 3*d*x*\text{sqrt}(a + b/x)*(2*a*d - 3*b*c)/(4*c**2*(c + d/x)*(a*d - b*c)**2) + d**(3/2)$

$$2) * (24 * a^{**2} * d^{**2} - 56 * a * b * c * d + 35 * b^{**2} * c^{**2}) * \operatorname{atanh}(\operatorname{sqrt}(d) * \operatorname{sqrt}(a + b/x) / \operatorname{sqrt}(a * d - b * c)) / (4 * c^{**4} * (a * d - b * c)^{(5/2)}) + x * \operatorname{sqrt}(a + b/x) * (3 * a * d - 4 * b * c) * (4 * a * d - b * c) / (4 * a * c^{**3} * (a * d - b * c)^{(3/2)}) - (6 * a * d + b * c) * \operatorname{atanh}(\operatorname{sqrt}(a + b/x) / \operatorname{sqrt}(a)) / (a^{(3/2)} * c^{**4})$$

**Mathematica [C]** time = 0.950155, size = 301, normalized size = 1.2

$$\frac{4(ad+bc)\log\left(2\sqrt{ax}\sqrt{a+\frac{b}{x}+2ax+b}\right)}{a^{3/2}} + \frac{2cx\sqrt{a+\frac{b}{x}}(2a^2d^2(2c^2x^2+9cdx+6d^2)-abcd(8c^2x^2+29cdx+19d^2)+4b^2c^2(cx+d)^2)}{a(cx+d)^2(bc-ad)^2} - \frac{id^{3/2}(24a^2d^2-56abcd+35b^2c^2)}{8c^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b/x] \* (c + d/x)^3), x]

[Out] ((2\*c\*Sqrt[a + b/x]\*x\*(4\*b^2\*c^2\*(d + c\*x)^2 + 2\*a^2\*d^2\*(6\*d^2 + 9\*c\*d\*x + 2\*c^2\*x^2) - a\*b\*c\*d\*(19\*d^2 + 29\*c\*d\*x + 8\*c^2\*x^2)) / (a\*(b\*c - a\*d)^2\*(d + c\*x)^2) - (4\*(b\*c + 6\*a\*d)\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]\*x]) / a^(3/2) - (I\*d^(3/2)\*(35\*b^2\*c^2 - 56\*a\*b\*c\*d + 24\*a^2\*d^2)\*Log[(8\*c^5\*(b\*c - a\*d)^(3/2)\*((-2\*I)\*a\*d\*x + 2\*Sqrt[d]\*Sqrt[b\*c - a\*d]\*Sqrt[a + b/x]\*x - I\*b\*(d - c\*x))] / (d^(5/2)\*(35\*b^2\*c^2 - 56\*a\*b\*c\*d + 24\*a^2\*d^2)\*(d + c\*x))) / (b\*c - a\*d)^(5/2)) / (8\*c^4)

**Maple [B]** time = 0.025, size = 2269, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+d/x)^3/(a+b/x)^(1/2), x)

[Out] 1/8\*((a\*x+b)/x)^(1/2)\*x\*(-12\*(x\*(a\*x+b))^(1/2)\*a^(9/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*x^3\*c^5\*d^2+12\*(x\*(a\*x+b))^(3/2)\*a^(7/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*x\*c^5\*d^2+68\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^4\*((a\*d-b\*c)\*d/c^2)^(1/2)\*b\*c^2\*d^5+80\*ln((2\*(x\*(a\*x+b))^(1/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*c-2\*a\*d\*x+b\*c\*x-b\*d)/(c\*x+d))\*a^(9/2)\*x^2\*b\*c^3\*d^4-91\*ln((2\*(x\*(a\*x+b))^(1/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*c-2\*a\*d\*x+b\*c\*x-b\*d)/(c\*x+d))\*a^(7/2)\*x^2\*b^2\*c^4\*d^3+35\*ln((2\*(x\*(a\*x+b))^(1/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*c-2\*a\*d\*x+b\*c\*x-b\*d)/(c\*x+d))\*a^(5/2)\*x^2\*b^3\*c^5\*d^2-18\*(x\*(a\*x+b))^(3/2)\*a^(5/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*b\*c^5\*d^2+36\*(x\*(a\*x+b))^(1/2)\*a^(9/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*x\*c^3\*d^4-60\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^3\*((a\*d-b\*c)\*d/c^2)^(1/2)\*b^2\*c^3\*d^4+160\*ln((2\*(x\*(a\*x+b))^(1/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*c-2\*a\*d\*x+b\*c\*x-b\*d)/(c\*x+d))\*a^(9/2)\*x\*b\*c^2\*d^5-182\*ln((2\*(x\*(a\*x+b))^(1/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*c-2\*a\*d\*x+b\*c\*x-b\*d)/(c\*x+d))\*a^(7/2)\*x\*b^2\*c^3\*d^4+70\*ln((2\*(x\*(a\*x+b))^(1/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*c-2\*a\*d\*x+b\*c\*x-b\*d)/(c\*x+d))\*a^(5/2)\*x\*b^3\*c^4\*d^3-62\*(x\*(a\*x+b))^(1/2)\*a^(7/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*b\*c^3\*d^4-8\*(x\*(a\*x+b))^(1/2)\*a^(3/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*x^2\*b^3\*c^7+4\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*((a\*d-b\*c)\*d/c^2)^(1/2)\*x^2\*a\*b^4\*c^7-8\*(x\*(a\*x+b))^(1/2)\*a^(3/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*b^3\*c^5\*d^2+4\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*((a\*d-b\*c)\*d/c^2)^(1/2)\*a\*b^4\*c^5\*d^2-24\*ln((2\*(x\*(a\*x+b))^(1/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*c-2\*a\*d\*x+b\*c\*x-b\*d)/(c\*x+d))\*a^(11/2)\*d^7+12\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^2\*((a\*d-b\*c)\*d/c^2)^(1/2)\*b^3\*c^4\*d^3-24\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^5\*((a\*d-b\*c)\*d/c^2)^(1/2)\*x^2\*c^3\*d^4-48\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^5\*((a\*d-b\*c)\*d/c^2)^(1/2)\*x\*c^2\*d^5-18\*(x\*(a\*x+b))^(1/2)\*a^(7/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*x^2\*b\*c^5\*d^2+46\*(x\*(a\*x+b))^(1/2)\*a^(5/2)\*((a\*d-b\*c)\*d/c^2)^(1/2)\*x^2\*b^2\*c^6\*d+68\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^4\*((a\*d-b\*c)\*d/c^2)^(1/2)\*x^2\*b

$$\begin{aligned}
& c^4 d^3 + 22 (x (a x + b))^{1/2} a^{7/2} ((a d - b c) d / c^2)^{1/2} x^3 \\
& b^2 c^6 d + 136 \ln(1/2 (2 (x (a x + b))^{1/2} a^{1/2} + 2 a x + b) / a^{1/2}) \\
& ) a^4 ((a d - b c) d / c^2)^{1/2} x^2 b^2 c^3 d^4 - 60 \ln(1/2 (2 (x (a x + b)) \\
& )^{1/2} a^{1/2} + 2 a x + b) / a^{1/2}) a^3 ((a d - b c) d / c^2)^{1/2} x^2 \\
& b^2 c^5 d^2 - 22 (x (a x + b))^{3/2} a^{5/2} ((a d - b c) d / c^2)^{1/2} \\
& x^2 b^2 c^6 d + 92 (x (a x + b))^{1/2} a^{5/2} ((a d - b c) d / c^2)^{1/2} x \\
& b^2 c^5 d^2 + 24 \ln(1/2 (2 (x (a x + b))^{1/2} a^{1/2} + 2 a x + b) / a^{1/2}) \\
& ) a^2 ((a d - b c) d / c^2)^{1/2} x^2 b^3 c^5 d^2 - 120 \ln(1/2 (2 (x (a x + b)) \\
& )^{1/2} a^{1/2} + 2 a x + b) / a^{1/2}) a^3 ((a d - b c) d / c^2)^{1/2} \\
& x^2 b^2 c^4 d^3 + 12 \ln(1/2 (2 (x (a x + b))^{1/2} a^{1/2} + 2 a x + b) / \\
& a^{1/2}) a^2 ((a d - b c) d / c^2)^{1/2} x^2 b^3 c^6 d - 102 (x (a x + b)) \\
& )^{1/2} a^{7/2} ((a d - b c) d / c^2)^{1/2} x^2 b^3 c^4 d^3 - 16 (x (a x + b)) \\
& )^{1/2} a^{3/2} ((a d - b c) d / c^2)^{1/2} x^2 b^3 c^6 d + 8 \ln(1/2 (2 (x (a x + b)) \\
& )^{1/2} a^{1/2} + 2 a x + b) / a^{1/2}) ((a d - b c) d / c^2)^{1/2} \\
& x^2 a^4 c^6 d - 24 \ln(1/2 (2 (x (a x + b))^{1/2} a^{1/2} + 2 a x + b) / a \\
& )^{1/2}) a^5 ((a d - b c) d / c^2)^{1/2} c^2 d^6 - 24 \ln((2 (x (a x + b))^{1/2} \\
& )^{1/2} ((a d - b c) d / c^2)^{1/2} c - 2 a^2 d x + b^2 c x - b^2 d) / (c x + d)) a^{11/2} \\
& ) x^2 c^2 d^5 + 8 (x (a x + b))^{3/2} a^{7/2} ((a d - b c) d / c^2)^{1/2} \\
& c^4 d^3 - 48 \ln((2 (x (a x + b))^{1/2} ((a d - b c) d / c^2)^{1/2} c - 2 a \\
& )^{1/2} a^2 d x + b^2 c x - b^2 d) / (c x + d)) a^{11/2} x^2 c^2 d^6 + 24 (x (a x + b))^{1/2} a^9 \\
& )^{1/2} ((a d - b c) d / c^2)^{1/2} c^2 d^5 + 80 \ln((2 (x (a x + b))^{1/2} \\
& )^{1/2} ((a d - b c) d / c^2)^{1/2} c - 2 a^2 d x + b^2 c x - b^2 d) / (c x + d)) a^{9/2} b^2 c \\
& )^{1/2} d^6 - 91 \ln((2 (x (a x + b))^{1/2} ((a d - b c) d / c^2)^{1/2} c - 2 a^2 d x \\
& )^{1/2} a^2 d x + b^2 c x - b^2 d) / (c x + d)) a^{7/2} b^2 c^2 d^5 + 35 \ln((2 (x (a x + b))^{1/2} \\
& )^{1/2} ((a d - b c) d / c^2)^{1/2} c - 2 a^2 d x + b^2 c x - b^2 d) / (c x + d)) a^{5/2} \\
& b^3 c^3 d^4 + 46 (x (a x + b))^{1/2} a^{5/2} ((a d - b c) d / c^2)^{1/2} \\
& b^2 c^4 d^3) / (x (a x + b))^{1/2} / c^5 / (a d - b c)^3 / (c x + d)^2 / ((a d - b c) \\
& )^{1/2} / a^{5/2}
\end{aligned}$$


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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(a + b/x)\*(c + d/x)^3),x, algorithm="maxima")

[Out] Exception raised: ValueError

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**Fricas [A]** time = 0.766872, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(a + b/x)\*(c + d/x)^3),x, algorithm="fricas")

[Out] 
$$\begin{aligned}
& [1/8 * ((35 * a^2 * b^2 * c^2 * d^3 - 56 * a^2 * b^2 * c^2 * d^4 + 24 * a^3 * d^5 + (35 * a^2 * b^2 * \\
& c^4 * d - 56 * a^2 * b^2 * c^3 * d^2 + 24 * a^3 * c^2 * d^3) * x^2 + 2 * (35 * a^2 * b^2 * c^3 * \\
& d^2 - 56 * a^2 * b^2 * c^2 * d^3 + 24 * a^3 * c^2 * d^4) * x) * \sqrt{a} * \sqrt{-d / (b * c - \\
& a * d)} * \log(-2 * (b * c - a * d) * x * \sqrt{-d / (b * c - a * d)}) * \sqrt{(a * x + b) / \\
& x) - b * d + (b * c - 2 * a * d) * x) / (c * x + d) + 2 * (4 * (b^2 * c^5 - 2 * a * b^2 * c^4 * \\
& d + a^2 * c^3 * d^2) * x^3 + (8 * b^2 * c^4 * d - 29 * a * b^2 * c^3 * d^2 + 18 * a^2 * c^2 * d^3) * x^2 + \\
& (4 * b^2 * c^3 * d^2 - 19 * a * b^2 * c^2 * d^3 + 12 * a^2 * c^2 * d^4) * x) * \\
& \sqrt{a} * \sqrt{(a * x + b) / x) + 4 * (b^3 * c^3 * d^2 + 4 * a * b^2 * c^2 * d^3 - 11 * \\
& a^2 * b^2 * c^2 * d^4 + 6 * a^3 * d^5 + (b^3 * c^5 + 4 * a * b^2 * c^4 * d - 11 * a^2 * b^2 * c^3 * \\
& d^2 + 6 * a^3 * c^2 * d^3) * x^2 + 2 * (b^3 * c^4 * d + 4 * a * b^2 * c^3 * d^2 - 11 * \\
& a^2 * b^2 * c^2 * d^3 + 6 * a^3 * c^2 * d^4) * x) * \log(-2 * a * x * \sqrt{(a * x + b) / x) + (2 * \\
& a * x + b) * \sqrt{a}) / ((a * b^2 * c^6 * d^2 - 2 * a^2 * b^2 * c^5 * d^3 + a^3 * c^4 * d^4 + \\
& (a * b^2 * c^8 - 2 * a^2 * b^2 * c^7 * d + a^3 * c^6 * d^2) * x^2 + 2 * (a * b^2 * c^7 * \\
& d - 2 * a^2 * b^2 * c^6 * d^2 + a^3 * c^5 * d^3) * x) * \sqrt{a}), 1/8 * ((35 * a^2 * b^2 * c^2 * \\
& d^3 - 56 * a^2 * b^2 * c^2 * d^4 + 24 * a^3 * d^5 + (35 * a^2 * b^2 * c^4 * d - 56 * a^2 * b^2 * \\
& c^3 * d^2 + 24 * a^3 * c^2 * d^3) * x^2 + 2 * (35 * a^2 * b^2 * c^3 * d^2 - 56 * a^2 * b^2 * c^2 * \\
& d^3 + 24 * a^3 * c^2 * d^4) * x) * \sqrt{-a} * \sqrt{-d / (b * c - a * d)} * \log(-2 * (b * c - \\
& a * d) * x * \sqrt{-d / (b * c - a * d)}) * \sqrt{(a * x + b) / x) - b * d + (b * c
\end{aligned}$$

$$\begin{aligned}
& - 2*a*d*x)/(c*x + d)) + 2*(4*(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*x^3 + (8*b^2*c^4*d - 29*a*b*c^3*d^2 + 18*a^2*c^2*d^3)*x^2 + (4*b^2*c^3*d^2 - 19*a*b*c^2*d^3 + 12*a^2*c*d^4)*x)*sqrt(-a)*sqrt((a*x + b)/x) + 8*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x)))/((a*b^2*c^6*d^2 - 2*a^2*b*c^5*d^3 + a^3*c^4*d^4 + (a*b^2*c^8 - 2*a^2*b*c^7*d + a^3*c^6*d^2)*x^2 + 2*(a*b^2*c^7*d - 2*a^2*b*c^6*d^2 + a^3*c^5*d^3)*x)*sqrt(-a)), -1/4*((35*a*b^2*c^2*d^3 - 56*a^2*b*c*d^4 + 24*a^3*d^5 + (35*a*b^2*c^4*d - 56*a^2*b*c^3*d^2 + 24*a^3*c^2*d^3)*x^2 + 2*(35*a*b^2*c^3*d^2 - 56*a^2*b*c^2*d^3 + 24*a^3*c*d^4)*x)*sqrt(a)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d/(b*c - a*d)))/(d*sqrt((a*x + b)/x))) - (4*(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*x^3 + (8*b^2*c^4*d - 29*a*b*c^3*d^2 + 18*a^2*c^2*d^3)*x^2 + (4*b^2*c^3*d^2 - 19*a*b*c^2*d^3 + 12*a^2*c*d^4)*x)*sqrt(a)*sqrt((a*x + b)/x) - 2*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*log(-2*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a))/((a*b^2*c^6*d^2 - 2*a^2*b*c^5*d^3 + a^3*c^4*d^4 + (a*b^2*c^8 - 2*a^2*b*c^7*d + a^3*c^6*d^2)*x^2 + 2*(a*b^2*c^7*d - 2*a^2*b*c^6*d^2 + a^3*c^5*d^3)*x)*sqrt(a)), -1/4*((35*a*b^2*c^2*d^3 - 56*a^2*b*c*d^4 + 24*a^3*d^5 + (35*a*b^2*c^4*d - 56*a^2*b*c^3*d^2 + 24*a^3*c^2*d^3)*x^2 + 2*(35*a*b^2*c^3*d^2 - 56*a^2*b*c^2*d^3 + 24*a^3*c*d^4)*x)*sqrt(-a)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d/(b*c - a*d)))/(d*sqrt((a*x + b)/x))) - (4*(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*x^3 + (8*b^2*c^4*d - 29*a*b*c^3*d^2 + 18*a^2*c^2*d^3)*x^2 + (4*b^2*c^3*d^2 - 19*a*b*c^2*d^3 + 12*a^2*c*d^4)*x)*sqrt(-a)*sqrt((a*x + b)/x) - 4*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x)))/((a*b^2*c^6*d^2 - 2*a^2*b*c^5*d^3 + a^3*c^4*d^4 + (a*b^2*c^8 - 2*a^2*b*c^7*d + a^3*c^6*d^2)*x^2 + 2*(a*b^2*c^7*d - 2*a^2*b*c^6*d^2 + a^3*c^5*d^3)*x)*sqrt(-a))]
\end{aligned}$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d/x)\*\*3/(a+b/x)\*\*(1/2),x)

[Out] Exception raised: ValueError

**GIAC/XCAS [A]** time = 0.261172, size = 459, normalized size = 1.84

$$-\frac{1}{4}b \left( \frac{(35b^2c^2d^2 - 56abcd^3 + 24a^2d^4) \arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{(b^3c^6 - 2ab^2c^5d + a^2bc^4d^2)\sqrt{bcd-ad^2}} + \frac{13b^2c^2d^2\sqrt{\frac{ax+b}{x}} - 21abcd^3\sqrt{\frac{ax+b}{x}} + 8a^2d^4\sqrt{\frac{ax+b}{x}} + \frac{11(ax+b)}{x}}{(b^2c^5 - 2abc^4d + a^2c^3d^2)(bc-ad+\frac{ax}{x})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(a + b/x)\*(c + d/x)^3),x, algorithm="giac")

[Out] -1/4\*b\*((35\*b^2\*c^2\*d^2 - 56\*a\*b\*c\*d^3 + 24\*a^2\*d^4)\*arctan(d\*sqrt((a\*x + b)/x)/sqrt(b\*c\*d - a\*d^2))/((b^3\*c^6 - 2\*a\*b^2\*c^5\*d + a^2\*b\*c^4\*d^2)\*sqrt(b\*c\*d - a\*d^2)) + (13\*b^2\*c^2\*d^2\*sqrt((a\*x + b)/x) - 21\*a\*b\*c\*d^3\*sqrt((a\*x + b)/x) + 8\*a^2\*d^4\*sqrt((a\*x + b)/x) + 11\*(a\*x + b)\*b\*c\*d^3\*sqrt((a\*x + b)/x)/x - 8\*(a\*x + b)\*a\*d^4

$$4\sqrt{(ax+b)/x}/x)/((b^2c^5 - 2ab^4c^4d + a^2c^3d^2)(bc - ad + (ax+b)d/x)^2) + 4\sqrt{(ax+b)/x}/((a - (ax+b)/x)a^3c^3) - 4(bc + 6ad)\arctan(\sqrt{(ax+b)/x}/\sqrt{-a})/(\sqrt{-a}ab^4c^4)$$



$$3.153 \quad \int \frac{\left(c + \frac{d}{x}\right)^3}{\left(a + \frac{b}{x}\right)^{3/2}} dx$$

**Optimal.** Leaf size=132

$$-\frac{3c^2(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{(bc - 2ad)(2a^2d^2 - 2abcd + 3b^2c^2) - \frac{abd^2(2ad+bc)}{x}}{a^2b^2\sqrt{a + \frac{b}{x}}} + \frac{cx\left(c + \frac{d}{x}\right)^2}{a\sqrt{a + \frac{b}{x}}}$$

[Out]  $((b*c - 2*a*d)*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2) - (a*b*d^2*(b*c + 2*a*d))/x)/(a^2*b^2*\text{Sqrt}[a + b/x]) + (c*(c + d/x)^2*x)/(a*\text{Sqrt}[a + b/x]) - (3*c^2*(b*c - 2*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/a^{(5/2)}$

**Rubi [A]** time = 0.326522, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$-\frac{3c^2(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{(bc - 2ad)(2a^2d^2 - 2abcd + 3b^2c^2) - \frac{abd^2(2ad+bc)}{x}}{a^2b^2\sqrt{a + \frac{b}{x}}} + \frac{cx\left(c + \frac{d}{x}\right)^2}{a\sqrt{a + \frac{b}{x}}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d/x)^3/(a + b/x)^{(3/2)}, x]$

[Out]  $((b*c - 2*a*d)*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2) - (a*b*d^2*(b*c + 2*a*d))/x)/(a^2*b^2*\text{Sqrt}[a + b/x]) + (c*(c + d/x)^2*x)/(a*\text{Sqrt}[a + b/x]) - (3*c^2*(b*c - 2*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/a^{(5/2)}$

**Rubi in Sympy [A]** time = 27.2067, size = 122, normalized size = 0.92

$$\frac{cx\left(c + \frac{d}{x}\right)^2}{a\sqrt{a + \frac{b}{x}}} - \frac{4\left(\frac{abd^2(2ad+bc)}{4x} + \left(\frac{ad}{2} - \frac{bc}{4}\right)(2a^2d^2 - 2abcd + 3b^2c^2)\right)}{a^2b^2\sqrt{a + \frac{b}{x}}} + \frac{3c^2(2ad - bc) \text{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c+d/x)**3/(a+b/x)**(3/2), x)$

[Out]  $c*x*(c + d/x)**2/(a*\text{sqrt}(a + b/x)) - 4*(a*b*d**2*(2*a*d + b*c)/(4*x) + (a*d/2 - b*c/4)*(2*a**2*d**2 - 2*a*b*c*d + 3*b**2*c**2))/(a**2*b**2*\text{sqrt}(a + b/x)) + 3*c**2*(2*a*d - b*c)*\text{atanh}(\text{sqrt}(a + b/x)/\text{sqrt}(a))/a**(5/2)$

**Mathematica [A]** time = 0.179282, size = 123, normalized size = 0.93

$$\frac{3c^2(2ad - bc) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right)}{2a^{5/2}} + \frac{\sqrt{a + \frac{b}{x}}(-4a^3d^3x - 2a^2bd^2(d - 3cx) + ab^2c^2x(cx - 6d) + 3b^3c^3x)}{a^2b^2(ax + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d/x)^3/(a + b/x)^(3/2), x]

[Out] (Sqrt[a + b/x] \* (3\*b^3\*c^3\*x - 4\*a^3\*d^3\*x - 2\*a^2\*b\*d^2\*(d - 3\*c\*x) + a\*b^2\*c^2\*x\*(-6\*d + c\*x)))/(a^2\*b^2\*(b + a\*x)) + (3\*c^2\*(-(b\*c) + 2\*a\*d)\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]\*x])/(2\*a^(5/2))

**Maple [B]** time = 0.025, size = 976, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^3/(a+b/x)^(3/2), x)

[Out] 
$$-1/2 * ((a*x+b)/x)^{(1/2)} / x / a^{(9/2)} * (3 * \ln(1/2 * (2 * (x * (a*x+b))^{(1/2)} * a^{(1/2)} + 2 * a*x+b) / a^{(1/2)}) * x^4 * a^4 * b^4 * c^3 - 6 * a^{(5/2)} * (x * (a*x+b))^{(1/2)} * x^2 * b^5 * c^3 + 6 * \ln(1/2 * (2 * (x * (a*x+b))^{(1/2)} * a^{(1/2)} + 2 * a*x+b) / a^{(1/2)}) * x^3 * a^3 * b^5 * c^3 + 3 * \ln(1/2 * (2 * (x * (a*x+b))^{(1/2)} * a^{(1/2)} + 2 * a*x+b) / a^{(1/2)}) * x^2 * a^2 * b^6 * c^3 - 6 * a^{(9/2)} * (x * (a*x+b))^{(1/2)} * x^4 * b^3 * c^3 + 4 * a^{(7/2)} * (x * (a*x+b))^{(3/2)} * c^3 * x^2 * b^3 - 12 * a^{(7/2)} * (x * (a*x+b))^{(1/2)} * x^3 * b^4 * c^3 + 8 * a^{(11/2)} * (a*x^2 + b*x)^{(3/2)} * x * b * d^3 - 3 * \ln(1/2 * (2 * (a*x^2 + b*x)^{(1/2)} * a^{(1/2)} + 2 * a*x+b) / a^{(1/2)}) * a^4 * x^2 * b^4 * c * d^2 - 6 * a^{(9/2)} * (a*x^2 + b*x)^{(1/2)} * x^2 * b^3 * c * d^2 - 6 * a^{(9/2)} * (x * (a*x+b))^{(1/2)} * x^2 * b^3 * c * d^2 + 12 * a^{(7/2)} * (x * (a*x+b))^{(1/2)} * x^2 * b^4 * c^2 * d - 6 * \ln(1/2 * (2 * (x * (a*x+b))^{(1/2)} * a^{(1/2)} + 2 * a*x+b) / a^{(1/2)}) * a^3 * x^2 * b^5 * c^2 * d - 12 * a^{(11/2)} * (x * (a*x+b))^{(1/2)} * x^3 * b^2 * c * d^2 + 24 * a^{(9/2)} * (x * (a*x+b))^{(1/2)} * x^3 * b^3 * c^2 * d + 3 * \ln(1/2 * (2 * (x * (a*x+b))^{(1/2)} * a^{(1/2)} + 2 * a*x+b) / a^{(1/2)}) * a^4 * x^2 * b^4 * c * d^2 - 12 * \ln(1/2 * (2 * (x * (a*x+b))^{(1/2)} * a^{(1/2)} + 2 * a*x+b) / a^{(1/2)}) * a^4 * x^3 * b^4 * c^2 * d + 4 * a^{(13/2)} * (a*x^2 + b*x)^{(3/2)} * x^2 * d^3 - 4 * a^{(13/2)} * (x * (a*x+b))^{(3/2)} * d^3 * x^2 + 4 * a^{(9/2)} * (a*x^2 + b*x)^{(3/2)} * b^2 * d^3 - 12 * a^{(9/2)} * (x * (a*x+b))^{(3/2)} * c^2 * d * x^2 * b^2 + 3 * \ln(1/2 * (2 * (x * (a*x+b))^{(1/2)} * a^{(1/2)} + 2 * a*x+b) / a^{(1/2)}) * a^6 * x^4 * b^2 * c * d^2 - 3 * \ln(1/2 * (2 * (a*x^2 + b*x)^{(1/2)} * a^{(1/2)} + 2 * a*x+b) / a^{(1/2)}) * a^6 * x^4 * b^2 * c * d^2 + 12 * a^{(11/2)} * (x * (a*x+b))^{(3/2)} * c * d^2 * x^2 * b - 12 * a^{(11/2)} * (a*x^2 + b*x)^{(1/2)} * x^3 * b^2 * c * d^2 - 6 * a^{(13/2)} * (a*x^2 + b*x)^{(1/2)} * x^4 * b * c * d^2 - 6 * a^{(13/2)} * (x * (a*x+b))^{(1/2)} * x^4 * b * c * d^2 + 12 * a^{(11/2)} * (x * (a*x+b))^{(1/2)} * x^4 * b^2 * c^2 * d + 6 * \ln(1/2 * (2 * (x * (a*x+b))^{(1/2)} * a^{(1/2)} + 2 * a*x+b) / a^{(1/2)}) * a^5 * x^4 * b^3 * c^2 * d - 6 * \ln(1/2 * (2 * (a*x^2 + b*x)^{(1/2)} * a^{(1/2)} + 2 * a*x+b) / a^{(1/2)}) * a^5 * x^4 * b^3 * c^2 * d^2) / (x * (a*x+b))^{(1/2)} / b^3 / (a*x+b)^2$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)^3/(a + b/x)^(3/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.256538, size = 1, normalized size = 0.01

$$\frac{3(b^3c^3 - 2ab^2c^2d)x\sqrt{\frac{ax+b}{x}}\log\left(2ax\sqrt{\frac{ax+b}{x}} + (2ax+b)\sqrt{a}\right) - 2(ab^2c^3x^2 - 2a^2bd^3 + (3b^3c^3 - 6ab^2c^2d + 6a^2bcd^2 - 4a^2b^2c^2d^2))\sqrt{\frac{ax+b}{x}}}{2a^{\frac{5}{2}}b^2x\sqrt{\frac{ax+b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)^3/(a + b/x)^(3/2),x, algorithm="fricas")

[Out] [-1/2\*(3\*(b^3\*c^3 - 2\*a\*b^2\*c^2\*d)\*x\*sqrt((a\*x + b)/x)\*log(2\*a\*x\*sqrt((a\*x + b)/x) + (2\*a\*x + b)\*sqrt(a)) - 2\*(a\*b^2\*c^3\*x^2 - 2\*a^2\*b\*d^3 + (3\*b^3\*c^3 - 6\*a\*b^2\*c^2\*d + 6\*a^2\*b\*c\*d^2 - 4\*a^3\*d^3)\*x)\*sqrt(a))/(a^(5/2)\*b^2\*x\*sqrt((a\*x + b)/x)), (3\*(b^3\*c^3 - 2\*a\*b^2\*c^2\*d)\*x\*sqrt((a\*x + b)/x)\*arctan(a/(sqrt(-a)\*sqrt((a\*x + b)/x))) + (a\*b^2\*c^3\*x^2 - 2\*a^2\*b\*d^3 + (3\*b^3\*c^3 - 6\*a\*b^2\*c^2\*d + 6\*a^2\*b\*c\*d^2 - 4\*a^3\*d^3)\*x)\*sqrt(-a))/(sqrt(-a)\*a^2\*b^2\*x\*sqrt((a\*x + b)/x))]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx + d)^3}{x^3 \left(a + \frac{b}{x}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*\*3/(a+b/x)\*\*(3/2),x)

[Out] Integral((c\*x + d)\*\*3/(x\*\*3\*(a + b/x)\*\*(3/2)), x)

**GIAC/XCAS [A]** time = 0.252264, size = 297, normalized size = 2.25

$$-b \left( \frac{2d^3 \sqrt{\frac{ax+b}{x}}}{b^3} - \frac{3(bc^3 - 2ac^2d) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}b} - \frac{2ab^3c^3 - 6a^2b^2c^2d + 6a^3bcd^2 - 2a^4d^3 - \frac{3(ax+b)b^3c^3}{x} + \frac{6(ax+b)ab^2c^2d}{x}}{\left(a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)} a^2b^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)^3/(a + b/x)^(3/2),x, algorithm="giac")

[Out] -b\*(2\*d^3\*sqrt((a\*x + b)/x)/b^3 - 3\*(b\*c^3 - 2\*a\*c^2\*d)\*arctan(sqrt((a\*x + b)/x)/sqrt(-a))/(sqrt(-a)\*a^2\*b) - (2\*a\*b^3\*c^3 - 6\*a^2\*b^2\*c^2\*d + 6\*a^3\*b\*c\*d^2 - 2\*a^4\*d^3 - 3\*(a\*x + b)\*b^3\*c^3/x + 6\*(a\*x + b)\*a\*b^2\*c^2\*d/x - 6\*(a\*x + b)\*a^2\*b\*c\*d^2/x + 2\*(a\*x + b)\*a^3\*d^3/x)/((a\*sqrt((a\*x + b)/x) - (a\*x + b)\*sqrt((a\*x + b)/x)\*a^2\*b^3))

$$3.154 \quad \int \frac{\left(c + \frac{d}{x}\right)^2}{\left(a + \frac{b}{x}\right)^{3/2}} dx$$

**Optimal.** Leaf size=94

$$-\frac{c(3bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{2a^2d^2 + bc(3bc - 4ad)}{a^2b\sqrt{a + \frac{b}{x}}} + \frac{c^2x}{a\sqrt{a + \frac{b}{x}}}$$

[Out]  $(2*a^2*d^2 + b*c*(3*b*c - 4*a*d))/(a^2*b*\text{Sqrt}[a + b/x]) + (c^2*x)/(a*\text{Sqrt}[a + b/x]) - (c*(3*b*c - 4*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/a^{(5/2)}$

**Rubi [A]** time = 0.247795, antiderivative size = 90, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$-\frac{c(3bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{\frac{c(3bc-4ad)}{a^2} + \frac{2d^2}{b}}{\sqrt{a + \frac{b}{x}}} + \frac{c^2x}{a\sqrt{a + \frac{b}{x}}}$$

Antiderivative was successfully verified.

[In] Int[(c + d/x)^2/(a + b/x)^(3/2), x]

[Out]  $((2*d^2)/b + (c*(3*b*c - 4*a*d))/a^2)/\text{Sqrt}[a + b/x] + (c^2*x)/(a*\text{Sqrt}[a + b/x]) - (c*(3*b*c - 4*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/a^{(5/2)}$

**Rubi in Sympy [A]** time = 18.8213, size = 82, normalized size = 0.87

$$\frac{c^2x}{a\sqrt{a + \frac{b}{x}}} + \frac{2\left(a^2d^2 - \frac{bc(4ad-3bc)}{2}\right)}{a^2b\sqrt{a + \frac{b}{x}}} + \frac{c(4ad - 3bc) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d/x)\*\*2/(a+b/x)\*\*(3/2), x)

[Out]  $c^2*x/(a*\text{sqrt}(a + b/x)) + 2*(a^2*d^2 - b*c*(4*a*d - 3*b*c)/2)/(a^2*b*\text{sqrt}(a + b/x)) + c*(4*a*d - 3*b*c)*\text{atanh}(\text{sqrt}(a + b/x)/\text{sqrt}(a))/a^{(5/2)}$

**Mathematica [A]** time = 0.187898, size = 100, normalized size = 1.06

$$\frac{c(4ad - 3bc) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right)}{2a^{5/2}} + \frac{x\sqrt{a + \frac{b}{x}}(2a^2d^2 + abc(cx - 4d) + 3b^2c^2)}{a^2b(ax + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d/x)^2/(a + b/x)^(3/2), x]

[Out]  $(\text{Sqrt}[a + b/x] * x * (3 * b^2 * c^2 + 2 * a^2 * d^2 + a * b * c * (-4 * d + c * x))) / (a^2 * b * (b + a * x)) + (c * (-3 * b * c + 4 * a * d) * \text{Log}[b + 2 * a * x + 2 * \text{Sqrt}[a] * \text{Sqrt}[a + b/x] * x]) / (2 * a^{(5/2)})$

**Maple [B]** time = 0.022, size = 796, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c+d/x)^2/(a+b/x)^(3/2), x)`

[Out]  $\frac{1}{2} * ((a * x + b) / x)^{(1/2)} * x / a^{(9/2)} * (6 * a^{(9/2)} * (x * (a * x + b))^{(1/2)} * x^2 * b^2 * c^2 + a^6 * \ln(1/2 * (2 * (a * x^2 + b * x))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x^2 * b^2 * d^2 - a^6 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x^2 * b^2 * d^2 + 4 * a^{(11/2)} * (a * x^2 + b * x)^{(1/2)} * x * b^2 * d^2 + 8 * a^{(9/2)} * (x * (a * x + b))^{(3/2)} * c * d * b + 4 * a^{(11/2)} * (x * (a * x + b))^{(1/2)} * x * b^2 * d^2 - 3 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x^2 * a^4 * b^3 * c^2 - 6 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x * a^3 * b^4 * c^2 + 6 * a^{(5/2)} * (x * (a * x + b))^{(1/2)} * b^4 * c^2 - 3 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * a^2 * b^5 * c^2 + 2 * a^{(13/2)} * (a * x^2 + b * x)^{(1/2)} * x^2 * d^2 + 2 * a^{(13/2)} * (x * (a * x + b))^{(1/2)} * x^2 * d^2 - 4 * a^{(7/2)} * (x * (a * x + b))^{(3/2)} * c^2 * b^2 + 2 * a^{(9/2)} * (a * x^2 + b * x)^{(1/2)} * b^2 * d^2 + 2 * a^{(9/2)} * (x * (a * x + b))^{(1/2)} * b^2 * d^2 + a^4 * \ln(1/2 * (2 * (a * x^2 + b * x))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * b^3 * d^2 - a^4 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * b^3 * d^2 - 4 * a^{(11/2)} * (x * (a * x + b))^{(3/2)} * d^2 + 12 * a^{(7/2)} * (x * (a * x + b))^{(1/2)} * x * b^3 * c^2 + 2 * a^5 * \ln(1/2 * (2 * (a * x^2 + b * x))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x * b^2 * d^2 - 2 * a^5 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x * b^2 * d^2 - 8 * a^{(7/2)} * (x * (a * x + b))^{(1/2)} * b^3 * c * d + 4 * a^3 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * b^4 * c * d - 16 * a^{(9/2)} * (x * (a * x + b))^{(1/2)} * x * b^2 * c * d + 4 * a^5 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x^2 * b^2 * c * d + 8 * a^4 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x * b^3 * c * d - 8 * a^{(11/2)} * (x * (a * x + b))^{(1/2)} * x^2 * b * c * d) / (x * (a * x + b))^{(1/2)} / b^2 / (a * x + b)^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c + d/x)^2/(a + b/x)^(3/2), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.243, size = 1, normalized size = 0.01

$$\left[ \frac{(3 b^2 c^2 - 4 a b c d) \sqrt{\frac{a x + b}{x}} \log \left( 2 a x \sqrt{\frac{a x + b}{x}} + (2 a x + b) \sqrt{a} \right) - 2 (a b c^2 x + 3 b^2 c^2 - 4 a b c d + 2 a^2 d^2) \sqrt{a} (3 b^2 c^2 - 4 a b c d) \sqrt{\frac{a x + b}{x}}}{2 a^{\frac{5}{2}} b \sqrt{\frac{a x + b}{x}}}, \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c + d/x)^2/(a + b/x)^(3/2), x, algorithm="fricas")`

[Out]  $[-1/2 * ((3 * b^2 * c^2 - 4 * a * b * c * d) * \text{sqrt}((a * x + b) / x) * \log(2 * a * x * \text{sqrt}((a * x + b) / x) + (2 * a * x + b) * \text{sqrt}(a))) - 2 * (a * b * c^2 * x + 3 * b^2 * c^2 - 4$

```
*a*b*c*d + 2*a^2*d^2)*sqrt(a))/(a^(5/2)*b*sqrt((a*x + b)/x)), ((3
*b^2*c^2 - 4*a*b*c*d)*sqrt((a*x + b)/x)*arctan(a/(sqrt(-a)*sqrt((
a*x + b)/x))) + (a*b*c^2*x + 3*b^2*c^2 - 4*a*b*c*d + 2*a^2*d^2)*s
qrt(-a))/(sqrt(-a)*a^2*b*sqrt((a*x + b)/x))]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx + d)^2}{x^2 \left(a + \frac{b}{x}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*\*2/(a+b/x)\*\*(3/2), x)

[Out] Integral((c\*x + d)\*\*2/(x\*\*2\*(a + b/x)\*\*(3/2)), x)

---

**GIAC/XCAS [A]** time = 0.251661, size = 217, normalized size = 2.31

$$b \left( \frac{(3bc^2 - 4acd) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-aa^2b}} + \frac{2ab^2c^2 - 4a^2bcd + 2a^3d^2 - \frac{3(ax+b)b^2c^2}{x} + \frac{4(ax+b)abcd}{x} - \frac{2(ax+b)a^2d^2}{x}}{\left(a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^2b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)^2/(a + b/x)^(3/2), x, algorithm="giac")

[Out] b\*((3\*b\*c^2 - 4\*a\*c\*d)\*arctan(sqrt((a\*x + b)/x)/sqrt(-a))/(sqrt(-a)\*a^2\*b) + (2\*a\*b^2\*c^2 - 4\*a^2\*b\*c\*d + 2\*a^3\*d^2 - 3\*(a\*x + b)\*b^2\*c^2/x + 4\*(a\*x + b)\*a\*b\*c\*d/x - 2\*(a\*x + b)\*a^2\*d^2/x)/((a\*sqrt((a\*x + b)/x) - (a\*x + b)\*sqrt((a\*x + b)/x)/x)\*a^2\*b^2)

$$3.155 \quad \int \frac{c + \frac{d}{x}}{\left(a + \frac{b}{x}\right)^{3/2}} dx$$

**Optimal.** Leaf size=76

$$-\frac{(3bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{3bc - 2ad}{a^2 \sqrt{a + \frac{b}{x}}} + \frac{cx}{a \sqrt{a + \frac{b}{x}}}$$

[Out]  $(3*b*c - 2*a*d)/(a^2*\text{Sqrt}[a + b/x]) + (c*x)/(a*\text{Sqrt}[a + b/x]) - ((3*b*c - 2*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/a^{(5/2)}$

**Rubi [A]** time = 0.154825, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$-\frac{(3bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{3bc - 2ad}{a^2 \sqrt{a + \frac{b}{x}}} + \frac{cx}{a \sqrt{a + \frac{b}{x}}}$$

Antiderivative was successfully verified.

[In] Int[(c + d/x)/(a + b/x)^(3/2), x]

[Out]  $(3*b*c - 2*a*d)/(a^2*\text{Sqrt}[a + b/x]) + (c*x)/(a*\text{Sqrt}[a + b/x]) - ((3*b*c - 2*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/a^{(5/2)}$

**Rubi in Sympy [A]** time = 12.5111, size = 66, normalized size = 0.87

$$\frac{cx}{a \sqrt{a + \frac{b}{x}}} - \frac{2(ad - \frac{3bc}{2})}{a^2 \sqrt{a + \frac{b}{x}}} + \frac{2(ad - \frac{3bc}{2}) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d/x)/(a+b/x)\*\*(3/2), x)

[Out]  $c*x/(a*\text{sqrt}(a + b/x)) - 2*(a*d - 3*b*c/2)/(a**2*\text{sqrt}(a + b/x)) + 2*(a*d - 3*b*c/2)*\text{atanh}(\text{sqrt}(a + b/x)/\text{sqrt}(a))/a^{(5/2)}$

**Mathematica [A]** time = 0.116594, size = 81, normalized size = 1.07

$$\frac{(2ad - 3bc) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right)}{2a^{5/2}} + \frac{x\sqrt{a + \frac{b}{x}}(acx - 2ad + 3bc)}{a^2(ax + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d/x)/(a + b/x)^(3/2), x]

[Out]  $(\text{Sqrt}[a + b/x]*x*(3*b*c - 2*a*d + a*c*x))/(a^2*(b + a*x)) + ((-3*b*c + 2*a*d)*\text{Log}[b + 2*a*x + 2*\text{Sqrt}[a]*\text{Sqrt}[a + b/x]*x])/(2*a^{(5/2)}$

2))

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**Maple [B]** time = 0.018, size = 394, normalized size = 5.2

$$-\frac{x}{2b(ax+b)^2} \sqrt{\frac{ax+b}{x}} \left( 4a^{11/2} \sqrt{x(ax+b)} x^2 d - 6a^{9/2} \sqrt{x(ax+b)} x^2 bc - 4a^{9/2} (x(ax+b))^{3/2} d + 8a^{9/2} \sqrt{x(ax+b)} xbd + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)/(a+b/x)^(3/2), x)

[Out] 
$$-1/2 * ((a * x + b) / x)^{(1/2)} * x * (4 * a^{(11/2)} * (x * (a * x + b))^{(1/2)} * x^2 * d - 6 * a^{(9/2)} * (x * (a * x + b))^{(1/2)} * x^2 * b * c - 4 * a^{(9/2)} * (x * (a * x + b))^{(3/2)} * d + 8 * a^{(9/2)} * (x * (a * x + b))^{(1/2)} * x * b * d + 4 * a^{(7/2)} * (x * (a * x + b))^{(3/2)} * c * b - 12 * a^{(7/2)} * (x * (a * x + b))^{(1/2)} * x * b^2 * c + 4 * a^{(7/2)} * (x * (a * x + b))^{(1/2)} * b^2 * d - 6 * a^{(5/2)} * (x * (a * x + b))^{(1/2)} * b^3 * c - 2 * a^{(5/2)} * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x^2 * b * d + 3 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x^2 * a^4 * b^2 * c - 4 * a^4 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x * b^2 * d + 6 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x * a^3 * b^3 * c - 2 * a^3 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * b^3 * d + 3 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * a^2 * b^4 * c) / a^{(9/2)} / (x * (a * x + b))^{(1/2)} / b / (a * x + b)^2$$

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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)/(a + b/x)^(3/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

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**Fricas [A]** time = 0.24653, size = 1, normalized size = 0.01

$$\left[ \frac{(3bc - 2ad) \sqrt{\frac{ax+b}{x}} \log\left(2ax \sqrt{\frac{ax+b}{x}} + (2ax+b)\sqrt{a}\right) - 2(acx + 3bc - 2ad)\sqrt{a} (3bc - 2ad) \sqrt{\frac{ax+b}{x}} \arctan\left(\frac{a}{\sqrt{-a} \sqrt{\frac{ax+b}{x}}}\right)}{2a^{\frac{5}{2}} \sqrt{\frac{ax+b}{x}}}, \frac{(3bc - 2ad) \sqrt{\frac{ax+b}{x}} \arctan\left(\frac{a}{\sqrt{-a} \sqrt{\frac{ax+b}{x}}}\right)}{\sqrt{-aa^2} \sqrt{\frac{ax+b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)/(a + b/x)^(3/2), x, algorithm="fricas")

[Out] 
$$[-1/2 * ((3 * b * c - 2 * a * d) * \sqrt{(a * x + b) / x} * \log(2 * a * x * \sqrt{(a * x + b) / x} + (2 * a * x + b) * \sqrt{a}) - 2 * (a * c * x + 3 * b * c - 2 * a * d) * \sqrt{a}) / (a^{(5/2)} * \sqrt{(a * x + b) / x}), ((3 * b * c - 2 * a * d) * \sqrt{(a * x + b) / x} * \arctan(a / (\sqrt{-a} * \sqrt{(a * x + b) / x})) + (a * c * x + 3 * b * c - 2 * a * d) * \sqrt{a}) / (\sqrt{-a} * a^2 * \sqrt{(a * x + b) / x})]$$

---



**Sympy [A]** time = 21.4323, size = 224, normalized size = 2.95

$$c \left( \frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}} + \frac{3\sqrt{b}\sqrt{x}}{a^2\sqrt{\frac{ax}{b}+1}} - \frac{3b \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}} \right) + d \left( -\frac{2a^3x\sqrt{1+\frac{b}{ax}}}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b} - \frac{a^3x \log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b} \right. \\ \left. + \frac{2a^3x \log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b} - \frac{a^2b \log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b} + \frac{2a^2b \log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)/(a+b/x)\*\*(3/2), x)

[Out] c\*(x\*\*(3/2)/(a\*sqrt(b)\*sqrt(a\*x/b + 1)) + 3\*sqrt(b)\*sqrt(x)/(a\*\*2\*sqrt(a\*x/b + 1)) - 3\*b\*asinh(sqrt(a)\*sqrt(x)/sqrt(b))/a\*\*(5/2)) + d\*(-2\*a\*\*3\*x\*sqrt(1 + b/(a\*x))/(a\*\*(9/2)\*x + a\*\*(7/2)\*b) - a\*\*3\*x\*log(b/(a\*x))/(a\*\*(9/2)\*x + a\*\*(7/2)\*b) + 2\*a\*\*3\*x\*log(sqrt(1 + b/(a\*x)) + 1)/(a\*\*(9/2)\*x + a\*\*(7/2)\*b) - a\*\*2\*b\*log(b/(a\*x))/(a\*\*(9/2)\*x + a\*\*(7/2)\*b) + 2\*a\*\*2\*b\*log(sqrt(1 + b/(a\*x)) + 1)/(a\*\*(9/2)\*x + a\*\*(7/2)\*b))

**GIAC/XCAS [A]** time = 0.254238, size = 165, normalized size = 2.17

$$b \left( \frac{(3bc - 2ad) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}^2b} + \frac{2abc - 2a^2d - \frac{3(ax+b)bc}{x} + \frac{2(ax+b)ad}{x}}{\left(a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^2b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)/(a + b/x)^(3/2), x, algorithm="giac")

[Out] b\*((3\*b\*c - 2\*a\*d)\*arctan(sqrt((a\*x + b)/x)/sqrt(-a))/(sqrt(-a)\*a^2\*b) + (2\*a\*b\*c - 2\*a^2\*d - 3\*(a\*x + b)\*b\*c/x + 2\*(a\*x + b)\*a\*d/x)/((a\*sqrt((a\*x + b)/x) - (a\*x + b)\*sqrt((a\*x + b)/x)/x)\*a^2\*b)

$$3.156 \quad \int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2}} dx$$

**Optimal.** Leaf size=61

$$-\frac{3b \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{3x\sqrt{a+\frac{b}{x}}}{a^2} - \frac{2x}{a\sqrt{a+\frac{b}{x}}}$$

[Out]  $(-2*x)/(a*\text{Sqrt}[a + b/x]) + (3*\text{Sqrt}[a + b/x]*x)/a^2 - (3*b*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/a^{(5/2)}$

**Rubi [A]** time = 0.0834458, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$-\frac{3b \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{3x\sqrt{a+\frac{b}{x}}}{a^2} - \frac{2x}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(-3/2), x]

[Out]  $(-2*x)/(a*\text{Sqrt}[a + b/x]) + (3*\text{Sqrt}[a + b/x]*x)/a^2 - (3*b*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/a^{(5/2)}$

**Rubi in Sympy [A]** time = 8.025, size = 51, normalized size = 0.84

$$-\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3x\sqrt{a+\frac{b}{x}}}{a^2} - \frac{3b \operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a+b/x)\*\*(3/2), x)

[Out]  $-2*x/(a*\text{sqrt}(a + b/x)) + 3*x*\text{sqrt}(a + b/x)/a**2 - 3*b*\text{atanh}(\text{sqrt}(a + b/x)/\text{sqrt}(a))/a**(5/2)$

**Mathematica [A]** time = 0.100055, size = 67, normalized size = 1.1

$$\frac{x\sqrt{a+\frac{b}{x}}(ax+3b)}{a^2(ax+b)} - \frac{3b \log\left(2\sqrt{ax}\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2a^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(-3/2), x]

[Out]  $(\text{Sqrt}[a + b/x]*x*(3*b + a*x))/(a^2*(b + a*x)) - (3*b*\text{Log}[b + 2*a*x + 2*\text{Sqrt}[a]*\text{Sqrt}[a + b/x]*x])/(2*a^{(5/2)})$

**Maple [B]** time = 0.005, size = 203, normalized size = 3.3

$$-\frac{x}{2(ax+b)^2} \sqrt{\frac{ax+b}{x}} \left( -6a^{9/2} \sqrt{x(ax+b)} x^2 + 4a^{7/2} (x(ax+b))^{3/2} - 12a^{7/2} \sqrt{x(ax+b)} xb + 3 \ln \left( \frac{1}{2} \frac{2\sqrt{x(ax+b)}\sqrt{a} + \sqrt{a}}{\sqrt{a}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b/x)^(3/2), x)

[Out]  $-1/2 * ((a*x+b)/x)^{(1/2)} * x/a^{(9/2)} * (-6 * a^{(9/2)} * (x*(a*x+b))^{(1/2)} * x^{2+4*a^{(7/2)} * (x*(a*x+b))^{(3/2)} - 12 * a^{(7/2)} * (x*(a*x+b))^{(1/2)} * x*b + 3 * \ln(1/2 * (2 * (x*(a*x+b))^{(1/2)} * a^{(1/2)} + 2 * a*x+b)/a^{(1/2)}) * x^{2*a^{(4)} * b - 6 * a^{(5/2)} * (x*(a*x+b))^{(1/2)} * b^2 + 6 * \ln(1/2 * (2 * (x*(a*x+b))^{(1/2)} * a^{(1/2)} + 2 * a*x+b)/a^{(1/2)}) * x * a^3 * b^2 + 3 * \ln(1/2 * (2 * (x*(a*x+b))^{(1/2)} * a^{(1/2)} + 2 * a*x+b)/a^{(1/2)}) * a^2 * b^3) / (x*(a*x+b))^{(1/2)} / (a*x+b)^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(-3/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.251474, size = 1, normalized size = 0.02

$$\left[ \frac{3b\sqrt{\frac{ax+b}{x}} \log\left(-2ax\sqrt{\frac{ax+b}{x}} + (2ax+b)\sqrt{a}\right) + 2(ax+3b)\sqrt{a}}{2a^{\frac{5}{2}}\sqrt{\frac{ax+b}{x}}}, \frac{3b\sqrt{\frac{ax+b}{x}} \arctan\left(\frac{a}{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}\right) + (ax+3b)\sqrt{-a}}{\sqrt{-aa^2}\sqrt{\frac{ax+b}{x}}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(-3/2), x, algorithm="fricas")

[Out]  $[1/2 * (3*b*\sqrt{(a*x+b)/x}) * \log(-2*a*x*\sqrt{(a*x+b)/x} + (2*a*x+b)*\sqrt{a}) + 2*(a*x+3*b)*\sqrt{a}) / (a^{(5/2)}*\sqrt{(a*x+b)/x}), (3*b*\sqrt{(a*x+b)/x}) * \arctan(a/(\sqrt{-a}*\sqrt{(a*x+b)/x})) + (a*x+3*b)*\sqrt{-a}) / (\sqrt{-a}*a^2*\sqrt{(a*x+b)/x})]$

**Sympy [A]** time = 11.4526, size = 71, normalized size = 1.16

$$\frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}} + \frac{3\sqrt{b}\sqrt{x}}{a^2\sqrt{\frac{ax}{b}+1}} - \frac{3b \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)\*\*(3/2), x)

[Out]  $x^{(3/2)} / (a*\sqrt{b}) * \sqrt{a*x/b + 1}) + 3*\sqrt{b} * \sqrt{x} / (a^{**2} * \sqrt{a*x/b + 1}) - 3*b * \operatorname{asinh}(\sqrt{a} * \sqrt{x} / \sqrt{b}) / a^{** (5/2)}$

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**GIAC/XCAS [A]** time = 0.250133, size = 116, normalized size = 1.9

$$b \left( \frac{3 \arctan \left( \frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}} \right)}{\sqrt{-aa^2}} + \frac{2a - \frac{3(ax+b)}{x}}{\left( a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x} \right) a^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(-3/2),x, algorithm="giac")

[Out] b\*(3\*arctan(sqrt((a\*x + b)/x)/sqrt(-a))/(sqrt(-a)\*a^2) + (2\*a - 3\*(a\*x + b)/x)/((a\*sqrt((a\*x + b)/x) - (a\*x + b)\*sqrt((a\*x + b)/x)/x)\*a^2))

$$3.157 \quad \int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)} dx$$

**Optimal.** Leaf size=147

$$-\frac{(2ad + 3bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2} + \frac{b(3bc - ad)}{a^2c\sqrt{a + \frac{b}{x}}(bc - ad)} + \frac{2d^{5/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc - ad)^{3/2}} + \frac{x}{ac\sqrt{a + \frac{b}{x}}}$$

[Out] (b\*(3\*b\*c - a\*d))/(a^2\*c\*(b\*c - a\*d)\*Sqrt[a + b/x]) + x/(a\*c\*Sqrt[a + b/x]) + (2\*d^(5/2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(c^2\*(b\*c - a\*d)^(3/2)) - ((3\*b\*c + 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(5/2)\*c^2)

**Rubi [A]** time = 0.618832, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{(2ad + 3bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2} + \frac{b(3bc - ad)}{a^2c\sqrt{a + \frac{b}{x}}(bc - ad)} + \frac{2d^{5/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc - ad)^{3/2}} + \frac{x}{ac\sqrt{a + \frac{b}{x}}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b/x)^(3/2)\*(c + d/x)), x]

[Out] (b\*(3\*b\*c - a\*d))/(a^2\*c\*(b\*c - a\*d)\*Sqrt[a + b/x]) + x/(a\*c\*Sqrt[a + b/x]) + (2\*d^(5/2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(c^2\*(b\*c - a\*d)^(3/2)) - ((3\*b\*c + 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(5/2)\*c^2)

**Rubi in Sympy [A]** time = 67.6655, size = 122, normalized size = 0.83

$$\frac{2d^{5/2} \operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{c^2(ad - bc)^{3/2}} + \frac{x}{ac\sqrt{a + \frac{b}{x}}} + \frac{b(ad - 3bc)}{a^2c\sqrt{a + \frac{b}{x}}(ad - bc)} - \frac{(2ad + 3bc) \operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a+b/x)\*\*(3/2)/(c+d/x), x)

[Out] 2\*d\*\*(5/2)\*atanh(sqrt(d)\*sqrt(a + b/x)/sqrt(a\*d - b\*c))/(c\*\*2\*(a\*d - b\*c)\*\*(3/2)) + x/(a\*c\*sqrt(a + b/x)) + b\*(a\*d - 3\*b\*c)/(a\*\*2\*c\*sqrt(a + b/x)\*(a\*d - b\*c)) - (2\*a\*d + 3\*b\*c)\*atanh(sqrt(a + b/x)/sqrt(a))/(a\*\*(5/2)\*c\*\*2)

**Mathematica [A]** time = 1.22763, size = 197, normalized size = 1.34

$$-\frac{(2ad+3bc)\log\left(2\sqrt{ax}\sqrt{a+\frac{b}{x}}+2ax+b\right)}{a^{5/2}} + \frac{2cx\sqrt{a+\frac{b}{x}}(a^2dx+ab(d-cx)-3b^2c)}{a^2(ax+b)(ad-bc)} + \frac{2d^{5/2}\log(cx+d)}{(ad-bc)^{3/2}} - \frac{2d^{5/2}\log\left(2\sqrt{dx}\sqrt{a+\frac{b}{x}}\sqrt{ad-bc}-2adx+bcx-bd\right)}{(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b/x)^(3/2)\*(c + d/x)),x]

[Out] 
$$\frac{((2*c*\text{Sqrt}[a + b/x]*x*(-3*b^2*c + a^2*d*x + a*b*(d - c*x)))/((a^2*(-(b*c) + a*d)*(b + a*x)) + (2*d^{5/2}*\text{Log}[d + c*x])/(-(b*c) + a*d)^{3/2} - ((3*b*c + 2*a*d)*\text{Log}[b + 2*a*x + 2*\text{Sqrt}[a]*\text{Sqrt}[a + b/x]*x])/a^{5/2} - (2*d^{5/2}*\text{Log}[-(b*d) + b*c*x - 2*a*d*x + 2*\text{Sqrt}[d]*\text{Sqrt}[-(b*c) + a*d]*\text{Sqrt}[a + b/x]*x])/(-(b*c) + a*d)^{3/2}))/((2*c^2)$$

**Maple [B]** time = 0.026, size = 1480, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b/x)^(3/2)/(c+d/x),x)

[Out] 
$$\frac{1}{2} * \left( \frac{(a*x+b)}{x} \right)^{1/2} * \frac{x}{a^{9/2}} * (-3 * \ln(1/2 * (2 * (x * (a*x+b))^{1/2} * a^{1/2} + 2 * a*x+b)/a^{1/2})) * ((a*d-b*c) * d/c^2)^{1/2} * a^2 * b^5 * c^4 - 4 * (x * (a*x+b))^{3/2} * ((a*d-b*c) * d/c^2)^{1/2} * a^{7/2} * b^2 * c^4 - 4 * \ln((2 * (x * (a*x+b))^{1/2} * ((a*d-b*c) * d/c^2)^{1/2} * c - 2 * a*d*x + b*c*x - b*d)/(c * x+d)) * a^{13/2} * x * b * d^4 + 2 * \ln((2 * (x * (a*x+b))^{1/2} * ((a*d-b*c) * d/c^2)^{1/2} * c - 2 * a*d*x + b*c*x - b*d)/(c * x+d)) * a^{9/2} * b^3 * c * d^3 - 2 * \ln((2 * (x * (a*x+b))^{1/2} * ((a*d-b*c) * d/c^2)^{1/2} * c - 2 * a*d*x + b*c*x - b*d)/(c * x+d)) * a^{15/2} * x^2 * d^4 - 2 * \ln((2 * (x * (a*x+b))^{1/2} * ((a*d-b*c) * d/c^2)^{1/2} * c - 2 * a*d*x + b*c*x - b*d)/(c * x+d)) * a^{11/2} * b^2 * d^4 + 4 * (x * (a*x+b))^{3/2} * ((a*d-b*c) * d/c^2)^{1/2} * a^{9/2} * b * c^3 * d + 12 * (x * (a*x+b))^{1/2} * ((a*d-b*c) * d/c^2)^{1/2} * a^{7/2} * x * b^3 * c^4 - 2 * \ln(1/2 * (2 * (x * (a*x+b))^{1/2} * a^{1/2} + 2 * a*x+b)/a^{1/2})) * ((a*d-b*c) * d/c^2)^{1/2} * a^3 * b^4 * c^3 * d + 4 * \ln((2 * (x * (a*x+b))^{1/2} * ((a*d-b*c) * d/c^2)^{1/2} * c - 2 * a*d*x + b*c*x - b*d)/(c * x+d)) * a^{11/2} * x * b^2 * c * d^3 + 2 * (x * (a*x+b))^{1/2} * ((a*d-b*c) * d/c^2)^{1/2} * a^{9/2} * b^2 * c^2 * d^2 - 8 * (x * (a*x+b))^{1/2} * ((a*d-b*c) * d/c^2)^{1/2} * a^{7/2} * b^3 * c^3 * d + 2 * \ln((2 * (x * (a*x+b))^{1/2} * ((a*d-b*c) * d/c^2)^{1/2} * c - 2 * a*d*x + b*c*x - b*d)/(c * x+d)) * a^{13/2} * x^2 * b * c * d^3 + 6 * (x * (a*x+b))^{1/2} * ((a*d-b*c) * d/c^2)^{1/2} * a^{5/2} * b^4 * c^4 + 4 * (x * (a*x+b))^{1/2} * ((a*d-b*c) * d/c^2)^{1/2} * a^{11/2} * x * b * c^2 * d^2 + \ln(1/2 * (2 * (x * (a*x+b))^{1/2} * a^{1/2} + 2 * a*x+b)/a^{1/2})) * ((a*d-b*c) * d/c^2)^{1/2} * a^6 * x^2 * b * c^2 * d^2 - 4 * \ln(1/2 * (2 * (x * (a*x+b))^{1/2} * a^{1/2} + 2 * a*x+b)/a^{1/2})) * ((a*d-b*c) * d/c^2)^{1/2} * a^6 * x * b * c * d^3 - 8 * (x * (a*x+b))^{1/2} * ((a*d-b*c) * d/c^2)^{1/2} * a^{11/2} * x^2 * b * c^3 * d - 16 * (x * (a*x+b))^{1/2} * ((a*d-b*c) * d/c^2)^{1/2} * a^{9/2} * x * b^2 * c^3 * d + 2 * \ln(1/2 * (2 * (x * (a*x+b))^{1/2} * a^{1/2} + 2 * a*x+b)/a^{1/2})) * ((a*d-b*c) * d/c^2)^{1/2} * a^5 * x * b^2 * c^2 * d^2 + 4 * \ln(1/2 * (2 * (x * (a*x+b))^{1/2} * a^{1/2} + 2 * a*x+b)/a^{1/2})) * ((a*d-b*c) * d/c^2)^{1/2} * a^5 * x^2 * b^2 * c^3 * d + 8 * \ln(1/2 * (2 * (x * (a*x+b))^{1/2} * a^{1/2} + 2 * a*x+b)/a^{1/2})) * ((a*d-b*c) * d/c^2)^{1/2} * a^4 * x * b^3 * c^3 * d + \ln(1/2 * (2 * (x * (a*x+b))^{1/2} * a^{1/2} + 2 * a*x+b)/a^{1/2})) * ((a*d-b*c) * d/c^2)^{1/2} * a^4 * b^3 * c^2 * d^2 - 3 * \ln(1/2 * (2 * (x * (a*x+b))^{1/2} * a^{1/2} + 2 * a*x+b)/a^{1/2})) * ((a*d-b*c) * d/c^2)^{1/2} * x^2 * a^4 * b^3 * c^4 - 6 * \ln(1/2 * (2 * (x * (a*x+b))^{1/2} * a^{1/2} + 2 * a*x+b)/a^{1/2})) * ((a*d-b*c) * d/c^2)^{1/2} * x^2 * a^4 * b^3 * c^4 - 6 * \ln(1/2 * (2 * (x * (a*x+b))^{1/2} * a^{1/2} + 2 * a*x+b)/a^{1/2})) * ((a*d-b*c) * d/c^2)^{1/2} * a^7 * x^2 * c * d^3 + 2 * (x * (a*x+b))^{1/2} * ((a*d-b*c) * d/c^2)^{1/2} * a^{13/2} * x^2 * c^2 * d^2 + 6 * (x * (a*x+b))^{1/2} * ((a*d-b*c) * d/c^2)^{1/2} * a^{9/2} * x^2 * b^2 * c^4)/(x * (a*x+b))^{1/2}/(a*d-b*c)^2/c^3/((a*d-b*c) * d/c^2)^{1/2}/(a*x+b)^2$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a + b/x)^(3/2)\*(c + d/x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.38584, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a + b/x)^(3/2)\*(c + d/x)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/2*(2*a^{5/2}*d^2*\sqrt{-d/(b*c - a*d)})*\sqrt{(a*x + b)/x}*\log(- \\ & (2*(b*c - a*d)*x*\sqrt{-d/(b*c - a*d)})*\sqrt{(a*x + b)/x} - b*d + ( \\ & b*c - 2*a*d)*x)/(c*x + d)) - (3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*\sqrt{ \\ & (a*x + b)/x}*\log(-2*a*x*\sqrt{(a*x + b)/x} + (2*a*x + b)*\sqrt{a \\ & )) - 2*(3*b^2*c^2 - a*b*c*d + (a*b*c^2 - a^2*c*d)*x)*\sqrt{a})/((a \\ & ^2*b*c^3 - a^3*c^2*d)*\sqrt{a}*\sqrt{(a*x + b)/x}), -(\sqrt{-a}*a^2* \\ & d^2*\sqrt{-d/(b*c - a*d)})*\sqrt{(a*x + b)/x}*\log(-2*(b*c - a*d)*x* \\ & \sqrt{-d/(b*c - a*d)})*\sqrt{(a*x + b)/x} - b*d + (b*c - 2*a*d)*x)/( \\ & c*x + d)) - (3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*\sqrt{(a*x + b)/x}*a \\ & rctan(a/(\sqrt{-a}*\sqrt{(a*x + b)/x})) - (3*b^2*c^2 - a*b*c*d + (a \\ & *b*c^2 - a^2*c*d)*x)*\sqrt{-a})/((a^2*b*c^3 - a^3*c^2*d)*\sqrt{-a}* \\ & \sqrt{(a*x + b)/x}), 1/2*(4*a^{5/2}*d^2*\sqrt{d/(b*c - a*d)})*\sqrt{(( \\ & a*x + b)/x)*\arctan(-(b*c - a*d)*\sqrt{d/(b*c - a*d)})/(d*\sqrt{(a*x \\ & + b)/x}))} + (3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*\sqrt{(a*x + b)/x}* \\ & \log(-2*a*x*\sqrt{(a*x + b)/x} + (2*a*x + b)*\sqrt{a}) + 2*(3*b^2*c^2 \\ & - a*b*c*d + (a*b*c^2 - a^2*c*d)*x)*\sqrt{a})/((a^2*b*c^3 - a^3*c^2 \\ & ^2*d)*\sqrt{a}*\sqrt{(a*x + b)/x}), (2*\sqrt{-a}*a^2*d^2*\sqrt{d/(b*c \\ & - a*d)})*\sqrt{(a*x + b)/x}*\arctan(-(b*c - a*d)*\sqrt{d/(b*c - a*d)}) \\ & /((d*\sqrt{(a*x + b)/x}))} + (3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*\sqrt{ \\ & (a*x + b)/x}*\arctan(a/(\sqrt{-a}*\sqrt{(a*x + b)/x}))} + (3*b^2*c^2 \\ & - a*b*c*d + (a*b*c^2 - a^2*c*d)*x)*\sqrt{-a})/((a^2*b*c^3 - a^3*c^2 \\ & ^2*d)*\sqrt{-a}*\sqrt{(a*x + b)/x})] \end{aligned}$$

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a + \frac{b}{x}\right)^{\frac{3}{2}}(cx + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)\*\*(3/2)/(c+d/x),x)

[Out] Integral(x/((a + b/x)\*\*(3/2)\*(c\*x + d)), x)

**GIAC/XCAS** [A] time = 0.254063, size = 261, normalized size = 1.78

$$\left( \frac{2d^3 \arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{(b^2c^3 - abc^2d)\sqrt{bcd-ad^2}} + \frac{2abc - \frac{3(ax+b)bc}{x} + \frac{(ax+b)ad}{x}}{(a^2bc^2 - a^3cd)\left(a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}}\right)} + \frac{(3bc + 2ad) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-aa^2bc^2}} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a + b/x)^(3/2)\*(c + d/x)),x, algorithm="giac")

[Out] 
$$(2*d^3*\arctan(d*\sqrt{(a*x + b)/x}/\sqrt{b*c*d - a*d^2}))/((b^2*c^3 - a*b*c^2*d)*\sqrt{b*c*d - a*d^2}) + (2*a*b*c - 3*(a*x + b)*b*c/x$$

$$+ (a^2 x + b) a^2 d/x / ((a^2 b c^2 - a^3 c d) (a \sqrt{(a^2 x + b)/x} - (a^2 x + b) \sqrt{(a^2 x + b)/x}/x)) + (3 b^2 c + 2 a^2 d) \arctan(\sqrt{(a^2 x + b)/x}/\sqrt{-a}) / (\sqrt{-a} a^2 b c^2) b$$



$$3.158 \quad \int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2} dx$$

**Optimal.** Leaf size=224

$$\begin{aligned} & -\frac{(4ad + 3bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3} + \frac{b(2a^2d^2 - 2abcd + 3b^2c^2)}{a^2c^2\sqrt{a + \frac{b}{x}}(bc - ad)^2} \\ & + \frac{d^{5/2}(7bc - 4ad) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^3(bc - ad)^{5/2}} + \frac{d(bc - 2ad)}{ac^2\sqrt{a + \frac{b}{x}}\left(c + \frac{d}{x}\right)(bc - ad)} + \frac{x}{ac\sqrt{a + \frac{b}{x}}\left(c + \frac{d}{x}\right)} \end{aligned}$$

[Out] (b\*(3\*b^2\*c^2 - 2\*a\*b\*c\*d + 2\*a^2\*d^2))/(a^2\*c^2\*(b\*c - a\*d)^2\*sqrt[a + b/x]) + (d\*(b\*c - 2\*a\*d))/(a\*c^2\*(b\*c - a\*d)\*sqrt[a + b/x]\*(c + d/x)) + x/(a\*c\*sqrt[a + b/x]\*(c + d/x)) + (d^(5/2)\*(7\*b\*c - 4\*a\*d)\*ArcTan[(sqrt[d]\*sqrt[a + b/x])/sqrt[b\*c - a\*d]])/(c^3\*(b\*c - a\*d)^(5/2)) - ((3\*b\*c + 4\*a\*d)\*ArcTanh[sqrt[a + b/x]/sqrt[a]])/(a^(5/2)\*c^3)

**Rubi [A]** time = 0.96161, antiderivative size = 224, normalized size of antiderivative = 1., number of rules used = 9, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$\begin{aligned} & -\frac{(4ad + 3bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3} + \frac{b(2a^2d^2 - 2abcd + 3b^2c^2)}{a^2c^2\sqrt{a + \frac{b}{x}}(bc - ad)^2} \\ & + \frac{d^{5/2}(7bc - 4ad) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^3(bc - ad)^{5/2}} + \frac{d(bc - 2ad)}{ac^2\sqrt{a + \frac{b}{x}}\left(c + \frac{d}{x}\right)(bc - ad)} + \frac{x}{ac\sqrt{a + \frac{b}{x}}\left(c + \frac{d}{x}\right)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b/x)^(3/2)\*(c + d/x)^2), x]

[Out] (b\*(3\*b^2\*c^2 - 2\*a\*b\*c\*d + 2\*a^2\*d^2))/(a^2\*c^2\*(b\*c - a\*d)^2\*sqrt[a + b/x]) + (d\*(b\*c - 2\*a\*d))/(a\*c^2\*(b\*c - a\*d)\*sqrt[a + b/x]\*(c + d/x)) + x/(a\*c\*sqrt[a + b/x]\*(c + d/x)) + (d^(5/2)\*(7\*b\*c - 4\*a\*d)\*ArcTan[(sqrt[d]\*sqrt[a + b/x])/sqrt[b\*c - a\*d]])/(c^3\*(b\*c - a\*d)^(5/2)) - ((3\*b\*c + 4\*a\*d)\*ArcTanh[sqrt[a + b/x]/sqrt[a]])/(a^(5/2)\*c^3)

**Rubi in Sympy [A]** time = 110.149, size = 194, normalized size = 0.87

$$\begin{aligned} & -\frac{dx}{c\sqrt{a + \frac{b}{x}}\left(c + \frac{d}{x}\right)(ad - bc)} + \frac{d^{5/2}(4ad - 7bc) \operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{ad - bc}}\right)}{c^3(ad - bc)^{5/2}} \\ & + \frac{x(2ad - bc)}{ac^2\sqrt{a + \frac{b}{x}}(ad - bc)} + \frac{b(2a^2d^2 - 2abcd + 3b^2c^2)}{a^2c^2\sqrt{a + \frac{b}{x}}(ad - bc)^2} - \frac{(4ad + 3bc) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a+b/x)\*\*(3/2)/(c+d/x)\*\*2, x)



$$\begin{aligned}
& a^{(1/2)+2*a*x+b}/a^{(1/2)} * a^6 * ((a*d-b*c)*d/c^2)^{(1/2)} * x^2 * b^2 * c^3 \\
& * d^3 - 3 * \ln(1/2 * (2 * (x * (a*x+b))^{(1/2)} * a^{(1/2)+2*a*x+b}/a^{(1/2)})) * a^6 * \\
& ((a*d-b*c)*d/c^2)^{(1/2)} * x^3 * b^2 * c^4 * d^2 + 4 * (x * (a*x+b))^{(3/2)} * a^{(11/2)} * \\
& ((a*d-b*c)*d/c^2)^{(1/2)} * x * b * c^4 * d^2 + 10 * (x * (a*x+b))^{(1/2)} * a^{(11/2)} * \\
& ((a*d-b*c)*d/c^2)^{(1/2)} * x^3 * b^2 * c^5 * d - 4 * (x * (a*x+b))^{(3/2)} * a^{(9/2)} * \\
& ((a*d-b*c)*d/c^2)^{(1/2)} * x * b^2 * c^5 * d - 4 * (x * (a*x+b))^{(1/2)} * a^{(13/2)} * \\
& ((a*d-b*c)*d/c^2)^{(1/2)} * x^2 * b * c^3 * d^3 - 8 * (x * (a*x+b))^{(1/2)} * a^{(11/2)} * \\
& ((a*d-b*c)*d/c^2)^{(1/2)} * x^2 * b^2 * c^4 * d^2 - 5 * \ln(1/2 * (2 * (x * (a*x+b))^{(1/2)} * \\
& a^{(1/2)+2*a*x+b}/a^{(1/2)})) * a^3 * ((a*d-b*c)*d/c^2)^{(1/2)} * b^5 * c^4 * d^2 - 4 * \\
& \ln((2 * (x * (a*x+b))^{(1/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / \\
& (c * x + d)) * a^{(17/2)} * x^3 * c * d^5 - 8 * \ln((2 * (x * (a*x+b))^{(1/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * \\
& c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(15/2)} * x * b * d^6 + 11 * \ln((2 * (x * (a*x+b))^{(1/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * \\
& c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(11/2)} * b^3 * c * d^5 - 7 * \ln((2 * (x * (a*x+b))^{(1/2)} * \\
& ((a*d-b*c)*d/c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * \\
& a^{(9/2)} * b^4 * c^2 * d^4 - 12 * (x * (a*x+b))^{(1/2)} * a^{(7/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * x^2 * b^4 * c^6 - 4 * \\
& \ln(1/2 * (2 * (x * (a*x+b))^{(1/2)} * a^{(1/2)+2*a*x+b}/a^{(1/2)})) * a^6 * ((a*d-b*c)*d/c^2)^{(1/2)} * b^2 * c * d^5 + 3 * \ln((2 * (x * (a*x+b))^{(1/2)} * \\
& ((a*d-b*c)*d/c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(15/2)} * x^2 * b * c * d^5 + 15 * \\
& \ln((2 * (x * (a*x+b))^{(1/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(13/2)} * x^2 * b^2 * c^2 * d^4 - 14 * \\
& \ln((2 * (x * (a*x+b))^{(1/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(11/2)} * x^2 * b^3 * c^3 * d^3 - 2 * (x * (a*x+b))^{(3/2)} * a^{(9/2)} * \\
& ((a*d-b*c)*d/c^2)^{(1/2)} * b^2 * c^4 * d^2 + 4 * (x * (a*x+b))^{(3/2)} * a^{(7/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * b^3 * c^5 * d + 9 * \\
& \ln(1/2 * (2 * (x * (a*x+b))^{(1/2)} * a^{(1/2)+2*a*x+b}/a^{(1/2)})) * a^5 * ((a*d-b*c)*d/c^2)^{(1/2)} * b^3 * c^2 * d^4 - 4 * \\
& \ln(1/2 * (2 * (x * (a*x+b))^{(1/2)} * a^{(1/2)+2*a*x+b}/a^{(1/2)})) * a^8 * ((a*d-b*c)*d/c^2)^{(1/2)} * x^3 * c^2 * d^4 - 2 * (x * (a*x+b))^{(1/2)} * a^{(15/2)} * \\
& ((a*d-b*c)*d/c^2)^{(1/2)} * x^4 * c^4 * d^2 + 18 * \ln((2 * (x * (a*x+b))^{(1/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(13/2)} * x * b^2 * c * d^5 - 3 * \\
& \ln((2 * (x * (a*x+b))^{(1/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(11/2)} * x * b^3 * c^2 * d^4 - 7 * \ln((2 * (x * (a*x+b))^{(1/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(9/2)} * x * b^4 * c^3 * d^3 + 4 * (x * (a*x+b))^{(1/2)} * a^{(11/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * b^2 * c^2 * d^4 - 8 * (x * (a*x+b))^{(1/2)} * a^{(9/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * b^3 * c^3 * d^3 + 10 * (x * (a*x+b))^{(1/2)} * a^{(7/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * b^4 * c^4 * d^2 - 3 * \ln(1/2 * (2 * (x * (a*x+b))^{(1/2)} * a^{(1/2)+2*a*x+b}/a^{(1/2)})) * a^4 * ((a*d-b*c)*d/c^2)^{(1/2)} * b^4 * c^3 * d^3 + 8 * (x * (a*x+b))^{(1/2)} * a^{(13/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * x * b * c^2 * d^4 - 14 * (x * (a*x+b))^{(1/2)} * a^{(11/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * x * b^2 * c^3 * d^3 + 12 * (x * (a*x+b))^{(1/2)} * a^{(9/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * x * b^3 * c^4 * d^2 - 2 * (x * (a*x+b))^{(1/2)} * a^{(7/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * x * b^4 * c^5 * d - 13 * \ln(1/2 * (2 * (x * (a*x+b))^{(1/2)} * a^{(1/2)+2*a*x+b}/a^{(1/2)})) * a^4 * ((a*d-b*c)*d/c^2)^{(1/2)} * x * b^4 * c^4 * d^2 + \ln(1/2 * (2 * (x * (a*x+b))^{(1/2)} * a^{(1/2)+2*a*x+b}/a^{(1/2)})) * a^7 * ((a*d-b*c)*d/c^2)^{(1/2)} * x^2 * b * c^2 * d^4 - 6 * (x * (a*x+b))^{(1/2)} * a^{(5/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * x * b^5 * c^6 + 3 * \ln(1/2 * (2 * (x * (a*x+b))^{(1/2)} * a^{(1/2)+2*a*x+b}/a^{(1/2)})) * ((a*d-b*c)*d/c^2)^{(1/2)} * x^3 * a^4 * b^4 * c^6 - 4 * \ln((2 * (x * (a*x+b))^{(1/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(17/2)} * x^2 * d^6 - 4 * \ln((2 * (x * (a*x+b))^{(1/2)} * ((a*d-b*c)*d/c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(13/2)} * b^2 * d^6 / (x * (a*x+b))^{(1/2)} / (a*d-b*c)^3 / c^4 / ((a*d-b*c)*d/c^2)^{(1/2)} / (a*x+b)^2 / (c*x+d)
\end{aligned}$$


---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a + b/x)^(3/2) \* (c + d/x)^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.812128, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a + b/x)^(3/2)\*(c + d/x)^2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/2*((7*a^2*b*c*d^3 - 4*a^3*d^4 + (7*a^2*b*c^2*d^2 - 4*a^3*c*d^3)*x)*\sqrt{a}*\sqrt{-d/(b*c - a*d)}*\sqrt{(a*x + b)/x}*\log(-(2*(b*c - a*d)*x*\sqrt{-d/(b*c - a*d)}*\sqrt{(a*x + b)/x} - b*d + (b*c - 2*a*d)*x)/(c*x + d)) - (3*b^3*c^3*d - 2*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 + 4*a^3*d^4 + (3*b^3*c^4 - 2*a*b^2*c^3*d - 5*a^2*b*c^2*d^2 + 4*a^3*c*d^3)*x)*\sqrt{(a*x + b)/x}*\log(-2*a*x*\sqrt{(a*x + b)/x} + (2*a*x + b)*\sqrt{a}) - 2*(3*b^3*c^3*d - 2*a*b^2*c^2*d^2 + 2*a^2*b*c*d^3 + (a*b^2*c^4 - 2*a^2*b*c^3*d + a^3*c^2*d^2)*x^2 + (3*b^3*c^4 - a*b^2*c^3*d - a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x)*\sqrt{a}]/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x)*\sqrt{a}*\sqrt{(a*x + b)/x}), 1/2*(2*(7*a^2*b*c*d^3 - 4*a^3*d^4 + (7*a^2*b*c^2*d^2 - 4*a^3*c*d^3)*x)*\sqrt{a}*\sqrt{d/(b*c - a*d)}*\sqrt{(a*x + b)/x}*\arctan(-(b*c - a*d)*\sqrt{d/(b*c - a*d)})/(d*\sqrt{(a*x + b)/x})) + (3*b^3*c^3*d - 2*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 + 4*a^3*d^4 + (3*b^3*c^4 - 2*a*b^2*c^3*d - 5*a^2*b*c^2*d^2 + 4*a^3*c*d^3)*x)*\sqrt{(a*x + b)/x}*\log(-2*a*x*\sqrt{(a*x + b)/x} + (2*a*x + b)*\sqrt{a}) + 2*(3*b^3*c^3*d - 2*a*b^2*c^2*d^2 + 2*a^2*b*c*d^3 + (a*b^2*c^4 - 2*a^2*b*c^3*d + a^3*c^2*d^2)*x^2 + (3*b^3*c^4 - a*b^2*c^3*d - a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x)*\sqrt{a}]/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x)*\sqrt{a}*\sqrt{(a*x + b)/x}), -1/2*((7*a^2*b*c*d^3 - 4*a^3*d^4 + (7*a^2*b*c^2*d^2 - 4*a^3*c*d^3)*x)*\sqrt{-a}*\sqrt{-d/(b*c - a*d)}*\sqrt{(a*x + b)/x}*\log(-(2*(b*c - a*d)*x*\sqrt{-d/(b*c - a*d)}*\sqrt{(a*x + b)/x} - b*d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(3*b^3*c^3*d - 2*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 + 4*a^3*d^4 + (3*b^3*c^4 - 2*a*b^2*c^3*d - 5*a^2*b*c^2*d^2 + 4*a^3*c*d^3)*x)*\sqrt{(a*x + b)/x}*\arctan(a/(\sqrt{-a}*\sqrt{(a*x + b)/x})) - 2*(3*b^3*c^3*d - 2*a*b^2*c^2*d^2 + 2*a^2*b*c*d^3 + (a*b^2*c^4 - 2*a^2*b*c^3*d + a^3*c^2*d^2)*x^2 + (3*b^3*c^4 - a*b^2*c^3*d - a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x)*\sqrt{-a}]/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x)*\sqrt{-a}*\sqrt{(a*x + b)/x}), ((7*a^2*b*c*d^3 - 4*a^3*d^4 + (7*a^2*b*c^2*d^2 - 4*a^3*c*d^3)*x)*\sqrt{-a}*\sqrt{d/(b*c - a*d)}*\sqrt{(a*x + b)/x}*\arctan(-(b*c - a*d)*\sqrt{d/(b*c - a*d)})/(d*\sqrt{(a*x + b)/x})) + (3*b^3*c^3*d - 2*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 + 4*a^3*d^4 + (3*b^3*c^4 - 2*a*b^2*c^3*d - 5*a^2*b*c^2*d^2 + 4*a^3*c*d^3)*x)*\sqrt{(a*x + b)/x}*\arctan(a/(\sqrt{-a}*\sqrt{(a*x + b)/x})) + (3*b^3*c^3*d - 2*a*b^2*c^2*d^2 + 2*a^2*b*c*d^3 + (a*b^2*c^4 - 2*a^2*b*c^3*d + a^3*c^2*d^2)*x^2 + (3*b^3*c^4 - a*b^2*c^3*d - a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x)*\sqrt{-a}]/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x)*\sqrt{-a}*\sqrt{(a*x + b)/x})] \end{aligned}$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)\*\*(3/2)/(c+d/x)\*\*2,x)

[Out] Exception raised: ValueError

**GIAC/XCAS [A]** time = 0.256391, size = 560, normalized size = 2.5

$$b \left( \frac{(7bcd^3 - 4ad^4) \arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{(b^3c^5 - 2ab^2c^4d + a^2bc^3d^2)\sqrt{bcd-ad^2}} + \frac{2ab^3c^3 - 2a^2b^2c^2d - \frac{3(ax+b)b^3c^3}{x} + \frac{7(ax+b)ab^2c^2d}{x} - \frac{3(ax+b)a^2bcd^2}{x} + \frac{2(ax+b)a^3d^3}{x}}{(a^2b^2c^4 - 2a^3bc^3d + a^4c^2d^2)} \left( abc\sqrt{\frac{ax+b}{x}} - a^2d\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)bc\sqrt{\frac{ax+b}{x}}}{x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a + b/x)^(3/2)\*(c + d/x)^2),x, algorithm="giac")

[Out] 
$$\frac{b \cdot ((7 \cdot b \cdot c \cdot d^3 - 4 \cdot a \cdot d^4) \cdot \arctan(d \cdot \sqrt{(a \cdot x + b)/x}) / \sqrt{b \cdot c \cdot d - a \cdot d^2})}{(b^3 \cdot c^5 - 2 \cdot a \cdot b^2 \cdot c^4 \cdot d + a^2 \cdot b \cdot c^3 \cdot d^2) \cdot \sqrt{b \cdot c \cdot d - a \cdot d^2}} + \frac{(2 \cdot a \cdot b^3 \cdot c^3 - 2 \cdot a^2 \cdot b^2 \cdot c^2 \cdot d - 3 \cdot (a \cdot x + b) \cdot b^3 \cdot c^3 / x + 7 \cdot (a \cdot x + b) \cdot a \cdot b^2 \cdot c^2 \cdot d / x - 3 \cdot (a \cdot x + b) \cdot a^2 \cdot b \cdot c \cdot d^2 / x + 2 \cdot (a \cdot x + b) \cdot a^3 \cdot d^3 / x - 3 \cdot (a \cdot x + b)^2 \cdot b^2 \cdot c^2 \cdot d / x^2 + 2 \cdot (a \cdot x + b)^2 \cdot a \cdot b \cdot c \cdot d^2 / x^2 - 2 \cdot (a \cdot x + b)^2 \cdot a^2 \cdot d^3 / x^2)}{(a^2 \cdot b^2 \cdot c^4 - 2 \cdot a^3 \cdot b \cdot c^3 \cdot d + a^4 \cdot c^2 \cdot d^2) \cdot (a \cdot b \cdot c \cdot \sqrt{(a \cdot x + b)/x} - a^2 \cdot d \cdot \sqrt{(a \cdot x + b)/x}) - (a \cdot x + b) \cdot b \cdot c \cdot \sqrt{(a \cdot x + b)/x} / x + 2 \cdot (a \cdot x + b) \cdot a \cdot d \cdot \sqrt{(a \cdot x + b)/x} / x - (a \cdot x + b)^2 \cdot d \cdot \sqrt{(a \cdot x + b)/x} / x^2)} + (3 \cdot b \cdot c + 4 \cdot a \cdot d) \cdot \arctan(\sqrt{(a \cdot x + b)/x} / \sqrt{-a}) / (\sqrt{-a} \cdot a^2 \cdot b \cdot c^3)$$

$$3.159 \quad \int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^3} dx$$

**Optimal.** Leaf size=320

$$\begin{aligned} & \frac{3(2ad + bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^4} + \frac{3d^{5/2}(8a^2d^2 - 24abcd + 21b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{7/2}} \\ & + \frac{d(12a^2d^2 - 21abcd + 4b^2c^2)}{4ac^3\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)^2} + \frac{3b(2bc-ad)(4a^2d^2 - abcd + 2b^2c^2)}{4a^2c^3\sqrt{a+\frac{b}{x}}(bc-ad)^3} \\ & + \frac{d(2bc-3ad)}{2ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2(bc-ad)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} \end{aligned}$$

[Out] (3\*b\*(2\*b\*c - a\*d)\*(2\*b^2\*c^2 - a\*b\*c\*d + 4\*a^2\*d^2))/(4\*a^2\*c^3\*(b\*c - a\*d)^3\*Sqrt[a + b/x]) + (d\*(2\*b\*c - 3\*a\*d))/(2\*a\*c^2\*(b\*c - a\*d)\*Sqrt[a + b/x]\*(c + d/x)^2) + (d\*(4\*b^2\*c^2 - 21\*a\*b\*c\*d + 12\*a^2\*d^2))/(4\*a\*c^3\*(b\*c - a\*d)^2\*Sqrt[a + b/x]\*(c + d/x)) + x/(a\*c\*Sqrt[a + b/x]\*(c + d/x)^2) + (3\*d^(5/2)\*(21\*b^2\*c^2 - 24\*a\*b\*c\*d + 8\*a^2\*d^2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(4\*c^4\*(b\*c - a\*d)^(7/2)) - (3\*(b\*c + 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(5/2)\*c^4)

**Rubi [A]** time = 1.52398, antiderivative size = 320, normalized size of antiderivative = 1., number of rules used = 10, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$\begin{aligned} & \frac{3(2ad + bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^4} + \frac{3d^{5/2}(8a^2d^2 - 24abcd + 21b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{7/2}} \\ & + \frac{d(12a^2d^2 - 21abcd + 4b^2c^2)}{4ac^3\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)^2} + \frac{3b(2bc-ad)(4a^2d^2 - abcd + 2b^2c^2)}{4a^2c^3\sqrt{a+\frac{b}{x}}(bc-ad)^3} \\ & + \frac{d(2bc-3ad)}{2ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2(bc-ad)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b/x)^(3/2)\*(c + d/x)^3), x]

[Out] (3\*b\*(2\*b\*c - a\*d)\*(2\*b^2\*c^2 - a\*b\*c\*d + 4\*a^2\*d^2))/(4\*a^2\*c^3\*(b\*c - a\*d)^3\*Sqrt[a + b/x]) + (d\*(2\*b\*c - 3\*a\*d))/(2\*a\*c^2\*(b\*c - a\*d)\*Sqrt[a + b/x]\*(c + d/x)^2) + (d\*(4\*b^2\*c^2 - 21\*a\*b\*c\*d + 12\*a^2\*d^2))/(4\*a\*c^3\*(b\*c - a\*d)^2\*Sqrt[a + b/x]\*(c + d/x)) + x/(a\*c\*Sqrt[a + b/x]\*(c + d/x)^2) + (3\*d^(5/2)\*(21\*b^2\*c^2 - 24\*a\*b\*c\*d + 8\*a^2\*d^2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(4\*c^4\*(b\*c - a\*d)^(7/2)) - (3\*(b\*c + 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(5/2)\*c^4)

**Rubi in Sympy [F-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a+b/x)\*\*(3/2)/(c+d/x)\*\*3, x)

[Out] Timed out

**Mathematica [C]** time = 1.82433, size = 385, normalized size = 1.2

$$\frac{12(2ad+bc)\log\left(2\sqrt{ax}\sqrt{a+\frac{b}{x}}+2ax+b\right)}{a^{5/2}} + \frac{3id^{5/2}(8a^2d^2-24abcd+21b^2c^2)\log\left(-\frac{8ic^5(bc-ad)^{5/2}\left(-2i\sqrt{dx}\sqrt{a+\frac{b}{x}}\sqrt{bc-ad}-2adx+bcx-bd\right)}{3d^{7/2}(cx+d)(8a^2d^2-24abcd+21b^2c^2)}\right)}{(bc-ad)^{7/2}} + \frac{2cx\sqrt{a+\frac{b}{x}}(2a^4d^3x)}{8c^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b/x)^(3/2)\*(c + d/x)^3), x]

[Out] 
$$\left(\frac{(2c\sqrt{a+b/x})^2 x^2 (-12b^4c^3(d+cx)^2 - 4ab^3c^2(-3d+cx)(d+cx)^2 + 2a^4d^3x^2(6d^2+9cdx+2c^2x^2) + a^3b^2d^2(12d^3-9cd^2x-37c^2d^2x^2-12c^3x^3) + a^2b^2cd^2(-27d^3-29cd^2x+12c^2d^2x^2+12c^3x^3))}{a^2(-bc+ad)^3(b+ax)(d+cx)^2} - (12(bc+2ad)\log[b+2ax+2\sqrt{a}\sqrt{a+b/x}x])/a^{5/2} + ((3I)d^{5/2}(21b^2c^2-24ab^2cd+8a^2d^2)\log[\frac{(-8I)^3c^5(bc-ad)^{5/2}(-bd+bcx-2ad^2x-(2I)\sqrt{d}\sqrt{bc-ad})}{d^{7/2}(21b^2c^2-24ab^2cd+8a^2d^2)(d+cx)}] + (8c^4))\right)$$

**Maple [B]** time = 0.027, size = 5164, normalized size = 16.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b/x)^(3/2)/(c+d/x)^3, x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a + b/x)^(3/2)\*(c + d/x)^3), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.24754, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a + b/x)^(3/2)\*(c + d/x)^3), x, algorithm="fricas")

[Out] 
$$\left[-\frac{1}{8}(3(21a^2b^2c^2d^4 - 24a^3b^2cd^5 + 8a^4d^6 + (21a^2b^2c^4d^2 - 24a^3b^2c^3d^3 + 8a^4c^2d^4)x^2 + 2(21a^2b^2c^3d^3 - 24a^3b^2c^2d^4 + 8a^4c^2d^5)x)\sqrt{a}\sqrt{-d/(bc-ad)}\sqrt{(ax+b)/x}\log\left(-\frac{2(bc-ad)x\sqrt{-d/(bc-ad)}\sqrt{(ax+b)/x} - bd + (bc-2ad)x}{(cx+d)}\right) - 12(b^4c^4d^2 - ab^3c^3d^3 - 3a^2b^2c^2d^4 + 5a^3b^2c^2d^4)\right)$$

$$\begin{aligned}
& d^5 - 2a^4d^6 + (b^4c^6 - ab^3c^5d - 3a^2b^2c^4d^2 + 5 \\
& a^3b^2c^3d^3 - 2a^4c^2d^4) x^2 + 2(b^4c^5d - ab^3c^4d^2 \\
& - 3a^2b^2c^3d^3 + 5a^3b^2c^2d^4 - 2a^4c^2d^5) x) \sqrt{(a \\
& x + b)/x} \log(-2ax \sqrt{(ax + b)/x} + (2ax + b) \sqrt{a}) - \\
& 2(12b^4c^4d^2 - 12ab^3c^3d^3 + 27a^2b^2c^2d^4 - 12a^3 \\
& b^2c^2d^5 + 4(ab^3c^6 - 3a^2b^2c^5d + 3a^3b^2c^4d^2 - a^4 \\
& c^3d^3) x^3 + (12b^4c^6 - 4ab^3c^5d - 12a^2b^2c^4d^2 \\
& + 37a^3b^2c^3d^3 - 18a^4c^2d^4) x^2 + (24b^4c^5d - 20a^3 \\
& b^3c^4d^2 + 29a^2b^2c^3d^3 + 9a^3b^2c^2d^4 - 12a^4c^2d^5) \\
& ) x) \sqrt{a}) / ((a^2b^3c^7d^2 - 3a^3b^2c^6d^3 + 3a^4b^2c^5 \\
& d^4 - a^5c^4d^5 + (a^2b^3c^9 - 3a^3b^2c^8d + 3a^4b^2c^7 \\
& d^2 - a^5c^6d^3) x^2 + 2(a^2b^3c^8d - 3a^3b^2c^7d^2 + \\
& 3a^4b^2c^6d^3 - a^5c^5d^4) x) \sqrt{a}) \sqrt{(ax + b)/x}), -1/ \\
& 8(3(21a^2b^2c^2d^4 - 24a^3b^2c^2d^5 + 8a^4d^6 + (21a^2b^2 \\
& c^4d^2 - 24a^3b^2c^3d^3 + 8a^4c^2d^4) x^2 + 2(21a^2b^2 \\
& c^3d^3 - 24a^3b^2c^2d^4 + 8a^4c^2d^5) x) \sqrt{-a}) \sqrt{-d/( \\
& bc - ad)} \sqrt{(ax + b)/x} \log(-(2(bc - ad) x \sqrt{-d/(bc \\
& - ad)}) \sqrt{(ax + b)/x} - bd + (bc - 2ad) x)/(cx + d)) - 2 \\
& 4(b^4c^4d^2 - ab^3c^3d^3 - 3a^2b^2c^2d^4 + 5a^3b^2c^2d^5 - 2a^4d^6 + (b^4c^6 - ab^3c^5d - 3a^2b^2c^4d^2 + 5a^3 \\
& b^2c^3d^3 - 2a^4c^2d^4) x^2 + 2(b^4c^5d - ab^3c^4d^2 - \\
& 3a^2b^2c^3d^3 + 5a^3b^2c^2d^4 - 2a^4c^2d^5) x) \sqrt{(ax \\
& + b)/x} \arctan(a/(\sqrt{-a}) \sqrt{(ax + b)/x})) - 2(12b^4c^4d^2 \\
& - 12ab^3c^3d^3 + 27a^2b^2c^2d^4 - 12a^3b^2c^2d^5 + 4(a \\
& b^3c^6 - 3a^2b^2c^5d + 3a^3b^2c^4d^2 - a^4c^3d^3) x^3 + \\
& (12b^4c^6 - 4ab^3c^5d - 12a^2b^2c^4d^2 + 37a^3b^2c^3 \\
& d^3 - 18a^4c^2d^4) x^2 + (24b^4c^5d - 20ab^3c^4d^2 + 29 \\
& a^2b^2c^3d^3 + 9a^3b^2c^2d^4 - 12a^4c^2d^5) x) \sqrt{-a}) / ( \\
& (a^2b^3c^7d^2 - 3a^3b^2c^6d^3 + 3a^4b^2c^5d^4 - a^5c^4d^5 \\
& + (a^2b^3c^9 - 3a^3b^2c^8d + 3a^4b^2c^7d^2 - a^5c^6d^3) \\
& d^3) x^2 + 2(a^2b^3c^8d - 3a^3b^2c^7d^2 + 3a^4b^2c^6d^3 \\
& - a^5c^5d^4) x) \sqrt{-a}) \sqrt{(ax + b)/x}), 1/4(3(21a^2b^2 \\
& c^2d^4 - 24a^3b^2c^2d^5 + 8a^4d^6 + (21a^2b^2c^4d^2 - 24 \\
& a^3b^2c^3d^3 + 8a^4c^2d^4) x^2 + 2(21a^2b^2c^3d^3 - 24 \\
& a^3b^2c^2d^4 + 8a^4c^2d^5) x) \sqrt{a}) \sqrt{d/(bc - ad)}) \sqrt{ \\
& (ax + b)/x} \arctan(-(bc - ad) \sqrt{d/(bc - ad)}) / (d \sqrt{(ax \\
& + b)/x})) + 6(b^4c^4d^2 - ab^3c^3d^3 - 3a^2b^2c^2d^4 + \\
& 5a^3b^2c^2d^5 - 2a^4d^6 + (b^4c^6 - ab^3c^5d - 3a^2b^2c^4 \\
& d^2 + 5a^3b^2c^3d^3 - 2a^4c^2d^4) x^2 + 2(b^4c^5d - a^3 \\
& b^3c^4d^2 - 3a^2b^2c^3d^3 + 5a^3b^2c^2d^4 - 2a^4c^2d^5) x) \\
& \sqrt{(ax + b)/x} \log(-2ax \sqrt{(ax + b)/x} + (2ax + b) \sqrt{a}) + \\
& (12b^4c^4d^2 - 12ab^3c^3d^3 + 27a^2b^2c^2d^4 - 12a^3b^2c^2d^5 + 4 \\
& (ab^3c^6 - 3a^2b^2c^5d + 3a^3b^2c^4d^2 - a^4c^3d^3) x^3 + \\
& (12b^4c^6 - 4ab^3c^5d - 12a^2b^2c^4d^2 + 37a^3b^2c^3d^3 - 18a^4c^2d^4) \\
& x^2 + (24b^4c^5d - 20ab^3c^4d^2 + 29a^2b^2c^3d^3 + 9a^3b^2c^2d^4 - 12a^4c^2d^5) \\
& ) x) \sqrt{a}) / ((a^2b^3c^7d^2 - 3a^3b^2c^6d^3 + 3a^4b^2c^5d^4 - a^5c^4d^5 + (a^2b^3c^9 - 3a^3b^2c^8d + 3a^4b^2c^7d^2 - a^5c^6d^3) \\
& ) x^2 + 2(a^2b^3c^8d - 3a^3b^2c^7d^2 + 3a^4b^2c^6d^3 - a^5c^5d^4) x) \sqrt{a}) \sqrt{(ax + b)/ \\
& x}), 1/4(3(21a^2b^2c^2d^4 - 24a^3b^2c^2d^5 + 8a^4d^6 + (2 \\
& 1a^2b^2c^4d^2 - 24a^3b^2c^3d^3 + 8a^4c^2d^4) x^2 + 2(21 \\
& a^2b^2c^3d^3 - 24a^3b^2c^2d^4 + 8a^4c^2d^5) x) \sqrt{-a}) \sqrt{d/(bc - ad)}) \sqrt{(ax + b)/x} \arctan(-(bc - ad) \sqrt{d/(bc \\
& - ad)}) / (d \sqrt{(ax + b)/x})) + 12(b^4c^4d^2 - ab^3c^3d^3 - 3a^2b^2c^2d^4 + 5a^3b^2c^2d^5 - 2a^4d^6 + (b^4c^6 - ab^3c^5d - 3a^2b^2c^4d^2 + 5a^3b^2c^3d^3 - 2a^4c^2d^4) \\
& ) x^2 + 2(b^4c^5d - ab^3c^4d^2 - 3a^2b^2c^3d^3 + 5a^3b^2c^2d^4 - 2a^4c^2d^5) x) \sqrt{(ax + b)/x} \arctan(a/(\sqrt{-a}) \sqrt{(ax + b)/x})) + (12b^4c^4d^2 - 12ab^3c^3d^3 + 27a^2b^2c^2d^4 - 12a^3b^2c^2d^5 + 4 \\
& (ab^3c^6 - 3a^2b^2c^5d + 3a^3b^2c^4d^2 - a^4c^3d^3) x^3 + (12b^4c^6 - 4ab^3c^5d - 12a^2b^2c^4d^2 + 37a^3b^2c^3d^3 - 18a^4c^2d^4) x^2 + (24b^4c^5d - 20ab^3c^4d^2 + 29a^2b^2c^3d^3 + 9a^3b^2c^2d^4 - 12a^4c^2d^5) \\
& ) x) \sqrt{-a}) / ((a^2b^3c^7d^2 - 3a^3b^2c^6d^3 + 3a^4b^2c^5d^4 - a^5c^4d^5 + (a^2b^3c^9 - 3a^3b^2c^8d + 3a^4b^2c^7d^2 - a^5c^6d^3) x^2 + 2(a^2b^3c^8d - 3a^3b^2c^7d^2 + 3a^4b^2c^6d^3 - a^5c^5d^4) x) \sqrt{-a}) \sqrt{ \\
& (ax + b)/x})]
\end{aligned}$$



**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)\*\*(3/2)/(c+d/x)\*\*3,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.261271, size = 678, normalized size = 2.12

$$\frac{1}{4}b \left( \frac{3(21b^2c^2d^3 - 24abcd^4 + 8a^2d^5) \arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{(b^4c^7 - 3ab^3c^6d + 3a^2b^2c^5d^2 - a^3bc^4d^3)\sqrt{bcd-ad^2}} + \frac{4\left(2ab^3c^3 - \frac{3(ax+b)b^3c^3}{x} + \frac{3(ax+b)ab^2c^2d}{x} - \frac{3(ax+b)a^2bcd^2}{x} + \frac{(ax+b)\sqrt{ax+b}}{x}\right)}{(a^2b^3c^6 - 3a^3b^2c^5d + 3a^4bc^4d^2 - a^5c^3d^3)} \left( a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{ax+b}}{x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a + b/x)^(3/2)\*(c + d/x)^3),x, algorithm="giac")

[Out] 1/4\*b\*(3\*(21\*b^2\*c^2\*d^3 - 24\*a\*b\*c\*d^4 + 8\*a^2\*d^5)\*arctan(d\*sqrt((a\*x + b)/x)/sqrt(b\*c\*d - a\*d^2))/((b^4\*c^7 - 3\*a\*b^3\*c^6\*d + 3\*a^2\*b^2\*c^5\*d^2 - a^3\*b\*c^4\*d^3)\*sqrt(b\*c\*d - a\*d^2)) + 4\*(2\*a\*b^3\*c^3 - 3\*(a\*x + b)\*b^3\*c^3/x + 3\*(a\*x + b)\*a\*b^2\*c^2\*d/x - 3\*(a\*x + b)\*a^2\*b\*c\*d^2/x + (a\*x + b)\*a^3\*d^3/x)/((a^2\*b^3\*c^6 - 3\*a^3\*b^2\*c^5\*d + 3\*a^4\*b\*c^4\*d^2 - a^5\*c^3\*d^3)\*(a\*sqrt((a\*x + b)/x) - (a\*x + b)\*sqrt((a\*x + b)/x)/x) + (17\*b^2\*c^2\*d^3\*sqrt((a\*x + b)/x) - 25\*a\*b\*c\*d^4\*sqrt((a\*x + b)/x) + 8\*a^2\*d^5\*sqrt((a\*x + b)/x) + 15\*(a\*x + b)\*b\*c\*d^4\*sqrt((a\*x + b)/x)/x - 8\*(a\*x + b)\*a\*d^5\*sqrt((a\*x + b)/x)/x)/((b^3\*c^6 - 3\*a\*b^2\*c^5\*d + 3\*a^2\*b\*c^4\*d^2 - a^3\*c^3\*d^3)\*(b\*c - a\*d + (a\*x + b)\*d/x)^2) + 12\*(b\*c + 2\*a\*d)\*arctan(sqrt((a\*x + b)/x)/sqrt(-a))/(sqrt(-a)\*a^2\*b\*c^4))

$$3.160 \quad \int \frac{\left(c + \frac{d}{x}\right)^3}{\left(a + \frac{b}{x}\right)^{5/2}} dx$$

**Optimal.** Leaf size=143

$$\frac{c^2(5bc - 6ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{(bc - ad)(-4a^3d^2x - 2a^2bd(5cx + 3d) + ab^2c(20cx - 3d) + 15b^3c^2)}{3a^3b^2x\left(a + \frac{b}{x}\right)^{3/2}} + \frac{cx\left(c + \frac{d}{x}\right)^2}{a\left(a + \frac{b}{x}\right)^{3/2}}$$

[Out]  $(c*(c + d/x)^2*x)/(a*(a + b/x)^(3/2)) + ((b*c - a*d)*(15*b^3*c^2 - 4*a^3*d^2*x - 2*a^2*b*d*(3*d + 5*c*x) + a*b^2*c*(-3*d + 20*c*x)))/(3*a^3*b^2*(a + b/x)^(3/2)*x) - (c^2*(5*b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)$

**Rubi [A]** time = 0.428991, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{c^2(5bc - 6ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{(bc - ad)(-4a^3d^2x - 2a^2bd(5cx + 3d) - ab^2c(3d - 20cx) + 15b^3c^2)}{3a^3b^2x\left(a + \frac{b}{x}\right)^{3/2}} + \frac{cx\left(c + \frac{d}{x}\right)^2}{a\left(a + \frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c + d/x)^3/(a + b/x)^(5/2), x]

[Out]  $(c*(c + d/x)^2*x)/(a*(a + b/x)^(3/2)) + ((b*c - a*d)*(15*b^3*c^2 - 4*a^3*d^2*x - a*b^2*c*(3*d - 20*c*x) - 2*a^2*b*d*(3*d + 5*c*x)))/(3*a^3*b^2*(a + b/x)^(3/2)*x) - (c^2*(5*b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)$

**Rubi in Sympy [A]** time = 27.8366, size = 138, normalized size = 0.97

$$\frac{cx\left(c + \frac{d}{x}\right)^2}{a\left(a + \frac{b}{x}\right)^{3/2}} + \frac{4(ad - bc)\left(\frac{a(2a^2d^2 + 5abcd - 10b^2c^2)}{2} + \frac{3b(2a^2d^2 + abcd - 5b^2c^2)}{4x}\right)}{3a^3b^2\left(a + \frac{b}{x}\right)^{3/2}} + \frac{c^2(6ad - 5bc) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d/x)\*\*3/(a+b/x)\*\*(5/2), x)

[Out]  $c*x*(c + d/x)**2/(a*(a + b/x)**(3/2)) + 4*(a*d - b*c)*(a*(2*a**2*d**2 + 5*a*b*c*d - 10*b**2*c**2)/2 + 3*b*(2*a**2*d**2 + a*b*c*d - 5*b**2*c**2)/(4*x))/(3*a**3*b**2*(a + b/x)**(3/2)) + c**2*(6*a*d - 5*b*c)*atanh(sqrt(a + b/x)/sqrt(a))/a**(7/2)$

**Mathematica [A]** time = 0.242433, size = 145, normalized size = 1.01

$$\frac{c^2(6ad - 5bc) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right)}{2a^{7/2}} + \frac{x\sqrt{a + \frac{b}{x}}(4a^4d^3x + 6a^3bd^2(cx + d) + 3a^2b^2c^2x(cx - 8d) + 2ab^3c^2(10cx - 9d) + 15b^4c^3)}{3a^3b^2(ax + b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d/x)^3/(a + b/x)^(5/2), x]

[Out] (Sqrt[a + b/x]\*x\*(15\*b^4\*c^3 + 4\*a^4\*d^3\*x + 3\*a^2\*b^2\*c^2\*x\*(-8\*d + c\*x) + 6\*a^3\*b\*d^2\*(d + c\*x) + 2\*a\*b^3\*c^2\*(-9\*d + 10\*c\*x)))/(3\*a^3\*b^2\*(b + a\*x)^2) + (c^2\*(-5\*b\*c + 6\*a\*d)\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]\*x])/(2\*a^(7/2))

**Maple [B]** time = 0.023, size = 1157, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^3/(a+b/x)^(5/2), x)

[Out] 1/6\*((a\*x+b)/x)^(1/2)\*x/a^(13/2)\*(90\*a^(11/2)\*(x\*(a\*x+b))^(1/2)\*x^2\*b^4\*c^3+9\*ln(1/2\*(2\*(a\*x^2+b\*x))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^7\*x\*b^3\*d^3-9\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^7\*x\*b^3\*d^3+18\*a^(15/2)\*(a\*x^2+b\*x)^(1/2)\*x\*b^2\*d^3+12\*a^(13/2)\*(x\*(a\*x+b))^(3/2)\*b^2\*c\*d^2+24\*a^(11/2)\*(x\*(a\*x+b))^(3/2)\*b^3\*c^2\*d+18\*a^(15/2)\*(x\*(a\*x+b))^(1/2)\*x\*b^2\*d^3+90\*a^(9/2)\*(x\*(a\*x+b))^(1/2)\*x\*b^5\*c^3-36\*a^(9/2)\*(x\*(a\*x+b))^(1/2)\*b^5\*c^2\*d+18\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^4\*b^6\*c^2\*d-15\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*x^2\*d-15\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*x^2\*a^5\*b^5\*c^3-45\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*x^2\*a^5\*b^5\*c^3-45\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*x^2\*a^4\*b^6\*c^3+3\*ln(1/2\*(2\*(a\*x^2+b\*x))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^9\*x^3\*b\*d^3-3\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^9\*x^3\*b\*d^3+30\*a^(13/2)\*(x\*(a\*x+b))^(1/2)\*x^3\*b^3\*c^3+9\*ln(1/2\*(2\*(a\*x^2+b\*x))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^8\*x^2\*b^2\*d^3-9\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^8\*x^2\*b^2\*d^3+18\*a^(17/2)\*(a\*x^2+b\*x)^(1/2)\*x^2\*b\*d^3-24\*a^(11/2)\*(x\*(a\*x+b))^(3/2)\*x\*b^3\*c^3+18\*a^(17/2)\*(x\*(a\*x+b))^(1/2)\*x^2\*b\*d^3+30\*a^(7/2)\*(x\*(a\*x+b))^(1/2)\*b^6\*c^3-15\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^3\*b^7\*c^3+6\*a^(19/2)\*(a\*x^2+b\*x)^(1/2)\*x^3\*d^3+6\*a^(19/2)\*(x\*(a\*x+b))^(1/2)\*x^3\*d^3-12\*a^(17/2)\*(x\*(a\*x+b))^(3/2)\*x\*d^3-16\*a^(15/2)\*(x\*(a\*x+b))^(3/2)\*b\*d^3-20\*a^(9/2)\*(x\*(a\*x+b))^(3/2)\*b^4\*c^3+3\*ln(1/2\*(2\*(a\*x^2+b\*x))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^6\*b^4\*d^3-3\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^6\*b^4\*d^3+6\*a^(13/2)\*(a\*x^2+b\*x)^(1/2)\*b^3\*d^3+6\*a^(13/2)\*(x\*(a\*x+b))^(1/2)\*b^3\*d^3+54\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^5\*x\*b^5\*c^2\*d-36\*a^(15/2)\*(x\*(a\*x+b))^(1/2)\*x^3\*b^2\*c^2\*d+36\*a^(13/2)\*(x\*(a\*x+b))^(3/2)\*x\*b^2\*c^2\*d-108\*a^(13/2)\*(x\*(a\*x+b))^(1/2)\*x^2\*b^3\*c^2\*d+18\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^7\*x^3\*b^3\*c^2\*d+54\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^6\*x^2\*b^4\*c^2\*d-108\*a^(11/2)\*(x\*(a\*x+b))^(1/2)\*x\*b^4\*c^2\*d)/(x\*(a\*x+b))^(1/2)/b^3/(a\*x+b)^3

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)^3/(a + b/x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.254988, size = 1, normalized size = 0.01

$$\frac{3(5b^4c^3 - 6ab^3c^2d + (5ab^3c^3 - 6a^2b^2c^2d)x)\sqrt{\frac{ax+b}{x}} \log\left(2ax\sqrt{\frac{ax+b}{x}} + (2ax+b)\sqrt{a}\right) - 2(3a^2b^2c^3x^2 + 15b^4c^3 - 18ab^3c^2d)x}{6(a^4b^2x + a^3b^3)\sqrt{a}\sqrt{\frac{ax+b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)^3/(a + b/x)^(5/2),x, algorithm="fricas")

[Out] [-1/6\*(3\*(5\*b^4\*c^3 - 6\*a\*b^3\*c^2\*d + (5\*a\*b^3\*c^3 - 6\*a^2\*b^2\*c^2\*d)\*x)\*sqrt((a\*x + b)/x)\*log(2\*a\*x\*sqrt((a\*x + b)/x) + (2\*a\*x + b)\*sqrt(a)) - 2\*(3\*a^2\*b^2\*c^3\*x^2 + 15\*b^4\*c^3 - 18\*a\*b^3\*c^2\*d + 6\*a^3\*b\*d^3 + 2\*(10\*a\*b^3\*c^3 - 12\*a^2\*b^2\*c^2\*d + 3\*a^3\*b\*c\*d^2 + 2\*a^4\*d^3)\*x)\*sqrt(a))/((a^4\*b^2\*x + a^3\*b^3)\*sqrt(a)\*sqrt((a\*x + b)/x)), 1/3\*(3\*(5\*b^4\*c^3 - 6\*a\*b^3\*c^2\*d + (5\*a\*b^3\*c^3 - 6\*a^2\*b^2\*c^2\*d)\*x)\*sqrt((a\*x + b)/x)\*arctan(a/(sqrt(-a)\*sqrt((a\*x + b)/x))) + (3\*a^2\*b^2\*c^3\*x^2 + 15\*b^4\*c^3 - 18\*a\*b^3\*c^2\*d + 6\*a^3\*b\*d^3 + 2\*(10\*a\*b^3\*c^3 - 12\*a^2\*b^2\*c^2\*d + 3\*a^3\*b\*c\*d^2 + 2\*a^4\*d^3)\*x)\*sqrt(-a))/((a^4\*b^2\*x + a^3\*b^3)\*sqrt(-a)\*sqrt((a\*x + b)/x))]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx + d)^3}{x^3 \left(a + \frac{b}{x}\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*\*3/(a+b/x)\*\*(5/2),x)

[Out] Integral((c\*x + d)\*\*3/(x\*\*3\*(a + b/x)\*\*(5/2)), x)

**GIAC/XCAS [A]** time = 0.259834, size = 267, normalized size = 1.87

$$-\frac{1}{3}b \left( \frac{3c^3\sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right)a^3} - \frac{3(5bc^3 - 6ac^2d) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3b} - \frac{2\left(ab^3c^3 - 3a^2b^2c^2d + 3a^3bcd^2 - a^4d^3 + \frac{6(ax+b)b^3c^3}{x} - \frac{9(ax+b)}{x}\right)}{(ax+b)a^3b^3\sqrt{\frac{ax+b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)^3/(a + b/x)^(5/2),x, algorithm="giac")

[Out] -1/3\*b\*(3\*c^3\*sqrt((a\*x + b)/x)/((a - (a\*x + b)/x)\*a^3) - 3\*(5\*b^3\*c^3 - 6\*a\*c^2\*d)\*arctan(sqrt((a\*x + b)/x)/sqrt(-a))/((sqrt(-a))\*a^3\*b) - 2\*(a\*b^3\*c^3 - 3\*a^2\*b^2\*c^2\*d + 3\*a^3\*b\*c\*d^2 - a^4\*d^3 + 6\*(a\*x + b)\*b^3\*c^3/x - 9\*(a\*x + b)\*a\*b^2\*c^2\*d/x + 3\*(a\*x + b)\*a^3\*d^3/x)\*x/((a\*x + b)\*a^3\*b^3\*sqrt((a\*x + b)/x))

$$3.161 \quad \int \frac{\left(c + \frac{d}{x}\right)^2}{\left(a + \frac{b}{x}\right)^{5/2}} dx$$

**Optimal.** Leaf size=122

$$-\frac{c(5bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{c(5bc - 4ad)}{a^3 \sqrt{a + \frac{b}{x}}} + \frac{2a^2 d^2 + bc(5bc - 4ad)}{3a^2 b \left(a + \frac{b}{x}\right)^{3/2}} + \frac{c^2 x}{a \left(a + \frac{b}{x}\right)^{3/2}}$$

[Out]  $(2*a^2*d^2 + b*c*(5*b*c - 4*a*d))/(3*a^2*b*(a + b/x)^{(3/2)}) + (c*(5*b*c - 4*a*d))/(a^3*\text{Sqrt}[a + b/x]) + (c^2*x)/(a*(a + b/x)^{(3/2)}) - (c*(5*b*c - 4*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/a^{(7/2)}$

**Rubi [A]** time = 0.296017, antiderivative size = 118, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{c(5bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{c(5bc - 4ad)}{a^3 \sqrt{a + \frac{b}{x}}} + \frac{\frac{c(5bc - 4ad)}{a^2} + \frac{2d^2}{b}}{3 \left(a + \frac{b}{x}\right)^{3/2}} + \frac{c^2 x}{a \left(a + \frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c + d/x)^2/(a + b/x)^(5/2), x]

[Out]  $((2*d^2)/b + (c*(5*b*c - 4*a*d))/a^2)/(3*(a + b/x)^{(3/2)}) + (c*(5*b*c - 4*a*d))/(a^3*\text{Sqrt}[a + b/x]) + (c^2*x)/(a*(a + b/x)^{(3/2)}) - (c*(5*b*c - 4*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/a^{(7/2)}$

**Rubi in Sympy [A]** time = 22.7247, size = 107, normalized size = 0.88

$$\frac{c^2 x}{a \left(a + \frac{b}{x}\right)^{3/2}} + \frac{2 \left(a^2 d^2 - \frac{bc(4ad - 5bc)}{2}\right)}{3a^2 b \left(a + \frac{b}{x}\right)^{3/2}} - \frac{c(4ad - 5bc)}{a^3 \sqrt{a + \frac{b}{x}}} + \frac{c(4ad - 5bc) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d/x)\*\*2/(a+b/x)\*\*(5/2), x)

[Out]  $c**2*x/(a*(a + b/x)**(3/2)) + 2*(a**2*d**2 - b*c*(4*a*d - 5*b*c))/(3*a**2*b*(a + b/x)**(3/2)) - c*(4*a*d - 5*b*c)/(a**3*\text{sqrt}(a + b/x)) + c*(4*a*d - 5*b*c)*\text{atanh}(\text{sqrt}(a + b/x)/\text{sqrt}(a))/a**(7/2)$

**Mathematica [A]** time = 0.189269, size = 123, normalized size = 1.01

$$\frac{c(4ad - 5bc) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right)}{2a^{7/2}} + \frac{x\sqrt{a + \frac{b}{x}}(2a^3 d^2 x + a^2 bcx(3cx - 16d) + 4ab^2 c(5cx - 3d) + 15b^3 c^2)}{3a^3 b(ax + b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d/x)^2/(a + b/x)^(5/2),x]

[Out] (Sqrt[a + b/x]\*x\*(15\*b^3\*c^2 + 2\*a^3\*d^2\*x + a^2\*b\*c\*x\*(-16\*d + 3\*c\*x) + 4\*a\*b^2\*c\*(-3\*d + 5\*c\*x)))/(3\*a^3\*b\*(b + a\*x)^2) + (c\*(-5\*b\*c + 4\*a\*d)\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]\*x])/(2\*a^(7/2))

**Maple [B]** time = 0.02, size = 595, normalized size = 4.9

$$-\frac{x}{6b(ax+b)^3}\sqrt{\frac{ax+b}{x}}\left(24a^{15/2}\sqrt{x(ax+b)}x^3cd - 30a^{13/2}\sqrt{x(ax+b)}x^3bc^2 - 24a^{13/2}(x(ax+b))^{3/2}xcd + 72a^{13/2}\sqrt{x(ax+b)}x^3cd - 30a^{13/2}\sqrt{x(ax+b)}x^3bc^2 - 24a^{13/2}(x(ax+b))^{3/2}xcd + 72a^{13/2}\sqrt{x(ax+b)}x^3cd\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^2/(a+b/x)^(5/2),x)

[Out] -1/6\*((a\*x+b)/x)^(1/2)\*x\*(24\*a^(15/2)\*(x\*(a\*x+b))^(1/2)\*x^3\*c\*d-30\*a^(13/2)\*(x\*(a\*x+b))^(1/2)\*x^3\*b\*c^2-24\*a^(13/2)\*(x\*(a\*x+b))^(3/2)\*x\*c\*d+72\*a^(13/2)\*(x\*(a\*x+b))^(1/2)\*x^2\*b\*c\*d-4\*a^(13/2)\*(x\*(a\*x+b))^(3/2)\*d^2+24\*a^(11/2)\*(x\*(a\*x+b))^(3/2)\*x\*b\*c^2-90\*a^(11/2)\*(x\*(a\*x+b))^(1/2)\*x^2\*b^2\*c^2-16\*a^(11/2)\*(x\*(a\*x+b))^(3/2)\*b\*c\*d+72\*a^(11/2)\*(x\*(a\*x+b))^(1/2)\*x\*b^2\*c\*d+20\*a^(9/2)\*(x\*(a\*x+b))^(3/2)\*b^2\*c^2-90\*a^(9/2)\*(x\*(a\*x+b))^(1/2)\*x\*b^3\*c^2+24\*a^(9/2)\*(x\*(a\*x+b))^(1/2)\*b^3\*c\*d-30\*a^(7/2)\*(x\*(a\*x+b))^(1/2)\*b^4\*c^2-12\*a^7\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*x^3\*b\*c\*d+15\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*x^3\*a^6\*b^2\*c^2-36\*a^6\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*x^2\*b^2\*c\*d+45\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*x^2\*a^5\*b^3\*c^2-36\*a^5\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*x\*b^3\*c\*d+45\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*x\*a^4\*b^4\*c^2-12\*a^4\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*b^4\*c\*d+15\*ln(1/2\*(2\*(x\*(a\*x+b))^(1/2)\*a^(1/2)+2\*a\*x+b)/a^(1/2))\*a^3\*b^5\*c^2/a^(13/2)/(x\*(a\*x+b))^(1/2)/b/(a\*x+b)^3

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)^2/(a + b/x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.254442, size = 1, normalized size = 0.01

$$\frac{3(5b^3c^2 - 4ab^2cd + (5ab^2c^2 - 4a^2bcd)x)\sqrt{\frac{ax+b}{x}}\log\left(2ax\sqrt{\frac{ax+b}{x}} + (2ax+b)\sqrt{a}\right) - 2(3a^2bc^2x^2 + 15b^3c^2 - 12ab^2cd)}{6(a^4bx + a^3b^2)\sqrt{a}\sqrt{\frac{ax+b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)^2/(a + b/x)^(5/2),x, algorithm="fricas")

[Out] [-1/6\*(3\*(5\*b^3\*c^2 - 4\*a\*b^2\*c\*d + (5\*a\*b^2\*c^2 - 4\*a^2\*b\*c\*d)\*x)\*sqrt((a\*x + b)/x)\*log(2\*a\*x\*sqrt((a\*x + b)/x) + (2\*a\*x + b)\*sqrt

$t(a) - 2 \cdot (3 \cdot a^2 \cdot b \cdot c^2 \cdot x^2 + 15 \cdot b^3 \cdot c^2 - 12 \cdot a \cdot b^2 \cdot c \cdot d + 2 \cdot (10 \cdot a \cdot b^2 \cdot c^2 - 8 \cdot a^2 \cdot b \cdot c \cdot d + a^3 \cdot d^2) \cdot x) \cdot \sqrt{a} / ((a^4 \cdot b \cdot x + a^3 \cdot b^2) \cdot \sqrt{a} \cdot \sqrt{(a \cdot x + b)/x}), 1/3 \cdot (3 \cdot (5 \cdot b^3 \cdot c^2 - 4 \cdot a \cdot b^2 \cdot c \cdot d + (5 \cdot a \cdot b^2 \cdot c^2 - 4 \cdot a^2 \cdot b \cdot c \cdot d) \cdot x) \cdot \sqrt{(a \cdot x + b)/x} \cdot \arctan(a/\sqrt{-a} \cdot \sqrt{(a \cdot x + b)/x})) + (3 \cdot a^2 \cdot b \cdot c^2 \cdot x^2 + 15 \cdot b^3 \cdot c^2 - 12 \cdot a \cdot b^2 \cdot c \cdot d + 2 \cdot (10 \cdot a \cdot b^2 \cdot c^2 - 8 \cdot a^2 \cdot b \cdot c \cdot d + a^3 \cdot d^2) \cdot x) \cdot \sqrt{-a} / ((a^4 \cdot b \cdot x + a^3 \cdot b^2) \cdot \sqrt{-a} \cdot \sqrt{(a \cdot x + b)/x})]$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx + d)^2}{x^2 \left(a + \frac{b}{x}\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*\*2/(a+b/x)\*\*(5/2),x)

[Out] Integral((c\*x + d)\*\*2/(x\*\*2\*(a + b/x)\*\*(5/2)), x)

**GIAC/XCAS [A]** time = 0.257682, size = 217, normalized size = 1.78

$$-\frac{1}{3} b \left( \frac{3 c^2 \sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right) a^3} - \frac{3 (5 b c^2 - 4 a c d) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a} a^3 b} - \frac{2 \left(ab^2 c^2 - 2 a^2 b c d + a^3 d^2 + \frac{6(ax+b)b^2 c^2}{x} - \frac{6(ax+b)abcd}{x}\right) x}{(ax + b) a^3 b^2 \sqrt{\frac{ax+b}{x}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)^2/(a + b/x)^(5/2),x, algorithm="giac")

[Out] -1/3\*b\*(3\*c^2\*sqrt((a\*x + b)/x)/((a - (a\*x + b)/x)\*a^3) - 3\*(5\*b\*c^2 - 4\*a\*c\*d)\*arctan(sqrt((a\*x + b)/x)/sqrt(-a))/(sqrt(-a)\*a^3\*b) - 2\*(a\*b^2\*c^2 - 2\*a^2\*b\*c\*d + a^3\*d^2 + 6\*(a\*x + b)\*b^2\*c^2/x - 6\*(a\*x + b)\*a\*b\*c\*d/x)\*x/((a\*x + b)\*a^3\*b^2\*sqrt((a\*x + b)/x))

$$3.162 \quad \int \frac{c + \frac{d}{x}}{\left(a + \frac{b}{x}\right)^{5/2}} dx$$

**Optimal.** Leaf size=103

$$-\frac{(5bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{5bc - 2ad}{a^3 \sqrt{a + \frac{b}{x}}} + \frac{5bc - 2ad}{3a^2 \left(a + \frac{b}{x}\right)^{3/2}} + \frac{cx}{a \left(a + \frac{b}{x}\right)^{3/2}}$$

[Out] (5\*b\*c - 2\*a\*d)/(3\*a^2\*(a + b/x)^(3/2)) + (5\*b\*c - 2\*a\*d)/(a^3\*sqrt[a + b/x]) + (c\*x)/(a\*(a + b/x)^(3/2)) - ((5\*b\*c - 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)

**Rubi [A]** time = 0.198709, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$-\frac{(5bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{5bc - 2ad}{a^3 \sqrt{a + \frac{b}{x}}} + \frac{5bc - 2ad}{3a^2 \left(a + \frac{b}{x}\right)^{3/2}} + \frac{cx}{a \left(a + \frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c + d/x)/(a + b/x)^(5/2), x]

[Out] (5\*b\*c - 2\*a\*d)/(3\*a^2\*(a + b/x)^(3/2)) + (5\*b\*c - 2\*a\*d)/(a^3\*sqrt[a + b/x]) + (c\*x)/(a\*(a + b/x)^(3/2)) - ((5\*b\*c - 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)

**Rubi in Sympy [A]** time = 16.3615, size = 90, normalized size = 0.87

$$\frac{cx}{a \left(a + \frac{b}{x}\right)^{3/2}} - \frac{2 \left(ad - \frac{5bc}{2}\right)}{3a^2 \left(a + \frac{b}{x}\right)^{3/2}} - \frac{2 \left(ad - \frac{5bc}{2}\right)}{a^3 \sqrt{a + \frac{b}{x}}} + \frac{2 \left(ad - \frac{5bc}{2}\right) \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d/x)/(a+b/x)\*\*(5/2), x)

[Out] c\*x/(a\*(a + b/x)\*\*(3/2)) - 2\*(a\*d - 5\*b\*c/2)/(3\*a\*\*2\*(a + b/x)\*\*(3/2)) - 2\*(a\*d - 5\*b\*c/2)/(a\*\*3\*sqrt(a + b/x)) + 2\*(a\*d - 5\*b\*c/2)\*atanh(sqrt(a + b/x)/sqrt(a))/a\*\*(7/2)

**Mathematica [A]** time = 0.140561, size = 102, normalized size = 0.99

$$\frac{(2ad - 5bc) \log\left(2\sqrt{ax}\sqrt{a + \frac{b}{x}} + 2ax + b\right)}{2a^{7/2}} + \frac{x\sqrt{a + \frac{b}{x}}(a^2x(3cx - 8d) + ab(20cx - 6d) + 15b^2c)}{3a^3(ax + b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d/x)/(a + b/x)^(5/2), x]



[Out]  $(\text{Sqrt}[a + b/x] * x * (15 * b^2 * c + a^2 * x * (-8 * d + 3 * c * x) + a * b * (-6 * d + 2 * 0 * c * x))) / (3 * a^3 * (b + a * x)^2) + ((-5 * b * c + 2 * a * d) * \text{Log}[b + 2 * a * x + 2 * \text{Sqrt}[a] * \text{Sqrt}[a + b/x] * x]) / (2 * a^{(7/2)})$

**Maple [B]** time = 0.019, size = 548, normalized size = 5.3

$$-\frac{x}{6b(ax+b)^3} \sqrt{\frac{ax+b}{x}} \left( 12a^{15/2} \sqrt{x(ax+b)} x^3 d - 30a^{13/2} \sqrt{x(ax+b)} x^3 bc - 12a^{13/2} (x(ax+b))^{3/2} xd + 36a^{13/2} \sqrt{x(ax+b)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c+d/x)/(a+b/x)^(5/2), x)`

[Out]  $-1/6 * ((a * x + b) / x)^{(1/2)} * x * (12 * a^{(15/2)} * (x * (a * x + b))^{(1/2)} * x^3 * d - 30 * a^{(13/2)} * (x * (a * x + b))^{(1/2)} * x^3 * b * c - 12 * a^{(13/2)} * (x * (a * x + b))^{(3/2)} * x * d + 36 * a^{(13/2)} * (x * (a * x + b))^{(1/2)} * x^2 * b * d + 24 * a^{(11/2)} * (x * (a * x + b))^{(3/2)} * x * b * c - 90 * a^{(11/2)} * (x * (a * x + b))^{(1/2)} * x^2 * b^2 * c - 8 * a^{(11/2)} * (x * (a * x + b))^{(3/2)} * b * d + 36 * a^{(11/2)} * (x * (a * x + b))^{(1/2)} * x * b^2 * d + 20 * a^{(9/2)} * (x * (a * x + b))^{(3/2)} * b^2 * c - 90 * a^{(9/2)} * (x * (a * x + b))^{(1/2)} * x * b^3 * c + 12 * a^{(9/2)} * (x * (a * x + b))^{(1/2)} * b^3 * d - 30 * a^{(7/2)} * (x * (a * x + b))^{(1/2)} * b^4 * c - 6 * a^7 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x^3 * b * d + 15 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x^3 * a^6 * b^2 * c - 18 * a^6 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x^2 * b^2 * d + 45 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x^2 * a^5 * b^3 * c - 18 * a^5 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x * b^3 * d + 45 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * x * a^4 * b^4 * c - 6 * a^4 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * b^4 * d + 15 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * a^3 * b^5 * c) / a^{(13/2)} / (x * (a * x + b))^{(1/2)} / b / (a * x + b)^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c + d/x)/(a + b/x)^(5/2), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.241657, size = 1, normalized size = 0.01

$$\frac{3(5b^2c - 2abd + (5abc - 2a^2d)x) \sqrt{\frac{ax+b}{x}} \log\left(2ax \sqrt{\frac{ax+b}{x}} + (2ax+b)\sqrt{a}\right) - 2(3a^2cx^2 + 15b^2c - 6abd + 4(5abc - 2a^2d)x) \sqrt{\frac{ax+b}{x}}}{6(a^4x + a^3b)\sqrt{a}\sqrt{\frac{ax+b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c + d/x)/(a + b/x)^(5/2), x, algorithm="fricas")`

[Out]  $[-1/6 * (3 * (5 * b^2 * c - 2 * a * b * d + (5 * a * b * c - 2 * a^2 * d) * x) * \text{sqrt}((a * x + b) / x) * \log(2 * a * x * \text{sqrt}((a * x + b) / x) + (2 * a * x + b) * \text{sqrt}(a)) - 2 * (3 * a^2 * c * x^2 + 15 * b^2 * c - 6 * a * b * d + 4 * (5 * a * b * c - 2 * a^2 * d) * x) * \text{sqrt}(a)) / ((a^4 * x + a^3 * b) * \text{sqrt}(a) * \text{sqrt}((a * x + b) / x)), 1/3 * (3 * (5 * b^2 * c - 2 * a * b * d + (5 * a * b * c - 2 * a^2 * d) * x) * \text{sqrt}((a * x + b) / x) * \arctan(a / (\text{sqrt}(-a) * \text{sqrt}((a * x + b) / x))) + (3 * a^2 * c * x^2 + 15 * b^2 * c - 6 * a * b * d + 4 * ($

$$5*a*b*c - 2*a^2*d)*x)*\sqrt{-a})/((a^4*x + a^3*b)*\sqrt{-a})*\sqrt{(a*x + b)/x)}}$$

**Sympy [A]** time = 39.1088, size = 1479, normalized size = 14.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)/(a+b/x)\*\*(5/2),x)

[Out]  $c*(6*a^{17}*x^4*\sqrt{1 + b/(a*x)})/(6*a^{39/2}*x^3 + 18*a^{37/2}*b*x^2 + 18*a^{35/2}*b^2*x + 6*a^{33/2}*b^3) + 46*a^{16}*b*x^3*\sqrt{1 + b/(a*x)})/(6*a^{39/2}*x^3 + 18*a^{37/2}*b*x^2 + 18*a^{35/2}*b^2*x + 6*a^{33/2}*b^3) + 15*a^{16}*b*x^3*\log(b/(a*x))/(6*a^{39/2}*x^3 + 18*a^{37/2}*b*x^2 + 18*a^{35/2}*b^2*x + 6*a^{33/2}*b^3) - 30*a^{16}*b*x^3*\log(\sqrt{1 + b/(a*x)}) + 1)/(6*a^{39/2}*x^3 + 18*a^{37/2}*b*x^2 + 18*a^{35/2}*b^2*x + 6*a^{33/2}*b^3) + 70*a^{15}*b^2*x^2*\sqrt{1 + b/(a*x)})/(6*a^{39/2}*x^3 + 18*a^{37/2}*b*x^2 + 18*a^{35/2}*b^2*x + 6*a^{33/2}*b^3) + 45*a^{15}*b^2*x^2*\log(b/(a*x))/(6*a^{39/2}*x^3 + 18*a^{37/2}*b*x^2 + 18*a^{35/2}*b^2*x + 6*a^{33/2}*b^3) - 90*a^{15}*b^2*x^2*\log(\sqrt{1 + b/(a*x)}) + 1)/(6*a^{39/2}*x^3 + 18*a^{37/2}*b*x^2 + 18*a^{35/2}*b^2*x + 6*a^{33/2}*b^3) + 30*a^{14}*b^3*x*\sqrt{1 + b/(a*x)})/(6*a^{39/2}*x^3 + 18*a^{37/2}*b*x^2 + 18*a^{35/2}*b^2*x + 6*a^{33/2}*b^3) + 45*a^{14}*b^3*x*\log(b/(a*x))/(6*a^{39/2}*x^3 + 18*a^{37/2}*b*x^2 + 18*a^{35/2}*b^2*x + 6*a^{33/2}*b^3) - 90*a^{14}*b^3*x*\log(\sqrt{1 + b/(a*x)}) + 1)/(6*a^{39/2}*x^3 + 18*a^{37/2}*b*x^2 + 18*a^{35/2}*b^2*x + 6*a^{33/2}*b^3) + 15*a^{13}*b^4*\log(b/(a*x))/(6*a^{39/2}*x^3 + 18*a^{37/2}*b*x^2 + 18*a^{35/2}*b^2*x + 6*a^{33/2}*b^3) - 30*a^{13}*b^4*\log(\sqrt{1 + b/(a*x)}) + 1)/(6*a^{39/2}*x^3 + 18*a^{37/2}*b*x^2 + 18*a^{35/2}*b^2*x + 6*a^{33/2}*b^3) + d*(-8*a^7*x^3*\sqrt{1 + b/(a*x)})/(3*a^{19/2}*x^3 + 9*a^{17/2}*b*x^2 + 9*a^{15/2}*b^2*x + 3*a^{13/2}*b^3) - 3*a^7*x^3*\log(b/(a*x))/(3*a^{19/2}*x^3 + 9*a^{17/2}*b*x^2 + 9*a^{15/2}*b^2*x + 3*a^{13/2}*b^3) + 6*a^7*x^3*\log(\sqrt{1 + b/(a*x)}) + 1)/(3*a^{19/2}*x^3 + 9*a^{17/2}*b*x^2 + 9*a^{15/2}*b^2*x + 3*a^{13/2}*b^3) - 14*a^6*b*x^2*\sqrt{1 + b/(a*x)})/(3*a^{19/2}*x^3 + 9*a^{17/2}*b*x^2 + 9*a^{15/2}*b^2*x + 3*a^{13/2}*b^3) - 9*a^6*b*x^2*\log(b/(a*x))/(3*a^{19/2}*x^3 + 9*a^{17/2}*b*x^2 + 9*a^{15/2}*b^2*x + 3*a^{13/2}*b^3) + 18*a^6*b*x^2*\log(\sqrt{1 + b/(a*x)}) + 1)/(3*a^{19/2}*x^3 + 9*a^{17/2}*b*x^2 + 9*a^{15/2}*b^2*x + 3*a^{13/2}*b^3) - 6*a^5*b^2*x*\sqrt{1 + b/(a*x)})/(3*a^{19/2}*x^3 + 9*a^{17/2}*b*x^2 + 9*a^{15/2}*b^2*x + 3*a^{13/2}*b^3) - 9*a^5*b^2*x*\log(b/(a*x))/(3*a^{19/2}*x^3 + 9*a^{17/2}*b*x^2 + 9*a^{15/2}*b^2*x + 3*a^{13/2}*b^3) + 18*a^5*b^2*x*\log(\sqrt{1 + b/(a*x)}) + 1)/(3*a^{19/2}*x^3 + 9*a^{17/2}*b*x^2 + 9*a^{15/2}*b^2*x + 3*a^{13/2}*b^3) - 3*a^4*b^3*\log(b/(a*x))/(3*a^{19/2}*x^3 + 9*a^{17/2}*b*x^2 + 9*a^{15/2}*b^2*x + 3*a^{13/2}*b^3) + 6*a^4*b^3*\log(\sqrt{1 + b/(a*x)}) + 1)/(3*a^{19/2}*x^3 + 9*a^{17/2}*b*x^2 + 9*a^{15/2}*b^2*x + 3*a^{13/2}*b^3)$

**GIAC/XCAS [A]** time = 0.255601, size = 185, normalized size = 1.8

$$-\frac{1}{3}b \left( \frac{3c\sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right)a^3} - \frac{3(5bc - 2ad) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-aa^3b}} - \frac{2\left(abc - a^2d + \frac{6(ax+b)bc}{x} - \frac{3(ax+b)ad}{x}\right)x}{(ax+b)a^3b\sqrt{\frac{ax+b}{x}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c + d/x)/(a + b/x)^(5/2),x, algorithm="giac")

```
[Out] -1/3*b*(3*c*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3) - 3*(5*b*c
- 2*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3*b) - 2*
(a*b*c - a^2*d + 6*(a*x + b)*b*c/x - 3*(a*x + b)*a*d/x)*x/((a*x +
b)*a^3*b*sqrt((a*x + b)/x))
```

$$3.163 \quad \int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2}} dx$$

**Optimal.** Leaf size=82

$$-\frac{5b \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{5x\sqrt{a+\frac{b}{x}}}{a^3} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} - \frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

[Out]  $(-2*x)/(3*a*(a + b/x)^{(3/2)}) - (10*x)/(3*a^2*\text{Sqrt}[a + b/x]) + (5*\text{Sqrt}[a + b/x]*x)/a^3 - (5*b*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/a^{(7/2)}$

**Rubi [A]** time = 0.112652, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$-\frac{5b \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{5x\sqrt{a+\frac{b}{x}}}{a^3} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} - \frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(-5/2), x]

[Out]  $(-2*x)/(3*a*(a + b/x)^{(3/2)}) - (10*x)/(3*a^2*\text{Sqrt}[a + b/x]) + (5*\text{Sqrt}[a + b/x]*x)/a^3 - (5*b*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/a^{(7/2)}$

**Rubi in Sympy [A]** time = 11.5393, size = 70, normalized size = 0.85

$$-\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{5x\sqrt{a+\frac{b}{x}}}{a^3} - \frac{5b \operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a+b/x)\*\*(5/2), x)

[Out]  $-2*x/(3*a*(a + b/x)**(3/2)) - 10*x/(3*a**2*\text{sqrt}(a + b/x)) + 5*x*\text{sqrt}(a + b/x)/a**3 - 5*b*\text{atanh}(\text{sqrt}(a + b/x)/\text{sqrt}(a))/a**(7/2)$

**Mathematica [A]** time = 0.133915, size = 82, normalized size = 1.

$$\frac{x\sqrt{a+\frac{b}{x}}(3a^2x^2+20abx+15b^2)}{3a^3(ax+b)^2} - \frac{5b \log\left(2\sqrt{ax}\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2a^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(-5/2), x]

[Out] (Sqrt[a + b/x]\*x\*(15\*b^2 + 20\*a\*b\*x + 3\*a^2\*x^2))/(3\*a^3\*(b + a\*x)^2) - (5\*b\*Log[b + 2\*a\*x + 2\*Sqrt[a]\*Sqrt[a + b/x]\*x])/(2\*a^(7/2))

**Maple [B]** time = 0.006, size = 276, normalized size = 3.4

$$-\frac{x}{6(ax+b)^3} \sqrt{\frac{ax+b}{x}} \left( -30 a^{13/2} \sqrt{x(ax+b)} x^3 + 24 a^{11/2} (x(ax+b))^{3/2} x - 90 a^{11/2} \sqrt{x(ax+b)} x^2 b + 20 b a^{9/2} (x(ax+b))^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b/x)^(5/2), x)

[Out] 
$$-1/6 * ((a*x+b)/x)^{(1/2)} * x/a^{(13/2)} * (-30*a^{(13/2)} * (x*(a*x+b))^{(1/2)} * x^3 + 24*a^{(11/2)} * (x*(a*x+b))^{(3/2)} * x - 90*a^{(11/2)} * (x*(a*x+b))^{(1/2)} * x^2 * b + 20*b*a^{(9/2)} * (x*(a*x+b))^{(3/2)} - 90*a^{(9/2)} * (x*(a*x+b))^{(1/2)} * x * b^2 + 15*\ln(1/2*(2*(x*(a*x+b))^{(1/2)} * a^{(1/2)} + 2*a*x+b)/a^{(1/2)}) * x^3 * a^6 * b - 30*a^{(7/2)} * (x*(a*x+b))^{(1/2)} * b^3 + 45*\ln(1/2*(2*(x*(a*x+b))^{(1/2)} * a^{(1/2)} + 2*a*x+b)/a^{(1/2)}) * x^2 * a^5 * b^2 + 45*\ln(1/2*(2*(x*(a*x+b))^{(1/2)} * a^{(1/2)} + 2*a*x+b)/a^{(1/2)}) * x * a^4 * b^3 + 15*\ln(1/2*(2*(x*(a*x+b))^{(1/2)} * a^{(1/2)} + 2*a*x+b)/a^{(1/2)}) * a^3 * b^4) / (x*(a*x+b))^{(1/2)} / (a*x+b)^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(-5/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.242169, size = 1, normalized size = 0.01

$$\left[ \frac{15(abx + b^2) \sqrt{\frac{ax+b}{x}} \log\left(-2ax\sqrt{\frac{ax+b}{x}} + (2ax + b)\sqrt{a}\right) + 2(3a^2x^2 + 20abx + 15b^2)\sqrt{a}}{6(a^4x + a^3b)\sqrt{a}\sqrt{\frac{ax+b}{x}}}, \frac{15(abx + b^2) \sqrt{\frac{ax+b}{x}} \arctan\left(\frac{1}{\sqrt{a}}\right)}{3(a^4x + a^3b)\sqrt{a}\sqrt{\frac{ax+b}{x}}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(-5/2), x, algorithm="fricas")

[Out] 
$$\left[ \frac{1}{6} * (15 * (a*b*x + b^2) * \sqrt{(a*x + b)/x} * \log(-2*a*x*\sqrt{(a*x + b)/x} + (2*a*x + b)*\sqrt{a}) + 2 * (3*a^2*x^2 + 20*a*b*x + 15*b^2) * \sqrt{a}) / ((a^4*x + a^3*b) * \sqrt{a} * \sqrt{(a*x + b)/x}), \frac{1}{3} * (15 * (a*b*x + b^2) * \sqrt{(a*x + b)/x} * \arctan(a/(\sqrt{-a} * \sqrt{(a*x + b)/x})) + (3*a^2*x^2 + 20*a*b*x + 15*b^2) * \sqrt{-a}) / ((a^4*x + a^3*b) * \sqrt{-a} * \sqrt{(a*x + b)/x}) \right]$$

**Sympy [A]** time = 17.8996, size = 774, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)\*\*(5/2),x)

[Out]  $6*a^{17}*x^4*\sqrt{1+b/(a*x)} / (6*a^{(39/2)}*x^3 + 18*a^{(37/2)}*b*x^2 + 18*a^{(35/2)}*b^2*x + 6*a^{(33/2)}*b^3) + 46*a^{16}*b*x^3*\sqrt{1+b/(a*x)} / (6*a^{(39/2)}*x^3 + 18*a^{(37/2)}*b*x^2 + 18*a^{(35/2)}*b^2*x + 6*a^{(33/2)}*b^3) + 15*a^{16}*b*x^3*\log(b/(a*x)) / (6*a^{(39/2)}*x^3 + 18*a^{(37/2)}*b*x^2 + 18*a^{(35/2)}*b^2*x + 6*a^{(33/2)}*b^3) - 30*a^{16}*b*x^3*\log(\sqrt{1+b/(a*x)} + 1) / (6*a^{(39/2)}*x^3 + 18*a^{(37/2)}*b*x^2 + 18*a^{(35/2)}*b^2*x + 6*a^{(33/2)}*b^3) + 70*a^{15}*b^2*x^2*\sqrt{1+b/(a*x)} / (6*a^{(39/2)}*x^3 + 18*a^{(37/2)}*b*x^2 + 18*a^{(35/2)}*b^2*x + 6*a^{(33/2)}*b^3) + 45*a^{15}*b^2*x^2*\log(b/(a*x)) / (6*a^{(39/2)}*x^3 + 18*a^{(37/2)}*b*x^2 + 18*a^{(35/2)}*b^2*x + 6*a^{(33/2)}*b^3) - 90*a^{15}*b^2*x^2*\log(\sqrt{1+b/(a*x)} + 1) / (6*a^{(39/2)}*x^3 + 18*a^{(37/2)}*b*x^2 + 18*a^{(35/2)}*b^2*x + 6*a^{(33/2)}*b^3) + 30*a^{14}*b^3*x*\sqrt{1+b/(a*x)} / (6*a^{(39/2)}*x^3 + 18*a^{(37/2)}*b*x^2 + 18*a^{(35/2)}*b^2*x + 6*a^{(33/2)}*b^3) + 45*a^{14}*b^3*x*\log(b/(a*x)) / (6*a^{(39/2)}*x^3 + 18*a^{(37/2)}*b*x^2 + 18*a^{(35/2)}*b^2*x + 6*a^{(33/2)}*b^3) - 90*a^{14}*b^3*x*\log(\sqrt{1+b/(a*x)} + 1) / (6*a^{(39/2)}*x^3 + 18*a^{(37/2)}*b*x^2 + 18*a^{(35/2)}*b^2*x + 6*a^{(33/2)}*b^3) + 15*a^{13}*b^4*\log(b/(a*x)) / (6*a^{(39/2)}*x^3 + 18*a^{(37/2)}*b*x^2 + 18*a^{(35/2)}*b^2*x + 6*a^{(33/2)}*b^3) - 30*a^{13}*b^4*\log(\sqrt{1+b/(a*x)} + 1) / (6*a^{(39/2)}*x^3 + 18*a^{(37/2)}*b*x^2 + 18*a^{(35/2)}*b^2*x + 6*a^{(33/2)}*b^3)$

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**GIAC/XCAS [A]** time = 0.254127, size = 132, normalized size = 1.61

$$\frac{1}{3}b \left( \frac{2 \left( a + \frac{6(ax+b)}{x} \right) x}{(ax+b)a^3 \sqrt{\frac{ax+b}{x}}} + \frac{15 \arctan \left( \frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}} \right)}{\sqrt{-aa^3}} - \frac{3 \sqrt{\frac{ax+b}{x}}}{\left( a - \frac{ax+b}{x} \right) a^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^(-5/2),x, algorithm="giac")

[Out]  $\frac{1}{3}b*(2*(a + 6*(a*x + b)/x)*x/((a*x + b)*a^3*\sqrt{(a*x + b)/x}) + 15*\arctan(\sqrt{(a*x + b)/x}/\sqrt{-a})/(\sqrt{-a}*a^3) - 3*\sqrt{(a*x + b)/x}/((a - (a*x + b)/x)*a^3)$

$$3.164 \quad \int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)} dx$$

**Optimal.** Leaf size=201

$$\begin{aligned} & -\frac{(2ad + 5bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2} + \frac{b(5bc - 3ad)}{3a^2c \left(a + \frac{b}{x}\right)^{3/2} (bc - ad)} \\ & + \frac{b(a^2d^2 - 8abcd + 5b^2c^2)}{a^3c\sqrt{a + \frac{b}{x}}(bc - ad)^2} - \frac{2d^{7/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc - ad)^{5/2}} + \frac{x}{ac \left(a + \frac{b}{x}\right)^{3/2}} \end{aligned}$$

[Out] (b\*(5\*b\*c - 3\*a\*d))/(3\*a^2\*c\*(b\*c - a\*d)\*(a + b/x)^(3/2)) + (b\*(5\*b^2\*c^2 - 8\*a\*b\*c\*d + a^2\*d^2))/(a^3\*c\*(b\*c - a\*d)^2\*Sqrt[a + b/x]) + x/(a\*c\*(a + b/x)^(3/2)) - (2\*d^(7/2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(c^2\*(b\*c - a\*d)^(5/2)) - ((5\*b\*c + 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)\*c^2)

**Rubi [A]** time = 0.946317, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned} & -\frac{(2ad + 5bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2} + \frac{b(5bc - 3ad)}{3a^2c \left(a + \frac{b}{x}\right)^{3/2} (bc - ad)} \\ & + \frac{b(a^2d^2 - 8abcd + 5b^2c^2)}{a^3c\sqrt{a + \frac{b}{x}}(bc - ad)^2} - \frac{2d^{7/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc - ad)^{5/2}} + \frac{x}{ac \left(a + \frac{b}{x}\right)^{3/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b/x)^(5/2)\*(c + d/x)), x]

[Out] (b\*(5\*b\*c - 3\*a\*d))/(3\*a^2\*c\*(b\*c - a\*d)\*(a + b/x)^(3/2)) + (b\*(5\*b^2\*c^2 - 8\*a\*b\*c\*d + a^2\*d^2))/(a^3\*c\*(b\*c - a\*d)^2\*Sqrt[a + b/x]) + x/(a\*c\*(a + b/x)^(3/2)) - (2\*d^(7/2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(c^2\*(b\*c - a\*d)^(5/2)) - ((5\*b\*c + 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)\*c^2)

**Rubi in Sympy [A]** time = 106.153, size = 173, normalized size = 0.86

$$\begin{aligned} & \frac{2d^{7/2} \operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{c^2(ad - bc)^{5/2}} + \frac{x}{ac \left(a + \frac{b}{x}\right)^{3/2}} + \frac{b(3ad - 5bc)}{3a^2c \left(a + \frac{b}{x}\right)^{3/2} (ad - bc)} \\ & + \frac{b(a^2d^2 - 8abcd + 5b^2c^2)}{a^3c\sqrt{a + \frac{b}{x}}(ad - bc)^2} - \frac{(2ad + 5bc) \operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a+b/x)\*\*(5/2)/(c+d/x), x)

[Out]  $2*d^{7/2}*atanh(sqrt(d)*sqrt(a+b/x)/sqrt(a*d-b*c))/(c^{2*(a*d-b*c)^{5/2}})+x/(a*c*(a+b/x)^{3/2})+b*(3*a*d-5*b*c)/(3*a^{2*c*(a+b/x)^{3/2}}*(a*d-b*c))+b*(a^{2*d^2}-8*a*b*c*d+5*b^{2*c^2})/(a^{3*c*sqrt(a+b/x)}*(a*d-b*c)^2)-(2*a*d+5*b*c)*atanh(sqrt(a+b/x)/sqrt(a))/(a^{7/2}*c^2)$

**Mathematica [A]** time = 0.963246, size = 250, normalized size = 1.24

$$\frac{3(2ad+5bc)\log\left(\frac{2\sqrt{ax}\sqrt{a+\frac{b}{x}}+2ax+b}{a^{7/2}}\right)+\frac{2cx\sqrt{a+\frac{b}{x}}(3a^4d^2x^2+6a^3bdx(d-cx)+a^2b^2(3c^2x^2-32cdx+3d^2)+4ab^3c(5cx-6d)+15b^4c^2)}{a^3(ax+b)^2(bc-ad)^2}}{6c^2}+\frac{6d^{7/2}\log(cx+d)}{(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a+b/x)^(5/2)\*(c+d/x)),x]

[Out]  $((2*c*sqrt[a+b/x]*x*(15*b^4*c^2+3*a^4*d^2*x^2+6*a^3*b*d*x*(d-c*x)+4*a*b^3*c*(-6*d+5*c*x)+a^2*b^2*(3*d^2-32*c*d*x+3*c^2*x^2)))/(a^3*(b*c-a*d)^2*(b+a*x)^2)+(6*d^{7/2}*Log[d+c*x])/(-b*c+a*d)^{5/2}-(3*(5*b*c+2*a*d)*Log[b+2*a*x+2*sqrt[a]*sqrt[a+b/x]*x])/a^{7/2}-(6*d^{7/2}*Log[-b*d+b*c*x-2*a*d*x+2*sqrt[d]*sqrt[-b*c+a*d]*sqrt[a+b/x]*x])/(-b*c+a*d)^{5/2})/(6*c^2)$

**Maple [B]** time = 0.024, size = 2490, normalized size = 12.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b/x)^(5/2)/(c+d/x),x)

[Out]  $1/6*((a*x+b)/x)^{1/2}*x/a^{13/2}*(-54*(x*(a*x+b))^{1/2}*a^{17/2}*((a*d-b*c)*d/c^2)^{1/2}*x^3*b^3*c^3*d^2+78*(x*(a*x+b))^{1/2}*a^{15/2}*((a*d-b*c)*d/c^2)^{1/2}*x^3*b^2*c^4*d-18*a^9*\ln(1/2*(2*(x*(a*x+b))^{1/2}*a^{1/2}+2*a*x+b)/a^{1/2}))*((a*d-b*c)*d/c^2)^{1/2}*x^2*b*c*d^4+3*a^9*\ln(1/2*(2*(x*(a*x+b))^{1/2}*a^{1/2}+2*a*x+b)/a^{1/2}))*((a*d-b*c)*d/c^2)^{1/2}*x^3*b^2*c^2*d^3+36*(x*(a*x+b))^{3/2}*a^{15/2}*((a*d-b*c)*d/c^2)^{1/2}*x*b^3*c^3*d^2-60*(x*(a*x+b))^{3/2}*a^{13/2}*((a*d-b*c)*d/c^2)^{1/2}*x*b^2*c^4*d+18*(x*(a*x+b))^{1/2}*a^{17/2}*((a*d-b*c)*d/c^2)^{1/2}*x^2*b^2*c^2*d^3+3*a^6*\ln(1/2*(2*(x*(a*x+b))^{1/2}*a^{1/2}+2*a*x+b)/a^{1/2}))*((a*d-b*c)*d/c^2)^{1/2}*b^4*c^2*d^3+27*a^5*\ln(1/2*(2*(x*(a*x+b))^{1/2}*a^{1/2}+2*a*x+b)/a^{1/2}))*((a*d-b*c)*d/c^2)^{1/2}*b^5*c^3*d^2-39*a^4*\ln(1/2*(2*(x*(a*x+b))^{1/2}*a^{1/2}+2*a*x+b)/a^{1/2}))*((a*d-b*c)*d/c^2)^{1/2}*b^6*c^4*d-6*a^{10}*\ln(1/2*(2*(x*(a*x+b))^{1/2}*a^{1/2}+2*a*x+b)/a^{1/2}))*((a*d-b*c)*d/c^2)^{1/2}*x^3*c^2*d^4+6*(x*(a*x+b))^{1/2}*a^{19/2}*((a*d-b*c)*d/c^2)^{1/2}*x^3*c^2*d^3-30*(x*(a*x+b))^{1/2}*a^{13/2}*((a*d-b*c)*d/c^2)^{1/2}*x^3*b^3*c^5+6*a^{19/2}*\ln((2*(x*(a*x+b))^{1/2}*((a*d-b*c)*d/c^2)^{1/2}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^3*b^3*c^2*d^4+24*(x*(a*x+b))^{3/2}*a^{11/2}*((a*d-b*c)*d/c^2)^{1/2}*x^2*b^4*c^5+15*\ln(1/2*(2*(x*(a*x+b))^{1/2}*a^{1/2}+2*a*x+b)/a^{1/2}))*((a*d-b*c)*d/c^2)^{1/2}*x^3*a^6*b^4*c^5+45*\ln(1/2*(2*(x*(a*x+b))^{1/2}*a^{1/2}+2*a*x+b)/a^{1/2}))*((a*d-b*c)*d/c^2)^{1/2}*x^2*a^5*b^5*c^5+45*\ln(1/2*(2*(x*(a*x+b))^{1/2}*a^{1/2}+2*a*x+b)/a^{1/2}))*((a*d-b*c)*d/c^2)^{1/2}*x^4*b^6*c^5-162*(x*(a*x+b))^{1/2}*a^{13/2}*((a*d-b*c)*d/c^2)^{1/2}*x^3*b^3*c^3*d^2+234*(x*(a*x+b))^{1/2}*a^{11/2}*((a*d-b*c)*d/c^2)^{1/2}*x^2*b^4*c^4*d+18*a^{17/2}*\ln((2*(x*(a*x+b))^{1/2}*((a*d-b*c)*d/c^2)^{1/2}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*b^2*c^2*d^4+32*(x*(a*x+b))^{3/2}*a^{13/2}*((a*d-b*c)*d/c^2)^{1/2}*b^2*c^3*d^2-52*(x*(a*x+b))^{3/2}*a^{11/2}*((a*d-b*c)*d/c^2)^{1/2}*b^3*c^4*d-90*(x*(a*x+b))^{1/2}*a^{9/2}*((a*d-b*c)*d/c^2)^{1/2}*x^2*b^5*c^5-6*a^7*\ln(1/2*(2*(x*(a*x+b))^{1/2}*a^{1/2}+2*a*x+b)/a^{1/2}))*((a*d-b*c)*d/c^2)^{1/2}*b^3*c^2*d^4+18*a^{15/2}*\ln((2*$



$$\begin{aligned} & (x^*(a*x+b))^{(1/2)} * ((a*d-b*c)^*d/c^2)^{(1/2)} * c-2*a*d*x+b*c*x-b*d)/(c \\ & *x+d) * x^*b^3*c^*d^4+6*(x^*(a*x+b))^{(1/2)} * a^{(13/2)} * ((a*d-b*c)^*d/c^2) \\ & ^{(1/2)} * b^3*c^2*d^3-54*(x^*(a*x+b))^{(1/2)} * a^{(11/2)} * ((a*d-b*c)^*d/c^2) \\ & ^{(1/2)} * b^4*c^3*d^2+78*(x^*(a*x+b))^{(1/2)} * a^{(9/2)} * ((a*d-b*c)^*d/c^2) \\ & ^{(1/2)} * b^5*c^4*d-6*a^{(21/2)} * \ln((2*(x^*(a*x+b))^{(1/2)} * ((a*d-b*c)^*d \\ & /c^2)^{(1/2)} * c-2*a*d*x+b*c*x-b*d)/(c*x+d) * x^3*d^5-6*a^{(15/2)} * \ln(( \\ & 2*(x^*(a*x+b))^{(1/2)} * ((a*d-b*c)^*d/c^2)^{(1/2)} * c-2*a*d*x+b*c*x-b*d)/ \\ & (c*x+d) * b^3*d^5+9*a^7 * \ln(1/2*(2*(x^*(a*x+b))^{(1/2)} * a^{(1/2)}+2*a*x+ \\ & b)/a^{(1/2)}) * ((a*d-b*c)^*d/c^2)^{(1/2)} * x^*b^3*c^2*d^3+81*a^7 * \ln(1/2*( \\ & 2*(x^*(a*x+b))^{(1/2)} * a^{(1/2)}+2*a*x+b)/a^{(1/2)}) * ((a*d-b*c)^*d/c^2)^{(1/2)} \\ & * x^2*b^3*c^3*d^2-39*a^7 * \ln(1/2*(2*(x^*(a*x+b))^{(1/2)} * a^{(1/2)}+2 \\ & *a*x+b)/a^{(1/2)}) * ((a*d-b*c)^*d/c^2)^{(1/2)} * x^3*b^3*c^4*d+81*a^6 * \ln( \\ & 1/2*(2*(x^*(a*x+b))^{(1/2)} * a^{(1/2)}+2*a*x+b)/a^{(1/2)}) * ((a*d-b*c)^*d/c \\ & ^2)^{(1/2)} * x^*b^4*c^3*d^2-117*a^6 * \ln(1/2*(2*(x^*(a*x+b))^{(1/2)} * a^{(1/2)} \\ & +2*a*x+b)/a^{(1/2)}) * ((a*d-b*c)^*d/c^2)^{(1/2)} * x^2*b^4*c^4*d-117*a^5 * \ln(1/2*(2*(x^*(a*x+b))^{(1/2)} * a^{(1/2)}+2*a*x+b)/a^{(1/2)}) * ((a*d-b*c)^*d/c^2)^{(1/2)} * x^*b^5*c^4*d-162*(x^*(a*x+b))^{(1/2)} * a^{(15/2)} * ((a*d-b*c)^*d/c^2)^{(1/2)} * x^2*b^2*c^3*d^2+234*(x^*(a*x+b))^{(1/2)} * a^{(13/2)} * ((a*d-b*c)^*d/c^2)^{(1/2)} * x^2*b^3*c^4*d-18*a^8 * \ln(1/2*(2*(x^*(a*x+b))^{(1/2)} * a^{(1/2)}+2*a*x+b)/a^{(1/2)}) * ((a*d-b*c)^*d/c^2)^{(1/2)} * x^*b^2*c^*d^4+9*a^8 * \ln(1/2*(2*(x^*(a*x+b))^{(1/2)} * a^{(1/2)}+2*a*x+b)/a^{(1/2)}) * ((a*d-b*c)^*d/c^2)^{(1/2)} * x^2*b^2*c^2*d^3+27*a^8 * \ln(1/2*(2*(x^*(a*x+b))^{(1/2)} * a^{(1/2)}+2*a*x+b)/a^{(1/2)}) * ((a*d-b*c)^*d/c^2)^{(1/2)} * x^3*b^2*c^3*d^2+18*(x^*(a*x+b))^{(1/2)} * a^{(15/2)} * ((a*d-b*c)^*d/c^2)^{(1/2)} * x^*b^2*c^2*d^3-18*a^{(19/2)} * \ln((2*(x^*(a*x+b))^{(1/2)} * ((a*d-b*c)^*d/c^2)^{(1/2)} * c-2*a*d*x+b*c*x-b*d)/(c*x+d) * x^2*b^*d^5+20*(x^*(a*x+b))^{(3/2)} * a^{(9/2)} * ((a*d-b*c)^*d/c^2)^{(1/2)} * b^4*c^5-18*a^{(17/2)} * \ln((2*(x^*(a*x+b))^{(1/2)} * ((a*d-b*c)^*d/c^2)^{(1/2)} * c-2*a*d*x+b*c*x-b*d)/(c*x+d) * x^*b^2*d^5+6*a^{(13/2)} * \ln((2*(x^*(a*x+b))^{(1/2)} * ((a*d-b*c)^*d/c^2)^{(1/2)} * c-2*a*d*x+b*c*x-b*d)/(c*x+d) * b^4*c^*d^4-30*(x^*(a*x+b))^{(1/2)} * a^{(7/2)} * ((a*d-b*c)^*d/c^2)^{(1/2)} * b^6*c^5+15 * \ln(1/2*(2*(x^*(a*x+b))^{(1/2)} * a^{(1/2)}+2*a*x+b)/a^{(1/2)}) * ((a*d-b*c)^*d/c^2)^{(1/2)} * a^3*b^7*c^5)/(x^*(a*x+b))^{(1/2)}/(a*d-b*c)^3/c^3/((a*d-b*c)^*d/c^2)^{(1/2)}/(a*x+b)^3 \end{aligned}$$


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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a + b/x)^(5/2)\*(c + d/x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

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**Fricas [A]** time = 0.903203, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a + b/x)^(5/2)\*(c + d/x)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [1/6*(6*(a^4*d^3*x + a^3*b*d^3)*\sqrt{a}*\sqrt{-d/(b*c - a*d)}*\sqrt{ \\ & (a*x + b)/x}*\log(-2*(b*c - a*d)*x*\sqrt{-d/(b*c - a*d)}*\sqrt{(a* \\ & x + b)/x} - b*d + (b*c - 2*a*d)*x)/(c*x + d) + 3*(5*b^4*c^3 - 8* \\ & a*b^3*c^2*d + a^2*b^2*c*d^2 + 2*a^3*b*d^3 + (5*a*b^3*c^3 - 8*a^2* \\ & b^2*c^2*d + a^3*b*c*d^2 + 2*a^4*d^3)*x)*\sqrt{(a*x + b)/x}*\log(-2* \\ & a*x*\sqrt{(a*x + b)/x} + (2*a*x + b)*\sqrt{a}) + 2*(15*b^4*c^3 - 24 \\ & *a*b^3*c^2*d + 3*a^2*b^2*c*d^2 + 3*(a^2*b^2*c^3 - 2*a^3*b*c^2*d + \\ & a^4*c*d^2)*x^2 + 2*(10*a*b^3*c^3 - 16*a^2*b^2*c^2*d + 3*a^3*b*c^* \\ & d^2)*x)*\sqrt{a})/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*d + a^5*b*c^2*d^2 \\ & + (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x)*\sqrt{a}*\sqrt{(a* \\ & x + b)/x}), 1/3*(3*(a^4*d^3*x + a^3*b*d^3)*\sqrt{-a}*\sqrt{-d/(b*c \\ & - a*d)}*\sqrt{(a*x + b)/x}*\log(-2*(b*c - a*d)*x*\sqrt{-d/(b*c - a* \end{aligned}$$

d))\*sqrt((a\*x + b)/x) - b\*d + (b\*c - 2\*a\*d)\*x)/(c\*x + d)) + 3\*(5\*b^4\*c^3 - 8\*a\*b^3\*c^2\*d + a^2\*b^2\*c\*d^2 + 2\*a^3\*b\*d^3 + (5\*a\*b^3\*c^3 - 8\*a^2\*b^2\*c^2\*d + a^3\*b\*c\*d^2 + 2\*a^4\*d^3)\*x)\*sqrt((a\*x + b)/x)\*arctan(a/(sqrt(-a)\*sqrt((a\*x + b)/x))) + (15\*b^4\*c^3 - 24\*a\*b^3\*c^2\*d + 3\*a^2\*b^2\*c\*d^2 + 3\*(a^2\*b^2\*c^3 - 2\*a^3\*b\*c^2\*d + a^4\*c\*d^2)\*x^2 + 2\*(10\*a\*b^3\*c^3 - 16\*a^2\*b^2\*c^2\*d + 3\*a^3\*b\*c\*d^2)\*x)\*sqrt(-a))/((a^3\*b^3\*c^4 - 2\*a^4\*b^2\*c^3\*d + a^5\*b\*c^2\*d^2 + (a^4\*b^2\*c^4 - 2\*a^5\*b\*c^3\*d + a^6\*c^2\*d^2)\*x)\*sqrt(-a)\*sqrt((a\*x + b)/x)), -1/6\*(12\*(a^4\*d^3\*x + a^3\*b\*d^3)\*sqrt(a)\*sqrt(d/(b\*c - a\*d))\*sqrt((a\*x + b)/x)\*arctan(-(b\*c - a\*d)\*sqrt(d/(b\*c - a\*d)))/(d\*sqrt((a\*x + b)/x))) - 3\*(5\*b^4\*c^3 - 8\*a\*b^3\*c^2\*d + a^2\*b^2\*c\*d^2 + 2\*a^3\*b\*d^3 + (5\*a\*b^3\*c^3 - 8\*a^2\*b^2\*c^2\*d + a^3\*b\*c\*d^2 + 2\*a^4\*d^3)\*x)\*sqrt((a\*x + b)/x)\*log(-2\*a\*x\*sqrt((a\*x + b)/x) + (2\*a\*x + b)\*sqrt(a)) - 2\*(15\*b^4\*c^3 - 24\*a\*b^3\*c^2\*d + 3\*a^2\*b^2\*c\*d^2 + 3\*(a^2\*b^2\*c^3 - 2\*a^3\*b\*c^2\*d + a^4\*c\*d^2)\*x^2 + 2\*(10\*a\*b^3\*c^3 - 16\*a^2\*b^2\*c^2\*d + 3\*a^3\*b\*c\*d^2)\*x)\*sqrt(a))/((a^3\*b^3\*c^4 - 2\*a^4\*b^2\*c^3\*d + a^5\*b\*c^2\*d^2 + (a^4\*b^2\*c^4 - 2\*a^5\*b\*c^3\*d + a^6\*c^2\*d^2)\*x)\*sqrt(a)\*sqrt((a\*x + b)/x)), -1/3\*(6\*(a^4\*d^3\*x + a^3\*b\*d^3)\*sqrt(-a)\*sqrt(d/(b\*c - a\*d))\*sqrt((a\*x + b)/x)\*arctan(-(b\*c - a\*d)\*sqrt(d/(b\*c - a\*d)))/(d\*sqrt((a\*x + b)/x))) - 3\*(5\*b^4\*c^3 - 8\*a\*b^3\*c^2\*d + a^2\*b^2\*c\*d^2 + 2\*a^3\*b\*d^3 + (5\*a\*b^3\*c^3 - 8\*a^2\*b^2\*c^2\*d + a^3\*b\*c\*d^2 + 2\*a^4\*d^3)\*x)\*sqrt((a\*x + b)/x)\*arctan(a/(sqrt(-a)\*sqrt((a\*x + b)/x))) - (15\*b^4\*c^3 - 24\*a\*b^3\*c^2\*d + 3\*a^2\*b^2\*c\*d^2 + 3\*(a^2\*b^2\*c^3 - 2\*a^3\*b\*c^2\*d + a^4\*c\*d^2)\*x^2 + 2\*(10\*a\*b^3\*c^3 - 16\*a^2\*b^2\*c^2\*d + 3\*a^3\*b\*c\*d^2)\*x)\*sqrt(-a))/((a^3\*b^3\*c^4 - 2\*a^4\*b^2\*c^3\*d + a^5\*b\*c^2\*d^2 + (a^4\*b^2\*c^4 - 2\*a^5\*b\*c^3\*d + a^6\*c^2\*d^2)\*x)\*sqrt(-a)\*sqrt((a\*x + b)/x))]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a + \frac{b}{x}\right)^{\frac{5}{2}} (cx + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)\*\*(5/2)/(c+d/x), x)

[Out] Integral(x/((a + b/x)\*\*(5/2)\*(c\*x + d)), x)

**GIAC/XCAS [A]** time = 0.258445, size = 332, normalized size = 1.65

$$-\frac{1}{3} \left( \frac{6d^4 \arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{(b^3c^4 - 2ab^2c^3d + a^2bc^2d^2)\sqrt{bcd-ad^2}} - \frac{2\left(ab^2c - a^2bd + \frac{6(ax+b)b^2c}{x} - \frac{9(ax+b)abd}{x}\right)x}{(a^3b^2c^2 - 2a^4bcd + a^5d^2)(ax+b)\sqrt{\frac{ax+b}{x}}} + \frac{3\sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right)a^3c} - \frac{3(5bc + 2d)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a + b/x)^(5/2)\*(c + d/x)), x, algorithm="giac")

[Out] -1/3\*(6\*d^4\*arctan(d\*sqrt((a\*x + b)/x)/sqrt(b\*c\*d - a\*d^2))/((b^3\*c^4 - 2\*a\*b^2\*c^3\*d + a^2\*b^2\*c^2\*d^2)\*sqrt(b\*c\*d - a\*d^2)) - 2\*(a\*b^2\*c - a^2\*b\*d + 6\*(a\*x + b)\*b^2\*c/x - 9\*(a\*x + b)\*a\*b\*d/x)\*x/((a^3\*b^2\*c^2 - 2\*a^4\*b\*c\*d + a^5\*d^2)\*(a\*x + b)\*sqrt((a\*x + b)/x)) + 3\*sqrt((a\*x + b)/x)/((a - (a\*x + b)/x)\*a^3\*c) - 3\*(5\*b\*c + 2\*a\*d)\*arctan(sqrt((a\*x + b)/x)/sqrt(-a))/(sqrt(-a)\*a^3\*b\*c^2))\*b

$$3.165 \quad \int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2} dx$$

**Optimal.** Leaf size=287

$$\begin{aligned} & \frac{(4ad + 5bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3} + \frac{b(6a^2d^2 - 6abcd + 5b^2c^2)}{3a^2c^2\left(a + \frac{b}{x}\right)^{3/2}(bc - ad)^2} + \frac{b(bc - 2ad)(a^2d^2 - abcd + 5b^2c^2)}{a^3c^2\sqrt{a + \frac{b}{x}}(bc - ad)^3} \\ & - \frac{d^{7/2}(9bc - 4ad) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc - ad)^{7/2}} + \frac{d(bc - 2ad)}{ac^2\left(a + \frac{b}{x}\right)^{3/2}\left(c + \frac{d}{x}\right)(bc - ad)} + \frac{x}{ac\left(a + \frac{b}{x}\right)^{3/2}\left(c + \frac{d}{x}\right)} \end{aligned}$$

[Out] (b\*(5\*b^2\*c^2 - 6\*a\*b\*c\*d + 6\*a^2\*d^2))/(3\*a^2\*c^2\*(b\*c - a\*d)^2\*(a + b/x)^(3/2)) + (b\*(b\*c - 2\*a\*d)\*(5\*b^2\*c^2 - a\*b\*c\*d + a^2\*d^2))/(a^3\*c^2\*(b\*c - a\*d)^3\*Sqrt[a + b/x]) + (d\*(b\*c - 2\*a\*d))/(a\*c^2\*(b\*c - a\*d)\*(a + b/x)^(3/2)\*(c + d/x)) + x/(a\*c\*(a + b/x)^(3/2)\*(c + d/x)) - (d^(7/2)\*(9\*bc - 4\*ad)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(c^3\*(b\*c - a\*d)^(7/2)) - ((5\*b\*c + 4\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)\*c^3)

**Rubi [A]** time = 1.40881, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$\begin{aligned} & \frac{(4ad + 5bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3} + \frac{b(6a^2d^2 - 6abcd + 5b^2c^2)}{3a^2c^2\left(a + \frac{b}{x}\right)^{3/2}(bc - ad)^2} + \frac{b(bc - 2ad)(a^2d^2 - abcd + 5b^2c^2)}{a^3c^2\sqrt{a + \frac{b}{x}}(bc - ad)^3} \\ & - \frac{d^{7/2}(9bc - 4ad) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc - ad)^{7/2}} + \frac{d(bc - 2ad)}{ac^2\left(a + \frac{b}{x}\right)^{3/2}\left(c + \frac{d}{x}\right)(bc - ad)} + \frac{x}{ac\left(a + \frac{b}{x}\right)^{3/2}\left(c + \frac{d}{x}\right)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b/x)^(5/2)\*(c + d/x)^2), x]

[Out] (b\*(5\*b^2\*c^2 - 6\*a\*b\*c\*d + 6\*a^2\*d^2))/(3\*a^2\*c^2\*(b\*c - a\*d)^2\*(a + b/x)^(3/2)) + (b\*(b\*c - 2\*a\*d)\*(5\*b^2\*c^2 - a\*b\*c\*d + a^2\*d^2))/(a^3\*c^2\*(b\*c - a\*d)^3\*Sqrt[a + b/x]) + (d\*(b\*c - 2\*a\*d))/(a\*c^2\*(b\*c - a\*d)\*(a + b/x)^(3/2)\*(c + d/x)) + x/(a\*c\*(a + b/x)^(3/2)\*(c + d/x)) - (d^(7/2)\*(9\*bc - 4\*ad)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(c^3\*(b\*c - a\*d)^(7/2)) - ((5\*b\*c + 4\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)\*c^3)

**Rubi in Sympy [A]** time = 160.074, size = 252, normalized size = 0.88

$$\begin{aligned} & \frac{dx}{c\left(a + \frac{b}{x}\right)^{3/2}\left(c + \frac{d}{x}\right)(ad - bc)} + \frac{d^{7/2}(4ad - 9bc) \operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{c^3(ad - bc)^{7/2}} + \frac{x(2ad - bc)}{ac^2\left(a + \frac{b}{x}\right)^{3/2}(ad - bc)} \\ & + \frac{b(6a^2d^2 - 6abcd + 5b^2c^2)}{3a^2c^2\left(a + \frac{b}{x}\right)^{3/2}(ad - bc)^2} + \frac{b(2ad - bc)(a^2d^2 - abcd + 5b^2c^2)}{a^3c^2\sqrt{a + \frac{b}{x}}(ad - bc)^3} - \frac{(4ad + 5bc) \operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(a+b/x)**(5/2)/(c+d/x)**2,x)`

[Out] 
$$-d^*x/(c*(a + b/x)**(3/2)*(c + d/x)*(a*d - b*c)) + d**(7/2)*(4*a*d - 9*b*c)*\operatorname{atanh}(\sqrt{d}*\sqrt{a + b/x}/\sqrt{a*d - b*c})/(c**3*(a*d - b*c)**(7/2)) + x*(2*a*d - b*c)/(a*c**2*(a + b/x)**(3/2)*(a*d - b*c)) + b*(6*a**2*d**2 - 6*a*b*c*d + 5*b**2*c**2)/(3*a**2*c**2*(a + b/x)**(3/2)*(a*d - b*c)**2) + b*(2*a*d - b*c)*(a**2*d**2 - a*b*c*d + 5*b**2*c**2)/(a**3*c**2*\sqrt{a + b/x}*(a*d - b*c)**3) - (4*a*d + 5*b*c)*\operatorname{atanh}(\sqrt{a + b/x}/\sqrt{a})/(a**(7/2)*c**3)$$

**Mathematica [C]** time = 1.84408, size = 364, normalized size = 1.27

$$-\frac{3(4ad+5bc)\log\left(2\sqrt{ax}\sqrt{a+\frac{b}{x}}+2ax+b\right)}{a^{7/2}} + \frac{2\sqrt{a+\frac{b}{x}}(-3a^4d^4(ax+b)^2(cx+d)+3a^4d^5(ax+b)^2+2b^5c^3(cx+d)(bc-ad)+14b^4c^4(ax+b)^2(cx+d)-4b^4c^3(ax+b)(cx+d)-4b^4c^2d^2(cx+d)+4b^4c^2d^2)}{a^4(ax+b)^2(cx+d)(bc-ad)^3}$$

6c<sup>3</sup>

Antiderivative was successfully verified.

[In] `Integrate[1/((a + b/x)^(5/2)*(c + d/x)^2),x]`

[Out] 
$$\left(\left(2*\sqrt{a + b/x}\right)^*(3*a^4*d^5*(b + a*x)^2 + 2*b^5*c^3*(b*c - a*d)*(d + c*x) - 4*b^4*c^3*(4*b*c - 7*a*d)*(b + a*x)*(d + c*x) + 14*b^4*c^4*(b + a*x)^2*(d + c*x) - 26*a*b^3*c^3*d*(b + a*x)^2*(d + c*x) - 3*a^4*d^4*(b + a*x)^2*(d + c*x) + 3*a*c*(b*c - a*d)^3*x*(b + a*x)^2*(d + c*x)\right)/(a^4*(b*c - a*d)^3*(b + a*x)^2*(d + c*x)) - (3*(5*b*c + 4*a*d)*\operatorname{Log}[b + 2*a*x + 2*\sqrt{a}*\sqrt{a + b/x}*x])/a^{7/2} + ((3*I)*d^{(7/2)}*(-9*b*c + 4*a*d)*\operatorname{Log}[(2*c^4*(b*c - a*d)^{(5/2)})*((-2*I)*a*d*x + 2*\sqrt{d}*\sqrt{b*c - a*d}*\sqrt{a + b/x}*x - I*b*(d - c*x))]/(d^{(9/2)}*(9*b*c - 4*a*d)*(d + c*x)))/(b*c - a*d)^{(7/2)}/(6*c^3)$$

**Maple [B]** time = 0.028, size = 4648, normalized size = 16.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b/x)^(5/2)/(c+d/x)^2,x)`

[Out] 
$$-1/6*((a*x+b)/x)^{(1/2)}*x/a^{(13/2)}*(27*\ln((2*(x*(a*x+b))^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))^a^{(19/2)}*x^4*b^2*c^3*d^4-6*((a*d-b*c)*d/c^2)^{(1/2)}*a^{(19/2)}*(x*(a*x+b))^{(3/2)}*x^3*c^4*d^3-6*((a*d-b*c)*d/c^2)^{(1/2)}*a^{(21/2)}*(x*(a*x+b))^{(1/2)}*x^4*c^3*d^4-30*((a*d-b*c)*d/c^2)^{(1/2)}*a^{(13/2)}*(x*(a*x+b))^{(1/2)}*x^4*b^4*c^7-3*\ln((2*(x*(a*x+b))^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))^a^{(21/2)}*x^3*b*c*d^6-90*\ln((2*(x*(a*x+b))^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))^a^{(19/2)}*x^3*b^2*c^2*d^5+81*\ln((2*(x*(a*x+b))^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))^a^{(17/2)}*x^3*b^3*c^3*d^4+24*((a*d-b*c)*d/c^2)^{(1/2)}*a^{(11/2)}*(x*(a*x+b))^{(3/2)}*x^2*b^4*c^7-12*((a*d-b*c)*d/c^2)^{(1/2)}*a^{(21/2)}*(x*(a*x+b))^{(1/2)}*x^3*c^2*d^5-90*((a*d-b*c)*d/c^2)^{(1/2)}*a^{(11/2)}*(x*(a*x+b))^{(1/2)}*x^3*b^5*c^7+12*\ln(1/2*(2*(x*(a*x+b))^{(1/2)}*a^{(1/2)}+2*a*x+b)/a^{(1/2)})*((a*d-b*c)*d/c^2)^{(1/2)}*a^8*b^3*c*d^6-81*\ln((2*(x*(a*x+b))^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))^a^{(19/2)}*x^2*b^2*c*d^6-36*\ln((2*(x*(a*x+b))^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))^a^{(17/2)}*x^2*b^3*c^2*d^5+81*\ln((2*(x*(a*x+b))^{(1/2)}*((a*d-b*c)*d/c^2)^{(1/2)}*c-2*a*d*x+b*c*x-b*d)/(c*x+d))^a^{(15/2)}*x^2*b^4*c^3*d^4+20*((a*d-b*c)*d/c^2)^{(1/2)}*a^{(9/2)}*(x*(a*x+b))^{(3/2)}*x*b^5*c^7-90*((a*d-b*c)*d/c^2)^{(1/2)}*a^{(9/2)}*(x*(a*x+b))^{(1/2)}*x^2*b^6*c^7-48*\ln(1/2*(2*(x*(a*x+b))^{(1/2)}*a^{(1/2)}+2*a*x+b)/a^{(1/2)})*((a*d-b*c)*d/c^2)^{(1/2)}*a^4*b^7*c^5*d^2+15*\ln(1/2*(2*(x*(a*x+b))^{(1/2)}*a^{(1/2)}+2*a*x+b)/a^{(1/2)})*((a*d-b*c)*d/c^2)^{(1/2)}$$

$$\begin{aligned}
& 1/2) * x^4 * a^6 * b^5 * c^7 - 39 * \ln((2 * (x * (a * x + b)))^{(1/2)} * ((a * d - b * c) * d / c^2) \\
& ^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(15/2)} * b^4 * c * d^6 + 27 * \ln((2 * \\
& (x * (a * x + b)))^{(1/2)} * ((a * d - b * c) * d / c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c \\
& * x + d)) * a^{(13/2)} * b^5 * c^2 * d^5 + 12 * \ln((2 * (x * (a * x + b)))^{(1/2)} * ((a * d - b * c) \\
& * d / c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(23/2)} * x^4 * c * d^6 + 36 \\
& * \ln((2 * (x * (a * x + b)))^{(1/2)} * ((a * d - b * c) * d / c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - \\
& b * d) / (c * x + d)) * a^{(21/2)} * x^2 * b * d^7 + 36 * \ln((2 * (x * (a * x + b)))^{(1/2)} * ((a * d \\
& - b * c) * d / c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(19/2)} * x * b^2 * d \\
& ^7 - 30 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(7/2)} * (x * (a * x + b))^{(1/2)} * x * b^7 * c^7 \\
& - 84 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(11/2)} * (x * (a * x + b))^{(1/2)} * b^5 * c^4 * d^3 \\
& + 96 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(9/2)} * (x * (a * x + b))^{(1/2)} * b^6 * c^5 * d^2 \\
& + 42 * \ln(1/2 * (2 * (x * (a * x + b)))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * ((a * d - \\
& b * c) * d / c^2)^{(1/2)} * a^5 * b^6 * c^4 * d^3 + 12 * \ln(1/2 * (2 * (x * (a * x + b)))^{(1/2)} * \\
& a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{11} * x^4 * c^2 * d^5 \\
& + 12 * \ln(1/2 * (2 * (x * (a * x + b)))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * ((a * d - \\
& b * c) * d / c^2)^{(1/2)} * a^{11} * x^3 * c * d^6 + 6 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(21/ \\
& 2)} * (x * (a * x + b))^{(1/2)} * x^5 * c^4 * d^3 - 39 * \ln((2 * (x * (a * x + b)))^{(1/2)} * ((a * d \\
& - b * c) * d / c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(21/2)} * x^4 * b * c \\
& ^2 * d^5 - 156 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(15/2)} * (x * (a * x + b))^{(1/2)} * x^3 \\
& * b^3 * c^5 * d^2 + 12 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(19/2)} * (x * (a * x + b))^{(1/2)} \\
& ) * x^3 * b * c^3 * d^4 + 258 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(13/2)} * (x * (a * x + b))^{(1/2)} \\
& * x^3 * b^4 * c^6 * d^3 + 3 * \ln(1/2 * (2 * (x * (a * x + b)))^{(1/2)} * a^{(1/2)} + 2 * a * x + b \\
& ) / a^{(1/2)}) * ((a * d - b * c) * d / c^2)^{(1/2)} * a^7 * x * b^4 * c^3 * d^4 - 48 * \ln(1/2 * (2 \\
& * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(1 \\
& / 2)} * a^7 * x^4 * b^4 * c^6 * d^3 + 30 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(15/2)} * (x * (a * x \\
& + b))^{(3/2)} * x * b^2 * c^4 * d^3 - 28 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(13/2)} * (x * ( \\
& a * x + b))^{(3/2)} * x * b^3 * c^5 * d^2 - 40 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(11/2)} * ( \\
& x * (a * x + b))^{(3/2)} * x * b^4 * c^6 * d^2 + 96 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(15/2)} * \\
& (x * (a * x + b))^{(1/2)} * x^4 * b^3 * c^6 * d - 84 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(17/ \\
& 2)} * (x * (a * x + b))^{(1/2)} * x^4 * b^2 * c^5 * d^2 + 42 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} \\
& ) * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * ((a * d - b * c) * d / c^2)^{(1/2)} * a^8 * x^4 * b^3 * \\
& c^5 * d^2 - 63 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * \\
& ((a * d - b * c) * d / c^2)^{(1/2)} * a^8 * x^2 * b^3 * c^3 * d^4 + 78 * \ln(1/2 * (2 * (x * (a * x + \\
& b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * ((a * d - b * c) * d / c^2)^{(1/2)} * a^8 * x \\
& ^3 * b^3 * c^4 * d^3 + 36 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * \\
& ((a * d - b * c) * d / c^2)^{(1/2)} * a^9 * x * b^2 * c * d^6 - 87 * \ln(1/2 * (2 * (x * (a \\
& * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * ((a * d - b * c) * d / c^2)^{(1/2)} * a^8 \\
& * x * b^3 * c^2 * d^5 + 36 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(13/2)} * (x * (a * x + b))^{(1/2)} \\
& * x^2 * b^4 * c^5 * d^2 + 12 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x \\
& + b) / a^{(1/2)}) * ((a * d - b * c) * d / c^2)^{(1/2)} * a^9 * x^4 * b^2 * c^4 * d^3 + 48 * ((a * d \\
& - b * c) * d / c^2)^{(1/2)} * a^{(19/2)} * (x * (a * x + b))^{(1/2)} * x^4 * b * c^4 * d^3 + 12 * \ln \\
& ((2 * (x * (a * x + b))^{(1/2)} * ((a * d - b * c) * d / c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d \\
& ) / (c * x + d)) * a^{(23/2)} * x^3 * d^7 + 12 * \ln((2 * (x * (a * x + b))^{(1/2)} * ((a * d - b * c) \\
& * d / c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(17/2)} * b^3 * d^7 + 204 * \\
& ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(11/2)} * (x * (a * x + b))^{(1/2)} * x * b^5 * c^5 * d^2 + \\
& 6 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(9/2)} * (x * (a * x + b))^{(1/2)} * x * b^6 * c^6 * d^4 \\
& + 5 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * ((a * d - b * c) \\
& ) * d / c^2)^{(1/2)} * x^3 * a^5 * b^6 * c^7 - 30 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(7/2)} \\
& * (x * (a * x + b))^{(1/2)} * b^7 * c^6 * d^4 + 45 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} \\
& ) + 2 * a * x + b) / a^{(1/2)}) * ((a * d - b * c) * d / c^2)^{(1/2)} * x^2 * a^4 * b^7 * c^7 + 15 * \ln \\
& (1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * ((a * d - b * c) * d \\
& / c^2)^{(1/2)} * x * a^3 * b^8 * c^7 + 15 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + \\
& 2 * a * x + b) / a^{(1/2)}) * ((a * d - b * c) * d / c^2)^{(1/2)} * a^3 * b^8 * c^6 * d - 33 * \ln(1/2 \\
& * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * ((a * d - b * c) * d / c^2) \\
& ^{(1/2)} * a^7 * b^4 * c^2 * d^5 - 105 * \ln((2 * (x * (a * x + b))^{(1/2)} * ((a * d - b * c) * d / c \\
& ^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(17/2)} * x * b^3 * c * d^6 + 42 * \ln \\
& ((2 * (x * (a * x + b))^{(1/2)} * ((a * d - b * c) * d / c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * \\
& d) / (c * x + d)) * a^{(15/2)} * x * b^4 * c^2 * d^5 + 27 * \ln((2 * (x * (a * x + b))^{(1/2)} * ((a \\
& * d - b * c) * d / c^2)^{(1/2)} * c - 2 * a * d * x + b * c * x - b * d) / (c * x + d)) * a^{(13/2)} * x * b^5 \\
& * c^3 * d^4 + 38 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(13/2)} * (x * (a * x + b))^{(3/2)} * b^3 \\
& * c^4 * d^3 - 64 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(11/2)} * (x * (a * x + b))^{(3/2)} * b^4 \\
& * c^5 * d^2 + 20 * ((a * d - b * c) * d / c^2)^{(1/2)} * a^{(9/2)} * (x * (a * x + b))^{(3/2)} * b^5 \\
& * c^6 * d^2 + 12 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) \\
& * ((a * d - b * c) * d / c^2)^{(1/2)} * a^6 * b^5 * c^3 * d^4 - 12 * ((a * d - b * c) * d / c^2)^{(1/2)} \\
& * a^{(15/2)} * (x * (a * x + b))^{(1/2)} * b^3 * c^2 * d^5 + 30 * ((a * d - b * c) * d / c^2)^{(1/2)} \\
& * a^{(13/2)} * (x * (a * x + b))^{(1/2)} * b^4 * c^3 * d^4 - 36 * ((a * d - b * c) * d / c^2)^{(1/2)} \\
& * a^{(17/2)} * (x * (a * x + b))^{(1/2)} * x * b^2 * c^2 * d^5 + 84 * ((a * d - b * c) * d / c^2) \\
& ^{(1/2)} * a^{(15/2)} * (x * (a * x + b))^{(1/2)} * x * b^3 * c^3 * d^4 - 18 * \ln(1/2 * (2 * (x * \\
& (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * ((a * d - b * c) * d / c^2)^{(1/2)} * \\
& a^6 * x^2 * b^5 * c^5 * d^2 - 129 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x \\
& + b) / a^{(1/2)}) * ((a * d - b * c) * d / c^2)^{(1/2)} * a^6 * x^3 * b^5 * c^6 * d - 3 * \ln(1/2 * ( \\
& 2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x + b) / a^{(1/2)}) * ((a * d - b * c) * d / c^2)^{(1/2)} \\
& * a^4 * x * b^7 * c^6 * d + 78 * \ln(1/2 * (2 * (x * (a * x + b))^{(1/2)} * a^{(1/2)} + 2 * a * x \\
& + b) / a^{(1/2)}) * ((a * d - b * c) * d / c^2)^{(1/2)} * a^7 * x^3 * b^4 * c^5 * d^2 - 72 * ((a * d
\end{aligned}$$

$$\begin{aligned}
& -b^*c^*d/c^2)^{(1/2)} * a^{(13/2)} * (x^*(a^*x+b))^{(3/2)} * x^2 * b^3 * c^6 * d - 87 * \ln \\
& (1/2 * (2 * (x^*(a^*x+b))^{(1/2)} * a^{(1/2)} + 2 * a^*x+b)/a^{(1/2)}) * ((a^*d-b^*c)^*d/ \\
& c^2)^{(1/2)} * a^9 * x^3 * b^2 * c^3 * d^4 - 99 * \ln(1/2 * (2 * (x^*(a^*x+b))^{(1/2)} * a^{(1/2)} \\
& + 2 * a^*x+b)/a^{(1/2)}) * ((a^*d-b^*c)^*d/c^2)^{(1/2)} * a^5 * x^2 * b^6 * c^6 * d + \\
& 198 * ((a^*d-b^*c)^*d/c^2)^{(1/2)} * a^{(11/2)} * (x^*(a^*x+b))^{(1/2)} * x^2 * b^5 * c^6 * d + \\
& 138 * \ln(1/2 * (2 * (x^*(a^*x+b))^{(1/2)} * a^{(1/2)} + 2 * a^*x+b)/a^{(1/2)}) * ((a^*d-b^*c)^*d/c^2)^{(1/2)} * a^6 * x * b^5 * c^4 * d^3 - 156 * ((a^*d-b^*c)^*d/c^2)^{(1/2)} * a^{(15/2)} * (x^*(a^*x+b))^{(1/2)} * x^2 * b^3 * c^4 * d^3 - 102 * \ln(1/2 * (2 * (x^*(a^*x+b))^{(1/2)} * a^{(1/2)} + 2 * a^*x+b)/a^{(1/2)}) * ((a^*d-b^*c)^*d/c^2)^{(1/2)} * a^5 * x * b^6 * c^5 * d^2 + 3 * \ln(1/2 * (2 * (x^*(a^*x+b))^{(1/2)} * a^{(1/2)} + 2 * a^*x+b)/a^{(1/2)}) * ((a^*d-b^*c)^*d/c^2)^{(1/2)} * a^{10} * x^3 * b * c^2 * d^5 - 33 * \ln(1/2 * (2 * (x^*(a^*x+b))^{(1/2)} * a^{(1/2)} + 2 * a^*x+b)/a^{(1/2)}) * ((a^*d-b^*c)^*d/c^2)^{(1/2)} * a^{10} * x^4 * b * c^3 * d^4 - 36 * ((a^*d-b^*c)^*d/c^2)^{(1/2)} * a^{(19/2)} * (x^*(a^*x+b))^{(1/2)} * x^2 * b * c^2 * d^5 + 72 * ((a^*d-b^*c)^*d/c^2)^{(1/2)} * a^{(17/2)} * (x^*(a^*x+b))^{(1/2)} * x^2 * b^2 * c^3 * d^4 + 36 * \ln(1/2 * (2 * (x^*(a^*x+b))^{(1/2)} * a^{(1/2)} + 2 * a^*x+b)/a^{(1/2)}) * ((a^*d-b^*c)^*d/c^2)^{(1/2)} * a^{10} * x^2 * b * c * d^6 - 63 * \ln(1/2 * (2 * (x^*(a^*x+b))^{(1/2)} * a^{(1/2)} + 2 * a^*x+b)/a^{(1/2)}) * ((a^*d-b^*c)^*d/c^2)^{(1/2)} * a^9 * x^2 * b^2 * c^2 * d^5 - 222 * ((a^*d-b^*c)^*d/c^2)^{(1/2)} * a^{(13/2)} * (x^*(a^*x+b))^{(1/2)} * x * b^4 * c^4 * d^3 - 18 * ((a^*d-b^*c)^*d/c^2)^{(1/2)} * a^{(17/2)} * (x^*(a^*x+b))^{(3/2)} * x^2 * b * c^4 * d^3 + 48 * ((a^*d-b^*c)^*d/c^2)^{(1/2)} * a^{(15/2)} * (x^*(a^*x+b))^{(3/2)} * x^2 * b^2 * c^5 * d^2 + 162 * \ln(1/2 * (2 * (x^*(a^*x+b))^{(1/2)} * a^{(1/2)} + 2 * a^*x+b)/a^{(1/2)}) * ((a^*d-b^*c)^*d/c^2)^{(1/2)} * a^7 * x^2 * b^4 * c^4 * d^3 + 24 * ((a^*d-b^*c)^*d/c^2)^{(1/2)} * a^{(17/2)} * (x^*(a^*x+b))^{(1/2)} * x^3 * b^2 * c^4 * d^3 / (x^*(a^*x+b))^{(1/2)} / (a^*d-b^*c)^4 / (a^*x+b)^3 / c^4 / ((a^*d-b^*c)^*d/c^2)^{(1/2)} / (c^*x+d)
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a + b/x)^(5/2) \* (c + d/x)^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.54242, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a + b/x)^(5/2) \* (c + d/x)^2), x, algorithm="fricas")

[Out] 
$$\begin{aligned}
& [1/6 * (3 * (9 * a^3 * b^2 * c^*d^4 - 4 * a^4 * b^*d^5 + (9 * a^4 * b^*c^2 * d^3 - 4 * a^5 * c^*d^4) * x^2 + (9 * a^3 * b^2 * c^2 * d^3 + 5 * a^4 * b^*c^*d^4 - 4 * a^5 * d^5) * x) * \\
& \text{sqrt}(a) * \text{sqrt}(-d/(b^*c - a^*d)) * \text{sqrt}((a^*x + b)/x) * \log(-(2 * (b^*c - a^*d) * x * \text{sqrt}(-d/(b^*c - a^*d)) * \text{sqrt}((a^*x + b)/x) - b^*d + (b^*c - 2 * a^*d) * x) / (c^*x + d)) + 3 * (5 * b^5 * c^4 * d - 11 * a * b^4 * c^3 * d^2 + 3 * a^2 * b^3 * c^2 * d^3 + 7 * a^3 * b^2 * c^*d^4 - 4 * a^4 * b^*d^5 + (5 * a * b^4 * c^5 - 11 * a^2 * b^3 * c^4 * d + 3 * a^3 * b^2 * c^3 * d^2 + 7 * a^4 * b^*c^2 * d^3 - 4 * a^5 * c^*d^4) * x^2 + (5 * b^5 * c^5 - 6 * a * b^4 * c^4 * d - 8 * a^2 * b^3 * c^3 * d^2 + 10 * a^3 * b^2 * c^2 * d^3 + 3 * a^4 * b^*c^*d^4 - 4 * a^5 * d^5) * x) * \text{sqrt}((a^*x + b)/x) * \log(-2 * a^*x * \text{sqrt}((a^*x + b)/x) + (2 * a^*x + b) * \text{sqrt}(a)) + 2 * (15 * b^5 * c^4 * d - 33 * a * b^4 * c^3 * d^2 + 9 * a^2 * b^3 * c^2 * d^3 - 6 * a^3 * b^2 * c^*d^4 + 3 * (a^2 * b^3 * c^5 - 3 * a^3 * b^2 * c^4 * d + 3 * a^4 * b^*c^3 * d^2 - a^5 * c^2 * d^3) * x^3 + (20 * a * b^4 * c^5 - 41 * a^2 * b^3 * c^4 * d + 9 * a^3 * b^2 * c^3 * d^2 + 3 * a^4 * b^*c^2 * d^3 - 6 * a^5 * c^*d^4) * x^2 + (15 * b^5 * c^5 - 13 * a * b^4 * c^4 * d - 35 * a^2 * b^3 * c^3 * d^2 + 15 * a^3 * b^2 * c^2 * d^3 - 12 * a^4 * b^*c^*d^4) * x) * \text{sqrt}(a)) / ((a^3 * b^4 * c^6 * d - 3 * a^4 * b^3 * c^5 * d^2 + 3 * a^5 * b^2 * c^4 * d^3 - a^6 * b^*c^3 * d^4 + (a^4 * b^3 * c^7 - 3 * a^5 * b^2 * c^6 * d + 3 * a^6 * b^*c^5 * d^2 - a^7 * c^4 * d^3) * x^2 + (a^3 * b^4 * c^7 - 2 * a^4 * b^3 * c^6 * d + 2 * a^6 * b^*c^4 * d^3 - a^7 * c^3 * d^4) * x) * \text{sqrt}(a) * \text{sqrt}((a^*x + b)/x)), -1/6 * (6 * (9 * a^3 * b^2 * c^*d^4 - 4 * a^4 * b^*d^5 + (9 * a^4 * b^*c^2 * d^3 - 4 * a^5 * c^*d^4) * x^2 + (9 * a^3 * b^2 * c^2 * d^3 + 5 * a^4 * b^*c^*d^4 - 4 * a^5 * d^5) * x) * \text{sqrt}(a) * \text{sqrt}(d/(b^*c - a^*d)) * s
\end{aligned}$$

$$\begin{aligned} & \text{sqrt}((a*x + b)/x) * \arctan(-(b*c - a*d) * \text{sqrt}(d/(b*c - a*d)) / (d * \text{sqrt}((a*x + b)/x))) - 3 * (5*b^5*c^4*d - 11*a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 + 7*a^3*b^2*c*d^4 - 4*a^4*b*d^5 + (5*a*b^4*c^5 - 11*a^2*b^3*c^4*d + 3*a^3*b^2*c^3*d^2 + 7*a^4*b*c^2*d^3 - 4*a^5*c*d^4) * x^2 + (5*b^5*c^5 - 6*a*b^4*c^4*d - 8*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 3*a^4*b*c*d^4 - 4*a^5*d^5) * x) * \text{sqrt}((a*x + b)/x) * \log(-2*a*x * \text{sqrt}((a*x + b)/x) + (2*a*x + b) * \text{sqrt}(a)) - 2 * (15*b^5*c^4*d - 33*a*b^4*c^3*d^2 + 9*a^2*b^3*c^2*d^3 - 6*a^3*b^2*c*d^4 + 3*(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3) * x^3 + (20*a*b^4*c^5 - 41*a^2*b^3*c^4*d + 9*a^3*b^2*c^3*d^2 + 3*a^4*b*c^2*d^3 - 6*a^5*c*d^4) * x^2 + (15*b^5*c^5 - 13*a*b^4*c^4*d - 35*a^2*b^3*c^3*d^2 + 15*a^3*b^2*c^2*d^3 - 12*a^4*b*c*d^4) * x) * \text{sqrt}(a) / ((a^3*b^4*c^6*d - 3*a^4*b^3*c^5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3*a^6*b*c^5*d^2 - a^7*c^4*d^3) * x^2 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*d + 2*a^6*b*c^4*d^3 - a^7*c^3*d^4) * x) * \text{sqrt}(a) * \text{sqrt}((a*x + b)/x)), 1/6 * (3*(9*a^3*b^2*c*d^4 - 4*a^4*b*d^5 + (9*a^4*b*c^2*d^3 - 4*a^5*c*d^4) * x^2 + (9*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - 4*a^5*d^5) * x) * \text{sqrt}(-a) * \text{sqrt}(-d/(b*c - a*d)) * \text{sqrt}((a*x + b)/x) * \log(-(2*(b*c - a*d) * x * \text{sqrt}(-d/(b*c - a*d)) * \text{sqrt}((a*x + b)/x) - b*d + (b*c - 2*a*d) * x) / (c*x + d)) + 6*(5*b^5*c^4*d - 11*a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 + 7*a^3*b^2*c*d^4 - 4*a^4*b*d^5 + (5*a*b^4*c^5 - 11*a^2*b^3*c^4*d + 3*a^3*b^2*c^3*d^2 + 7*a^4*b*c^2*d^3 - 4*a^5*c*d^4) * x^2 + (5*b^5*c^5 - 6*a*b^4*c^4*d - 8*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 3*a^4*b*c*d^4 - 4*a^5*d^5) * x) * \text{sqrt}((a*x + b)/x) * \arctan(a / (\text{sqrt}(-a) * \text{sqrt}((a*x + b)/x))) + 2*(15*b^5*c^4*d - 33*a*b^4*c^3*d^2 + 9*a^2*b^3*c^2*d^3 - 6*a^3*b^2*c*d^4 + 3*(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3) * x^3 + (20*a*b^4*c^5 - 41*a^2*b^3*c^4*d + 9*a^3*b^2*c^3*d^2 + 3*a^4*b*c^2*d^3 - 6*a^5*c*d^4) * x^2 + (15*b^5*c^5 - 13*a*b^4*c^4*d - 35*a^2*b^3*c^3*d^2 + 15*a^3*b^2*c^2*d^3 - 12*a^4*b*c*d^4) * x) * \text{sqrt}(-a) / ((a^3*b^4*c^6*d - 3*a^4*b^3*c^5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3*a^6*b*c^5*d^2 - a^7*c^4*d^3) * x^2 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*d + 2*a^6*b*c^4*d^3 - a^7*c^3*d^4) * x) * \text{sqrt}(-a) * \text{sqrt}((a*x + b)/x)), -1/3 * (3*(9*a^3*b^2*c*d^4 - 4*a^4*b*d^5 + (9*a^4*b*c^2*d^3 - 4*a^5*c*d^4) * x^2 + (9*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - 4*a^5*d^5) * x) * \text{sqrt}(-a) * \text{sqrt}(d/(b*c - a*d)) * \text{sqrt}((a*x + b)/x) * \arctan(-(b*c - a*d) * \text{sqrt}(d/(b*c - a*d)) / (d * \text{sqrt}((a*x + b)/x))) - 3*(5*b^5*c^4*d - 11*a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 + 7*a^3*b^2*c*d^4 - 4*a^4*b*d^5 + (5*a*b^4*c^5 - 11*a^2*b^3*c^4*d + 3*a^3*b^2*c^3*d^2 + 7*a^4*b*c^2*d^3 - 4*a^5*c*d^4) * x^2 + (5*b^5*c^5 - 6*a*b^4*c^4*d - 8*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 3*a^4*b*c*d^4 - 4*a^5*d^5) * x) * \text{sqrt}((a*x + b)/x) * \arctan(a / (\text{sqrt}(-a) * \text{sqrt}((a*x + b)/x))) - (15*b^5*c^4*d - 33*a*b^4*c^3*d^2 + 9*a^2*b^3*c^2*d^3 - 6*a^3*b^2*c*d^4 + 3*(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3) * x^3 + (20*a*b^4*c^5 - 41*a^2*b^3*c^4*d + 9*a^3*b^2*c^3*d^2 + 3*a^4*b*c^2*d^3 - 6*a^5*c*d^4) * x^2 + (15*b^5*c^5 - 13*a*b^4*c^4*d - 35*a^2*b^3*c^3*d^2 + 15*a^3*b^2*c^2*d^3 - 12*a^4*b*c*d^4) * x) * \text{sqrt}(-a) / ((a^3*b^4*c^6*d - 3*a^4*b^3*c^5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3*a^6*b*c^5*d^2 - a^7*c^4*d^3) * x^2 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*d + 2*a^6*b*c^4*d^3 - a^7*c^3*d^4) * x) * \text{sqrt}(-a) * \text{sqrt}((a*x + b)/x))] \end{aligned}$$


---

**Sympy** [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)\*\*(5/2)/(c+d/x)\*\*2,x)

[Out] Exception raised: ValueError

---

**GIAC/XCAS [A]** time = 0.280938, size = 779, normalized size = 2.71

$$-\frac{1}{3}b \left( \frac{3(9bcd^4 - 4ad^5) \arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{(b^4c^6 - 3ab^3c^5d + 3a^2b^2c^4d^2 - a^3bc^3d^3)\sqrt{bcd-ad^2}} - \frac{2\left(ab^3c - a^2b^2d + \frac{6(ax+b)b^3c}{x} - \frac{12(ax+b)ab^2d}{x}\right)x}{(a^3b^3c^3 - 3a^4b^2c^2d + 3a^5bcd^2 - a^6d^3)(ax+b)\sqrt{\frac{ax+b}{x}}} + \frac{3\left(b^4c^6 - 3ab^3c^5d + 3a^2b^2c^4d^2 - a^3bc^3d^3\right)\sqrt{bcd-ad^2}}{(a^3b^3c^3 - 3a^4b^2c^2d + 3a^5bcd^2 - a^6d^3)(ax+b)\sqrt{\frac{ax+b}{x}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a + b/x)^(5/2)\*(c + d/x)^2),x, algorithm="giac")

[Out] 
$$-1/3*b*(3*(9*b*c*d^4 - 4*a*d^5)*\arctan(d*\sqrt{(a*x + b)/x}/\sqrt{b*c*d - a*d^2}))/((b^4*c^6 - 3*a*b^3*c^5*d + 3*a^2*b^2*c^4*d^2 - a^3*b*c^3*d^3)*\sqrt{b*c*d - a*d^2}) - 2*(a*b^3*c - a^2*b^2*d + 6*(a*x + b)*b^3*c/x - 12*(a*x + b)*a*b^2*d/x)*x/((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*(a*x + b)*\sqrt{(a*x + b)/x}) + 3*(b^4*c^4*\sqrt{(a*x + b)/x} - 4*a*b^3*c^3*d*\sqrt{(a*x + b)/x} + 6*a^2*b^2*c^2*d^2*\sqrt{(a*x + b)/x} - 4*a^3*b*c*d^3*\sqrt{(a*x + b)/x} + 2*a^4*d^4*\sqrt{(a*x + b)/x} + (a*x + b)*b^3*c^3*d*\sqrt{(a*x + b)/x}/x - 3*(a*x + b)*a*b^2*c^2*d^2*\sqrt{(a*x + b)/x}/x + 3*(a*x + b)*a^2*b*c*d^3*\sqrt{(a*x + b)/x}/x - 2*(a*x + b)*a^3*d^4*\sqrt{(a*x + b)/x}/x)/((a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*d^3)*(a*b*c - a^2*d - (a*x + b)*b*c/x + 2*(a*x + b)*a*d/x - (a*x + b)^2*d/x^2)) - 3*(5*b*c + 4*a*d)*\arctan(\sqrt{(a*x + b)/x}/\sqrt{-a})/(\sqrt{-a}*a^3*b*c^3)$$



$$3.166 \quad \int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^3} dx$$

**Optimal.** Leaf size=409

$$\frac{(6ad + 5bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) d^{7/2} (24a^2d^2 - 88abcd + 99b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{a^{7/2}c^4} - \frac{4c^4(bc - ad)^{9/2}}{d(12a^2d^2 - 23abcd + 4b^2c^2)} + \frac{b(-36a^3d^3 + 87a^2bcd^2 - 36ab^2c^2d + 20b^3c^3)}{4ac^3\left(a + \frac{b}{x}\right)^{3/2}\left(c + \frac{d}{x}\right)(bc - ad)^2} + \frac{12a^2c^3\left(a + \frac{b}{x}\right)^{3/2}(bc - ad)^3}{b(12a^4d^4 - 35a^3bcd^3 + 24a^2b^2c^2d^2 - 56ab^3c^3d + 20b^4c^4)} + \frac{4a^3c^3\sqrt{a + \frac{b}{x}}(bc - ad)^4}{d(2bc - 3ad)} + \frac{x}{2ac^2\left(a + \frac{b}{x}\right)^{3/2}\left(c + \frac{d}{x}\right)^2(bc - ad)} + \frac{x}{ac\left(a + \frac{b}{x}\right)^{3/2}\left(c + \frac{d}{x}\right)^2}$$

[Out] (b\*(20\*b^3\*c^3 - 36\*a\*b^2\*c^2\*d + 87\*a^2\*b\*c\*d^2 - 36\*a^3\*d^3))/(12\*a^2\*c^3\*(b\*c - a\*d)^3\*(a + b/x)^(3/2)) + (b\*(20\*b^4\*c^4 - 56\*a\*b^3\*c^3\*d + 24\*a^2\*b^2\*c^2\*d^2 - 35\*a^3\*b\*c\*d^3 + 12\*a^4\*d^4))/(4\*a^3\*c^3\*(b\*c - a\*d)^4\*Sqrt[a + b/x]) + (d\*(2\*b\*c - 3\*a\*d))/(2\*a\*c^2\*(b\*c - a\*d)\*(a + b/x)^(3/2)\*(c + d/x)^2) + (d\*(4\*b^2\*c^2 - 2\*3\*a\*b\*c\*d + 12\*a^2\*d^2))/(4\*a\*c^3\*(b\*c - a\*d)^2\*(a + b/x)^(3/2)\*(c + d/x)) + x/(a\*c\*(a + b/x)^(3/2)\*(c + d/x)^2) - (d^(7/2)\*(99\*b^2\*c^2 - 88\*a\*b\*c\*d + 24\*a^2\*d^2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(4\*c^4\*(b\*c - a\*d)^(9/2)) - ((5\*b\*c + 6\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)\*c^4)

**Rubi [A]** time = 2.07224, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$\frac{(6ad + 5bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) d^{7/2} (24a^2d^2 - 88abcd + 99b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{a^{7/2}c^4} - \frac{4c^4(bc - ad)^{9/2}}{d(12a^2d^2 - 23abcd + 4b^2c^2)} + \frac{b(-36a^3d^3 + 87a^2bcd^2 - 36ab^2c^2d + 20b^3c^3)}{4ac^3\left(a + \frac{b}{x}\right)^{3/2}\left(c + \frac{d}{x}\right)(bc - ad)^2} + \frac{12a^2c^3\left(a + \frac{b}{x}\right)^{3/2}(bc - ad)^3}{b(12a^4d^4 - 35a^3bcd^3 + 24a^2b^2c^2d^2 - 56ab^3c^3d + 20b^4c^4)} + \frac{4a^3c^3\sqrt{a + \frac{b}{x}}(bc - ad)^4}{d(2bc - 3ad)} + \frac{x}{2ac^2\left(a + \frac{b}{x}\right)^{3/2}\left(c + \frac{d}{x}\right)^2(bc - ad)} + \frac{x}{ac\left(a + \frac{b}{x}\right)^{3/2}\left(c + \frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b/x)^(5/2)\*(c + d/x)^3), x]

[Out] (b\*(20\*b^3\*c^3 - 36\*a\*b^2\*c^2\*d + 87\*a^2\*b\*c\*d^2 - 36\*a^3\*d^3))/(12\*a^2\*c^3\*(b\*c - a\*d)^3\*(a + b/x)^(3/2)) + (b\*(20\*b^4\*c^4 - 56\*a\*b^3\*c^3\*d + 24\*a^2\*b^2\*c^2\*d^2 - 35\*a^3\*b\*c\*d^3 + 12\*a^4\*d^4))/(4\*a^3\*c^3\*(b\*c - a\*d)^4\*Sqrt[a + b/x]) + (d\*(2\*b\*c - 3\*a\*d))/(2\*a\*c^2\*(b\*c - a\*d)\*(a + b/x)^(3/2)\*(c + d/x)^2) + (d\*(4\*b^2\*c^2 - 2\*3\*a\*b\*c\*d + 12\*a^2\*d^2))/(4\*a\*c^3\*(b\*c - a\*d)^2\*(a + b/x)^(3/2)\*(c + d/x)) + x/(a\*c\*(a + b/x)^(3/2)\*(c + d/x)^2) - (d^(7/2)\*(99\*b^2\*c^2 - 88\*a\*b\*c\*d + 24\*a^2\*d^2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(4\*c^4\*(b\*c - a\*d)^(9/2)) - ((5\*b\*c + 6\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)\*c^4)

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(a+b/x)**(5/2)/(c+d/x)**3,x)`

[Out] Timed out

**Mathematica [C]** time = 3.81985, size = 465, normalized size = 1.14

$$\frac{12(6ad+5bc)\log\left(2\sqrt{ax}\sqrt{a+\frac{b}{x}+2ax+b}\right)}{a^{7/2}} + \frac{3id^{7/2}(24a^2d^2-88abcd+99b^2c^2)\log\left(\frac{8c^5(bc-ad)^{7/2}\left(2\sqrt{dx}\sqrt{a+\frac{b}{x}}\sqrt{bc-ad-2iadx-ib(d-cx)}\right)}{d^{9/2}(cx+d)(24a^2d^2-88abcd+99b^2c^2)}\right)}{(bc-ad)^{9/2}} + \frac{2\sqrt{a+\frac{b}{x}}(-30a^5d^5(a$$

Antiderivative was successfully verified.

[In] `Integrate[1/((a + b/x)^(5/2)*(c + d/x)^3),x]`

[Out] 
$$\begin{aligned} & -((2*\text{Sqrt}[a + b/x]*(6*a^4*d^6*(b*c - a*d)*(b + a*x)^2 + 3*a^4*d^5 \\ & *(-23*b*c + 12*a*d)*(b + a*x)^2*(d + c*x) - 8*b^6*c^4*(b*c - a*d) \\ & *(d + c*x)^2 + 8*b^5*c^4*(8*b*c - 17*a*d)*(b + a*x)*(d + c*x)^2 - \\ & 56*b^5*c^5*(b + a*x)^2*(d + c*x)^2 + 128*a*b^4*c^4*d*(b + a*x)^2 \\ & *(d + c*x)^2 + 63*a^4*b*c*d^4*(b + a*x)^2*(d + c*x)^2 - 30*a^5*d^4 \\ & 5*(b + a*x)^2*(d + c*x)^2 - 12*a*c*(b*c - a*d)^4*x*(b + a*x)^2*(d \\ & + c*x)^2))/ (a^4*(b*c - a*d)^4*(b + a*x)^2*(d + c*x)^2) + (12*(5* \\ & b*c + 6*a*d)*\text{Log}[b + 2*a*x + 2*\text{Sqrt}[a]*\text{Sqrt}[a + b/x]*x])/a^{(7/2)} \\ & + ((3*I)*d^{(7/2)}*(99*b^2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*\text{Log}[(8*c^4 \\ & 5*(b*c - a*d)^{(7/2)}*((-2*I)*a*d*x + 2*\text{Sqrt}[d]*\text{Sqrt}[b*c - a*d]*\text{Sqr} \\ & t[a + b/x]*x - I*b*(d - c*x))]/(d^{(9/2)}*(99*b^2*c^2 - 88*a*b*c*d \\ & + 24*a^2*d^2)*(d + c*x)))/(b*c - a*d)^{(9/2)})/(24*c^4) \end{aligned}$$

**Maple [B]** time = 0.031, size = 7306, normalized size = 17.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b/x)^(5/2)/(c+d/x)^3,x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((a + b/x)^(5/2)*(c + d/x)^3),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 6.63689, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a + b/x)^(5/2)\*(c + d/x)^3),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [1/24*(3*(99*a^3*b^3*c^2*d^5 - 88*a^4*b^2*c*d^6 + 24*a^5*b*d^7 + (99*a^4*b^2*c^4*d^3 - 88*a^5*b*c^3*d^4 + 24*a^6*c^2*d^5)*x^3 + (99*a^3*b^3*c^4*d^3 + 110*a^4*b^2*c^3*d^4 - 152*a^5*b*c^2*d^5 + 48*a^6*c*d^6)*x^2 + (198*a^3*b^3*c^3*d^4 - 77*a^4*b^2*c^2*d^5 - 40*a^5*b*c*d^6 + 24*a^6*d^7)*x)*\sqrt{a}\sqrt{-d/(b*c - a*d)}\sqrt{(a*x + b)/x} \\ & \log(-2*(b*c - a*d)*x*\sqrt{-d/(b*c - a*d)}\sqrt{(a*x + b)/x} - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 12*(5*b^6*c^5*d^2 - 14*a*b^5*c^4*d^3 + 6*a^2*b^4*c^3*d^4 + 16*a^3*b^3*c^2*d^5 - 19*a^4*b^2*c*d^6 + 6*a^5*b*d^7 + (5*a*b^5*c^7 - 14*a^2*b^4*c^6*d + 6*a^3*b^3*c^5*d^2 + 16*a^4*b^2*c^4*d^3 - 19*a^5*b*c^3*d^4 + 6*a^6*c^2*d^5)*x^3 + (5*b^6*c^7 - 4*a*b^5*c^6*d - 22*a^2*b^4*c^5*d^2 + 28*a^3*b^3*c^4*d^3 + 13*a^4*b^2*c^3*d^4 - 32*a^5*b*c^2*d^5 + 12*a^6*c*d^6)*x^2 + (10*b^6*c^6*d - 23*a*b^5*c^5*d^2 - 2*a^2*b^4*c^4*d^3 + 38*a^3*b^3*c^3*d^4 - 22*a^4*b^2*c^2*d^5 - 7*a^5*b*c*d^6 + 6*a^6*d^7)*x)*\sqrt{(a*x + b)/x} \\ & \log(-2*a*x*\sqrt{(a*x + b)/x} + (2*a*x + b)*\sqrt{a})) + 2*(60*b^6*c^5*d^2 - 168*a*b^5*c^4*d^3 + 72*a^2*b^4*c^3*d^4 - 105*a^3*b^3*c^2*d^5 + 36*a^4*b^2*c*d^6 + 12*(a^2*b^4*c^7 - 4*a^3*b^3*c^6*d + 6*a^4*b^2*c^5*d^2 - 4*a^5*b*c^4*d^3 + a^6*c^3*d^4)*x^4 + (80*a*b^5*c^7 - 200*a^2*b^4*c^6*d + 48*a^3*b^3*c^5*d^2 + 48*a^4*b^2*c^4*d^3 - 135*a^5*b*c^3*d^4 + 54*a^6*c^2*d^5)*x^3 + (60*b^6*c^7 - 8*a*b^5*c^6*d - 364*a^2*b^4*c^5*d^2 + 192*a^3*b^3*c^4*d^3 - 234*a^4*b^2*c^3*d^4 + 3*a^5*b*c^2*d^5 + 36*a^6*c*d^6)*x^2 + (120*b^6*c^6*d - 256*a*b^5*c^5*d^2 - 80*a^2*b^4*c^4*d^3 - 15*a^3*b^3*c^3*d^4 - 156*a^4*b^2*c^2*d^5 + 72*a^5*b*c*d^6)*x)*\sqrt{a} \\ & /((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*d^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6 + (a^4*b^4*c^10 - 4*a^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6*d^4)*x^3 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^3*c^8*d^2 + 8*a^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^2 + (2*a^3*b^5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a^7*b*c^5*d^5 + a^8*c^4*d^6)*x)*\sqrt{a}\sqrt{(a*x + b)/x}), 1/24*(3*(99*a^3*b^3*c^2*d^5 - 88*a^4*b^2*c*d^6 + 24*a^5*b*d^7 + (99*a^4*b^2*c^4*d^3 - 88*a^5*b*c^3*d^4 + 24*a^6*c^2*d^5)*x^3 + (99*a^3*b^3*c^4*d^3 + 110*a^4*b^2*c^3*d^4 - 152*a^5*b*c^2*d^5 + 48*a^6*c*d^6)*x^2 + (198*a^3*b^3*c^3*d^4 - 77*a^4*b^2*c^2*d^5 - 40*a^5*b*c*d^6 + 24*a^6*d^7)*x)*\sqrt{-a}\sqrt{-d/(b*c - a*d)}\sqrt{(a*x + b)/x} \\ & \log(-2*(b*c - a*d)*x*\sqrt{-d/(b*c - a*d)}\sqrt{(a*x + b)/x} - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 24*(5*b^6*c^5*d^2 - 14*a*b^5*c^4*d^3 + 6*a^2*b^4*c^3*d^4 + 16*a^3*b^3*c^2*d^5 - 19*a^4*b^2*c*d^6 + 6*a^5*b*d^7 + (5*a*b^5*c^7 - 14*a^2*b^4*c^6*d + 6*a^3*b^3*c^5*d^2 + 16*a^4*b^2*c^4*d^3 - 19*a^5*b*c^3*d^4 + 6*a^6*c^2*d^5)*x^3 + (5*b^6*c^7 - 4*a*b^5*c^6*d - 22*a^2*b^4*c^5*d^2 + 28*a^3*b^3*c^4*d^3 + 13*a^4*b^2*c^3*d^4 - 32*a^5*b*c^2*d^5 + 12*a^6*c*d^6)*x^2 + (10*b^6*c^6*d - 23*a*b^5*c^5*d^2 - 2*a^2*b^4*c^4*d^3 + 38*a^3*b^3*c^3*d^4 - 22*a^4*b^2*c^2*d^5 - 7*a^5*b*c*d^6 + 6*a^6*d^7)*x)*\sqrt{(a*x + b)/x} \\ & \arctan(a/(\sqrt{-a}\sqrt{(a*x + b)/x})) + 2*(60*b^6*c^5*d^2 - 168*a*b^5*c^4*d^3 + 72*a^2*b^4*c^3*d^4 - 105*a^3*b^3*c^2*d^5 + 36*a^4*b^2*c*d^6 + 12*(a^2*b^4*c^7 - 4*a^3*b^3*c^6*d + 6*a^4*b^2*c^5*d^2 - 4*a^5*b*c^4*d^3 + a^6*c^3*d^4)*x^4 + (80*a*b^5*c^7 - 200*a^2*b^4*c^6*d + 48*a^3*b^3*c^5*d^2 + 48*a^4*b^2*c^4*d^3 - 135*a^5*b*c^3*d^4 + 54*a^6*c^2*d^5)*x^3 + (60*b^6*c^7 - 8*a*b^5*c^6*d - 364*a^2*b^4*c^5*d^2 + 192*a^3*b^3*c^4*d^3 - 234*a^4*b^2*c^3*d^4 + 3*a^5*b*c^2*d^5 + 36*a^6*c*d^6)*x^2 + (120*b^6*c^6*d - 256*a*b^5*c^5*d^2 - 80*a^2*b^4*c^4*d^3 - 15*a^3*b^3*c^3*d^4 - 156*a^4*b^2*c^2*d^5 + 72*a^5*b*c*d^6)*x)*\sqrt{-a} \\ & /((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*d^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6 + (a^4*b^4*c^10 - 4*a^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6*d^4)*x^3 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^3*c^8*d^2 + 8*a^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^2 + (2*a^3*b^5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a^7*b*c^5*d^5 + a^8*c^4*d^6)*x)*\sqrt{-a}\sqrt{(a*x + b)/x}), -1/12*(3*(99*a^3*b^3*c^2*d^5 - 88*a^4*b^2*c*d^6 + 24*a^5*b*d^7 + (99*a^4*b^2*c^4*d^3 - 88*a^5*b*c^3*d^4 + 24*a^6*c^2*d^5)*x^3 + (99*a^3*b^3*c^4*d^3 + 110*a^4*b^2*c^3*d^4 - 152*a^5*b*c^2*d^5 + 48*a^6*c*d^6)*x^2 + (198*a^3*b^3*c^3*d^4 - 77*a^4*b^2*c^2*d^5 - 40*a^5*b*c*d^6 + 24*a^6*d^7)*x)*\sqrt{a}\sqrt{d/(b*c - a*d)}\sqrt{(a*x + b)/x} \\ & \arctan(-(b*c - a*d)*\sqrt{d/(b*c - a*d)})/(d*\sqrt{(a*x + b)/x})) - 6*(5*b^6*c^5*d^2 - 14*a*b^5*c^4*d^3 + 6*a^2*b^4*c^3*d^4 + 16*a^3*b^3*c^2*d^5 \end{aligned}$$

$$\begin{aligned}
& 5 - 19a^4b^2c^2d^6 + 6a^5b^2d^7 + (5ab^5c^7 - 14a^2b^4c^6d + 6a^3b^3c^5d^2 + 16a^4b^2c^4d^3 - 19a^5b^2c^3d^4 + \\
& 6a^6c^2d^5)x^3 + (5b^6c^7 - 4ab^5c^6d - 22a^2b^4c^5d^2 + 28a^3b^3c^4d^3 + 13a^4b^2c^3d^4 - 32a^5b^2c^2d^5 \\
& + 12a^6c^2d^6)x^2 + (10b^6c^6d - 23ab^5c^5d^2 - 2a^2b^4c^4d^3 + 38a^3b^3c^3d^4 - 22a^4b^2c^2d^5 - 7a^5b^2c^2d^6 + 6a^6d^7)x \\
& \sqrt{(ax + b)/x} \log(-2ax \sqrt{(ax + b)/x}) + (2ax + b) \sqrt{a} - (60b^6c^5d^2 - 168ab^5c^4d^3 + 72a^2b^4c^3d^4 - 105a^3b^3c^2d^5 + 36a^4b^2c^2d^6 + 12 \\
& (a^2b^4c^7 - 4a^3b^3c^6d + 6a^4b^2c^5d^2 - 4a^5b^2c^4d^3 + a^6c^3d^4)x^4 + (80ab^5c^7 - 200a^2b^4c^6d + 48a^3b^3c^5d^2 + 48a^4b^2c^4d^3 - 135a^5b^2c^3d^4 + 54a^6c^2d^5)x^3 + (60b^6c^7 - 8ab^5c^6d - 364a^2b^4c^5d^2 \\
& + 192a^3b^3c^4d^3 - 234a^4b^2c^3d^4 + 3a^5b^2c^2d^5 + 36a^6c^2d^6)x^2 + (120b^6c^6d - 256ab^5c^5d^2 - 80a^2b^4c^4d^3 - 15a^3b^3c^3d^4 - 156a^4b^2c^2d^5 + 72a^5b^2c^2d^6)x \\
& \sqrt{a}) / ((a^3b^5c^8d^2 - 4a^4b^4c^7d^3 + 6a^5b^3c^6d^4 - 4a^6b^2c^5d^5 + a^7b^2c^4d^6 + (a^4b^4c^{10} - 4a^5b^3c^9d + 6a^6b^2c^8d^2 - 4a^7b^2c^7d^3 + a^8c^6d^4) \\
& x^3 + (a^3b^5c^{10} - 2a^4b^4c^9d - 2a^5b^3c^8d^2 + 8a^6b^2c^7d^3 - 7a^7b^2c^6d^4 + 2a^8c^5d^5)x^2 + (2a^3b^5c^9d - 7a^4b^4c^8d^2 + 8a^5b^3c^7d^3 - 2a^6b^2c^6d^4 - 2a^7b^2c^5d^5 + a^8c^4d^6)x) \sqrt{a} \sqrt{(ax + b)/x} \\
& \left. \right), -1/12(3(99a^3b^3c^2d^5 - 88a^4b^2c^2d^6 + 24a^5b^2d^7 + (99a^4b^2c^4d^3 - 88a^5b^2c^3d^4 + 24a^6c^2d^5)x^3 + (99a^3b^3c^4d^3 + 110a^4b^2c^3d^4 - 152a^5b^2c^2d^5 + 48a^6c^2d^6)x^2 + (198a^3b^3c^3d^4 - 77a^4b^2c^2d^5 - 40a^5b^2c^2d^6 + 24a^6d^7)x) \sqrt{-a} \sqrt{d/(bc - ad)} \sqrt{(ax + b)/x} \arctan(-(bc - ad) \sqrt{d/(bc - ad)}) / (d \sqrt{(ax + b)/x})) - 12(5b^6c^5d^2 - 14ab^5c^4d^3 + 6a^2b^4c^3d^4 + 16a^3b^3c^2d^5 - 19a^4b^2c^2d^6 + 6a^5b^2d^7 + (5ab^5c^7 - 14a^2b^4c^6d + 6a^3b^3c^5d^2 + 16a^4b^2c^4d^3 - 19a^5b^2c^3d^4 + 6a^6c^2d^5)x^3 + (5b^6c^7 - 4ab^5c^6d - 22a^2b^4c^5d^2 + 28a^3b^3c^4d^3 + 13a^4b^2c^3d^4 - 32a^5b^2c^2d^5 + 12a^6c^2d^6)x^2 + (10b^6c^6d - 23ab^5c^5d^2 - 2a^2b^4c^4d^3 + 38a^3b^3c^3d^4 - 22a^4b^2c^2d^5 - 7a^5b^2c^2d^6 + 6a^6d^7)x) \sqrt{(ax + b)/x} \arctan(a/(\sqrt{-a} \sqrt{(ax + b)/x})) - (60b^6c^5d^2 - 168ab^5c^4d^3 + 72a^2b^4c^3d^4 - 105a^3b^3c^2d^5 + 36a^4b^2c^2d^6 + 12(a^2b^4c^7 - 4a^3b^3c^6d + 6a^4b^2c^5d^2 - 4a^5b^2c^4d^3 + a^6c^3d^4)x^4 + (80ab^5c^7 - 200a^2b^4c^6d + 48a^3b^3c^5d^2 + 48a^4b^2c^4d^3 - 135a^5b^2c^3d^4 + 54a^6c^2d^5)x^3 + (60b^6c^7 - 8ab^5c^6d - 364a^2b^4c^5d^2 + 192a^3b^3c^4d^3 - 234a^4b^2c^3d^4 + 3a^5b^2c^2d^5 + 36a^6c^2d^6)x^2 + (120b^6c^6d - 256ab^5c^5d^2 - 80a^2b^4c^4d^3 - 15a^3b^3c^3d^4 - 156a^4b^2c^2d^5 + 72a^5b^2c^2d^6)x) \sqrt{-a}) / ((a^3b^5c^8d^2 - 4a^4b^4c^7d^3 + 6a^5b^3c^6d^4 - 4a^6b^2c^5d^5 + a^7b^2c^4d^6 + (a^4b^4c^{10} - 4a^5b^3c^9d + 6a^6b^2c^8d^2 - 4a^7b^2c^7d^3 + a^8c^6d^4) x^3 + (a^3b^5c^{10} - 2a^4b^4c^9d - 2a^5b^3c^8d^2 + 8a^6b^2c^7d^3 - 7a^7b^2c^6d^4 + 2a^8c^5d^5) x^2 + (2a^3b^5c^9d - 7a^4b^4c^8d^2 + 8a^5b^3c^7d^3 - 2a^6b^2c^6d^4 - 2a^7b^2c^5d^5 + a^8c^4d^6) x) \sqrt{-a} \sqrt{(ax + b)/x})]
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)\*\*(5/2)/(c+d/x)\*\*3,x)

[Out] Timed out

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**GIAC/XCAS [A]** time = 0.266349, size = 703, normalized size = 1.72

$$-\frac{1}{12} b \left( \frac{3 (99 b^2 c^2 d^4 - 88 a b c d^5 + 24 a^2 d^6) \arctan \left( \frac{d \sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}} \right)}{(b^5 c^8 - 4 a b^4 c^7 d + 6 a^2 b^3 c^6 d^2 - 4 a^3 b^2 c^5 d^3 + a^4 b c^4 d^4) \sqrt{bcd-ad^2}} - \frac{8 \left( a b^4 c - a^2 b^3 d + \frac{6(ax+b)b^4 c}{x} - \frac{15(ax+b)^2 c^2 d}{x^2} \right)}{(a^3 b^4 c^4 - 4 a^4 b^3 c^3 d + 6 a^5 b^2 c^2 d^2 - 4 a^6 b c d^3 + a^7 d^4)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a + b/x)^(5/2)\*(c + d/x)^3),x, algorithm="giac")

[Out] -1/12\*b\*(3\*(99\*b^2\*c^2\*d^4 - 88\*a\*b\*c\*d^5 + 24\*a^2\*d^6)\*arctan(d\*sqrt((a\*x + b)/x)/sqrt(b\*c\*d - a\*d^2))/((b^5\*c^8 - 4\*a\*b^4\*c^7\*d + 6\*a^2\*b^3\*c^6\*d^2 - 4\*a^3\*b^2\*c^5\*d^3 + a^4\*b\*c^4\*d^4)\*sqrt(b\*c\*d - a\*d^2)) - 8\*(a\*b^4\*c - a^2\*b^3\*d + 6\*(a\*x + b)\*b^4\*c/x - 15\*(a\*x + b)\*a\*b^3\*d/x)\*x/((a^3\*b^4\*c^4 - 4\*a^4\*b^3\*c^3\*d + 6\*a^5\*b^2\*c^2\*d^2 - 4\*a^6\*b\*c\*d^3 + a^7\*d^4)\*(a\*x + b)\*sqrt((a\*x + b)/x)) + 3\*(21\*b^2\*c^2\*d^4\*sqrt((a\*x + b)/x) - 29\*a\*b\*c\*d^5\*sqrt((a\*x + b)/x) + 8\*a^2\*d^6\*sqrt((a\*x + b)/x) + 19\*(a\*x + b)\*b\*c\*d^5\*sqrt((a\*x + b)/x)/x - 8\*(a\*x + b)\*a\*d^6\*sqrt((a\*x + b)/x)/((b^4\*c^7 - 4\*a\*b^3\*c^6\*d + 6\*a^2\*b^2\*c^5\*d^2 - 4\*a^3\*b\*c^4\*d^3 + a^4\*c^3\*d^4)\*(b\*c - a\*d + (a\*x + b)\*d/x)^2) + 12\*sqrt((a\*x + b)/x)/((a - (a\*x + b)/x)\*a^3\*c^3) - 12\*(5\*b\*c + 6\*a\*d)\*arctan(sqrt((a\*x + b)/x)/sqrt(-a))/(sqrt(-a)\*a^3\*b\*c^4))

$$3.167 \quad \int \sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}} dx$$

**Optimal.** Leaf size=123

$$x\sqrt{a + \frac{b}{x}}\sqrt{c + \frac{d}{x}} + \frac{(ad + bc) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a + \frac{b}{x}}}{\sqrt{a}\sqrt{c + \frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}} - 2\sqrt{b}\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{b}\sqrt{c + \frac{d}{x}}}\right)$$

[Out] Sqrt[a + b/x]\*Sqrt[c + d/x]\*x + ((b\*c + a\*d)\*ArcTanh[(Sqrt[c]\*Sqrt[a + b/x])/(Sqrt[a]\*Sqrt[c + d/x])])/(Sqrt[a]\*Sqrt[c]) - 2\*Sqrt[b]\*Sqrt[d]\*ArcTanh[(Sqrt[d]\*Sqrt[a + b/x])/(Sqrt[b]\*Sqrt[c + d/x])]

**Rubi [A]** time = 0.353574, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$x\sqrt{a + \frac{b}{x}}\sqrt{c + \frac{d}{x}} + \frac{(ad + bc) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a + \frac{b}{x}}}{\sqrt{a}\sqrt{c + \frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}} - 2\sqrt{b}\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{b}\sqrt{c + \frac{d}{x}}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x]\*Sqrt[c + d/x], x]

[Out] Sqrt[a + b/x]\*Sqrt[c + d/x]\*x + ((b\*c + a\*d)\*ArcTanh[(Sqrt[c]\*Sqrt[a + b/x])/(Sqrt[a]\*Sqrt[c + d/x])])/(Sqrt[a]\*Sqrt[c]) - 2\*Sqrt[b]\*Sqrt[d]\*ArcTanh[(Sqrt[d]\*Sqrt[a + b/x])/(Sqrt[b]\*Sqrt[c + d/x])]

**Rubi in Sympy [A]** time = 32.876, size = 104, normalized size = 0.85

$$-2\sqrt{b}\sqrt{d} \operatorname{atanh}\left(\frac{\sqrt{b}\sqrt{c + \frac{d}{x}}}{\sqrt{d}\sqrt{a + \frac{b}{x}}}\right) + x\sqrt{a + \frac{b}{x}}\sqrt{c + \frac{d}{x}} + \frac{(ad + bc) \operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{a + \frac{b}{x}}}{\sqrt{a}\sqrt{c + \frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d/x)\*\*(1/2)\*(a+b/x)\*\*(1/2), x)

[Out] -2\*sqrt(b)\*sqrt(d)\*atanh(sqrt(b)\*sqrt(c + d/x)/(sqrt(d)\*sqrt(a + b/x))) + x\*sqrt(a + b/x)\*sqrt(c + d/x) + (a\*d + b\*c)\*atanh(sqrt(c)\*sqrt(a + b/x)/(sqrt(a)\*sqrt(c + d/x)))/(sqrt(a)\*sqrt(c))

**Mathematica [A]** time = 0.355393, size = 168, normalized size = 1.37

$$x\sqrt{a + \frac{b}{x}}\sqrt{c + \frac{d}{x}} - \sqrt{b}\sqrt{d} \log\left(2\sqrt{b}\sqrt{d}x\sqrt{a + \frac{b}{x}}\sqrt{c + \frac{d}{x}} + adx + bcx + 2bd\right) + \frac{(ad + bc) \log\left(2\sqrt{a}\sqrt{c}x\sqrt{a + \frac{b}{x}}\sqrt{c + \frac{d}{x}} + a(2cx + d) + bc\right)}{2\sqrt{a}\sqrt{c}} + \sqrt{b}\sqrt{d} \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x]\*Sqrt[c + d/x],x]

[Out] Sqrt[a + b/x]\*Sqrt[c + d/x]\*x + Sqrt[b]\*Sqrt[d]\*Log[x] - Sqrt[b]\*Sqrt[d]\*Log[2\*b\*d + b\*c\*x + a\*d\*x + 2\*Sqrt[b]\*Sqrt[d]\*Sqrt[a + b/x]\*Sqrt[c + d/x]\*x] + ((b\*c + a\*d)\*Log[b\*c + 2\*Sqrt[a]\*Sqrt[c]\*Sqrt[a + b/x]\*Sqrt[c + d/x]\*x + a\*(d + 2\*c\*x)))/(2\*Sqrt[a]\*Sqrt[c])

**Maple [B]** time = 0.033, size = 253, normalized size = 2.1

$$\frac{x}{2} \sqrt{\frac{cx+d}{x}} \sqrt{\frac{ax+b}{x}} \left( \sqrt{bd} \ln \left( \frac{1}{2} \left( 2acx + 2\sqrt{acx^2 + adx + bcx + bd}\sqrt{ac} + ad + bc \right) \frac{1}{\sqrt{ac}} \right) ad + \sqrt{bd} \ln \left( \frac{1}{2} \left( 2acx + 2\sqrt{acx^2} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^(1/2)\*(a+b/x)^(1/2),x)

[Out] 1/2\*((a\*x+b)/x)^(1/2)\*x\*((c\*x+d)/x)^(1/2)\*((b\*d)^(1/2)\*ln(1/2\*(2\*a\*c\*x+2\*(a\*c\*x^2+a\*d\*x+b\*c\*x+b\*d)^(1/2)\*(a\*c)^(1/2)+a\*d+b\*c)/(a\*c)^(1/2))\*a\*d+(b\*d)^(1/2)\*ln(1/2\*(2\*a\*c\*x+2\*(a\*c\*x^2+a\*d\*x+b\*c\*x+b\*d)^(1/2)\*(a\*c)^(1/2)+a\*d+b\*c)/(a\*c)^(1/2))\*b\*c-2\*b\*d\*ln((a\*d\*x+b\*c\*x+2\*(b\*d)^(1/2)\*(a\*c\*x^2+a\*d\*x+b\*c\*x+b\*d)^(1/2)+2\*b\*d)/x)\*(a\*c)^(1/2)+2\*(a\*c\*x^2+a\*d\*x+b\*c\*x+b\*d)^(1/2)\*(a\*c)^(1/2)\*(b\*d)^(1/2))/(a\*c\*x^2+a\*d\*x+b\*c\*x+b\*d)^(1/2)/(a\*c)^(1/2)/(b\*d)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)\*sqrt(c + d/x),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.728648, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)\*sqrt(c + d/x),x, algorithm="fricas")

[Out] [1/4\*(4\*sqrt(a\*c)\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) + (b\*c + a\*d)\*log(-4\*(2\*a^2\*c^2\*x^2 + (a\*b\*c^2 + a^2\*c\*d)\*x)\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) - (8\*a^2\*c^2\*x^2 + b^2\*c^2 + 6\*a\*b\*c\*d + a^2\*d^2 + 8\*(a\*b\*c^2 + a^2\*c\*d)\*x)\*sqrt(a\*c)) + 2\*sqrt(a\*c)\*sqrt(b\*d)\*log(-(8\*b^2\*d^2 + (b^2\*c^2 + 6\*a\*b\*c\*d + a^2\*d^2)\*x^2 - 4\*(2\*b\*d\*x + (b\*c + a\*d)\*x^2)\*sqrt(b\*d)\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) + 8\*(b^2\*c\*d + a\*b\*d^2)\*x/x^2))/sqrt(a\*c), 1/4\*(4\*sqrt(a\*c)\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) + 4\*sqrt(a\*c)\*sqrt(-b\*d)\*arctan(1/2\*(2\*b\*d + (b\*c + a\*d)\*x)\*sqrt(-b\*d)/(b\*d\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x))) + (b\*c + a\*d)\*log(-4\*(2\*a^2\*c^2\*x^2 + (a\*b\*c^2 + a^2\*c\*d)\*x)\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) - (8\*a^2\*c^2\*x^2 + b^2\*c^2 + 6\*a\*b\*c\*d + a^2\*d^2 + 8\*(a\*b\*c^2 + a^2\*c\*d)\*x)\*sqrt(a\*c))/sqrt(a\*c), 1/2\*(2\*sqrt(-a\*c)\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) + (b\*c + a\*d)\*arctan(2\*sqrt(-a\*c)\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x)/(2\*a\*c\*x + b\*c + a\*d)) + sqrt(-a\*c)\*sqrt(b\*d)\*log(-(8\*b^2\*d^2 + (b^2\*c^2 + 6\*a\*b\*c\*d + a^2\*d^2)\*x^2 - 4\*(2\*b\*d\*x + (b\*c + a\*d)\*x^2)\*sqrt(b\*d)\*sqrt((a\*x + b)/x)\*sqrt((c\*x

+ d)/x) + 8\*(b^2\*c\*d + a\*b\*d^2)\*x/x^2)/sqrt(-a\*c), 1/2\*(2\*sqrt(-a\*c)\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) + (b\*c + a\*d)\*arctan(2\*sqrt(-a\*c)\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x)/(2\*a\*c\*x + b\*c + a\*d) + 2\*sqrt(-a\*c)\*sqrt(-b\*d)\*arctan(1/2\*(2\*b\*d + (b\*c + a\*d)\*x)\*sqrt(-b\*d)/(b\*d\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x))))/sqrt(-a\*c]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*\*(1/2)\*(a+b/x)\*\*(1/2),x)

[Out] Integral(sqrt(a + b/x)\*sqrt(c + d/x), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)\*sqrt(c + d/x),x, algorithm="giac")

[Out] integrate(sqrt(a + b/x)\*sqrt(c + d/x), x)



$$3.168 \quad \int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}} dx$$

**Optimal.** Leaf size=81

$$\frac{(bc - ad) \tanh^{-1} \left( \frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}} \right)}{\sqrt{ac}^{3/2}} + \frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

[Out] (Sqrt[a + b/x]\*Sqrt[c + d/x]\*x)/c + ((b\*c - a\*d)\*ArcTanh[(Sqrt[c]\*Sqrt[a + b/x])/(Sqrt[a]\*Sqrt[c + d/x])])/(Sqrt[a]\*c^(3/2))

**Rubi [A]** time = 0.192844, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{(bc - ad) \tanh^{-1} \left( \frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}} \right)}{\sqrt{ac}^{3/2}} + \frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x]/Sqrt[c + d/x], x]

[Out] (Sqrt[a + b/x]\*Sqrt[c + d/x]\*x)/c + ((b\*c - a\*d)\*ArcTanh[(Sqrt[c]\*Sqrt[a + b/x])/(Sqrt[a]\*Sqrt[c + d/x])])/(Sqrt[a]\*c^(3/2))

**Rubi in Sympy [A]** time = 14.8743, size = 65, normalized size = 0.8

$$\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c} - \frac{(ad - bc) \operatorname{atanh} \left( \frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}} \right)}{\sqrt{ac}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b/x)\*\*(1/2)/(c+d/x)\*\*(1/2), x)

[Out] x\*sqrt(a + b/x)\*sqrt(c + d/x)/c - (a\*d - b\*c)\*atanh(sqrt(c)\*sqrt(a + b/x)/(sqrt(a)\*sqrt(c + d/x)))/(sqrt(a)\*c\*\*(3/2))

**Mathematica [A]** time = 0.19538, size = 98, normalized size = 1.21

$$\frac{(bc - ad) \log \left( 2\sqrt{a}\sqrt{c}\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}} + 2acx + ad + bc \right)}{2\sqrt{ac}^{3/2}} + \frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x]/Sqrt[c + d/x], x]

[Out] (Sqrt[a + b/x]\*Sqrt[c + d/x]\*x)/c + ((b\*c - a\*d)\*Log[b\*c + a\*d + 2\*a\*c\*x + 2\*Sqrt[a]\*Sqrt[c]\*Sqrt[a + b/x]\*Sqrt[c + d/x]\*x])/(2\*Sqrt[a]\*c^(3/2))

**Maple [B]** time = 0.034, size = 155, normalized size = 1.9

$$-\frac{x}{2c} \sqrt{\frac{ax+b}{x}} \sqrt{\frac{cx+d}{x}} \left( \ln \left( \frac{1}{2} \left( 2acx + 2\sqrt{(cx+d)(ax+b)}\sqrt{ac} + ad + bc \right) \frac{1}{\sqrt{ac}} \right) \right) ad - \ln \left( \frac{1}{2} \left( 2acx + 2\sqrt{(cx+d)(ax+b)}\sqrt{ac} + ad + bc \right) \frac{1}{\sqrt{ac}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(1/2)/(c+d/x)^(1/2), x)

[Out] -1/2\*((a\*x+b)/x)^(1/2)\*x\*((c\*x+d)/x)^(1/2)\*(ln(1/2\*(2\*a\*c\*x+2\*((c\*x+d)\*(a\*x+b))^(1/2)\*(a\*c)^(1/2)+a\*d+b\*c)/(a\*c)^(1/2))\*a\*d-ln(1/2\*(2\*a\*c\*x+2\*((c\*x+d)\*(a\*x+b))^(1/2)\*(a\*c)^(1/2)+a\*d+b\*c)/(a\*c)^(1/2))\*b\*c-2\*((c\*x+d)\*(a\*x+b))^(1/2)\*(a\*c)^(1/2))/((c\*x+d)\*(a\*x+b))^(1/2)/c/(a\*c)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)/sqrt(c + d/x), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.327013, size = 1, normalized size = 0.01

$$\frac{4\sqrt{acx}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} - (bc - ad)\log\left(4(2a^2c^2x^2 + (abc^2 + a^2cd)x)\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} - (8a^2c^2x^2 + b^2c^2 + 6abcd + a^2d^2 + 8abc^2x)\sqrt{ac}\right)}{4\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)/sqrt(c + d/x), x, algorithm="fricas")

[Out] [1/4\*(4\*sqrt(a\*c)\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) - (b\*c - a\*d)\*log(4\*(2\*a^2\*c^2\*x^2 + (a\*b\*c^2 + a^2\*c\*d)\*x)\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) - (8\*a^2\*c^2\*x^2 + b^2\*c^2 + 6\*a\*b\*c\*d + a^2\*d^2 + 8\*(a\*b\*c^2 + a^2\*c\*d)\*x)\*sqrt(a\*c))/(sqrt(a\*c)\*c), 1/2\*(2\*sqrt(-a\*c)\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) + (b\*c - a\*d)\*arctan(2\*sqrt(-a\*c)\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x)/(2\*a\*c\*x + b\*c + a\*d))/(sqrt(-a\*c)\*c)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(1/2)/(c+d/x)\*\*(1/2), x)

[Out] Integral(sqrt(a + b/x)/sqrt(c + d/x), x)

---

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)/sqrt(c + d/x),x, algorithm="giac")

[Out] integrate(sqrt(a + b/x)/sqrt(c + d/x), x)

$$3.169 \quad \int \frac{\sqrt{a + \frac{b}{x}}}{\left(c + \frac{d}{x}\right)^{3/2}} dx$$

**Optimal.** Leaf size=122

$$\frac{(bc - 3ad) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a + \frac{b}{x}}}{\sqrt{a}\sqrt{c + \frac{d}{x}}}\right)}{\sqrt{ac}^{5/2}} - \frac{\sqrt{a + \frac{b}{x}}(bc - 3ad)}{ac^2\sqrt{c + \frac{d}{x}}} + \frac{x\left(a + \frac{b}{x}\right)^{3/2}}{ac\sqrt{c + \frac{d}{x}}}$$

[Out] -(((b\*c - 3\*a\*d)\*Sqrt[a + b/x])/(a\*c^2\*Sqrt[c + d/x])) + ((a + b/x)^(3/2)\*x)/(a\*c\*Sqrt[c + d/x]) + ((b\*c - 3\*a\*d)\*ArcTanh[(Sqrt[c]\*Sqrt[a + b/x])/(Sqrt[a]\*Sqrt[c + d/x])])/(Sqrt[a]\*c^(5/2))

**Rubi [A]** time = 0.300756, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{(bc - 3ad) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a + \frac{b}{x}}}{\sqrt{a}\sqrt{c + \frac{d}{x}}}\right)}{\sqrt{ac}^{5/2}} - \frac{\sqrt{a + \frac{b}{x}}(bc - 3ad)}{ac^2\sqrt{c + \frac{d}{x}}} + \frac{x\left(a + \frac{b}{x}\right)^{3/2}}{ac\sqrt{c + \frac{d}{x}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x]/(c + d/x)^(3/2), x]

[Out] -(((b\*c - 3\*a\*d)\*Sqrt[a + b/x])/(a\*c^2\*Sqrt[c + d/x])) + ((a + b/x)^(3/2)\*x)/(a\*c\*Sqrt[c + d/x]) + ((b\*c - 3\*a\*d)\*ArcTanh[(Sqrt[c]\*Sqrt[a + b/x])/(Sqrt[a]\*Sqrt[c + d/x])])/(Sqrt[a]\*c^(5/2))

**Rubi in Sympy [A]** time = 24.1821, size = 114, normalized size = 0.93

$$-\frac{2dx\left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{c\sqrt{c + \frac{d}{x}}(ad - bc)} + \frac{x\sqrt{a + \frac{b}{x}}\sqrt{c + \frac{d}{x}}(3ad - bc)}{c^2(ad - bc)} - \frac{(3ad - bc) \operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{a + \frac{b}{x}}}{\sqrt{a}\sqrt{c + \frac{d}{x}}}\right)}{\sqrt{ac}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b/x)\*\*(1/2)/(c+d/x)\*\*(3/2), x)

[Out] -2\*d\*x\*(a + b/x)\*\*(3/2)/(c\*sqrt(c + d/x)\*(a\*d - b\*c)) + x\*sqrt(a + b/x)\*sqrt(c + d/x)\*(3\*a\*d - b\*c)/(c\*\*2\*(a\*d - b\*c)) - (3\*a\*d - b\*c)\*atanh(sqrt(c)\*sqrt(a + b/x)/(sqrt(a)\*sqrt(c + d/x)))/(sqrt(a)\*c\*\*(5/2))

**Mathematica [A]** time = 0.214013, size = 112, normalized size = 0.92

$$\frac{(bc - 3ad) \log\left(2\sqrt{a}\sqrt{c}\sqrt{a + \frac{b}{x}}\sqrt{c + \frac{d}{x}} + 2acx + ad + bc\right)}{2\sqrt{ac}^{5/2}} + \frac{x\sqrt{a + \frac{b}{x}}\sqrt{c + \frac{d}{x}}(cx + 3d)}{c^2(cx + d)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x]/(c + d/x)^(3/2), x]

[Out]  $(\sqrt{a + b/x} \sqrt{c + d/x} x^2 (3d + cx)) / (c^2 (d + cx)) + ((b^2 c - 3a^2 d) \operatorname{Log}[b^2 c + a^2 d + 2a^2 c x + 2\sqrt{a} \sqrt{c} \sqrt{a + b/x} \sqrt{c + d/x} x]) / (2\sqrt{a} c^{5/2})$

**Maple [B]** time = 0.049, size = 280, normalized size = 2.3

$$\frac{x}{(2cx + 2d)c^2} \sqrt{\frac{ax+b}{x}} \sqrt{\frac{cx+d}{x}} \left( -3 \ln \left( \frac{1}{2} \frac{2acx + 2\sqrt{(cx+d)(ax+b)}\sqrt{ac} + ad + bc}{\sqrt{ac}} \right) xacd + \ln \left( \frac{1}{2} (2acx + 2\sqrt{(cx+d)(ax+b)}) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x)^(1/2)/(c+d/x)^(3/2), x)`

[Out]  $\frac{1}{2} \left( \frac{(a^2 x + b^2) \sqrt{c^2 x + d^2}}{(c^2 x + d^2) \sqrt{a^2 x + b^2}} \right)^{1/2} x^2 \left( \frac{(c^2 x + d^2) \sqrt{a^2 x + b^2}}{(c^2 x + d^2) \sqrt{a^2 x + b^2}} \right)^{1/2} \left( -3 \ln \left( \frac{1}{2} (2a^2 c^2 x + 2(c^2 x + d^2) \sqrt{a^2 x + b^2}) / (a^2 c^2 + a^2 d + b^2 c) \right) x^2 a^2 c^2 d + \ln \left( \frac{1}{2} (2a^2 c^2 x + 2(c^2 x + d^2) \sqrt{a^2 x + b^2}) / (a^2 c^2 + a^2 d + b^2 c) \right) x^2 b^2 c^2 + 2x^2 c^2 \left( \frac{(c^2 x + d^2) \sqrt{a^2 x + b^2}}{(c^2 x + d^2) \sqrt{a^2 x + b^2}} \right)^{1/2} (a^2 c^2 + a^2 d + b^2 c) / (a^2 c^2 + a^2 d + b^2 c) - 3 \ln \left( \frac{1}{2} (2a^2 c^2 x + 2(c^2 x + d^2) \sqrt{a^2 x + b^2}) / (a^2 c^2 + a^2 d + b^2 c) \right) x^2 a^2 d^2 + \ln \left( \frac{1}{2} (2a^2 c^2 x + 2(c^2 x + d^2) \sqrt{a^2 x + b^2}) / (a^2 c^2 + a^2 d + b^2 c) \right) x^2 b^2 c^2 d + 6x^2 d^2 \left( \frac{(c^2 x + d^2) \sqrt{a^2 x + b^2}}{(c^2 x + d^2) \sqrt{a^2 x + b^2}} \right)^{1/2} (a^2 c^2 + a^2 d + b^2 c) / (a^2 c^2 + a^2 d + b^2 c) \right) / (c^2 x + d^2) / (a^2 c^2 + a^2 d + b^2 c) / ((c^2 x + d^2) \sqrt{a^2 x + b^2})^{1/2} / c^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(a + b/x)/(c + d/x)^(3/2), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.328971, size = 1, normalized size = 0.01

$$\frac{4(c^2 x^2 + 3dx) \sqrt{ac} \sqrt{\frac{ax+b}{x}} \sqrt{\frac{cx+d}{x}} - (bcd - 3ad^2 + (bc^2 - 3acd)x) \log \left( 4(2a^2 c^2 x^2 + (abc^2 + a^2 cd)x) \sqrt{\frac{ax+b}{x}} \sqrt{\frac{cx+d}{x}} - (8a^2 c^2 x^2 + 4a^2 c^2 x + b^2 c^2) \sqrt{ac} \right)}{4(c^3 x + c^2 d) \sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(a + b/x)/(c + d/x)^(3/2), x, algorithm="fricas")`

[Out]  $\left[ \frac{1}{4} (4(c^2 x^2 + 3d^2 x) \sqrt{a^2 c} \sqrt{(a^2 x + b^2)/x} \sqrt{(c^2 x + d^2)/x} - (b^2 c^2 d - 3a^2 d^2 + (b^2 c^2 - 3a^2 c^2 d)x) \log(4(2a^2 c^2 x^2 + (abc^2 + a^2 cd)x) \sqrt{\frac{ax+b}{x}} \sqrt{\frac{cx+d}{x}} - (8a^2 c^2 x^2 + 4a^2 c^2 x + b^2 c^2) \sqrt{ac})) / ((c^3 x + c^2 d) \sqrt{a^2 c}) \right], \frac{1}{2} (2(c^2 x^2 + 3d^2 x) \sqrt{-a^2 c} \sqrt{(a^2 x + b^2)/x} \sqrt{(c^2 x + d^2)/x} + (b^2 c^2 d - 3a^2 d^2 + (b^2 c^2 - 3a^2 c^2 d)x) \arctan(2\sqrt{-a^2 c} x \sqrt{(a^2 x + b^2)/x} \sqrt{(c^2 x + d^2)/x}) / (2a^2 c^2 x + b^2 c + a^2 d)) / ((c^3 x + c^2 d) \sqrt{-a^2 c}) \right]$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\left(c + \frac{d}{x}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(1/2)/(c+d/x)\*\*(3/2),x)

[Out] Integral(sqrt(a + b/x)/(c + d/x)\*\*(3/2), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x)/(c + d/x)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.170 \quad \int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$$

**Optimal.** Leaf size=96

$$\frac{b \left(a + \frac{b}{x}\right)^{p+1} \left(c + \frac{d}{x}\right)^q \left(\frac{b\left(c + \frac{d}{x}\right)}{bc - ad}\right)^{-q} F_1\left(p + 1; -q, 2; p + 2; -\frac{d\left(a + \frac{b}{x}\right)}{bc - ad}, \frac{a + \frac{b}{x}}{a}\right)}{a^2(p + 1)}$$

[Out]  $-\left(\left(b \cdot \left(a + \frac{b}{x}\right)^{(1 + p)} \cdot \left(c + \frac{d}{x}\right)^q \cdot \text{AppellF1}\left[1 + p, -q, 2, 2 + p, -\left(\frac{d \cdot \left(a + \frac{b}{x}\right)}{b \cdot c - a \cdot d}\right), \left(a + \frac{b}{x}\right)/a\right]\right) / \left(a^{2 \cdot (1 + p)} \cdot \left(b \cdot \left(c + \frac{d}{x}\right)\right) / \left(b \cdot c - a \cdot d\right)^q\right)$

**Rubi [A]** time = 0.180777, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{b \left(a + \frac{b}{x}\right)^{p+1} \left(c + \frac{d}{x}\right)^q \left(\frac{b\left(c + \frac{d}{x}\right)}{bc - ad}\right)^{-q} F_1\left(p + 1; -q, 2; p + 2; -\frac{d\left(a + \frac{b}{x}\right)}{bc - ad}, \frac{a + \frac{b}{x}}{a}\right)}{a^2(p + 1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^p\*(c + d/x)^q,x]

[Out]  $-\left(\left(b \cdot \left(a + \frac{b}{x}\right)^{(1 + p)} \cdot \left(c + \frac{d}{x}\right)^q \cdot \text{AppellF1}\left[1 + p, -q, 2, 2 + p, -\left(\frac{d \cdot \left(a + \frac{b}{x}\right)}{b \cdot c - a \cdot d}\right), \left(a + \frac{b}{x}\right)/a\right]\right) / \left(a^{2 \cdot (1 + p)} \cdot \left(b \cdot \left(c + \frac{d}{x}\right)\right) / \left(b \cdot c - a \cdot d\right)^q\right)$

**Rubi in Sympy [A]** time = 23.2342, size = 66, normalized size = 0.69

$$\frac{b \left(\frac{b\left(-c - \frac{d}{x}\right)}{ad - bc}\right)^{-q} \left(a + \frac{b}{x}\right)^{p+1} \left(c + \frac{d}{x}\right)^q \text{appellf1}\left(p + 1, 2, -q, p + 2, \frac{a + \frac{b}{x}}{a}, \frac{d\left(a + \frac{b}{x}\right)}{ad - bc}\right)}{a^2(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b/x)\*\*p\*(c+d/x)\*\*q,x)

[Out]  $-b \cdot \left(b \cdot \left(-c - \frac{d}{x}\right) / \left(a \cdot d - b \cdot c\right)\right)^{-q} \cdot \left(a + \frac{b}{x}\right)^{p+1} \cdot \left(c + \frac{d}{x}\right)^q \cdot \text{appellf1}\left(p + 1, 2, -q, p + 2, \left(a + \frac{b}{x}\right) / a, \frac{d \cdot \left(a + \frac{b}{x}\right)}{a \cdot d - b \cdot c}\right) / \left(a^{2 \cdot (p + 1)}\right)$

**Mathematica [B]** time = 0.493105, size = 206, normalized size = 2.15

$$\frac{bdx(p + q - 2) \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q F_1\left(-p - q + 1; -p, -q; -p - q + 2; -\frac{d\left(a + \frac{b}{x}\right)}{bc - ad}\right) + (p + q - 1) \left(x \left(adp F_1\left(-p - q + 2; 1 - p, -q; -p - q + 3; -\frac{ax}{b}, -\frac{cx}{d}\right) + bcq F_1\left(-p - q + 2; -p, 1 - q; -p - q + 3; -\frac{ax}{b}, -\frac{cx}{d}\right)\right)\right)}{a^2(p + 1)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b/x)^p\*(c + d/x)^q,x]

[Out]  $\left(b \cdot d \cdot \left(-2 + p + q\right) \cdot \left(a + \frac{b}{x}\right)^p \cdot \left(c + \frac{d}{x}\right)^q \cdot \text{AppellF1}\left[1 - p - q, -p, -q, 2 - p - q, -\left(\frac{a \cdot x}{b}\right), -\left(\frac{c \cdot x}{d}\right)\right]\right) / \left(\left(-1 + p + q\right) \cdot \left(-b \cdot d \cdot \left(-2 + p + q\right) \cdot \text{AppellF1}\left[1 - p - q, -p, -q, 2 - p - q, -\left(\frac{a \cdot x}{b}\right), -\left(\frac{c \cdot x}{d}\right)\right]\right)\right)$

$(c*x)/d)) + x*(a*d*p*AppellF1[2 - p - q, 1 - p, -q, 3 - p - q, -((a*x)/b), -((c*x)/d)] + b*c*q*AppellF1[2 - p - q, -p, 1 - q, 3 - p - q, -((a*x)/b), -((c*x)/d)]))$

**Maple [F]** time = 0.142, size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^p\*(c+d/x)^q,x)

[Out] int((a+b/x)^p\*(c+d/x)^q,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^p\*(c + d/x)^q,x, algorithm="maxima")

[Out] integrate((a + b/x)^p\*(c + d/x)^q, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{ax + b}{x}\right)^p \left(\frac{cx + d}{x}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x)^p\*(c + d/x)^q,x, algorithm="fricas")

[Out] integral(((a\*x + b)/x)^p\*((c\*x + d)/x)^q, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*p\*(c+d/x)\*\*q,x)

[Out] Integral((a + b/x)\*\*p\*(c + d/x)\*\*q, x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a + b/x)^p*(c + d/x)^q,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^p*(c + d/x)^q, x)
```

$$3.171 \quad \int \frac{a + \frac{b}{x^2}}{c + \frac{d}{x^2}} dx$$

**Optimal.** Leaf size=39

$$\frac{(bc - ad) \tan^{-1} \left( \frac{\sqrt{cx}}{\sqrt{d}} \right)}{c^{3/2} \sqrt{d}} + \frac{ax}{c}$$

[Out] (a\*x)/c + ((b\*c - a\*d)\*ArcTan[(Sqrt[c]\*x)/Sqrt[d]])/(c^(3/2)\*Sqrt[d])

**Rubi [A]** time = 0.0718176, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\frac{(bc - ad) \tan^{-1} \left( \frac{\sqrt{cx}}{\sqrt{d}} \right)}{c^{3/2} \sqrt{d}} + \frac{ax}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/(c + d/x^2), x]

[Out] (a\*x)/c + ((b\*c - a\*d)\*ArcTan[(Sqrt[c]\*x)/Sqrt[d]])/(c^(3/2)\*Sqrt[d])

**Rubi in Sympy [A]** time = 11.336, size = 34, normalized size = 0.87

$$\frac{ax}{c} - \frac{(ad - bc) \operatorname{atan} \left( \frac{\sqrt{cx}}{\sqrt{d}} \right)}{c^{3/2} \sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b/x\*\*2)/(c+d/x\*\*2), x)

[Out] a\*x/c - (a\*d - b\*c)\*atan(sqrt(c)\*x/sqrt(d))/(c\*\*(3/2)\*sqrt(d))

**Mathematica [A]** time = 0.0442703, size = 40, normalized size = 1.03

$$\frac{ax}{c} - \frac{(ad - bc) \tan^{-1} \left( \frac{\sqrt{cx}}{\sqrt{d}} \right)}{c^{3/2} \sqrt{d}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/(c + d/x^2), x]

[Out] (a\*x)/c - ((-b\*c) + a\*d)\*ArcTan[(Sqrt[c]\*x)/Sqrt[d]]/(c^(3/2)\*Sqrt[d])

**Maple [A]** time = 0.007, size = 45, normalized size = 1.2

$$\frac{ax}{c} - \frac{ad}{c} \arctan \left( cx \frac{1}{\sqrt{cd}} \right) \frac{1}{\sqrt{cd}} + b \arctan \left( cx \frac{1}{\sqrt{cd}} \right) \frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)/(c+d/x^2),x)`

[Out] `a*x/c-1/c/(c*d)^(1/2)*arctan(c*x/(c*d)^(1/2))*a*d+1/(c*d)^(1/2)*arctan(c*x/(c*d)^(1/2))*b`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a + b/x^2)/(c + d/x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.234813, size = 1, normalized size = 0.03

$$\left[ \frac{2\sqrt{-cd}ax - (bc - ad)\log\left(-\frac{2cdx - (cx^2 - d)\sqrt{-cd}}{cx^2 + d}\right)}{2\sqrt{-cdc}}, \frac{\sqrt{cd}ax + (bc - ad)\arctan\left(\frac{\sqrt{cd}x}{d}\right)}{\sqrt{cdc}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a + b/x^2)/(c + d/x^2),x, algorithm="fricas")`

[Out] `[1/2*(2*sqrt(-c*d)*a*x - (b*c - a*d)*log(-(2*c*d*x - (c*x^2 - d)*sqrt(-c*d))/(c*x^2 + d)))/(sqrt(-c*d)*c), (sqrt(c*d)*a*x + (b*c - a*d)*arctan(sqrt(c*d)*x/d))/(sqrt(c*d)*c)]`

**Sympy [A]** time = 1.66471, size = 82, normalized size = 2.1

$$\frac{ax}{c} + \frac{\sqrt{-\frac{1}{c^3d}}(ad - bc)\log\left(-cd\sqrt{-\frac{1}{c^3d}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{c^3d}}(ad - bc)\log\left(cd\sqrt{-\frac{1}{c^3d}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)/(c+d/x**2),x)`

[Out] `a*x/c + sqrt(-1/(c**3*d))*(a*d - b*c)*log(-c*d*sqrt(-1/(c**3*d)) + x)/2 - sqrt(-1/(c**3*d))*(a*d - b*c)*log(c*d*sqrt(-1/(c**3*d)) + x)/2`

**GIAC/XCAS [A]** time = 0.213993, size = 45, normalized size = 1.15

$$\frac{ax}{c} + \frac{(bc - ad)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cdc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a + b/x^2)/(c + d/x^2),x, algorithm="giac")`

```
[Out] a*x/c + (b*c - a*d)*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c)
```

$$3.172 \quad \int \sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} dx$$

**Optimal.** Leaf size=233

$$\begin{aligned} & -\frac{2d\sqrt{a + \frac{b}{x^2}}}{x\sqrt{c + \frac{d}{x^2}}} + x\sqrt{a + \frac{b}{x^2}}\sqrt{c + \frac{d}{x^2}} - \frac{\sqrt{c}\sqrt{a + \frac{b}{x^2}}(ad + bc)F\left(\cot^{-1}\left(\frac{\sqrt{cx}}{\sqrt{d}}\right) \middle| 1 - \frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c + \frac{d}{x^2}}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}} \\ & + \frac{2\sqrt{c}\sqrt{d}\sqrt{a + \frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{cx}}{\sqrt{d}}\right) \middle| 1 - \frac{bc}{ad}\right)}{\sqrt{c + \frac{d}{x^2}}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}} \end{aligned}$$

[Out]  $(-2*d*\text{Sqrt}[a + b/x^2]) / (\text{Sqrt}[c + d/x^2]*x) + \text{Sqrt}[a + b/x^2]*\text{Sqrt}[c + d/x^2]*x + (2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[a + b/x^2]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]], 1 - (b*c)/(a*d)]) / (\text{Sqrt}[(c*(a + b/x^2))/(a*(c + d/x^2))]*\text{Sqrt}[c + d/x^2]) - (\text{Sqrt}[c]*(b*c + a*d)*\text{Sqrt}[a + b/x^2]*\text{EllipticF}[\text{ArcCot}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]], 1 - (b*c)/(a*d)]) / (a*\text{Sqrt}[d]*\text{Sqrt}[(c*(a + b/x^2))/(a*(c + d/x^2))])* \text{Sqrt}[c + d/x^2]$

**Rubi [A]** time = 0.633792, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$\begin{aligned} & -\frac{2d\sqrt{a + \frac{b}{x^2}}}{x\sqrt{c + \frac{d}{x^2}}} + x\sqrt{a + \frac{b}{x^2}}\sqrt{c + \frac{d}{x^2}} - \frac{\sqrt{c}\sqrt{a + \frac{b}{x^2}}(ad + bc)F\left(\cot^{-1}\left(\frac{\sqrt{cx}}{\sqrt{d}}\right) \middle| 1 - \frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c + \frac{d}{x^2}}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}} \\ & + \frac{2\sqrt{c}\sqrt{d}\sqrt{a + \frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{cx}}{\sqrt{d}}\right) \middle| 1 - \frac{bc}{ad}\right)}{\sqrt{c + \frac{d}{x^2}}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x^2]\*Sqrt[c + d/x^2], x]

[Out]  $(-2*d*\text{Sqrt}[a + b/x^2]) / (\text{Sqrt}[c + d/x^2]*x) + \text{Sqrt}[a + b/x^2]*\text{Sqrt}[c + d/x^2]*x + (2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[a + b/x^2]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]], 1 - (b*c)/(a*d)]) / (\text{Sqrt}[(c*(a + b/x^2))/(a*(c + d/x^2))]*\text{Sqrt}[c + d/x^2]) - (\text{Sqrt}[c]*(b*c + a*d)*\text{Sqrt}[a + b/x^2]*\text{EllipticF}[\text{ArcCot}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]], 1 - (b*c)/(a*d)]) / (a*\text{Sqrt}[d]*\text{Sqrt}[(c*(a + b/x^2))/(a*(c + d/x^2))])* \text{Sqrt}[c + d/x^2]$

**Rubi in Sympy [A]** time = 62.9457, size = 202, normalized size = 0.87

$$\begin{aligned} & -\frac{\sqrt{a}\sqrt{c + \frac{d}{x^2}}(ad + bc)F\left(\text{atan}\left(\frac{\sqrt{b}}{\sqrt{ax}}\right) \middle| -\frac{ad}{bc} + 1\right)}{\sqrt{bc}\sqrt{\frac{a\left(c + \frac{d}{x^2}\right)}{c\left(a + \frac{b}{x^2}\right)}}\sqrt{a + \frac{b}{x^2}}} \\ & + \frac{2\sqrt{c}\sqrt{d}\sqrt{a + \frac{b}{x^2}}E\left(\text{atan}\left(\frac{\sqrt{d}}{\sqrt{cx}}\right) \middle| 1 - \frac{bc}{ad}\right)}{\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}\sqrt{c + \frac{d}{x^2}}} - \frac{2d\sqrt{a + \frac{b}{x^2}}}{x\sqrt{c + \frac{d}{x^2}}} + x\sqrt{a + \frac{b}{x^2}}\sqrt{c + \frac{d}{x^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d/x\*\*2)\*\*(1/2)\*(a+b/x\*\*2)\*\*(1/2), x)

[Out]  $-\sqrt{a} \sqrt{c + d/x^2} (a d + b c) \operatorname{elliptic}_f(\operatorname{atan}(\sqrt{b}/(\sqrt{a} x)), -a d/(b c) + 1)/(\sqrt{b} c \sqrt{a(c + d/x^2)}/(c(a + b/x^2))) \sqrt{a + b/x^2} + 2 \sqrt{c} \sqrt{d} \sqrt{a + b/x^2} \operatorname{elliptic}_e(\operatorname{atan}(\sqrt{d}/(\sqrt{c} x)), 1 - b c/(a d))/(\sqrt{c} (a + b/x^2)/(a(c + d/x^2))) \sqrt{c + d/x^2} - 2 d \sqrt{a + b/x^2} / (x \sqrt{c + d/x^2}) + x \sqrt{a + b/x^2} \sqrt{c + d/x^2}$

**Mathematica [C]** time = 0.519844, size = 205, normalized size = 0.88

$$\frac{x \sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} \left( \sqrt{\frac{a}{b}} (ax^2 + b) (cx^2 + d) + ix \sqrt{\frac{ax^2}{b} + 1} \sqrt{\frac{cx^2}{d} + 1} (bc - ad) F \left( i \sinh^{-1} \left( \sqrt{\frac{a}{b}} x \right) \middle| \frac{bc}{ad} \right) + 2iadx \sqrt{\frac{ax^2}{b} + 1} \sqrt{\frac{cx^2}{d} + 1} \right)}{\sqrt{\frac{a}{b}} (ax^2 + b) (cx^2 + d)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x^2]\*Sqrt[c + d/x^2],x]

[Out]  $-\left( \left( \operatorname{Sqrt}[a + b/x^2] \operatorname{Sqrt}[c + d/x^2] x \left( \operatorname{Sqrt}[a/b] (b + a x^2) (d + c x^2) + (2 I) a d x \operatorname{Sqrt}[1 + (a x^2)/b] \operatorname{Sqrt}[1 + (c x^2)/d] \operatorname{EllipticE}[I \operatorname{ArcSinh}[\operatorname{Sqrt}[a/b] x], (b c)/(a d)] + I (b c - a d) x \operatorname{Sqrt}[1 + (a x^2)/b] \operatorname{Sqrt}[1 + (c x^2)/d] \operatorname{EllipticF}[I \operatorname{ArcSinh}[\operatorname{Sqrt}[a/b] x], (b c)/(a d)] \right) \right) / \left( \operatorname{Sqrt}[a/b] (b + a x^2) (d + c x^2) \right)$

**Maple [A]** time = 0.05, size = 277, normalized size = 1.2

$$\frac{x}{acx^4 + adx^2 + bcx^2 + bd} \sqrt{\frac{cx^2 + d}{x^2}} \sqrt{\frac{ax^2 + b}{x^2}} \left( -\sqrt{-\frac{c}{d}} x^4 ac + \sqrt{\frac{cx^2 + d}{d}} \sqrt{\frac{ax^2 + b}{b}} \operatorname{EllipticF} \left( x \sqrt{-\frac{c}{d}}, \sqrt{\frac{ad}{bc}} \right) xad - cb \sqrt{\frac{cx^2 + d}{x^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x^2)^(1/2)\*(a+b/x^2)^(1/2),x)

[Out]  $\left( \frac{(a x^2 + b) \sqrt{c + d/x^2} x \left( \sqrt{c + d/x^2} \left( -(-c/d)^{1/2} x^4 a c + \sqrt{\frac{cx^2 + d}{d}} \sqrt{\frac{ax^2 + b}{b}} \operatorname{EllipticF} \left( x \sqrt{-c/d}, \sqrt{\frac{ad}{bc}} \right) xad - cb \sqrt{\frac{cx^2 + d}{x^2}} \right) + (a d/b/c)^{1/2} x \operatorname{Sqrt}[1 + (a x^2)/b] \operatorname{Sqrt}[1 + (c x^2)/d] \operatorname{EllipticE} \left( \operatorname{ArcSinh} \left( \sqrt{\frac{a}{b}} x \right), \frac{bc}{ad} \right) + (b c - a d) x \operatorname{Sqrt}[1 + (a x^2)/b] \operatorname{Sqrt}[1 + (c x^2)/d] \operatorname{EllipticF} \left( \operatorname{ArcSinh} \left( \sqrt{\frac{a}{b}} x \right), \frac{bc}{ad} \right) \right)}{(a x^2 + b) \sqrt{c + d/x^2} (d + c x^2)} \right)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(a + b/x^2)\*sqrt(c + d/x^2),x, algorithm="maxima")

[Out] integrate(sqrt(a + b/x^2)\*sqrt(c + d/x^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \sqrt{\frac{ax^2 + b}{x^2}} \sqrt{\frac{cx^2 + d}{x^2}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2),x, algorithm="fricas")`

[Out] `integral(sqrt((a*x^2 + b)/x^2)*sqrt((c*x^2 + d)/x^2), x)`

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d/x**2)**(1/2)*(a+b/x**2)**(1/2),x)`

[Out] `Integral(sqrt(a + b/x**2)*sqrt(c + d/x**2), x)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2),x, algorithm="giac")`

[Out] `integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2), x)`

$$3.173 \quad \int \frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}}} dx$$

**Optimal.** Leaf size=232

$$\begin{aligned} & -\frac{d\sqrt{a + \frac{b}{x^2}}}{cx\sqrt{c + \frac{d}{x^2}}} + \frac{x\sqrt{a + \frac{b}{x^2}}\sqrt{c + \frac{d}{x^2}}}{c} - \frac{b\sqrt{c}\sqrt{a + \frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{cx}}{\sqrt{d}}\right) \middle| 1 - \frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c + \frac{d}{x^2}}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}} \\ & + \frac{\sqrt{d}\sqrt{a + \frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{cx}}{\sqrt{d}}\right) \middle| 1 - \frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c + \frac{d}{x^2}}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}} \end{aligned}$$

[Out]  $-\left(\frac{d\sqrt{a + b/x^2}}{c\sqrt{c + d/x^2}}\right) + \left(\frac{\sqrt{a + b/x^2}\sqrt{c + d/x^2}}{c} + \left(\frac{\sqrt{d}\sqrt{a + b/x^2}}{\sqrt{c}\sqrt{c + d/x^2}}\right) \operatorname{EllipticE}\left[\operatorname{ArcCot}\left[\frac{\sqrt{c}x/\sqrt{d}}{1 - (b^*c)/(a^*d)}\right], 1 - (b^*c)/(a^*d)\right]\right) - \left(\frac{b\sqrt{c}\sqrt{a + b/x^2}}{a\sqrt{d}\sqrt{c + d/x^2}}\right) \operatorname{EllipticF}\left[\operatorname{ArcCot}\left[\frac{\sqrt{c}x/\sqrt{d}}{1 - (b^*c)/(a^*d)}\right], 1 - (b^*c)/(a^*d)\right]$

**Rubi [A]** time = 0.587952, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$

$$\begin{aligned} & -\frac{d\sqrt{a + \frac{b}{x^2}}}{cx\sqrt{c + \frac{d}{x^2}}} + \frac{x\sqrt{a + \frac{b}{x^2}}\sqrt{c + \frac{d}{x^2}}}{c} - \frac{b\sqrt{c}\sqrt{a + \frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{cx}}{\sqrt{d}}\right) \middle| 1 - \frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c + \frac{d}{x^2}}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}} \\ & + \frac{\sqrt{d}\sqrt{a + \frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{cx}}{\sqrt{d}}\right) \middle| 1 - \frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c + \frac{d}{x^2}}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x^2]/Sqrt[c + d/x^2], x]

[Out]  $-\left(\frac{d\sqrt{a + b/x^2}}{c\sqrt{c + d/x^2}}\right) + \left(\frac{\sqrt{a + b/x^2}\sqrt{c + d/x^2}}{c} + \left(\frac{\sqrt{d}\sqrt{a + b/x^2}}{\sqrt{c}\sqrt{c + d/x^2}}\right) \operatorname{EllipticE}\left[\operatorname{ArcCot}\left[\frac{\sqrt{c}x/\sqrt{d}}{1 - (b^*c)/(a^*d)}\right], 1 - (b^*c)/(a^*d)\right]\right) - \left(\frac{b\sqrt{c}\sqrt{a + b/x^2}}{a\sqrt{d}\sqrt{c + d/x^2}}\right) \operatorname{EllipticF}\left[\operatorname{ArcCot}\left[\frac{\sqrt{c}x/\sqrt{d}}{1 - (b^*c)/(a^*d)}\right], 1 - (b^*c)/(a^*d)\right]$

**Rubi in Sympy [A]** time = 56.6809, size = 199, normalized size = 0.86

$$\begin{aligned} & \frac{\sqrt{a}\sqrt{b}\sqrt{c + \frac{d}{x^2}}E\left(\operatorname{atan}\left(\frac{\sqrt{b}}{\sqrt{ax}}\right) \middle| -\frac{ad}{bc} + 1\right)}{c\sqrt{\frac{a\left(c + \frac{d}{x^2}\right)}{c\left(a + \frac{b}{x^2}\right)}}\sqrt{a + \frac{b}{x^2}}} - \frac{b\sqrt{c + \frac{d}{x^2}}}{cx\sqrt{a + \frac{b}{x^2}}} \\ & + \frac{x\sqrt{a + \frac{b}{x^2}}\sqrt{c + \frac{d}{x^2}}}{c} - \frac{b\sqrt{c}\sqrt{a + \frac{b}{x^2}}F\left(\operatorname{atan}\left(\frac{\sqrt{d}}{\sqrt{cx}}\right) \middle| 1 - \frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}\sqrt{c + \frac{d}{x^2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `rubi_integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(1/2),x)`

[Out] `sqrt(a)*sqrt(b)*sqrt(c + d/x**2)*elliptic_e(atan(sqrt(b)/(sqrt(a)*x)), -a*d/(b*c) + 1)/(c*sqrt(a*(c + d/x**2)/(c*(a + b/x**2))))*sqrt(a + b/x**2) - b*sqrt(c + d/x**2)/(c*x*sqrt(a + b/x**2)) + x*sqrt(a + b/x**2)*sqrt(c + d/x**2)/c - b*sqrt(c)*sqrt(a + b/x**2)*elliptic_f(atan(sqrt(d)/(sqrt(c)*x)), 1 - b*c/(a*d))/(a*sqrt(d)*sqrt(c*(a + b/x**2)/(a*(c + d/x**2)))*sqrt(c + d/x**2)`

**Mathematica [A]** time = 0.101855, size = 86, normalized size = 0.37

$$\frac{\sqrt{a + \frac{b}{x^2}} \sqrt{\frac{cx^2+d}{d}} E\left(\sin^{-1}\left(\sqrt{-\frac{c}{d}}x\right) \middle| \frac{ad}{bc}\right)}{\sqrt{-\frac{c}{d}} \sqrt{\frac{ax^2+b}{b}} \sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[a + b/x^2]/Sqrt[c + d/x^2],x]`

[Out] `(Sqrt[a + b/x^2]*Sqrt[(d + c*x^2)/d]*EllipticE[ArcSin[Sqrt[-(c/d)]*x], (a*d)/(b*c)])/(Sqrt[-(c/d)]*Sqrt[c + d/x^2]*Sqrt[(b + a*x^2)/b])`

**Maple [A]** time = 0.029, size = 94, normalized size = 0.4

$$\frac{b}{ax^2 + b} \sqrt{\frac{ax^2 + b}{x^2}} \text{EllipticE}\left(x \sqrt{-\frac{c}{d}}, \sqrt{\frac{ad}{bc}}\right) \sqrt{\frac{ax^2 + b}{b}} \sqrt{\frac{cx^2 + d}{d}} \frac{1}{\sqrt{-\frac{c}{d}}} \frac{1}{\sqrt{\frac{cx^2+d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x)`

[Out] `((a*x^2+b)/x^2)^(1/2)/(a*x^2+b)*EllipticE(x*(-c/d)^(1/2), (a*d/b/c)^(1/2))*((a*x^2+b)/b)^(1/2)*((c*x^2+d)/d)^(1/2)*b/(-c/d)^(1/2)/(c*x^2+d)/x^2)^(1/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2),x, algorithm="maxima")`

[Out] `integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\frac{ax^2+b}{x^2}}}{\sqrt{\frac{cx^2+d}{x^2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2),x, algorithm="fricas")`

[Out] `integral(sqrt((a*x^2 + b)/x^2)/sqrt((c*x^2 + d)/x^2), x)`

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(1/2),x)`

[Out] `Integral(sqrt(a + b/x**2)/sqrt(c + d/x**2), x)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2),x, algorithm="giac")`

[Out] `integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)`

$$3.174 \quad \int \frac{\sqrt{a + \frac{b}{x^2}}}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$$

**Optimal.** Leaf size=262

$$\frac{2\sqrt{d}\sqrt{a + \frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{cx}}{\sqrt{d}}\right) \middle| 1 - \frac{bc}{ad}\right)}{c^{3/2}\sqrt{c + \frac{d}{x^2}}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}} - \frac{2d\sqrt{a + \frac{b}{x^2}}}{c^2x\sqrt{c + \frac{d}{x^2}}} + \frac{2x\sqrt{a + \frac{b}{x^2}}\sqrt{c + \frac{d}{x^2}}}{c^2}$$

$$- \frac{x\sqrt{a + \frac{b}{x^2}}}{c\sqrt{c + \frac{d}{x^2}}} - \frac{b\sqrt{a + \frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{cx}}{\sqrt{d}}\right) \middle| 1 - \frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{d}\sqrt{c + \frac{d}{x^2}}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}}$$

[Out]  $(-2*d*\text{Sqrt}[a + b/x^2])/(c^2*\text{Sqrt}[c + d/x^2]*x) - (\text{Sqrt}[a + b/x^2]*x)/(c*\text{Sqrt}[c + d/x^2]) + (2*\text{Sqrt}[a + b/x^2]*\text{Sqrt}[c + d/x^2]*x)/c^2 + (2*\text{Sqrt}[d]*\text{Sqrt}[a + b/x^2]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]], 1 - (b*c)/(a*d)])/(c^{3/2}*\text{Sqrt}[(c*(a + b/x^2))/(a*(c + d/x^2)])*\text{Sqrt}[c + d/x^2]) - (b*\text{Sqrt}[a + b/x^2]*\text{EllipticF}[\text{ArcCot}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]], 1 - (b*c)/(a*d)]/(a*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[(c*(a + b/x^2))/(a*(c + d/x^2))])*\text{Sqrt}[c + d/x^2])$

**Rubi [A]** time = 0.835678, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$

$$\frac{2\sqrt{d}\sqrt{a + \frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{cx}}{\sqrt{d}}\right) \middle| 1 - \frac{bc}{ad}\right)}{c^{3/2}\sqrt{c + \frac{d}{x^2}}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}} - \frac{2d\sqrt{a + \frac{b}{x^2}}}{c^2x\sqrt{c + \frac{d}{x^2}}} + \frac{2x\sqrt{a + \frac{b}{x^2}}\sqrt{c + \frac{d}{x^2}}}{c^2}$$

$$- \frac{x\sqrt{a + \frac{b}{x^2}}}{c\sqrt{c + \frac{d}{x^2}}} - \frac{b\sqrt{a + \frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{cx}}{\sqrt{d}}\right) \middle| 1 - \frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{d}\sqrt{c + \frac{d}{x^2}}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[a + b/x^2]/(c + d/x^2)^{(3/2)}, x]$

[Out]  $(-2*d*\text{Sqrt}[a + b/x^2])/(c^2*\text{Sqrt}[c + d/x^2]*x) - (\text{Sqrt}[a + b/x^2]*x)/(c*\text{Sqrt}[c + d/x^2]) + (2*\text{Sqrt}[a + b/x^2]*\text{Sqrt}[c + d/x^2]*x)/c^2 + (2*\text{Sqrt}[d]*\text{Sqrt}[a + b/x^2]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]], 1 - (b*c)/(a*d)])/(c^{3/2}*\text{Sqrt}[(c*(a + b/x^2))/(a*(c + d/x^2)])*\text{Sqrt}[c + d/x^2]) - (b*\text{Sqrt}[a + b/x^2]*\text{EllipticF}[\text{ArcCot}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]], 1 - (b*c)/(a*d)]/(a*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[(c*(a + b/x^2))/(a*(c + d/x^2))])*\text{Sqrt}[c + d/x^2])$

**Rubi in Sympy [A]** time = 80.5002, size = 230, normalized size = 0.88

$$\frac{\sqrt{a}\sqrt{b}\sqrt{c + \frac{d}{x^2}}F\left(\text{atan}\left(\frac{\sqrt{b}}{\sqrt{ax}}\right) \middle| -\frac{ad}{bc} + 1\right)}{c^2\sqrt{\frac{a\left(c + \frac{d}{x^2}\right)}{c\left(a + \frac{b}{x^2}\right)}}\sqrt{a + \frac{b}{x^2}}} - \frac{x\sqrt{a + \frac{b}{x^2}}}{c\sqrt{c + \frac{d}{x^2}}} - \frac{2d\sqrt{a + \frac{b}{x^2}}}{c^2x\sqrt{c + \frac{d}{x^2}}}$$

$$+ \frac{2x\sqrt{a + \frac{b}{x^2}}\sqrt{c + \frac{d}{x^2}}}{c^2} + \frac{2\sqrt{d}\sqrt{a + \frac{b}{x^2}}E\left(\text{atan}\left(\frac{\sqrt{d}}{\sqrt{cx}}\right) \middle| 1 - \frac{bc}{ad}\right)}{c^{\frac{3}{2}}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}\sqrt{c + \frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(3/2),x)`

[Out]  $-\sqrt{a} \sqrt{b} \sqrt{c + d/x^2} \operatorname{elliptic}_f(\operatorname{atan}(\sqrt{b}/(\sqrt{a}x)), -a^2d/(b^2c) + 1)/(c^2 \sqrt{a} \sqrt{c + d/x^2}/(c(a + b/x^2)) \sqrt{a + b/x^2}) - x \sqrt{a + b/x^2}/(c \sqrt{c + d/x^2}) - 2^2 d \sqrt{a + b/x^2}/(c^2 x \sqrt{c + d/x^2}) + 2^2 x \sqrt{a + b/x^2} \sqrt{c + d/x^2}/c^2 + 2^2 \sqrt{d} \sqrt{a + b/x^2} \operatorname{elliptic}_e(\operatorname{atan}(\sqrt{d}/(\sqrt{c}x)), 1 - b^2c/(a^2d))/(c^{3/2} \sqrt{c} \sqrt{a + b/x^2}/(a \sqrt{c + d/x^2})) \sqrt{c + d/x^2}$

**Mathematica [C]** time = 0.376759, size = 191, normalized size = 0.73

$$\frac{\sqrt{a + \frac{b}{x^2}} \left( i \sqrt{\frac{ax^2}{b} + 1} \sqrt{\frac{cx^2}{d} + 1} (bc - 2ad) F \left( i \sinh^{-1} \left( \sqrt{\frac{a}{b}} x \right) \middle| \frac{bc}{ad} \right) + 2iad \sqrt{\frac{ax^2}{b} + 1} \sqrt{\frac{cx^2}{d} + 1} E \left( i \sinh^{-1} \left( \sqrt{\frac{a}{b}} x \right) \middle| \frac{bc}{ad} \right) + cx \sqrt{\frac{ax^2}{b} + 1} \right)}{c^2 \sqrt{\frac{a}{b}} (ax^2 + b) \sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[a + b/x^2]/(c + d/x^2)^(3/2),x]`

[Out]  $-\left( \left( \sqrt{a + b/x^2} \right) \left( \sqrt{a/b} c x (b + a x^2) + (2^2 I) a^2 d \sqrt{1 + (a x^2)/b} \sqrt{1 + (c x^2)/d} \operatorname{EllipticE}[I \operatorname{ArcSinh}[\sqrt{a/b} x], (b^2 c)/(a^2 d)] + I (b^2 c - 2^2 a^2 d) \sqrt{1 + (a x^2)/b} \sqrt{1 + (c x^2)/d} \operatorname{EllipticF}[I \operatorname{ArcSinh}[\sqrt{a/b} x], (b^2 c)/(a^2 d)] \right) \right) / \left( \sqrt{a/b} c^2 \sqrt{c + d/x^2} (b + a x^2) \right)$

**Maple [A]** time = 0.065, size = 185, normalized size = 0.7

$$-\frac{cx^2 + d}{x^2 c (ax^2 + b)} \sqrt{\frac{ax^2 + b}{x^2}} \left( x^3 a \sqrt{-\frac{c}{d}} + \operatorname{EllipticF} \left( x \sqrt{-\frac{c}{d}}, \sqrt{\frac{ad}{bc}} \right) b \sqrt{\frac{cx^2 + d}{d}} \sqrt{\frac{ax^2 + b}{b}} - 2 \operatorname{EllipticE} \left( x \sqrt{-\frac{c}{d}}, \sqrt{\frac{ad}{bc}} \right) b \sqrt{\frac{cx^2 + d}{d}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x)`

[Out]  $-\left( (a x^2 + b) / x^2 \right)^{1/2} / x^2 / \left( a x^2 + b \right) \left( x^3 a \left( -c/d \right)^{1/2} + \operatorname{EllipticF} \left( x \left( -c/d \right)^{1/2}, \left( a^2 d / b / c \right)^{1/2} \right) b \left( \left( c x^2 + d \right) / d \right)^{1/2} \left( \left( a x^2 + b \right) / b \right)^{1/2} - 2 \operatorname{EllipticE} \left( x \left( -c/d \right)^{1/2}, \left( a^2 d / b / c \right)^{1/2} \right) b \left( \left( c x^2 + d \right) / d \right)^{1/2} \left( \left( a x^2 + b \right) / b \right)^{1/2} + x^3 b \left( -c/d \right)^{1/2} \left( c x^2 + d \right) / \left( -c/d \right)^{1/2} / c / \left( \left( c x^2 + d \right) / x^2 \right)^{3/2} \right)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\left( c + \frac{d}{x^2} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2 \sqrt{\frac{ax^2+b}{x^2}}}{(cx^2+d)\sqrt{\frac{cx^2+d}{x^2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x, algorithm="fricas")`

[Out] `integral(x^2*sqrt((a*x^2 + b)/x^2)/((c*x^2 + d)*sqrt((c*x^2 + d)/x^2)), x)`

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(3/2), x)`

[Out] `Integral(sqrt(a + b/x**2)/(c + d/x**2)**(3/2), x)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x, algorithm="giac")`

[Out] `integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)`

$$3.175 \quad \int \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q dx$$

**Optimal.** Leaf size=79

$$x \left(a + \frac{b}{x^2}\right)^p \left(\frac{b}{ax^2} + 1\right)^{-p} \left(c + \frac{d}{x^2}\right)^q \left(\frac{d}{cx^2} + 1\right)^{-q} F_1\left(-\frac{1}{2}; -p, -q; \frac{1}{2}; -\frac{b}{ax^2}, -\frac{d}{cx^2}\right)$$

[Out]  $((a + b/x^2)^p (c + d/x^2)^q x \text{AppellF1}[-1/2, -p, -q, 1/2, -(b/(a * x^2)), -(d/(c * x^2))]) / ((1 + b/(a * x^2))^p (1 + d/(c * x^2))^q)$

**Rubi [A]** time = 0.233549, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$x \left(a + \frac{b}{x^2}\right)^p \left(\frac{b}{ax^2} + 1\right)^{-p} \left(c + \frac{d}{x^2}\right)^q \left(\frac{d}{cx^2} + 1\right)^{-q} F_1\left(-\frac{1}{2}; -p, -q; \frac{1}{2}; -\frac{b}{ax^2}, -\frac{d}{cx^2}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)^p (c + d/x^2)^q, x]

[Out]  $((a + b/x^2)^p (c + d/x^2)^q x \text{AppellF1}[-1/2, -p, -q, 1/2, -(b/(a * x^2)), -(d/(c * x^2))]) / ((1 + b/(a * x^2))^p (1 + d/(c * x^2))^q)$

**Rubi in Sympy [A]** time = 31.4201, size = 63, normalized size = 0.8

$$x \left(1 + \frac{b}{ax^2}\right)^{-p} \left(1 + \frac{d}{cx^2}\right)^{-q} \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q \text{appellf1}\left(-\frac{1}{2}, -p, -q, \frac{1}{2}, -\frac{b}{ax^2}, -\frac{d}{cx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b/x\*\*2)\*\*p\*(c+d/x\*\*2)\*\*q,x)

[Out]  $x*(1 + b/(a*x**2))**(-p)*(1 + d/(c*x**2))**(-q)*(a + b/x**2)**p*(c + d/x**2)**q*\text{appellf1}(-1/2, -p, -q, 1/2, -b/(a*x**2), -d/(c*x**2))$

**Mathematica [B]** time = 0.700548, size = 252, normalized size = 3.19

$$\frac{bdx(2p + 2q - 3) \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q F_1\left(-p - q + \frac{1}{2}; -p, -q; -p - q + \frac{1}{2}\right)}{(2p + 2q - 1) \left(2x^2 \left(adpF_1\left(-p - q + \frac{3}{2}; 1 - p, -q; -p - q + \frac{5}{2}; -\frac{ax^2}{b}, -\frac{cx^2}{d}\right) + bcqF_1\left(-p - q + \frac{3}{2}; -p, 1 - q; -p - q + \frac{5}{2}; -\frac{ax^2}{b}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b/x^2)^p (c + d/x^2)^q, x]

[Out]  $(b*d*(-3 + 2*p + 2*q)*(a + b/x^2)^p (c + d/x^2)^q x \text{AppellF1}[1/2 - p - q, -p, -q, 3/2 - p - q, -((a*x^2)/b), -((c*x^2)/d)]) / ((-1 + 2*p + 2*q)*(b*d*(3 - 2*p - 2*q)*\text{AppellF1}[1/2 - p - q, -p, -q, 3/2 - p - q, -((a*x^2)/b), -((c*x^2)/d)] + 2*x^2*(a*d*p*\text{AppellF1}[3/2 - p - q, 1 - p, -q, 5/2 - p - q, -((a*x^2)/b), -((c*x^2)/d)] + b*c*q*\text{AppellF1}[3/2 - p - q, -p, 1 - q, 5/2 - p - q, -((a*x^2)/b), -((c*x^2)/d)]))$

**Maple [F]** time = 0.141, size = 0, normalized size = 0.

$$\int \left( a + \frac{b}{x^2} \right)^p \left( c + \frac{d}{x^2} \right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x^2)^p*(c+d/x^2)^q,x)`

[Out] `int((a+b/x^2)^p*(c+d/x^2)^q,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( a + \frac{b}{x^2} \right)^p \left( c + \frac{d}{x^2} \right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a + b/x^2)^p*(c + d/x^2)^q,x, algorithm="maxima")`

[Out] `integrate((a + b/x^2)^p*(c + d/x^2)^q, x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \left( \frac{ax^2 + b}{x^2} \right)^p \left( \frac{cx^2 + d}{x^2} \right)^q, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a + b/x^2)^p*(c + d/x^2)^q,x, algorithm="fricas")`

[Out] `integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)**p*(c+d/x**2)**q,x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( a + \frac{b}{x^2} \right)^p \left( c + \frac{d}{x^2} \right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a + b/x^2)^p*(c + d/x^2)^q,x, algorithm="giac")`

[Out] `integrate((a + b/x^2)^p*(c + d/x^2)^q, x)`

$$3.176 \quad \int \frac{a + \frac{b}{x^3}}{c + \frac{d}{x^3}} dx$$

**Optimal.** Leaf size=145

$$-\frac{(bc - ad) \log\left(c^{2/3}x^2 - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}\right)}{6c^{4/3}d^{2/3}} + \frac{(bc - ad) \log\left(\sqrt[3]{cx} + \sqrt[3]{d}\right)}{3c^{4/3}d^{2/3}} - \frac{(bc - ad) \tan^{-1}\left(\frac{\sqrt[3]{d-2\sqrt[3]{cx}}}{\sqrt[3]{3}\sqrt[3]{d}}\right)}{\sqrt[3]{3}c^{4/3}d^{2/3}} + \frac{ax}{c}$$

[Out] (a\*x)/c - ((b\*c - a\*d)\*ArcTan[(d^(1/3) - 2\*c^(1/3)\*x)/(Sqrt[3]\*d^(1/3))]/(Sqrt[3]\*c^(4/3)\*d^(2/3)) + ((b\*c - a\*d)\*Log[d^(1/3) + c^(1/3)\*x]/(3\*c^(4/3)\*d^(2/3)) - ((b\*c - a\*d)\*Log[d^(2/3) - c^(1/3)\*d^(1/3)\*x + c^(2/3)\*x^2]/(6\*c^(4/3)\*d^(2/3)))

**Rubi [A]** time = 0.257555, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$

$$-\frac{(bc - ad) \log\left(c^{2/3}x^2 - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}\right)}{6c^{4/3}d^{2/3}} + \frac{(bc - ad) \log\left(\sqrt[3]{cx} + \sqrt[3]{d}\right)}{3c^{4/3}d^{2/3}} - \frac{(bc - ad) \tan^{-1}\left(\frac{\sqrt[3]{d-2\sqrt[3]{cx}}}{\sqrt[3]{3}\sqrt[3]{d}}\right)}{\sqrt[3]{3}c^{4/3}d^{2/3}} + \frac{ax}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^3)/(c + d/x^3), x]

[Out] (a\*x)/c - ((b\*c - a\*d)\*ArcTan[(d^(1/3) - 2\*c^(1/3)\*x)/(Sqrt[3]\*d^(1/3))]/(Sqrt[3]\*c^(4/3)\*d^(2/3)) + ((b\*c - a\*d)\*Log[d^(1/3) + c^(1/3)\*x]/(3\*c^(4/3)\*d^(2/3)) - ((b\*c - a\*d)\*Log[d^(2/3) - c^(1/3)\*d^(1/3)\*x + c^(2/3)\*x^2]/(6\*c^(4/3)\*d^(2/3)))

**Rubi in Sympy [A]** time = 37.5772, size = 134, normalized size = 0.92

$$\frac{ax}{c} - \frac{(ad - bc) \log\left(\sqrt[3]{cx} + \sqrt[3]{d}\right)}{3c^{4/3}d^{2/3}} + \frac{(ad - bc) \log\left(c^{2/3}x^2 - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}\right)}{6c^{4/3}d^{2/3}} + \frac{\sqrt[3]{3}(ad - bc) \operatorname{atan}\left(\frac{\sqrt[3]{3}\left(-2\sqrt[3]{cx} + \sqrt[3]{d}\right)}{\sqrt[3]{d}}\right)}{3c^{4/3}d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b/x\*\*3)/(c+d/x\*\*3), x)

[Out] a\*x/c - (a\*d - b\*c)\*log(c\*\*(1/3)\*x + d\*\*(1/3))/(3\*c\*\*(4/3)\*d\*\*(2/3)) + (a\*d - b\*c)\*log(c\*\*(2/3)\*x\*\*2 - c\*\*(1/3)\*d\*\*(1/3)\*x + d\*\*(2/3))/(6\*c\*\*(4/3)\*d\*\*(2/3)) + sqrt(3)\*(a\*d - b\*c)\*atan(sqrt(3)\*(-2\*c\*\*(1/3)\*x/3 + d\*\*(1/3)/3)/d\*\*(1/3))/(3\*c\*\*(4/3)\*d\*\*(2/3))

**Mathematica [A]** time = 0.165813, size = 129, normalized size = 0.89

$$-\frac{(bc - ad) \log\left(c^{2/3}x^2 - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}\right) + 2(bc - ad) \log\left(\sqrt[3]{cx} + \sqrt[3]{d}\right) - 2\sqrt[3]{3}(bc - ad) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{cx}}{\sqrt[3]{3}}\right)}{6c^{4/3}d^{2/3}} + 6a\sqrt[3]{cd^{2/3}}x$$

Antiderivative was successfully verified.



[In] Integrate[(a + b/x^3)/(c + d/x^3), x]

[Out]  $(6*a*c^{1/3}*d^{2/3}*x - 2*\sqrt{3}*(b*c - a*d)*\text{ArcTan}[(1 - (2*c^{1/3}*x)/d^{1/3})/\sqrt{3}]) + 2*(b*c - a*d)*\text{Log}[d^{1/3} + c^{1/3}*x] - (b*c - a*d)*\text{Log}[d^{2/3} - c^{1/3}*d^{1/3}*x + c^{2/3}*x^2]/(6*c^{4/3}*d^{2/3})$

**Maple [A]** time = 0.007, size = 195, normalized size = 1.3

$$\begin{aligned} & \frac{ax}{c} - \frac{ad}{3c^2} \ln\left(x + \sqrt[3]{\frac{d}{c}}\right) \left(\frac{d}{c}\right)^{-\frac{2}{3}} + \frac{b}{3c} \ln\left(x + \sqrt[3]{\frac{d}{c}}\right) \left(\frac{d}{c}\right)^{-\frac{2}{3}} \\ & + \frac{ad}{6c^2} \ln\left(x^2 - x\sqrt[3]{\frac{d}{c}} + \left(\frac{d}{c}\right)^{\frac{2}{3}}\right) \left(\frac{d}{c}\right)^{-\frac{2}{3}} - \frac{b}{6c} \ln\left(x^2 - x\sqrt[3]{\frac{d}{c}} + \left(\frac{d}{c}\right)^{\frac{2}{3}}\right) \left(\frac{d}{c}\right)^{-\frac{2}{3}} \\ & - \frac{\sqrt{3}ad}{3c^2} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{c}}} - 1\right)\right) \left(\frac{d}{c}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}b}{3c} \arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{c}}} - 1\right)\right) \left(\frac{d}{c}\right)^{-\frac{2}{3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^3)/(c+d/x^3), x)

[Out]  $a*x/c - 1/3/c^{2/3}/(d/c)^{2/3} * \ln(x + (d/c)^{1/3}) + a*d + 1/3/c/(d/c)^{2/3} * \ln(x + (d/c)^{1/3}) + b + 1/6/c^{2/3}/(d/c)^{2/3} * \ln(x^2 - x*(d/c)^{1/3} + (d/c)^{2/3}) + a*d - 1/6/c/(d/c)^{2/3} * \ln(x^2 - x*(d/c)^{1/3} + (d/c)^{2/3}) + b - 1/3/c^{2/3}/(d/c)^{2/3} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2/(d/c)^{1/3} * x - 1)) + a*d + 1/3/c/(d/c)^{2/3} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2/(d/c)^{1/3} * x - 1)) + b$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x^3)/(c + d/x^3), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.239295, size = 184, normalized size = 1.27

$$\frac{\sqrt{3}\left(6\sqrt{3}(-cd^2)^{\frac{1}{3}}ax + \sqrt{3}(bc - ad)\log\left((-cd^2)^{\frac{2}{3}}x^2 + (-cd^2)^{\frac{1}{3}}dx + d^2\right) - 2\sqrt{3}(bc - ad)\log\left((-cd^2)^{\frac{1}{3}}x - d\right) + 6(bc - ad)\right)}{18(-cd^2)^{\frac{1}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x^3)/(c + d/x^3), x, algorithm="fricas")

[Out]  $1/18*\sqrt{3}*(6*\sqrt{3}*(-c*d^2)^{1/3}*a*x + \sqrt{3}*(b*c - a*d)*\log((-c*d^2)^{2/3}*x^2 + (-c*d^2)^{1/3}*d*x + d^2) - 2*\sqrt{3}*(b*c - a*d)*\log((-c*d^2)^{1/3}*x - d) + 6*(b*c - a*d)*\arctan(1/3*(2*\sqrt{3}*(-c*d^2)^{1/3}*x + \sqrt{3}*d)/d))/((-c*d^2)^{1/3}*c)$

**Sympy [A]** time = 2.22337, size = 71, normalized size = 0.49

$$\frac{ax}{c} + \text{RootSum}\left(27t^3c^4d^2 + a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3, \left(t \mapsto t \log\left(-\frac{3tcd}{ad - bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x\*\*3)/(c+d/x\*\*3), x)

[Out] a\*x/c + RootSum(27\*\_t\*\*3\*c\*\*4\*d\*\*2 + a\*\*3\*d\*\*3 - 3\*a\*\*2\*b\*c\*d\*\*2 + 3\*a\*b\*\*2\*c\*\*2\*d - b\*\*3\*c\*\*3, Lambda(\_t, \_t\*log(-3\*\_t\*c\*d/(a\*d - b\*c) + x)))

**GIAC/XCAS [A]** time = 0.219929, size = 217, normalized size = 1.5

$$\frac{ax}{c} - \frac{(bc - ad)\left(-\frac{d}{c}\right)^{\frac{1}{3}} \ln\left(\left|x - \left(-\frac{d}{c}\right)^{\frac{1}{3}}\right|\right)}{3cd} + \frac{\sqrt{3}\left((-c^2d)^{\frac{1}{3}}bc - (-c^2d)^{\frac{1}{3}}ad\right) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{c}\right)^{\frac{1}{3}}}\right)}{3c^2d} + \frac{\left((-c^2d)^{\frac{1}{3}}bc - (-c^2d)^{\frac{1}{3}}ad\right) \ln\left(x^2 + x\left(-\frac{d}{c}\right)^{\frac{1}{3}} + \left(-\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a + b/x^3)/(c + d/x^3), x, algorithm="giac")

[Out] a\*x/c - 1/3\*(b\*c - a\*d)\*(-d/c)^(1/3)\*ln(abs(x - (-d/c)^(1/3)))/(c\*d) + 1/3\*sqrt(3)\*((-c^2\*d)^(1/3)\*b\*c - (-c^2\*d)^(1/3)\*a\*d)\*arctan(1/3\*sqrt(3)\*(2\*x + (-d/c)^(1/3))/(-d/c)^(1/3))/(c^2\*d) + 1/6\*((-c^2\*d)^(1/3)\*b\*c - (-c^2\*d)^(1/3)\*a\*d)\*ln(x^2 + x\*(-d/c)^(1/3) + (-d/c)^(2/3))/(c^2\*d)

$$3.177 \quad \int \frac{a+b\sqrt{x}}{c+d\sqrt{x}} dx$$

**Optimal.** Leaf size=49

$$\frac{2c(bc-ad)\log(c+d\sqrt{x})}{d^3} - \frac{2\sqrt{x}(bc-ad)}{d^2} + \frac{bx}{d}$$

[Out]  $(-2*(b*c - a*d)*\text{Sqrt}[x])/d^2 + (b*x)/d + (2*c*(b*c - a*d)*\text{Log}[c + d*\text{Sqrt}[x]])/d^3$

**Rubi [A]** time = 0.122266, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{2c(bc-ad)\log(c+d\sqrt{x})}{d^3} - \frac{2\sqrt{x}(bc-ad)}{d^2} + \frac{bx}{d}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*Sqrt[x])/(c + d*Sqrt[x]), x]`

[Out]  $(-2*(b*c - a*d)*\text{Sqrt}[x])/d^2 + (b*x)/d + (2*c*(b*c - a*d)*\text{Log}[c + d*\text{Sqrt}[x]])/d^3$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2b \int^{\sqrt{x}} x dx}{d} - \frac{2c(ad-bc)\log(c+d\sqrt{x})}{d^3} + (2ad-2bc) \int^{\sqrt{x}} \frac{1}{d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a+b*x**(1/2))/(c+d*x**(1/2)), x)`

[Out]  $2*b*\text{Integral}(x, (x, \text{sqrt}(x)))/d - 2*c*(a*d - b*c)*\log(c + d*\text{sqrt}(x))/d**3 + (2*a*d - 2*b*c)*\text{Integral}(d**(-2), (x, \text{sqrt}(x)))$

**Mathematica [A]** time = 0.0436009, size = 51, normalized size = 1.04

$$\frac{2(bc^2 - acd)\log(c+d\sqrt{x})}{d^3} + \frac{2\sqrt{x}(ad-bc)}{d^2} + \frac{bx}{d}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*Sqrt[x])/(c + d*Sqrt[x]), x]`

[Out]  $(2*(-(b*c) + a*d)*\text{Sqrt}[x])/d^2 + (b*x)/d + (2*(b*c^2 - a*c*d)*\text{Log}[c + d*\text{Sqrt}[x]])/d^3$

**Maple [A]** time = 0.007, size = 59, normalized size = 1.2

$$\frac{bx}{d} + 2\frac{a\sqrt{x}}{d} - 2\frac{b\sqrt{xc}}{d^2} - 2\frac{c\ln(c+d\sqrt{x})}{d^2} + 2\frac{c^2\ln(c+d\sqrt{x})}{d^3} b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*x^(1/2))/(c+d*x^(1/2)),x)`

[Out]  $b*x/d+2/d*a*x^{(1/2)}-2/d^2*b*x^{(1/2)}*c-2*c/d^2*\ln(c+d*x^{(1/2)})*a+2*c^2/d^3*\ln(c+d*x^{(1/2)})*b$

**Maxima [A]** time = 1.70584, size = 63, normalized size = 1.29

$$\frac{bdx - 2(bc - ad)\sqrt{x}}{d^2} + \frac{2(bc^2 - acd) \log(d\sqrt{x} + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sqrt(x) + a)/(d*sqrt(x) + c),x, algorithm="maxima")`

[Out]  $(b*d*x - 2*(b*c - a*d)*\sqrt{x})/d^2 + 2*(b*c^2 - a*c*d)*\log(d*\sqrt{x} + c)/d^3$

**Fricas [A]** time = 0.239749, size = 65, normalized size = 1.33

$$\frac{bd^2x + 2(bc^2 - acd) \log(d\sqrt{x} + c) - 2(bcd - ad^2)\sqrt{x}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sqrt(x) + a)/(d*sqrt(x) + c),x, algorithm="fricas")`

[Out]  $(b*d^2*x + 2*(b*c^2 - a*c*d)*\log(d*\sqrt{x} + c) - 2*(b*c*d - a*d^2)*\sqrt{x})/d^3$

**Sympy [A]** time = 0.817719, size = 82, normalized size = 1.67

$$\begin{cases} -\frac{2ac \log\left(\frac{c}{d} + \sqrt{x}\right)}{d^2} + \frac{2a\sqrt{x}}{d} + \frac{2bc^2 \log\left(\frac{c}{d} + \sqrt{x}\right)}{d^3} - \frac{2bc\sqrt{x}}{d^2} + \frac{bx}{d} & \text{for } d \neq 0 \\ \frac{ax + \frac{2bx^{\frac{3}{2}}}{3}}{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**(1/2))/(c+d*x**(1/2)),x)`

[Out] `Piecewise((-2*a*c*log(c/d + sqrt(x))/d**2 + 2*a*sqrt(x)/d + 2*b*c**2*log(c/d + sqrt(x))/d**3 - 2*b*c*sqrt(x)/d**2 + b*x/d, Ne(d, 0)), ((a*x + 2*b*x**(3/2)/3)/c, True))`

**GIAC/XCAS [A]** time = 0.215998, size = 66, normalized size = 1.35

$$\frac{bdx - 2bc\sqrt{x} + 2ad\sqrt{x}}{d^2} + \frac{2(bc^2 - acd) \ln(|d\sqrt{x} + c|)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sqrt(x) + a)/(d*sqrt(x) + c),x, algorithm="giac")`

[Out]  $(b*d*x - 2*b*c*sqrt(x) + 2*a*d*sqrt(x))/d^2 + 2*(b*c^2 - a*c*d)*\ln(\text{abs}(d*\sqrt{x} + c))/d^3$

$$3.178 \quad \int \frac{-1 + \sqrt[3]{x}}{1 + \sqrt[3]{x}} dx$$

**Optimal.** Leaf size=26

$$-3x^{2/3} + x + 6\sqrt[3]{x} - 6 \log(\sqrt[3]{x} + 1)$$

[Out]  $6 * x^{(1/3)} - 3 * x^{(2/3)} + x - 6 * \text{Log}[1 + x^{(1/3)}]$

**Rubi [A]** time = 0.0462667, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$-3x^{2/3} + x + 6\sqrt[3]{x} - 6 \log(\sqrt[3]{x} + 1)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(-1 + x^{(1/3)})/(1 + x^{(1/3)}), x]$

[Out]  $6 * x^{(1/3)} - 3 * x^{(2/3)} + x - 6 * \text{Log}[1 + x^{(1/3)}]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$6\sqrt[3]{x} + x - 6 \log(\sqrt[3]{x} + 1) - 6 \int^{\sqrt[3]{x}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-1+x^{(1/3)})/(1+x^{(1/3)}), x)$

[Out]  $6 * x^{(1/3)} + x - 6 * \log(x^{(1/3)} + 1) - 6 * \text{Integral}(x, (x, x^{(1/3)}))$

**Mathematica [A]** time = 0.0100471, size = 26, normalized size = 1.

$$-3x^{2/3} + x + 6\sqrt[3]{x} - 6 \log(\sqrt[3]{x} + 1)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(-1 + x^{(1/3)})/(1 + x^{(1/3)}), x]$

[Out]  $6 * x^{(1/3)} - 3 * x^{(2/3)} + x - 6 * \text{Log}[1 + x^{(1/3)}]$

**Maple [A]** time = 0.004, size = 21, normalized size = 0.8

$$6\sqrt[3]{x} - 3x^{2/3} + x - 6 \ln(1 + \sqrt[3]{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((-1+x^{(1/3)})/(1+x^{(1/3)}), x)$

[Out]  $6 * x^{(1/3)} - 3 * x^{(2/3)} + x - 6 * \ln(1 + x^{(1/3)})$

---

**Maxima [A]** time = 1.40036, size = 27, normalized size = 1.04

$$x - 3x^{\frac{2}{3}} + 6x^{\frac{1}{3}} - 6 \log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(1/3) - 1)/(x^(1/3) + 1),x, algorithm="maxima")

[Out] x - 3\*x^(2/3) + 6\*x^(1/3) - 6\*log(x^(1/3) + 1)

---

**Fricas [A]** time = 0.229914, size = 27, normalized size = 1.04

$$x - 3x^{\frac{2}{3}} + 6x^{\frac{1}{3}} - 6 \log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(1/3) - 1)/(x^(1/3) + 1),x, algorithm="fricas")

[Out] x - 3\*x^(2/3) + 6\*x^(1/3) - 6\*log(x^(1/3) + 1)

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**Sympy [A]** time = 0.471564, size = 24, normalized size = 0.92

$$-3x^{\frac{2}{3}} + 6\sqrt[3]{x} + x - 6 \log\left(\sqrt[3]{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x\*\*(1/3))/(1+x\*\*(1/3)),x)

[Out] -3\*x\*\*(2/3) + 6\*x\*\*(1/3) + x - 6\*log(x\*\*(1/3) + 1)

---

**GIAC/XCAS [A]** time = 0.215143, size = 27, normalized size = 1.04

$$x - 3x^{\frac{2}{3}} + 6x^{\frac{1}{3}} - 6 \ln\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(1/3) - 1)/(x^(1/3) + 1),x, algorithm="giac")

[Out] x - 3\*x^(2/3) + 6\*x^(1/3) - 6\*ln(x^(1/3) + 1)

$$3.179 \quad \int \frac{1+x^{2/3}}{-1+x^{2/3}} dx$$

**Optimal.** Leaf size=17

$$x + 6\sqrt[3]{x} - 6 \tanh^{-1}(\sqrt[3]{x})$$

[Out]  $6 * x^{(1/3)} + x - 6 * \text{ArcTanh}[x^{(1/3)}]$

**Rubi [A]** time = 0.0500649, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$

$$x + 6\sqrt[3]{x} - 6 \tanh^{-1}(\sqrt[3]{x})$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(1 + x^{(2/3)})/(-1 + x^{(2/3)}), x]$

[Out]  $6 * x^{(1/3)} + x - 6 * \text{ArcTanh}[x^{(1/3)}]$

**Rubi in Sympy [A]** time = 10.3831, size = 15, normalized size = 0.88

$$6\sqrt[3]{x} + x - 6 \operatorname{atanh}(\sqrt[3]{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((1+x^{(2/3)})/(-1+x^{(2/3)}), x)$

[Out]  $6 * x^{(1/3)} + x - 6 * \operatorname{atanh}(x^{(1/3)})$

**Mathematica [A]** time = 0.0102708, size = 31, normalized size = 1.82

$$x + 6\sqrt[3]{x} + 3 \log(1 - \sqrt[3]{x}) - 3 \log(\sqrt[3]{x} + 1)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(1 + x^{(2/3)})/(-1 + x^{(2/3)}), x]$

[Out]  $6 * x^{(1/3)} + x + 3 * \text{Log}[1 - x^{(1/3)}] - 3 * \text{Log}[1 + x^{(1/3)}]$

**Maple [A]** time = 0.005, size = 24, normalized size = 1.4

$$x + 6\sqrt[3]{x} + 3 \ln(-1 + \sqrt[3]{x}) - 3 \ln(1 + \sqrt[3]{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((1+x^{(2/3)})/(-1+x^{(2/3)}), x)$

[Out]  $x+6 * x^{(1/3)}+3 * \ln(-1+x^{(1/3)})-3 * \ln(1+x^{(1/3)})$

**Maxima [A]** time = 1.39064, size = 31, normalized size = 1.82

$$x + 6x^{1/3} - 3 \log(x^{1/3} + 1) + 3 \log(x^{1/3} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^(2/3) + 1)/(x^(2/3) - 1),x, algorithm="maxima")`

[Out]  $x + 6x^{1/3} - 3\log(x^{1/3} + 1) + 3\log(x^{1/3} - 1)$

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**Fricas** [A] time = 0.226716, size = 31, normalized size = 1.82

$$x + 6x^{\frac{1}{3}} - 3\log\left(x^{\frac{1}{3}} + 1\right) + 3\log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^(2/3) + 1)/(x^(2/3) - 1),x, algorithm="fricas")`

[Out]  $x + 6x^{1/3} - 3\log(x^{1/3} + 1) + 3\log(x^{1/3} - 1)$

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**Sympy** [A] time = 0.649011, size = 27, normalized size = 1.59

$$6\sqrt[3]{x} + x + 3\log(\sqrt[3]{x} - 1) - 3\log(\sqrt[3]{x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x**(2/3))/(-1+x**(2/3)),x)`

[Out]  $6x^{1/3} + x + 3\log(x^{1/3} - 1) - 3\log(x^{1/3} + 1)$

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**GIAC/XCAS** [A] time = 0.216685, size = 32, normalized size = 1.88

$$x + 6x^{\frac{1}{3}} - 3\ln\left(x^{\frac{1}{3}} + 1\right) + 3\ln\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^(2/3) + 1)/(x^(2/3) - 1),x, algorithm="giac")`

[Out]  $x + 6x^{1/3} - 3\ln(x^{1/3} + 1) + 3\ln(\text{abs}(x^{1/3} - 1))$



$$3.180 \quad \int \frac{-16+x^{3/4}}{16+x^{3/4}} dx$$

**Optimal.** Leaf size=104

$$x - 128\sqrt[4]{x} + \frac{256}{3}\sqrt[3]{2}\log\left(\sqrt[4]{x} + 2\sqrt[3]{2}\right) - \frac{128}{3}\sqrt[3]{2}\log\left(\sqrt{x} - 2\sqrt[3]{2}\sqrt[4]{x} + 4 \cdot 2^{2/3}\right) - \frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}$$

[Out]  $-128*x^{(1/4)} + x - (256*2^{(1/3)}*ArcTan[(2^{(1/3)} - x^{(1/4)})/(2^{(1/3)}*\sqrt{3})])/sqrt{3} + (256*2^{(1/3)}*Log[2*2^{(1/3)} + x^{(1/4)}])/3 - (128*2^{(1/3)}*Log[4*2^{(2/3)} - 2*2^{(1/3)}*x^{(1/4)} + Sqrt[x]])/3$

**Rubi [A]** time = 0.203454, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$

$$x - 128\sqrt[4]{x} + \frac{256}{3}\sqrt[3]{2}\log\left(\sqrt[4]{x} + 2\sqrt[3]{2}\right) - \frac{128}{3}\sqrt[3]{2}\log\left(\sqrt{x} - 2\sqrt[3]{2}\sqrt[4]{x} + 4 \cdot 2^{2/3}\right) - \frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-16 + x^(3/4))/(16 + x^(3/4)), x]

[Out]  $-128*x^{(1/4)} + x - (256*2^{(1/3)}*ArcTan[(2^{(1/3)} - x^{(1/4)})/(2^{(1/3)}*\sqrt{3})])/sqrt{3} + (256*2^{(1/3)}*Log[2*2^{(1/3)} + x^{(1/4)}])/3 - (128*2^{(1/3)}*Log[4*2^{(2/3)} - 2*2^{(1/3)}*x^{(1/4)} + Sqrt[x]])/3$

**Rubi in Sympy [A]** time = 19.8801, size = 99, normalized size = 0.95

$$-128\sqrt[4]{x} + x + \frac{256\sqrt[3]{2}\log\left(\sqrt[4]{x} + 2\sqrt[3]{2}\right)}{3} - \frac{128\sqrt[3]{2}\log\left(-2\sqrt[3]{2}\sqrt[4]{x} + \sqrt{x} + 4 \cdot 2^{2/3}\right)}{3} - \frac{256\sqrt[3]{2}\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(-\frac{2^{2/3}\sqrt[4]{x}}{6} + \frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-16+x\*\*(3/4))/(16+x\*\*(3/4)), x)

[Out]  $-128*x^{(1/4)} + x + 256*2^{(1/3)}*\log(x^{(1/4)} + 2*2^{(1/3)})/3 - 128*2^{(1/3)}*\log(-2*2^{(1/3)}*x^{(1/4)} + \operatorname{sqrt}(x) + 4*2^{(2/3)})/3 - 256*2^{(1/3)}*\operatorname{sqrt}(3)*\operatorname{atan}(\operatorname{sqrt}(3)*(-2^{(2/3)}*x^{(1/4)}/6 + 1/3))/3$

**Mathematica [A]** time = 0.0496975, size = 102, normalized size = 0.98

$$x - 128\sqrt[4]{x} + \frac{256}{3}\sqrt[3]{2}\log\left(2^{2/3}\sqrt[4]{x} + 4\right) - \frac{128}{3}\sqrt[3]{2}\log\left(\sqrt[3]{2}\sqrt{x} - 2 \cdot 2^{2/3}\sqrt[4]{x} + 8\right) + \frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{2^{2/3}\sqrt[4]{x}-2}{2\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(-16 + x^(3/4))/(16 + x^(3/4)), x]

[Out]  $-128*x^{(1/4)} + x + (256*2^{(1/3)}*ArcTan[(-2 + 2^{(2/3)}*x^{(1/4)})/(2*\sqrt{3})])/sqrt{3} + (256*2^{(1/3)}*Log[4 + 2^{(2/3)}*x^{(1/4)}])/3 - ($

$$128 \cdot 2^{(1/3)} \cdot \text{Log}[8 - 2 \cdot 2^{(2/3)} \cdot x^{(1/4)} + 2^{(1/3)} \cdot \text{Sqrt}[x]]/3$$

**Maple [A]** time = 0.009, size = 66, normalized size = 0.6

$$x - 128 \sqrt[4]{x} + \frac{128 \sqrt[3]{16}}{3} \ln(\sqrt[4]{x} + \sqrt[3]{16}) - \frac{64 \sqrt[3]{16}}{3} \ln(\sqrt{x} - \sqrt[4]{x} \sqrt[3]{16} + 16^{2/3}) \\ + \frac{128 \sqrt[3]{16} \sqrt{3}}{3} \arctan\left(\frac{\sqrt{3}}{3} \left(\frac{16^{2/3}}{8} \sqrt[4]{x} - 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-16+x^(3/4))/(16+x^(3/4)),x)

[Out] x-128\*x^(1/4)+128/3\*16^(1/3)\*ln(x^(1/4)+16^(1/3))-64/3\*16^(1/3)\*ln(x^(1/2)-x^(1/4)\*16^(1/3)+16^(2/3))+128/3\*16^(1/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(1/8\*16^(2/3)\*x^(1/4)-1))

**Maxima [A]** time = 1.64799, size = 90, normalized size = 0.87

$$\frac{256}{3} \sqrt{3}^{1/3} \arctan\left(-\frac{1}{6} \sqrt{3}^{2/3} (2^{1/3} - x^{1/4})\right) - \frac{128}{3} \cdot 2^{1/3} \log\left(\left(4 \cdot 2^{2/3}\right) - 2 \cdot 2^{1/3} x^{1/4} + \sqrt{x}\right) \\ + \frac{256}{3} \cdot 2^{1/3} \log\left(\left(2 \cdot 2^{1/3}\right) + x^{1/4}\right) + x - 128 x^{1/4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(3/4) - 16)/(x^(3/4) + 16),x, algorithm="maxima")

[Out] 256/3\*sqrt(3)\*2^(1/3)\*arctan(-1/6\*sqrt(3)\*2^(2/3)\*(2^(1/3) - x^(1/4))) - 128/3\*2^(1/3)\*log((4\*2^(2/3)) - 2\*2^(1/3)\*x^(1/4) + sqrt(x)) + 256/3\*2^(1/3)\*log((2\*2^(1/3)) + x^(1/4)) + x - 128\*x^(1/4)

**Fricas [A]** time = 0.242223, size = 123, normalized size = 1.18

$$-\frac{1}{9} \sqrt{3} \left( 128 \sqrt{3}^{1/3} \log\left(4 \cdot 2^{2/3} - 2 \cdot 2^{1/3} x^{1/4} + \sqrt{x}\right) - 256 \sqrt{3}^{1/3} \log\left(2 \cdot 2^{1/3} + x^{1/4}\right) - 3 \sqrt{3} x - 768 \cdot 2^{1/3} \arctan\left(-\frac{1}{6} \cdot 2^{2/3} \left(\sqrt{3}^{1/3} - \sqrt{3}\right)\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(3/4) - 16)/(x^(3/4) + 16),x, algorithm="fricas")

[Out] -1/9\*sqrt(3)\*(128\*sqrt(3)\*2^(1/3)\*log(4\*2^(2/3) - 2\*2^(1/3)\*x^(1/4) + sqrt(x)) - 256\*sqrt(3)\*2^(1/3)\*log(2\*2^(1/3) + x^(1/4)) - 3\*sqrt(3)\*x - 768\*2^(1/3)\*arctan(-1/6\*2^(2/3)\*(sqrt(3)\*2^(1/3) - sqrt(3)\*x^(1/4))) + 384\*sqrt(3)\*x^(1/4))

**Sympy [A]** time = 20.2673, size = 102, normalized size = 0.98

$$-128 \sqrt[4]{x} + x + \frac{256 \sqrt[3]{2} \log(\sqrt[4]{x} + 2 \sqrt[3]{2})}{3} \\ - \frac{128 \sqrt[3]{2} \log(-2 \sqrt[3]{2} \sqrt[4]{x} + \sqrt{x} + 4 \cdot 2^{2/3})}{3} + \frac{256 \sqrt[3]{2} \sqrt{3} \operatorname{atan}\left(\frac{2^{2/3} \sqrt{3} \sqrt[4]{x}}{6} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16+x**(3/4))/(16+x**(3/4)),x)
```

```
[Out] -128*x**(1/4) + x + 256*2**(1/3)*log(x**(1/4) + 2*2**(1/3))/3 - 1
28*2**(1/3)*log(-2*2**(1/3)*x**(1/4) + sqrt(x) + 4*2**(2/3))/3 +
256*2**(1/3)*sqrt(3)*atan(2**(2/3)*sqrt(3)*x**(1/4)/6 - sqrt(3)/3
)/3
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(3/4) - 16)/(x^(3/4) + 16),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.181 \quad \int \frac{1 + \frac{1}{\sqrt[3]{x}}}{-1 + \frac{1}{\sqrt[3]{x}}} dx$$

**Optimal.** Leaf size=30

$$-3x^{2/3} - x - 6\sqrt[3]{x} - 6 \log(1 - \sqrt[3]{x})$$

[Out]  $-6 * x^{(1/3)} - 3 * x^{(2/3)} - x - 6 * \text{Log}[1 - x^{(1/3)}]$

**Rubi [A]** time = 0.0549251, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$-3x^{2/3} - x - 6\sqrt[3]{x} - 6 \log(1 - \sqrt[3]{x})$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(1 + x^{(-1/3)})/(-1 + x^{(-1/3)}), x]$

[Out]  $-6 * x^{(1/3)} - 3 * x^{(2/3)} - x - 6 * \text{Log}[1 - x^{(1/3)}]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-6\sqrt[3]{x} - x - 6 \log(-\sqrt[3]{x} + 1) - 6 \int \sqrt[3]{x} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((1+1/x^{**}(1/3))/(-1+1/x^{**}(1/3)), x)$

[Out]  $-6 * x^{**}(1/3) - x - 6 * \log(-x^{**}(1/3) + 1) - 6 * \text{Integral}(x, (x, x^{**}(1/3)))$

**Mathematica [A]** time = 0.0105902, size = 30, normalized size = 1.

$$-3x^{2/3} - x - 6\sqrt[3]{x} - 6 \log(1 - \sqrt[3]{x})$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(1 + x^{(-1/3)})/(-1 + x^{(-1/3)}), x]$

[Out]  $-6 * x^{(1/3)} - 3 * x^{(2/3)} - x - 6 * \text{Log}[1 - x^{(1/3)}]$

**Maple [A]** time = 0.004, size = 23, normalized size = 0.8

$$-x - 3x^{2/3} - 6\sqrt[3]{x} - 6 \ln(-1 + \sqrt[3]{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((1+1/x^{(1/3)})/(-1+1/x^{(1/3)}), x)$

[Out]  $-x - 3 * x^{(2/3)} - 6 * x^{(1/3)} - 6 * \ln(-1 + x^{(1/3)})$

---

**Maxima [A]** time = 1.37639, size = 30, normalized size = 1.

$$-x - 3x^{\frac{2}{3}} - 6x^{\frac{1}{3}} - 6 \log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/x^(1/3) + 1)/(1/x^(1/3) - 1), x, algorithm="maxima")

[Out] -x - 3\*x^(2/3) - 6\*x^(1/3) - 6\*log(x^(1/3) - 1)

---

**Fricas [A]** time = 0.2368, size = 30, normalized size = 1.

$$-x - 3x^{\frac{2}{3}} - 6x^{\frac{1}{3}} - 6 \log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/x^(1/3) + 1)/(1/x^(1/3) - 1), x, algorithm="fricas")

[Out] -x - 3\*x^(2/3) - 6\*x^(1/3) - 6\*log(x^(1/3) - 1)

---

**Sympy [A]** time = 0.405284, size = 26, normalized size = 0.87

$$-3x^{\frac{2}{3}} - 6\sqrt[3]{x} - x - 6 \log\left(\sqrt[3]{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x\*\*(1/3))/(-1+1/x\*\*(1/3)), x)

[Out] -3\*x\*\*(2/3) - 6\*x\*\*(1/3) - x - 6\*log(x\*\*(1/3) - 1)

---

**GIAC/XCAS [A]** time = 0.215655, size = 31, normalized size = 1.03

$$-x - 3x^{\frac{2}{3}} - 6x^{\frac{1}{3}} - 6 \ln\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/x^(1/3) + 1)/(1/x^(1/3) - 1), x, algorithm="giac")

[Out] -x - 3\*x^(2/3) - 6\*x^(1/3) - 6\*ln(abs(x^(1/3) - 1))

$$3.182 \quad \int (a - bx^n)^{3/2} (a + bx^n)^{3/2} dx$$

**Optimal.** Leaf size=79

$$\frac{a^2 x \sqrt{a - bx^n} \sqrt{a + bx^n} {}_2F_1\left(-\frac{3}{2}, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2 x^{2n}}{a^2}\right)}{\sqrt{1 - \frac{b^2 x^{2n}}{a^2}}}$$

[Out] (a^2\*x\*Sqrt[a - b\*x^n]\*Sqrt[a + b\*x^n]\*Hypergeometric2F1[-3/2, 1/(2\*n), (2 + n^(-1))/2, (b^2\*x^(2\*n))/a^2])/Sqrt[1 - (b^2\*x^(2\*n))/a^2]

**Rubi [A]** time = 0.0826049, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{a^2 x \sqrt{a - bx^n} \sqrt{a + bx^n} {}_2F_1\left(-\frac{3}{2}, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2 x^{2n}}{a^2}\right)}{\sqrt{1 - \frac{b^2 x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^n)^(3/2)\*(a + b\*x^n)^(3/2), x]

[Out] (a^2\*x\*Sqrt[a - b\*x^n]\*Sqrt[a + b\*x^n]\*Hypergeometric2F1[-3/2, 1/(2\*n), (2 + n^(-1))/2, (b^2\*x^(2\*n))/a^2])/Sqrt[1 - (b^2\*x^(2\*n))/a^2]

**Rubi in Sympy [A]** time = 19.6892, size = 66, normalized size = 0.84

$$\frac{a^2 x \sqrt{a - bx^n} \sqrt{a + bx^n} {}_2F_1\left(-\frac{3}{2}, \frac{1}{2n} \middle| \frac{b^2 x^{2n}}{a^2}\right)}{\sqrt{1 - \frac{b^2 x^{2n}}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a-b\*x\*\*n)\*\*(3/2)\*(a+b\*x\*\*n)\*\*(3/2), x)

[Out] a\*\*2\*x\*sqrt(a - b\*x\*\*n)\*sqrt(a + b\*x\*\*n)\*hyper((-3/2, 1/(2\*n)), (n + 1/2)/n, ), b\*\*2\*x\*\*(2\*n)/a\*\*2)/sqrt(1 - b\*\*2\*x\*\*(2\*n)/a\*\*2)

**Mathematica [A]** time = 0.262048, size = 151, normalized size = 1.91

$$\frac{x \sqrt{a - bx^n} \sqrt{a + bx^n} \left( (a^2 - b^2 x^{2n}) (a^2 (4n + 1) - b^2 (n + 1) x^{2n}) + 3a^4 n^2 \sqrt{1 - \frac{b^2 x^{2n}}{a^2}} {}_2F_1\left(\frac{1}{2}, \frac{1}{2n}; 1 + \frac{1}{2n}, \frac{b^2 x^{2n}}{a^2}\right) \right)}{(n + 1)(3n + 1)(a^2 - b^2 x^{2n})}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^n)^(3/2)\*(a + b\*x^n)^(3/2), x]

[Out] (x\*Sqrt[a - b\*x^n]\*Sqrt[a + b\*x^n]\*((a^2 - b^2\*x^(2\*n))\*(a^2\*(1 + 4\*n) - b^2\*(1 + n)\*x^(2\*n)) + 3\*a^4\*n^2\*Sqrt[1 - (b^2\*x^(2\*n))/a^2]\*Hypergeometric2F1[1/2, 1/(2\*n), 1 + 1/(2\*n), (b^2\*x^(2\*n))/a^2]))/((1 + n)\*(1 + 3\*n)\*(a^2 - b^2\*x^(2\*n)))

---

**Maple [F]** time = 0.118, size = 0, normalized size = 0.

$$\int (a - bx^n)^{\frac{3}{2}} (a + bx^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x)`

[Out] `int((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^{\frac{3}{2}} (-bx^n + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2), x)`

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a-b*x**n)**(3/2)*(a+b*x**n)**(3/2),x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^{\frac{3}{2}} (-bx^n + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2),x, algorithm="giac")`

[Out] `integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2), x)`

### 3.183 $\int \sqrt{a - bx^n} \sqrt{a + bx^n} dx$

**Optimal.** Leaf size=76

$$\frac{x\sqrt{a - bx^n}\sqrt{a + bx^n} {}_2F_1\left(-\frac{1}{2}, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1 - \frac{b^2x^{2n}}{a^2}}}$$

[Out] (x\*Sqrt[a - b\*x^n]\*Sqrt[a + b\*x^n]\*Hypergeometric2F1[-1/2, 1/(2\*n), (2 + n^(-1))/2, (b^2\*x^(2\*n))/a^2])/Sqrt[1 - (b^2\*x^(2\*n))/a^2]

**Rubi [A]** time = 0.075931, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{x\sqrt{a - bx^n}\sqrt{a + bx^n} {}_2F_1\left(-\frac{1}{2}, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1 - \frac{b^2x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - b\*x^n]\*Sqrt[a + b\*x^n], x]

[Out] (x\*Sqrt[a - b\*x^n]\*Sqrt[a + b\*x^n]\*Hypergeometric2F1[-1/2, 1/(2\*n), (2 + n^(-1))/2, (b^2\*x^(2\*n))/a^2])/Sqrt[1 - (b^2\*x^(2\*n))/a^2]

**Rubi in Sympy [A]** time = 19.4373, size = 63, normalized size = 0.83

$$\frac{x\sqrt{a - bx^n}\sqrt{a + bx^n} {}_2F_1\left(-\frac{1}{2}, \frac{1}{2n} \middle| \frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1 - \frac{b^2x^{2n}}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a-b\*x\*\*n)\*\*(1/2)\*(a+b\*x\*\*n)\*\*(1/2), x)

[Out] x\*sqrt(a - b\*x\*\*n)\*sqrt(a + b\*x\*\*n)\*hyper((-1/2, 1/(2\*n)), ((n + 1/2)/n, ), b\*\*2\*x\*\*(2\*n)/a\*\*2)/sqrt(1 - b\*\*2\*x\*\*(2\*n)/a\*\*2)

**Mathematica [A]** time = 0.147357, size = 116, normalized size = 1.53

$$\frac{x\sqrt{a - bx^n}\sqrt{a + bx^n} \left( a^2n\sqrt{1 - \frac{b^2x^{2n}}{a^2}} {}_2F_1\left(\frac{1}{2}, \frac{1}{2n}; 1 + \frac{1}{2n}; \frac{b^2x^{2n}}{a^2}\right) + a^2 - b^2x^{2n} \right)}{(n + 1)(a^2 - b^2x^{2n})}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a - b\*x^n]\*Sqrt[a + b\*x^n], x]

[Out] (x\*Sqrt[a - b\*x^n]\*Sqrt[a + b\*x^n]\*(a^2 - b^2\*x^(2\*n) + a^2\*n\*Sqrt[1 - (b^2\*x^(2\*n))/a^2]\*Hypergeometric2F1[1/2, 1/(2\*n), 1 + 1/(2\*n), (b^2\*x^(2\*n))/a^2]))/((1 + n)\*(a^2 - b^2\*x^(2\*n)))



---

**Maple [F]** time = 0.068, size = 0, normalized size = 0.

$$\int \sqrt{a - bx^n} \sqrt{a + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x)`

[Out] `int((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^n + a} \sqrt{-bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a), x)`

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a),x, algorithm="fricas")`

[Out] Exception raised: TypeError

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a - bx^n} \sqrt{a + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a-b*x**n)**(1/2)*(a+b*x**n)**(1/2),x)`

[Out] `Integral(sqrt(a - b*x**n)*sqrt(a + b*x**n), x)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^n + a} \sqrt{-bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a),x, algorithm="giac")`

[Out] `integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a), x)`

### 3.184 $\int (a - bx^n)^p (a + bx^n)^p dx$

**Optimal.** Leaf size=72

$$x (a - bx^n)^p (a + bx^n)^p \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2n}, -p; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2 x^{2n}}{a^2}\right)$$

[Out]  $(x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p$

**Rubi [A]** time = 0.0733766, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$x (a - bx^n)^p (a + bx^n)^p \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2n}, -p; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2 x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^n)^p\*(a + b\*x^n)^p, x]

[Out]  $(x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p$

**Rubi in Sympy [A]** time = 21.4, size = 56, normalized size = 0.78

$$x \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{-p} (a - bx^n)^p (a + bx^n)^p {}_2F_1\left(-p, \frac{1}{2n} \middle| \frac{b^2 x^{2n}}{a^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a-b\*x\*\*n)\*\*p\*(a+b\*x\*\*n)\*\*p, x)

[Out]  $x*(1 - b**2*x**(2*n)/a**2)**(-p)*(a - b*x**n)**p*(a + b*x**n)**p*hyper((-p, 1/(2*n)), ((n + 1/2)/n, ), b**2*x**(2*n)/a**2)$

**Mathematica [A]** time = 0.126143, size = 72, normalized size = 1.

$$x (a - bx^n)^p (a + bx^n)^p \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2n}, -p; 1 + \frac{1}{2n}; \frac{b^2 x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^n)^p\*(a + b\*x^n)^p, x]

[Out]  $(x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, 1 + 1/(2*n), (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p$

**Maple [F]** time = 0.242, size = 0, normalized size = 0.

$$\int (a - bx^n)^p (a + bx^n)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a-b*x^n)^p*(a+b*x^n)^p,x)`

[Out] `int((a-b*x^n)^p*(a+b*x^n)^p,x)`

---

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p(-bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="maxima")`

[Out] `integrate((b*x^n + a)^p*(-b*x^n + a)^p, x)`

---

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((bx^n + a)^p(-bx^n + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="fricas")`

[Out] `integral((b*x^n + a)^p*(-b*x^n + a)^p, x)`

---

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int (a - bx^n)^p (a + bx^n)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a-b*x**n)**p*(a+b*x**n)**p,x)`

[Out] `Integral((a - b*x**n)**p*(a + b*x**n)**p, x)`

---

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p(-bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="giac")`

[Out] `integrate((b*x^n + a)^p*(-b*x^n + a)^p, x)`

### 3.185 $\int (a + bx^n)(c + dx^n)^4 dx$

**Optimal.** Leaf size=132

$$\frac{c^3 x^{n+1}(4ad + bc)}{n + 1} + \frac{2c^2 dx^{2n+1}(3ad + 2bc)}{2n + 1} + \frac{d^3 x^{4n+1}(ad + 4bc)}{4n + 1} + \frac{2cd^2 x^{3n+1}(2ad + 3bc)}{3n + 1} + ac^4 x + \frac{bd^4 x^{5n+1}}{5n + 1}$$

[Out]  $a * c^4 * x + (c^3 * (b * c + 4 * a * d) * x^{(1 + n)}) / (1 + n) + (2 * c^2 * d * (2 * b * c + 3 * a * d) * x^{(1 + 2 * n)}) / (1 + 2 * n) + (2 * c * d^2 * (3 * b * c + 2 * a * d) * x^{(1 + 3 * n)}) / (1 + 3 * n) + (d^3 * (4 * b * c + a * d) * x^{(1 + 4 * n)}) / (1 + 4 * n) + (b * d^4 * x^{(1 + 5 * n)}) / (1 + 5 * n)$

**Rubi [A]** time = 0.251411, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{c^3 x^{n+1}(4ad + bc)}{n + 1} + \frac{2c^2 dx^{2n+1}(3ad + 2bc)}{2n + 1} + \frac{d^3 x^{4n+1}(ad + 4bc)}{4n + 1} + \frac{2cd^2 x^{3n+1}(2ad + 3bc)}{3n + 1} + ac^4 x + \frac{bd^4 x^{5n+1}}{5n + 1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)\*(c + d\*x^n)^4, x]

[Out]  $a * c^4 * x + (c^3 * (b * c + 4 * a * d) * x^{(1 + n)}) / (1 + n) + (2 * c^2 * d * (2 * b * c + 3 * a * d) * x^{(1 + 2 * n)}) / (1 + 2 * n) + (2 * c * d^2 * (3 * b * c + 2 * a * d) * x^{(1 + 3 * n)}) / (1 + 3 * n) + (d^3 * (4 * b * c + a * d) * x^{(1 + 4 * n)}) / (1 + 4 * n) + (b * d^4 * x^{(1 + 5 * n)}) / (1 + 5 * n)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^4 x^{5n+1}}{5n + 1} + c^4 \int a dx + \frac{c^3 x^{n+1}(4ad + bc)}{n + 1} + \frac{2c^2 dx^{2n+1}(3ad + 2bc)}{2n + 1} + \frac{2cd^2 x^{3n+1}(2ad + 3bc)}{3n + 1} + \frac{d^3 x^{4n+1}(ad + 4bc)}{4n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*(c+d\*x\*\*n)\*\*4, x)

[Out]  $b * d^4 * x^{(5 * n + 1)} / (5 * n + 1) + c^4 * \text{Integral}(a, x) + c^3 * x^{(n + 1)} * (4 * a * d + b * c) / (n + 1) + 2 * c^2 * d * x^{(2 * n + 1)} * (3 * a * d + 2 * b * c) / (2 * n + 1) + 2 * c * d^2 * x^{(3 * n + 1)} * (2 * a * d + 3 * b * c) / (3 * n + 1) + d^3 * x^{(4 * n + 1)} * (a * d + 4 * b * c) / (4 * n + 1)$

**Mathematica [A]** time = 0.190337, size = 123, normalized size = 0.93

$$x \left( \frac{c^3 x^n (4ad + bc)}{n + 1} + \frac{2c^2 dx^{2n} (3ad + 2bc)}{2n + 1} + \frac{d^3 x^{4n} (ad + 4bc)}{4n + 1} + \frac{2cd^2 x^{3n} (2ad + 3bc)}{3n + 1} + ac^4 + \frac{bd^4 x^{5n}}{5n + 1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)\*(c + d\*x^n)^4, x]

[Out]  $x * (a * c^4 + (c^3 * (b * c + 4 * a * d) * x^n) / (1 + n) + (2 * c^2 * d * (2 * b * c + 3 * a * d) * x^{(2 * n)}) / (1 + 2 * n) + (2 * c * d^2 * (3 * b * c + 2 * a * d) * x^{(3 * n)}) / (1 + 3 * n) + (d^3 * (4 * b * c + a * d) * x^{(4 * n)}) / (1 + 4 * n) + (b * d^4 * x^{(5 * n)}) / (1 + 5 * n))$

**Maple [A]** time = 0.018, size = 138, normalized size = 1.1

$$ac^4x + \frac{bd^4x(e^{n\ln(x)})^5}{1+5n} + \frac{c^3(4ad+bc)xe^{n\ln(x)}}{1+n} + \frac{d^3(ad+4bc)x(e^{n\ln(x)})^4}{1+4n} \\ + 2\frac{cd^2(2ad+3bc)x(e^{n\ln(x)})^3}{1+3n} + 2\frac{c^2d(3ad+2bc)x(e^{n\ln(x)})^2}{1+2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*x^n)*(c+d*x^n)^4,x)`

[Out] `a*c^4*x+b*d^4/(1+5*n)*x*exp(n*ln(x))^5+c^3*(4*a*d+b*c)/(1+n)*x*exp(n*ln(x))+d^3*(a*d+4*b*c)/(1+4*n)*x*exp(n*ln(x))^4+2*c*d^2*(2*a*d+3*b*c)/(1+3*n)*x*exp(n*ln(x))^3+2*c^2*d*(3*a*d+2*b*c)/(1+2*n)*x*exp(n*ln(x))^2`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)*(d*x^n + c)^4,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.262764, size = 711, normalized size = 5.39

$$(24bd^4n^4 + 50bd^4n^3 + 35bd^4n^2 + 10bd^4n + bd^4)xx^{5n} + (4bcd^3 + ad^4 + 30(4bcd^3 + ad^4)n^4 + 61(4bcd^3 + ad^4)n^3 + 41(4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)*(d*x^n + c)^4,x, algorithm="fricas")`

[Out] `((24*b*d^4*n^4 + 50*b*d^4*n^3 + 35*b*d^4*n^2 + 10*b*d^4*n + b*d^4)*x*x^(5*n) + (4*b*c*d^3 + a*d^4 + 30*(4*b*c*d^3 + a*d^4)*n^4 + 61*(4*b*c*d^3 + a*d^4)*n^3 + 41*(4*b*c*d^3 + a*d^4)*n^2 + 11*(4*b*c*d^3 + a*d^4)*n)*x*x^(4*n) + 2*(3*b*c^2*d^2 + 2*a*c*d^3 + 40*(3*b*c^2*d^2 + 2*a*c*d^3)*n^4 + 78*(3*b*c^2*d^2 + 2*a*c*d^3)*n^3 + 49*(3*b*c^2*d^2 + 2*a*c*d^3)*n^2 + 12*(3*b*c^2*d^2 + 2*a*c*d^3)*n)*x*x^(3*n) + 2*(2*b*c^3*d + 3*a*c^2*d^2 + 60*(2*b*c^3*d + 3*a*c^2*d^2)*n^4 + 107*(2*b*c^3*d + 3*a*c^2*d^2)*n^3 + 59*(2*b*c^3*d + 3*a*c^2*d^2)*n^2 + 13*(2*b*c^3*d + 3*a*c^2*d^2)*n)*x*x^(2*n) + (b*c^4 + 4*a*c^3*d + 120*(b*c^4 + 4*a*c^3*d)*n^4 + 154*(b*c^4 + 4*a*c^3*d)*n^3 + 71*(b*c^4 + 4*a*c^3*d)*n^2 + 14*(b*c^4 + 4*a*c^3*d)*n)*x*x^n + (120*a*c^4*n^5 + 274*a*c^4*n^4 + 225*a*c^4*n^3 + 85*a*c^4*n^2 + 15*a*c^4*n + a*c^4)*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)`

**Sympy [A]** time = 9.81876, size = 2744, normalized size = 20.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*(c+d\*x\*\*n)\*\*4,x)

[Out] Piecewise((a\*c\*\*4\*x + 4\*a\*c\*\*3\*d\*log(x) - 6\*a\*c\*\*2\*d\*\*2/x - 2\*a\*c\*d\*\*3/x\*\*2 - a\*d\*\*4/(3\*x\*\*3) + b\*c\*\*4\*log(x) - 4\*b\*c\*\*3\*d/x - 3\*b\*c\*\*2\*d\*\*2/x\*\*2 - 4\*b\*c\*d\*\*3/(3\*x\*\*3) - b\*d\*\*4/(4\*x\*\*4), Eq(n, -1)), (a\*c\*\*4\*x + 8\*a\*c\*\*3\*d\*sqrt(x) + 6\*a\*c\*\*2\*d\*\*2\*log(x) - 8\*a\*c\*d\*\*3/sqrt(x) - a\*d\*\*4/x + 2\*b\*c\*\*4\*sqrt(x) + 4\*b\*c\*\*3\*d\*log(x) - 12\*b\*c\*\*2\*d\*\*2/sqrt(x) - 4\*b\*c\*d\*\*3/x - 2\*b\*d\*\*4/(3\*x\*\*(3/2)), Eq(n, -1/2)), (a\*c\*\*4\*x + 6\*a\*c\*\*3\*d\*x\*\*(2/3) + 18\*a\*c\*\*2\*d\*\*2\*x\*\*(1/3) + 4\*a\*c\*d\*\*3\*log(x) - 3\*a\*d\*\*4/x\*\*(1/3) + 3\*b\*c\*\*4\*x\*\*(2/3)/2 + 12\*b\*c\*\*3\*d\*x\*\*(1/3) + 6\*b\*c\*\*2\*d\*\*2\*log(x) - 12\*b\*c\*d\*\*3/x\*\*(1/3) - 3\*b\*d\*\*4/(2\*x\*\*(2/3)), Eq(n, -1/3)), (a\*c\*\*4\*x + 16\*a\*c\*\*3\*d\*x\*\*(3/4)/3 + 12\*a\*c\*\*2\*d\*\*2\*sqrt(x) + 16\*a\*c\*d\*\*3\*x\*\*(1/4) + a\*d\*\*4\*log(x) + 4\*b\*c\*\*4\*x\*\*(3/4)/3 + 8\*b\*c\*\*3\*d\*sqrt(x) + 24\*b\*c\*\*2\*d\*\*2\*x\*\*(1/4) + 4\*b\*c\*d\*\*3\*log(x) - 4\*b\*d\*\*4/x\*\*(1/4), Eq(n, -1/4)), (a\*c\*\*4\*x + 5\*a\*c\*\*3\*d\*x\*\*(4/5) + 10\*a\*c\*\*2\*d\*\*2\*x\*\*(3/5) + 10\*a\*c\*d\*\*3\*x\*\*(2/5) + 5\*a\*d\*\*4\*x\*\*(1/5) + 5\*b\*c\*\*4\*x\*\*(4/5)/4 + 20\*b\*c\*\*3\*d\*x\*\*(3/5)/3 + 15\*b\*c\*\*2\*d\*\*2\*x\*\*(2/5) + 20\*b\*c\*d\*\*3\*x\*\*(1/5) + b\*d\*\*4\*log(x), Eq(n, -1/5)), (120\*a\*c\*\*4\*n\*\*5\*x/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 274\*a\*c\*\*4\*n\*\*4\*x/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 225\*a\*c\*\*4\*n\*\*3\*x/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 85\*a\*c\*\*4\*n\*\*2\*x/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 15\*a\*c\*\*4\*n\*x/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + a\*c\*\*4\*x/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 480\*a\*c\*\*3\*d\*n\*\*4\*x\*x\*\*n/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 616\*a\*c\*\*3\*d\*n\*\*3\*x\*x\*\*n/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 284\*a\*c\*\*3\*d\*n\*\*2\*x\*x\*\*n/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 56\*a\*c\*\*3\*d\*n\*x\*x\*\*n/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 4\*a\*c\*\*3\*d\*x\*x\*\*n/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 360\*a\*c\*\*2\*d\*\*2\*n\*\*4\*x\*x\*\*(2\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 642\*a\*c\*\*2\*d\*\*2\*n\*\*3\*x\*x\*\*(2\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 354\*a\*c\*\*2\*d\*\*2\*n\*\*2\*x\*x\*\*(2\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 78\*a\*c\*\*2\*d\*\*2\*n\*x\*x\*\*(2\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 6\*a\*c\*\*2\*d\*\*2\*x\*x\*\*(2\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 160\*a\*c\*d\*\*3\*n\*\*4\*x\*x\*\*(3\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 312\*a\*c\*d\*\*3\*n\*\*3\*x\*x\*\*(3\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 196\*a\*c\*d\*\*3\*n\*\*2\*x\*x\*\*(3\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 48\*a\*c\*d\*\*3\*n\*x\*x\*\*(3\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 4\*a\*c\*d\*\*3\*x\*x\*\*(3\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 30\*a\*d\*\*4\*n\*\*4\*x\*x\*\*(4\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 61\*a\*d\*\*4\*n\*\*3\*x\*x\*\*(4\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 41\*a\*d\*\*4\*n\*\*2\*x\*x\*\*(4\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 11\*a\*d\*\*4\*n\*x\*x\*\*(4\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + a\*d\*\*4\*x\*x\*\*(4\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 120\*b\*c\*\*4\*n\*\*4\*x\*x\*\*n/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 154\*b\*c\*\*4\*n\*\*3\*x\*x\*\*n/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 71\*b\*c\*\*4\*n\*\*2\*x\*x\*\*n/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 14\*b\*c\*\*4\*n\*x\*x\*\*n/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + b\*c\*\*4\*x\*x\*\*n/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 240\*b\*c\*\*3\*d\*n\*\*4\*x\*x\*\*(2\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 428\*b\*c\*\*3\*d\*n\*\*3\*x\*x\*\*(2\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 236\*b\*c\*\*3\*d\*n\*\*2\*x\*x\*\*(2\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 52\*b\*c\*\*3\*d\*n\*x\*x\*\*(2\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 4\*b\*c\*\*3\*d\*x\*x\*\*(2\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 240\*b\*c\*\*2\*d\*\*2\*n\*\*4\*x\*x\*\*(3\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 468\*b\*c\*\*2\*d\*\*2\*n\*\*3\*x\*x\*\*(3\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 294\*b\*c\*\*2\*d\*\*2\*n\*\*2\*x\*x\*\*(3\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 72\*b\*c\*\*2\*d\*\*2\*n\*x\*x\*\*(3\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 6\*b\*c\*\*2\*d\*\*2\*x\*x\*\*(3\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 120\*b\*c\*d\*\*3\*n\*\*4\*x\*x\*\*(4\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 244\*b\*c\*d\*\*3\*n\*\*3\*x\*x\*\*(4\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 164\*b\*c\*d\*\*3\*n\*\*2\*x\*x\*\*(4\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 72\*b\*c\*d\*\*3\*n\*x\*x\*\*(4\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + 16\*b\*c\*d\*\*3\*x\*x\*\*(4\*n)/(120\*n\*\*5 + 274\*n\*\*4 + 225\*n\*\*3 + 85\*n\*\*2 + 15\*n + 1) + b\*d\*\*4/(4\*x\*\*4), Eq(n, -1))

$$\begin{aligned}
& 15^n + 1) + 44*b*c*d^{*3}*n*x*x^{*(4*n)}/(120*n^{*5} + 274*n^{*4} + 225* \\
& n^{*3} + 85*n^{*2} + 15*n + 1) + 4*b*c*d^{*3}*x*x^{*(4*n)}/(120*n^{*5} + 27 \\
& 4*n^{*4} + 225*n^{*3} + 85*n^{*2} + 15*n + 1) + 24*b*d^{*4}*n^{*4}*x*x^{*(5* \\
& n)}/(120*n^{*5} + 274*n^{*4} + 225*n^{*3} + 85*n^{*2} + 15*n + 1) + 50*b*d \\
& ^{*4}*n^{*3}*x*x^{*(5*n)}/(120*n^{*5} + 274*n^{*4} + 225*n^{*3} + 85*n^{*2} + 1 \\
& 5*n + 1) + 35*b*d^{*4}*n^{*2}*x*x^{*(5*n)}/(120*n^{*5} + 274*n^{*4} + 225*n \\
& ^{*3} + 85*n^{*2} + 15*n + 1) + 10*b*d^{*4}*n*x*x^{*(5*n)}/(120*n^{*5} + 27 \\
& 4*n^{*4} + 225*n^{*3} + 85*n^{*2} + 15*n + 1) + b*d^{*4}*x*x^{*(5*n)}/(120* \\
& n^{*5} + 274*n^{*4} + 225*n^{*3} + 85*n^{*2} + 15*n + 1), True))
\end{aligned}$$

**GIAC/XCAS [A]** time = 0.228548, size = 1073, normalized size = 8.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)\*(d\*x^n + c)^4,x, algorithm="giac")

[Out] (120\*a\*c^4\*n^5\*x + 24\*b\*d^4\*n^4\*x\*e^(5\*n\*ln(x)) + 120\*b\*c\*d^3\*n^4\*x\*e^(4\*n\*ln(x)) + 30\*a\*d^4\*n^4\*x^2\*e^(4\*n\*ln(x)) + 240\*b\*c^2\*d^2\*n^4\*x^2\*e^(3\*n\*ln(x)) + 160\*a\*c\*d^3\*n^4\*x^2\*e^(3\*n\*ln(x)) + 240\*b\*c^3\*d^2\*n^4\*x^2\*e^(2\*n\*ln(x)) + 360\*a\*c^2\*d^2\*n^4\*x^2\*e^(2\*n\*ln(x)) + 120\*b\*c^4\*n^4\*x^2\*e^(n\*ln(x)) + 480\*a\*c^3\*d^2\*n^4\*x^2\*e^(n\*ln(x)) + 274\*a\*c^4\*n^4\*x + 50\*b\*d^4\*n^3\*x^2\*e^(5\*n\*ln(x)) + 244\*b\*c\*d^3\*n^3\*x^2\*e^(4\*n\*ln(x)) + 61\*a\*d^4\*n^3\*x^2\*e^(4\*n\*ln(x)) + 468\*b\*c^2\*d^2\*n^3\*x^2\*e^(3\*n\*ln(x)) + 312\*a\*c\*d^3\*n^3\*x^2\*e^(3\*n\*ln(x)) + 428\*b\*c^3\*d^2\*n^3\*x^2\*e^(2\*n\*ln(x)) + 642\*a\*c^2\*d^2\*n^3\*x^2\*e^(2\*n\*ln(x)) + 154\*b\*c^4\*n^3\*x^2\*e^(n\*ln(x)) + 616\*a\*c^3\*d^2\*n^3\*x^2\*e^(n\*ln(x)) + 225\*a\*c^4\*n^3\*x + 35\*b\*d^4\*n^2\*x^2\*e^(5\*n\*ln(x)) + 164\*b\*c\*d^3\*n^2\*x^2\*e^(4\*n\*ln(x)) + 41\*a\*d^4\*n^2\*x^2\*e^(4\*n\*ln(x)) + 294\*b\*c^2\*d^2\*n^2\*x^2\*e^(3\*n\*ln(x)) + 196\*a\*c\*d^3\*n^2\*x^2\*e^(3\*n\*ln(x)) + 236\*b\*c^3\*d^2\*n^2\*x^2\*e^(2\*n\*ln(x)) + 354\*a\*c^2\*d^2\*n^2\*x^2\*e^(2\*n\*ln(x)) + 71\*b\*c^4\*n^2\*x^2\*e^(n\*ln(x)) + 284\*a\*c^3\*d^2\*n^2\*x^2\*e^(n\*ln(x)) + 85\*a\*c^4\*n^2\*x + 10\*b\*d^4\*n^2\*x^2\*e^(5\*n\*ln(x)) + 44\*b\*c\*d^3\*n^2\*x^2\*e^(4\*n\*ln(x)) + 11\*a\*d^4\*n^2\*x^2\*e^(4\*n\*ln(x)) + 72\*b\*c^2\*d^2\*n^2\*x^2\*e^(3\*n\*ln(x)) + 48\*a\*c\*d^3\*n^2\*x^2\*e^(3\*n\*ln(x)) + 52\*b\*c^3\*d^2\*n^2\*x^2\*e^(2\*n\*ln(x)) + 78\*a\*c^2\*d^2\*n^2\*x^2\*e^(2\*n\*ln(x)) + 14\*b\*c^4\*n^2\*x^2\*e^(n\*ln(x)) + 56\*a\*c^3\*d^2\*n^2\*x^2\*e^(n\*ln(x)) + 15\*a\*c^4\*n^2\*x + b\*d^4\*x^2\*e^(5\*n\*ln(x)) + 4\*b\*c\*d^3\*x^2\*e^(4\*n\*ln(x)) + a\*d^4\*x^2\*e^(4\*n\*ln(x)) + 6\*b\*c^2\*d^2\*x^2\*e^(3\*n\*ln(x)) + 4\*a\*c\*d^3\*x^2\*e^(3\*n\*ln(x)) + 4\*b\*c^3\*d^2\*x^2\*e^(2\*n\*ln(x)) + 6\*a\*c^2\*d^2\*x^2\*e^(2\*n\*ln(x)) + b\*c^4\*x^2\*e^(n\*ln(x)) + 4\*a\*c^3\*d^2\*x^2\*e^(n\*ln(x)) + a\*c^4\*x)/(120\*n^5 + 274\*n^4 + 225\*n^3 + 85\*n^2 + 15\*n + 1)

### 3.186 $\int (a + bx^n)(c + dx^n)^3 dx$

**Optimal.** Leaf size=99

$$\frac{c^2x^{n+1}(3ad + bc)}{n + 1} + \frac{d^2x^{3n+1}(ad + 3bc)}{3n + 1} + \frac{3cdx^{2n+1}(ad + bc)}{2n + 1} + ac^3x + \frac{bd^3x^{4n+1}}{4n + 1}$$

[Out]  $a*c^3*x + (c^2*(b*c + 3*a*d)*x^{(1 + n)})/(1 + n) + (3*c*d*(b*c + a*d)*x^{(1 + 2*n)})/(1 + 2*n) + (d^2*(3*b*c + a*d)*x^{(1 + 3*n)})/(1 + 3*n) + (b*d^3*x^{(1 + 4*n)})/(1 + 4*n)$

**Rubi [A]** time = 0.167138, antiderivative size = 99, normalized size of antiderivative = 1., number of rules used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{c^2x^{n+1}(3ad + bc)}{n + 1} + \frac{d^2x^{3n+1}(ad + 3bc)}{3n + 1} + \frac{3cdx^{2n+1}(ad + bc)}{2n + 1} + ac^3x + \frac{bd^3x^{4n+1}}{4n + 1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)\*(c + d\*x^n)^3, x]

[Out]  $a*c^3*x + (c^2*(b*c + 3*a*d)*x^{(1 + n)})/(1 + n) + (3*c*d*(b*c + a*d)*x^{(1 + 2*n)})/(1 + 2*n) + (d^2*(3*b*c + a*d)*x^{(1 + 3*n)})/(1 + 3*n) + (b*d^3*x^{(1 + 4*n)})/(1 + 4*n)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^3x^{4n+1}}{4n + 1} + c^3 \int a dx + \frac{c^2x^{n+1}(3ad + bc)}{n + 1} + \frac{3cdx^{2n+1}(ad + bc)}{2n + 1} + \frac{d^2x^{3n+1}(ad + 3bc)}{3n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*(c+d\*x\*\*n)\*\*3, x)

[Out]  $b*d**3*x**(4*n + 1)/(4*n + 1) + c**3*Integral(a, x) + c**2*x**(n + 1)*(3*a*d + b*c)/(n + 1) + 3*c*d*x**(2*n + 1)*(a*d + b*c)/(2*n + 1) + d**2*x**(3*n + 1)*(a*d + 3*b*c)/(3*n + 1)$

**Mathematica [A]** time = 0.144884, size = 92, normalized size = 0.93

$$x \left( \frac{c^2x^n(3ad + bc)}{n + 1} + \frac{d^2x^{3n}(ad + 3bc)}{3n + 1} + \frac{3cdx^{2n}(ad + bc)}{2n + 1} + ac^3 + \frac{bd^3x^{4n}}{4n + 1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)\*(c + d\*x^n)^3, x]

[Out]  $x*(a*c^3 + (c^2*(b*c + 3*a*d)*x^n)/(1 + n) + (3*c*d*(b*c + a*d)*x^{(2*n)})/(1 + 2*n) + (d^2*(3*b*c + a*d)*x^{(3*n)})/(1 + 3*n) + (b*d^3*x^{(4*n)})/(1 + 4*n))$

**Maple [A]** time = 0.016, size = 104, normalized size = 1.1

$$ac^3x + \frac{bd^3x \left( e^{n \ln(x)} \right)^4}{1 + 4n} + \frac{c^2(3ad + bc)xe^{n \ln(x)}}{1 + n} + \frac{d^2(ad + 3bc)x \left( e^{n \ln(x)} \right)^3}{1 + 3n} + 3 \frac{cd(ad + bc)x \left( e^{n \ln(x)} \right)^2}{1 + 2n}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*x^n)*(c+d*x^n)^3,x)`

[Out]  $a^3c^3x + b^3d^3 / (1+4^n) * x * \exp(n * \ln(x))^{4+c^2 * (3*a*d+b*c)} / (1+n) * x * \exp(n * \ln(x)) + d^2 * (a*d+3*b*c) / (1+3^n) * x * \exp(n * \ln(x))^{3+c*d * (a*d+b*c)} / (1+2^n) * x * \exp(n * \ln(x))^{2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)*(d*x^n + c)^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.24791, size = 431, normalized size = 4.35

$$\frac{(6bd^3n^3 + 11bd^3n^2 + 6bd^3n + bd^3)xx^{4n} + (3bcd^2 + ad^3 + 8(3bcd^2 + ad^3)n^3 + 14(3bcd^2 + ad^3)n^2 + 7(3bcd^2 + ad^3)n)x^{3n} + (3bcd^2 + ad^3)n^3 + 14(3bcd^2 + ad^3)n^2 + 7(3bcd^2 + ad^3)n}{(24n^4 + 50n^3 + 35n^2 + 10n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)*(d*x^n + c)^3,x, algorithm="fricas")`

[Out]  $((6*b*d^3*n^3 + 11*b*d^3*n^2 + 6*b*d^3*n + b*d^3)*x*x^{(4*n)} + (3*b*c*d^2 + a*d^3 + 8*(3*b*c*d^2 + a*d^3)*n^3 + 14*(3*b*c*d^2 + a*d^3)*n^2 + 7*(3*b*c*d^2 + a*d^3)*n)*x^{(3*n)} + 3*(b*c^2*d + a*c*d^2 + 12*(b*c^2*d + a*c*d^2)*n^3 + 19*(b*c^2*d + a*c*d^2)*n^2 + 8*(b*c^2*d + a*c*d^2)*n)*x*x^{(2*n)} + (b*c^3 + 3*a*c^2*d + 24*(b*c^3 + 3*a*c^2*d)*n^3 + 26*(b*c^3 + 3*a*c^2*d)*n^2 + 9*(b*c^3 + 3*a*c^2*d)*n)*x*x^n + (24*a*c^3*n^4 + 50*a*c^3*n^3 + 35*a*c^3*n^2 + 10*a*c^3*n + a*c^3)*x / (24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)$

**Sympy [A]** time = 5.2599, size = 1540, normalized size = 15.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n)*(c+d*x**n)**3,x)`

[Out]  $\text{Piecewise}((a^3c^3x + 3a^2c^2d \log(x) - 3a^2c^2d/x - a^2d^3/(2x^2) + b^3c^3 \log(x) - 3b^2c^2d/x - 3b^2c^2d/(2x^2) - b^2d^3/(3x^3), \text{Eq}(n, -1)), (a^3c^3x + 6a^2c^2d \sqrt{x} + 3a^2c^2d^2 \log(x) - 2a^2d^3/\sqrt{x} + 2b^2c^3 \sqrt{x} + 3b^2c^2d \log(x) - 6b^2c^2d/\sqrt{x} - b^2d^3/x, \text{Eq}(n, -1/2)), (a^3c^3x + 9a^2c^2d^2x^{(2/3)}/2 + 9a^2c^2d^2x^{(1/3)} + a^2d^3 \log(x) + 3b^2c^3x^{(2/3)}/2 + 9b^2c^2d^2x^{(1/3)} + 3b^2c^2d^2 \log(x) - 3b^2d^3/x^{(1/3)}, \text{Eq}(n, -1/3)), (a^3c^3x + 4a^2c^2d^2x^{(3/4)} + 6a^2c^2d^2 \sqrt{x} + 4a^2d^3x^{(1/4)} + 4b^2c^3x^{(3/4)}/3 + 6b^2c^2d^2 \sqrt{x} + 12b^2c^2d^2x^{(1/4)} + b^2d^3 \log(x), \text{Eq}(n, -1/4)), (24a^3c^3n^4x/(24n^4 + 50n^3 + 35n^2 + 10n + 1) + 50a^3c^3n^3x/(24n^4 + 50n^3 + 35n^2 + 10n + 1) + 35a^3c^3n^2x/(24n^4 + 50n^3 + 35n^2 + 10n + 1) + 10a^3c^3nx/(24n^4 + 50n^3 + 35n^2 + 10n + 1) + a^3c^3x/(24n^4 + 50n^3 + 35n^2 + 10n + 1) + 72a^2c^2d^2n^3x^n/(24n^4 + 50n^3 + 35n^2 + 10n + 1))$

```

4 + 50*n**3 + 35*n**2 + 10*n + 1) + 78*a*c**2*d*n**2*x*x**n/(24*n
**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 27*a*c**2*d*n*x*x**n/(24*n
**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3*a*c**2*d*x*x**n/(24*n**4 +
50*n**3 + 35*n**2 + 10*n + 1) + 36*a*c*d**2*n**3*x*x**(2*n)/(24*
n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 57*a*c*d**2*n**2*x*x**(2*n
)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*a*c*d**2*n*x*x**(
2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3*a*c*d**2*x*x**(
2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*a*d**3*n**3*x*x
**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 14*a*d**3*n**2
*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 7*a*d**3*n
*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + a*d**3*x*x
**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*b*c**3*n**3
*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 26*b*c**3*n**2
*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 9*b*c**3*n*x*x
**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b*c**3*x*x**n/(24*
n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 36*b*c**2*d*n**3*x*x**(2*n
)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 57*b*c**2*d*n**2*x*x
**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*b*c**2*d*n*
x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3*b*c**2*d*
x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*b*c*d**2
*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 42*b*
c*d**2*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) +
21*b*c*d**2*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1
) + 3*b*c*d**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1
) + 6*b*d**3*n**3*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n
+ 1) + 11*b*d**3*n**2*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 1
0*n + 1) + 6*b*d**3*n*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 1
0*n + 1) + b*d**3*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n
+ 1), True))

```

---

**GIAC/XCAS [A]** time = 0.224958, size = 656, normalized size = 6.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n + a)*(d*x^n + c)^3,x, algorithm="giac")
```

```
[Out] (24*a*c^3*n^4*x + 6*b*d^3*n^3*x*e^(4*n*ln(x)) + 24*b*c*d^2*n^3*x*
e^(3*n*ln(x)) + 8*a*d^3*n^3*x*e^(3*n*ln(x)) + 36*b*c^2*d*n^3*x*e^
(2*n*ln(x)) + 36*a*c*d^2*n^3*x*e^(2*n*ln(x)) + 24*b*c^3*n^3*x*e^
(n*ln(x)) + 72*a*c^2*d*n^3*x*e^(n*ln(x)) + 50*a*c^3*n^3*x + 11*b*d
^3*n^2*x*e^(4*n*ln(x)) + 42*b*c*d^2*n^2*x*e^(3*n*ln(x)) + 14*a*d^
3*n^2*x*e^(3*n*ln(x)) + 57*b*c^2*d*n^2*x*e^(2*n*ln(x)) + 57*a*c*d
^2*n^2*x*e^(2*n*ln(x)) + 26*b*c^3*n^2*x*e^(n*ln(x)) + 78*a*c^2*d*
n^2*x*e^(n*ln(x)) + 35*a*c^3*n^2*x + 6*b*d^3*n*x*e^(4*n*ln(x)) +
21*b*c*d^2*n*x*e^(3*n*ln(x)) + 7*a*d^3*n*x*e^(3*n*ln(x)) + 24*b*c
^2*d*n*x*e^(2*n*ln(x)) + 24*a*c*d^2*n*x*e^(2*n*ln(x)) + 9*b*c^3*n
*x*e^(n*ln(x)) + 27*a*c^2*d*n*x*e^(n*ln(x)) + 10*a*c^3*n*x + b*d^
3*x*e^(4*n*ln(x)) + 3*b*c*d^2*x*e^(3*n*ln(x)) + a*d^3*x*e^(3*n*ln
(x)) + 3*b*c^2*d*x*e^(2*n*ln(x)) + 3*a*c*d^2*x*e^(2*n*ln(x)) + b*
c^3*x*e^(n*ln(x)) + 3*a*c^2*d*x*e^(n*ln(x)) + a*c^3*x)/(24*n^4 +
50*n^3 + 35*n^2 + 10*n + 1)

```

### 3.187 $\int (a + bx^n)(c + dx^n)^2 dx$

**Optimal.** Leaf size=70

$$\frac{cx^{n+1}(2ad + bc)}{n + 1} + \frac{dx^{2n+1}(ad + 2bc)}{2n + 1} + ac^2x + \frac{bd^2x^{3n+1}}{3n + 1}$$

[Out]  $a*c^2*x + (c*(b*c + 2*a*d)*x^(1 + n))/(1 + n) + (d*(2*b*c + a*d)*x^(1 + 2*n))/(1 + 2*n) + (b*d^2*x^(1 + 3*n))/(1 + 3*n)$

**Rubi [A]** time = 0.114146, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{cx^{n+1}(2ad + bc)}{n + 1} + \frac{dx^{2n+1}(ad + 2bc)}{2n + 1} + ac^2x + \frac{bd^2x^{3n+1}}{3n + 1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)\*(c + d\*x^n)^2, x]

[Out]  $a*c^2*x + (c*(b*c + 2*a*d)*x^(1 + n))/(1 + n) + (d*(2*b*c + a*d)*x^(1 + 2*n))/(1 + 2*n) + (b*d^2*x^(1 + 3*n))/(1 + 3*n)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^2x^{3n+1}}{3n + 1} + c^2 \int a dx + \frac{cx^{n+1}(2ad + bc)}{n + 1} + \frac{dx^{2n+1}(ad + 2bc)}{2n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*(c+d\*x\*\*n)\*\*2, x)

[Out]  $b*d**2*x**(3*n + 1)/(3*n + 1) + c**2*Integral(a, x) + c*x**(n + 1)*(2*a*d + b*c)/(n + 1) + d*x**(2*n + 1)*(a*d + 2*b*c)/(2*n + 1)$

**Mathematica [A]** time = 0.109143, size = 65, normalized size = 0.93

$$x \left( \frac{dx^{2n}(ad + 2bc)}{2n + 1} + \frac{cx^n(2ad + bc)}{n + 1} + ac^2 + \frac{bd^2x^{3n}}{3n + 1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)\*(c + d\*x^n)^2, x]

[Out]  $x*(a*c^2 + (c*(b*c + 2*a*d)*x^n)/(1 + n) + (d*(2*b*c + a*d)*x^(2*n))/(1 + 2*n) + (b*d^2*x^(3*n))/(1 + 3*n))$

**Maple [A]** time = 0.014, size = 74, normalized size = 1.1

$$ac^2x + \frac{bd^2x \left( e^{n \ln(x)} \right)^3}{1 + 3n} + \frac{c(2ad + bc)xe^{n \ln(x)}}{1 + n} + \frac{d(ad + 2bc)x \left( e^{n \ln(x)} \right)^2}{1 + 2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*x^n)*(c+d*x^n)^2,x)`

[Out]  $a*c^2*x+b*d^2/(1+3*n)*x*\exp(n*\ln(x))^3+c*(2*a*d+b*c)/(1+n)*x*\exp(n*\ln(x))+d*(a*d+2*b*c)/(1+2*n)*x*\exp(n*\ln(x))^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)*(d*x^n + c)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.257042, size = 236, normalized size = 3.37

$$\frac{(2bd^2n^2 + 3bd^2n + bd^2)xx^{3n} + (2bcd + ad^2 + 3(2bcd + ad^2)n^2 + 4(2bcd + ad^2)n)xx^{2n} + (bc^2 + 2acd + 6(bc^2 + 2acd))}{6n^3 + 11n^2 + 6n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)*(d*x^n + c)^2,x, algorithm="fricas")`

[Out]  $((2*b*d^2*n^2 + 3*b*d^2*n + b*d^2)*x*x^{(3*n)} + (2*b*c*d + a*d^2 + 3*(2*b*c*d + a*d^2)*n^2 + 4*(2*b*c*d + a*d^2)*n)*x*x^{(2*n)} + (b*c^2 + 2*a*c*d + 6*(b*c^2 + 2*a*c*d)*n^2 + 5*(b*c^2 + 2*a*c*d)*n)*x*x^n + (6*a*c^2*n^3 + 11*a*c^2*n^2 + 6*a*c^2*n + a*c^2)*x)/(6*n^3 + 11*n^2 + 6*n + 1)$

**Sympy [A]** time = 2.79227, size = 726, normalized size = 10.37

$$\left\{ \begin{array}{l} ac^2x + 2acd \log(x) - \frac{ad^2}{x} + bc^2 \log(x) - \frac{2bcd}{x} - \frac{bd^2}{2x^2} \\ ac^2x + 4acd\sqrt{x} + ad^2 \log(x) + 2bc^2\sqrt{x} + 2bcd \log(x) - \frac{2bd^2}{\sqrt{x}} \\ ac^2x + 3acd x^{\frac{2}{3}} + 3ad^2\sqrt[3]{x} + \frac{3bc^2x^{\frac{2}{3}}}{2} + 6bcd\sqrt[3]{x} + bd^2 \log(x) \\ \frac{6ac^2n^3x}{6n^3+11n^2+6n+1} + \frac{11ac^2n^2x}{6n^3+11n^2+6n+1} + \frac{6ac^2nx}{6n^3+11n^2+6n+1} + \frac{ac^2x}{6n^3+11n^2+6n+1} + \frac{12acdn^2xx^n}{6n^3+11n^2+6n+1} + \frac{10acdnxx^n}{6n^3+11n^2+6n+1} + \frac{2acdx^n}{6n^3+11n^2+6n+1} + \frac{3ad^2n^2xx^{2n}}{6n^3+11n^2+6n+1} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n)*(c+d*x**n)**2,x)`

[Out]  $\text{Piecewise}((a*c**2*x + 2*a*c*d*\log(x) - a*d**2/x + b*c**2*\log(x) - 2*b*c*d/x - b*d**2/(2*x**2), \text{Eq}(n, -1)), (a*c**2*x + 4*a*c*d*\text{sqrt}(x) + a*d**2*\log(x) + 2*b*c**2*\text{sqrt}(x) + 2*b*c*d*\log(x) - 2*b*d**2/\text{sqrt}(x), \text{Eq}(n, -1/2)), (a*c**2*x + 3*a*c*d*x**(2/3) + 3*a*d**2*x**(1/3) + 3*b*c**2*x**(2/3)/2 + 6*b*c*d*x**(1/3) + b*d**2*\log(x), \text{Eq}(n, -1/3)), (6*a*c**2*n**3*x/(6*n**3 + 11*n**2 + 6*n + 1) + 11*a*c**2*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*c**2*n*x/(6*n**3 + 11*n**2 + 6*n + 1) + a*c**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 12*a*c*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*c*d*n*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 2*a*c*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 3*a*d**2*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + a*d**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 6*b*c**2*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*b*c**2*n*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + b*c**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*b*c*d*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*b*c*d*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*b*c*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1))$

```

3 + 11*n**2 + 6*n + 1) + 2*b*d**2*n**2*x*x**(3*n)/(6*n**3 + 11*n*
*2 + 6*n + 1) + 3*b*d**2*n*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1
) + b*d**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1), True))

```

**GIAC/XCAS [A]** time = 0.221124, size = 342, normalized size = 4.89

$$6ac^2n^3x + 2bd^2n^2xe^{3n\ln(x)} + 6bcdn^2xe^{2n\ln(x)} + 3ad^2n^2xe^{2n\ln(x)} + 6bc^2n^2xe^{n\ln(x)} + 12acd n^2xe^{n\ln(x)} + 11ac^2n^2x + 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n + a)*(d*x^n + c)^2,x, algorithm="giac")
```

```
[Out] (6*a*c^2*n^3*x + 2*b*d^2*n^2*x*e^(3*n*ln(x)) + 6*b*c*d*n^2*x*e^(2
*n*ln(x)) + 3*a*d^2*n^2*x*e^(2*n*ln(x)) + 6*b*c^2*n^2*x*e^(n*ln(x)
)) + 12*a*c*d*n^2*x*e^(n*ln(x)) + 11*a*c^2*n^2*x + 3*b*d^2*n*x*e^
(3*n*ln(x)) + 8*b*c*d*n*x*e^(2*n*ln(x)) + 4*a*d^2*n*x*e^(2*n*ln(x)
)) + 5*b*c^2*n*x*e^(n*ln(x)) + 10*a*c*d*n*x*e^(n*ln(x)) + 6*a*c^2
*n*x + b*d^2*x*e^(3*n*ln(x)) + 2*b*c*d*x*e^(2*n*ln(x)) + a*d^2*x*
e^(2*n*ln(x)) + b*c^2*x*e^(n*ln(x)) + 2*a*c*d*x*e^(n*ln(x)) + a*c
^2*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

### 3.188 $\int (a + bx^n)(c + dx^n) dx$

**Optimal.** Leaf size=40

$$\frac{x^{n+1}(ad + bc)}{n + 1} + acx + \frac{bdx^{2n+1}}{2n + 1}$$

[Out]  $a*c*x + ((b*c + a*d)*x^{(1 + n)})/(1 + n) + (b*d*x^{(1 + 2*n)})/(1 + 2*n)$

**Rubi [A]** time = 0.05853, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{x^{n+1}(ad + bc)}{n + 1} + acx + \frac{bdx^{2n+1}}{2n + 1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)\*(c + d\*x^n), x]

[Out]  $a*c*x + ((b*c + a*d)*x^{(1 + n)})/(1 + n) + (b*d*x^{(1 + 2*n)})/(1 + 2*n)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bdx^{2n+1}}{2n + 1} + c \int a dx + \frac{x^{n+1}(ad + bc)}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*(c+d\*x\*\*n), x)

[Out]  $b*d*x^{(2*n + 1)}/(2*n + 1) + c*Integral(a, x) + x^{(n + 1)}*(a*d + b*c)/(n + 1)$

**Mathematica [A]** time = 0.0829009, size = 37, normalized size = 0.92

$$x \left( \frac{x^n(ad + bc)}{n + 1} + ac + \frac{bdx^{2n}}{2n + 1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)\*(c + d\*x^n), x]

[Out]  $x*(a*c + ((b*c + a*d)*x^n)/(1 + n) + (b*d*x^{(2*n)})/(1 + 2*n))$

**Maple [A]** time = 0.012, size = 43, normalized size = 1.1

$$acx + \frac{(ad + bc)xe^{n \ln(x)}}{1 + n} + \frac{bdx \left( e^{n \ln(x)} \right)^2}{1 + 2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)\*(c+d\*x^n), x)

[Out]  $a*c*x+(a*d+b*c)/(1+n)*x*\exp(n*\ln(x))+b*d/(1+2*n)*x*\exp(n*\ln(x))^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)*(d*x^n + c),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.249835, size = 93, normalized size = 2.32

$$\frac{(bdn + bd)xx^{2n} + (bc + ad + 2(bc + ad)n)xx^n + (2acn^2 + 3acn + ac)x}{2n^2 + 3n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)*(d*x^n + c),x, algorithm="fricas")`

[Out]  $((b*d*n + b*d)*x*x^{(2*n)} + (b*c + a*d + 2*(b*c + a*d)*n)*x*x^n + (2*a*c*n^2 + 3*a*c*n + a*c)*x)/(2*n^2 + 3*n + 1)$

**Sympy [A]** time = 1.31153, size = 236, normalized size = 5.9

$$\begin{cases} acx + ad \log(x) + bc \log(x) - \frac{bd}{x} & \text{for } n = -1 \\ acx + 2ad\sqrt{x} + 2bc\sqrt{x} + bd \log(x) & \text{for } n = -\frac{1}{2} \\ \frac{2acn^2x}{2n^2+3n+1} + \frac{3acnx}{2n^2+3n+1} + \frac{acx}{2n^2+3n+1} + \frac{2adnxx^n}{2n^2+3n+1} + \frac{adxx^n}{2n^2+3n+1} + \frac{2bcnxx^n}{2n^2+3n+1} + \frac{bcxx^n}{2n^2+3n+1} + \frac{bdnxx^{2n}}{2n^2+3n+1} + \frac{bdxx^{2n}}{2n^2+3n+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n)*(c+d*x**n),x)`

[Out] `Piecewise((a*c*x + a*d*log(x) + b*c*log(x) - b*d/x, Eq(n, -1)), (a*c*x + 2*a*d*sqrt(x) + 2*b*c*sqrt(x) + b*d*log(x), Eq(n, -1/2)), (2*a*c*n**2*x/(2*n**2 + 3*n + 1) + 3*a*c*n*x/(2*n**2 + 3*n + 1) + a*c*x/(2*n**2 + 3*n + 1) + 2*a*d*n*x*x**n/(2*n**2 + 3*n + 1) + a*d*x*x**n/(2*n**2 + 3*n + 1) + 2*b*c*n*x*x**n/(2*n**2 + 3*n + 1) + b*c*x*x**n/(2*n**2 + 3*n + 1) + b*d*n*x*x**(2*n)/(2*n**2 + 3*n + 1) + b*d*x*x**(2*n)/(2*n**2 + 3*n + 1), True))`

**GIAC/XCAS [A]** time = 0.213836, size = 126, normalized size = 3.15

$$\frac{2acn^2x + bdnxe^{(2n\ln(x))} + 2bcnxe^{(n\ln(x))} + 2adnxe^{(n\ln(x))} + 3acnx + bdx e^{(2n\ln(x))} + bcxe^{(n\ln(x))} + adxe^{(n\ln(x))} + acx}{2n^2 + 3n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)*(d*x^n + c),x, algorithm="giac")`

[Out]  $(2*a*c*n^2*x + b*d*n*x*e^{(2*n*\ln(x))} + 2*b*c*n*x*e^{(n*\ln(x))} + 2*a*d*n*x*e^{(n*\ln(x))} + 3*a*c*n*x + b*d*x*e^{(2*n*\ln(x))} + b*c*x*e^{(n*\ln(x))} + a*d*x*e^{(n*\ln(x))} + a*c*x)/(2*n^2 + 3*n + 1)$

$$3.189 \quad \int \frac{a+bx^n}{c+dx^n} dx$$

**Optimal.** Leaf size=43

$$\frac{bx}{d} - \frac{x(bc - ad) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{cd}$$

[Out] (b\*x)/d - ((b\*c - a\*d)\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d\*x^n)/c)])/(c\*d)

**Rubi [A]** time = 0.0551257, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{bx}{d} - \frac{x(bc - ad) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{cd}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)/(c + d\*x^n), x]

[Out] (b\*x)/d - ((b\*c - a\*d)\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d\*x^n)/c)])/(c\*d)

**Rubi in Sympy [A]** time = 7.20277, size = 31, normalized size = 0.72

$$\frac{bx}{d} + \frac{x(ad - bc) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)/(c+d\*x\*\*n), x)

[Out] b\*x/d + x\*(a\*d - b\*c)\*hyper((1, 1/n), (1 + 1/n, ), -d\*x\*\*n/c)/(c\*d)

**Mathematica [A]** time = 0.0369414, size = 40, normalized size = 0.93

$$\frac{x\left((ad - bc) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right) + bc\right)}{cd}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)/(c + d\*x^n), x]

[Out] (x\*(b\*c + -(b\*c) + a\*d)\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d\*x^n)/c)])/(c\*d)

**Maple [F]** time = 0.083, size = 0, normalized size = 0.

$$\int \frac{a + bx^n}{c + dx^n} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*x^n)/(c+d*x^n),x)`

[Out] `int((a+b*x^n)/(c+d*x^n),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$-(bc - ad) \int \frac{1}{d^2 x^n + cd} dx + \frac{bx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)/(d*x^n + c),x, algorithm="maxima")`

[Out] `-(b*c - a*d)*integrate(1/(d^2*x^n + c*d), x) + b*x/d`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^n + a}{dx^n + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)/(d*x^n + c),x, algorithm="fricas")`

[Out] `integral((b*x^n + a)/(d*x^n + c), x)`

**Sympy** [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n)/(c+d*x**n),x)`

[Out] Exception raised: TypeError

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^n + a}{dx^n + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)/(d*x^n + c),x, algorithm="giac")`

[Out] `integrate((b*x^n + a)/(d*x^n + c), x)`

$$3.190 \quad \int \frac{a+bx^n}{(c+dx^n)^2} dx$$

**Optimal.** Leaf size=73

$$\frac{x(bc - ad(1 - n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2 dn} - \frac{x(bc - ad)}{cdn(c + dx^n)}$$

[Out] -(((b\*c - a\*d)\*x)/(c\*d\*n\*(c + d\*x^n))) + ((b\*c - a\*d\*(1 - n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(d\*x^n)/c])/(c^2\*d\*n)

**Rubi [A]** time = 0.0910764, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x(bc - ad(1 - n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2 dn} - \frac{x(bc - ad)}{cdn(c + dx^n)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)/(c + d\*x^n)^2, x]

[Out] -(((b\*c - a\*d)\*x)/(c\*d\*n\*(c + d\*x^n))) + ((b\*c - a\*d\*(1 - n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(d\*x^n)/c])/(c^2\*d\*n)

**Rubi in Sympy [A]** time = 9.66263, size = 53, normalized size = 0.73

$$\frac{x(ad - bc)}{cdn(c + dx^n)} + \frac{x(-ad(-n + 1) + bc) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2 dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)/(c+d\*x\*\*n)\*\*2, x)

[Out] x\*(a\*d - b\*c)/(c\*d\*n\*(c + d\*x\*\*n)) + x\*(-a\*d\*(-n + 1) + b\*c)\*hyper((1, 1/n), (1 + 1/n, ), -d\*x\*\*n/c)/(c\*\*2\*d\*n)

**Mathematica [A]** time = 0.0790748, size = 68, normalized size = 0.93

$$\frac{x\left((c + dx^n)(ad(n - 1) + bc) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right) + c(ad - bc)\right)}{c^2 dn(c + dx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)/(c + d\*x^n)^2, x]

[Out] (x\*(c\*(-(b\*c) + a\*d) + (b\*c + a\*d\*(-1 + n))\*(c + d\*x^n)\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(d\*x^n)/c]))/(c^2\*d\*n\*(c + d\*x^n))

**Maple [F]** time = 0.064, size = 0, normalized size = 0.

$$\int \frac{a + bx^n}{(c + dx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*x^n)/(c+d*x^n)^2,x)`

[Out] `int((a+b*x^n)/(c+d*x^n)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$(ad(n-1) + bc) \int \frac{1}{cd^2nx^n + c^2dn} dx - \frac{(bc - ad)x}{cd^2nx^n + c^2dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)/(d*x^n + c)^2,x, algorithm="maxima")`

[Out] `(a*d*(n - 1) + b*c)*integrate(1/(c*d^2*n*x^n + c^2*d*n), x) - (b*c - a*d)*x/(c*d^2*n*x^n + c^2*d*n)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^n + a}{d^2x^{2n} + 2cdx^n + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)/(d*x^n + c)^2,x, algorithm="fricas")`

[Out] `integral((b*x^n + a)/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)`

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n)/(c+d*x**n)**2,x)`

[Out] Exception raised: TypeError

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^n + a}{(dx^n + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)/(d*x^n + c)^2,x, algorithm="giac")`

[Out] `integrate((b*x^n + a)/(d*x^n + c)^2, x)`

$$3.191 \quad \int \frac{a+bx^n}{(c+dx^n)^3} dx$$

**Optimal.** Leaf size=78

$$\frac{x(bc - ad(1 - 2n)) {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3dn} - \frac{x(bc - ad)}{2cdn(c + dx^n)^2}$$

[Out]  $-\frac{(b^*c - a^*d)^*x}{(2^*c^*d^*n^*(c + d^*x^n)^2)} + \frac{(b^*c - a^*d^*(1 - 2^*n))}{x^*} \text{Hypergeometric2F1}[2, n^*(-1), 1 + n^*(-1), -((d^*x^n)/c)] / (2^*c^3 d^*n)$

**Rubi [A]** time = 0.092494, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x(bc - ad(1 - 2n)) {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3dn} - \frac{x(bc - ad)}{2cdn(c + dx^n)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)/(c + d\*x^n)^3, x]

[Out]  $-\frac{(b^*c - a^*d)^*x}{(2^*c^*d^*n^*(c + d^*x^n)^2)} + \frac{(b^*c - a^*d^*(1 - 2^*n))}{x^*} \text{Hypergeometric2F1}[2, n^*(-1), 1 + n^*(-1), -((d^*x^n)/c)] / (2^*c^3 d^*n)$

**Rubi in Sympy [A]** time = 10.1555, size = 60, normalized size = 0.77

$$\frac{x(ad - bc)}{2cdn(c + dx^n)^2} + \frac{x(-ad(-2n + 1) + bc) {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)/(c+d\*x\*\*n)\*\*3, x)

[Out]  $x^*(a^*d - b^*c) / (2^*c^*d^*n^*(c + d^*x^n)^2) + x^*(-a^*d^*(-2^*n + 1) + b^*c) \text{hyper}((2, 1/n), (1 + 1/n), -d^*x^n/c) / (2^*c^3 d^*n)$

**Mathematica [A]** time = 0.107669, size = 96, normalized size = 1.23

$$\frac{x\left(-\frac{c^2n(bc-ad)}{(c+dx^n)^2} + (n-1)(ad(2n-1) + bc) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right) + \frac{c(ad(2n-1)+bc)}{c+dx^n}\right)}{2c^3dn^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)/(c + d\*x^n)^3, x]

[Out]  $\frac{x^*(-((c^2*(b^*c - a^*d)^*n)/(c + d^*x^n)^2) + (c^*(b^*c + a^*d^*(-1 + 2^*n))))}{(c + d^*x^n)} + (-1 + n)^*(b^*c + a^*d^*(-1 + 2^*n)) \text{Hypergeometric2F1}[1, n^*(-1), 1 + n^*(-1), -((d^*x^n)/c)] / (2^*c^3 d^*n^2)$

**Maple [F]** time = 0.076, size = 0, normalized size = 0.

$$\int \frac{a + bx^n}{(c + dx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)/(c+d\*x^n)^3, x)

[Out] int((a+b\*x^n)/(c+d\*x^n)^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & ((2n^2 - 3n + 1)ad + bc(n - 1)) \int \frac{1}{2(c^2d^2n^2x^n + c^3dn^2)} dx \\ & + \frac{(ad^2(2n - 1) + bcd)xx^n + (acd(3n - 1) - bc^2(n - 1))x}{2(c^2d^3n^2x^{2n} + 2c^3d^2n^2x^n + c^4dn^2)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)/(d\*x^n + c)^3, x, algorithm="maxima")

[Out] ((2\*n^2 - 3\*n + 1)\*a\*d + b\*c\*(n - 1))\*integrate(1/2/(c^2\*d^2\*n^2\*x^n + c^3\*d\*n^2), x) + 1/2\*((a\*d^2\*(2\*n - 1) + b\*c\*d)\*x\*x^n + (a\*c\*d\*(3\*n - 1) - b\*c^2\*(n - 1))\*x)/(c^2\*d^3\*n^2\*x^(2\*n) + 2\*c^3\*d^2\*n^2\*x^n + c^4\*d\*n^2)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^n + a}{d^3x^{3n} + 3cd^2x^{2n} + 3c^2dx^n + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)/(d\*x^n + c)^3, x, algorithm="fricas")

[Out] integral((b\*x^n + a)/(d^3\*x^(3\*n) + 3\*c\*d^2\*x^(2\*n) + 3\*c^2\*d\*x^n + c^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)/(c+d\*x\*\*n)\*\*3, x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^n + a}{(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n + a)/(d*x^n + c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c)^3, x)
```

$$3.192 \quad \int \frac{a+bx^n}{(c+dx^n)^4} dx$$

**Optimal.** Leaf size=78

$$\frac{x(bc - ad(1 - 3n)) {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{3c^4dn} - \frac{x(bc - ad)}{3cdn(c + dx^n)^3}$$

[Out]  $-\frac{(b*c - a*d)*x}{(3*c*d*n*(c + d*x^n)^3)} + \frac{(b*c - a*d*(1 - 3*n)) * x * \text{Hypergeometric2F1}[3, n^{(-1)}, 1 + n^{(-1)}, -((d*x^n)/c)]}{(3*c^4*d*n)}$

**Rubi [A]** time = 0.0904342, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x(bc - ad(1 - 3n)) {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{3c^4dn} - \frac{x(bc - ad)}{3cdn(c + dx^n)^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)/(c + d\*x^n)^4, x]

[Out]  $-\frac{(b*c - a*d)*x}{(3*c*d*n*(c + d*x^n)^3)} + \frac{(b*c - a*d*(1 - 3*n)) * x * \text{Hypergeometric2F1}[3, n^{(-1)}, 1 + n^{(-1)}, -((d*x^n)/c)]}{(3*c^4*d*n)}$

**Rubi in Sympy [A]** time = 10.1421, size = 60, normalized size = 0.77

$$\frac{x(ad - bc)}{3cdn(c + dx^n)^3} + \frac{x(-ad(-3n + 1) + bc) {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{3c^4dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)/(c+d\*x\*\*n)\*\*4, x)

[Out]  $x*(a*d - b*c)/(3*c*d*n*(c + d*x**n)**3) + x*(-a*d*(-3*n + 1) + b*c)*\text{hyper}((3, 1/n), (1 + 1/n), -d*x**n/c)/(3*c**4*d*n)$

**Mathematica [A]** time = 0.165302, size = 136, normalized size = 1.74

$$\frac{x\left(-\frac{2c^3n^2(bc-ad)}{(c+dx^n)^3} + \frac{c^2n(ad(3n-1)+bc)}{(c+dx^n)^2} + (2n^2 - 3n + 1)(ad(3n-1) + bc) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right) + \frac{c(2n-1)(ad(3n-1)+bc)}{c+dx^n}\right)}{6c^4dn^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)/(c + d\*x^n)^4, x]

[Out]  $\frac{(x*((-2*c^3*(b*c - a*d)*n^2)/(c + d*x^n)^3 + (c^2*n*(b*c + a*d*(-1 + 3*n)))/(c + d*x^n)^2 + (c*(-1 + 2*n)*(b*c + a*d*(-1 + 3*n)))/(c + d*x^n) + (1 - 3*n + 2*n^2)*(b*c + a*d*(-1 + 3*n))*\text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, -((d*x^n)/c)]))/(6*c^4*d*n^3)}$

**Maple [F]** time = 0.093, size = 0, normalized size = 0.

$$\int \frac{a + bx^n}{(c + dx^n)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)/(c+d\*x^n)^4, x)

[Out] int((a+b\*x^n)/(c+d\*x^n)^4, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & ((2n^2 - 3n + 1)bc + (6n^3 - 11n^2 + 6n - 1)ad) \int \frac{1}{6(c^3d^2n^3x^n + c^4dn^3)} dx \\ & + \frac{((6n^2 - 5n + 1)ad^3 + bcd^2(2n - 1))xx^{2n} + ((15n^2 - 11n + 2)acd^2 + bc^2d(5n - 2))xx^n - ((2n^2 - 3n + 1)bc^3 - (11n^2 - 5n + 1)ad^3)}{6(c^3d^4n^3x^{3n} + 3c^4d^3n^3x^{2n} + 3c^5d^2n^3x^n + c^6dn^3)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)/(d\*x^n + c)^4, x, algorithm="maxima")

[Out] ((2\*n^2 - 3\*n + 1)\*b\*c + (6\*n^3 - 11\*n^2 + 6\*n - 1)\*a\*d)\*integrate(1/6/(c^3\*d^2\*n^3\*x^n + c^4\*d\*n^3), x) + 1/6\*(((6\*n^2 - 5\*n + 1)\*a\*d^3 + b\*c\*d^2\*(2\*n - 1))\*x\*x^(2\*n) + ((15\*n^2 - 11\*n + 2)\*a\*c\*d^2 + b\*c^2\*d\*(5\*n - 2))\*x\*x^n - ((2\*n^2 - 3\*n + 1)\*b\*c^3 - (11\*n^2 - 5\*n + 1)\*a\*c^2\*d)\*x)/(c^3\*d^4\*n^3\*x^(3\*n) + 3\*c^4\*d^3\*n^3\*x^(2\*n) + 3\*c^5\*d^2\*n^3\*x^n + c^6\*d\*n^3)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^n + a}{d^4x^{4n} + 4cd^3x^{3n} + 6c^2d^2x^{2n} + 4c^3dx^n + c^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)/(d\*x^n + c)^4, x, algorithm="fricas")

[Out] integral((b\*x^n + a)/(d^4\*x^(4\*n) + 4\*c\*d^3\*x^(3\*n) + 6\*c^2\*d^2\*x^(2\*n) + 4\*c^3\*d\*x^n + c^4), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)/(c+d\*x\*\*n)\*\*4, x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^n + a}{(dx^n + c)^4} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n + a)/(d*x^n + c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c)^4, x)
```

### 3.193 $\int (a + bx^n)^2 (d + ex^n)^3 dx$

**Optimal.** Leaf size=158

$$\frac{dx^{2n+1} (3a^2e^2 + 6abde + b^2d^2)}{2n+1} + \frac{ex^{3n+1} (a^2e^2 + 6abde + 3b^2d^2)}{3n+1} + a^2d^3x + \frac{ad^2x^{n+1}(3ae + 2bd)}{n+1} + \frac{be^2x^{4n+1}(2ae + 3bd)}{4n+1} + \frac{b^2e^3x^{5n+1}}{5n+1}$$

[Out]  $a^2d^3x + (a^2d^2(2bd + 3ae)x^{n+1} + (d^2(b^2d^2 + 6abd^2 + 3a^2e^2) + 6abd^2e + 3a^2e^2))x^{2n+1}/(2n+1) + (d^2(b^2d^2 + 6abd^2 + 3a^2e^2) + 6abd^2e + 3a^2e^2)x^{3n+1}/(3n+1) + (b^2e^3x^{5n+1} + 6abd^2e + 3a^2e^2)x^{4n+1}/(4n+1) + (b^2e^3x^{5n+1} + 6abd^2e + 3a^2e^2)x^{5n+1}/(5n+1)$

**Rubi [A]** time = 0.291011, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{dx^{2n+1} (3a^2e^2 + 6abde + b^2d^2)}{2n+1} + \frac{ex^{3n+1} (a^2e^2 + 6abde + 3b^2d^2)}{3n+1} + a^2d^3x + \frac{ad^2x^{n+1}(3ae + 2bd)}{n+1} + \frac{be^2x^{4n+1}(2ae + 3bd)}{4n+1} + \frac{b^2e^3x^{5n+1}}{5n+1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^2\*(d + e\*x^n)^3,x]

[Out]  $a^2d^3x + (a^2d^2(2bd + 3ae)x^{n+1} + (d^2(b^2d^2 + 6abd^2 + 3a^2e^2) + 6abd^2e + 3a^2e^2))x^{2n+1}/(2n+1) + (d^2(b^2d^2 + 6abd^2 + 3a^2e^2) + 6abd^2e + 3a^2e^2)x^{3n+1}/(3n+1) + (b^2e^3x^{5n+1} + 6abd^2e + 3a^2e^2)x^{4n+1}/(4n+1) + (b^2e^3x^{5n+1} + 6abd^2e + 3a^2e^2)x^{5n+1}/(5n+1)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{ad^2x^{n+1}(3ae + 2bd)}{n+1} + \frac{b^2e^3x^{5n+1}}{5n+1} + \frac{be^2x^{4n+1}(2ae + 3bd)}{4n+1} + d^3 \int a^2 dx + \frac{dx^{2n+1}(3ae(ae + 2bd) + b^2d^2)}{2n+1} + \frac{ex^{3n+1}(a^2e^2 + 3bd(2ae + bd))}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*2\*(d+e\*x\*\*n)\*\*3,x)

[Out]  $a*d^3*x^{n+1}/(n+1) + b^2*e^3*x^{5n+1}/(5n+1) + b^2*e^2*x^{4n+1}/(4n+1) + d^3*Integral(a^2,x) + d*x^{2n+1}/(2n+1) + e*x^{3n+1}/(3n+1) + (a^2*d^2*(ae + 2bd) + b^2*d^2)/(2n+1) + (a^2*d^2*(ae + 2bd) + b^2*d^2)/(3n+1)$

**Mathematica [A]** time = 0.251017, size = 149, normalized size = 0.94

$$x \left( \frac{dx^{2n} (3a^2e^2 + 6abde + b^2d^2)}{2n+1} + \frac{ex^{3n} (a^2e^2 + 6abde + 3b^2d^2)}{3n+1} + a^2d^3 + \frac{ad^2x^n(3ae + 2bd)}{n+1} + \frac{be^2x^{4n}(2ae + 3bd)}{4n+1} + \frac{b^2e^3x^{5n}}{5n+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^2\*(d + e\*x^n)^3,x]

[Out]  $x*(a^2*d^3 + (a*d^2*(2*b*d + 3*a*e)*x^n)/(1+n) + (d*(b^2*d^2 + 6*a*b*d*e + 3*a^2*e^2)*x^{(2*n)})/(1+2*n) + (e*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2)*x^{(3*n)})/(1+3*n) + (b*e^2*(3*b*d + 2*a*e)*x^{(4*n)})/(1+4*n) + (b^2*e^3*x^{(5*n)})/(1+5*n))$

**Maple [A]** time = 0.019, size = 164, normalized size = 1.

$$a^2 d^3 x + \frac{b^2 e^3 x \left( e^{n \ln(x)} \right)^5}{1+5n} + \frac{d \left( 3 a^2 e^2 + 6 a b d e + b^2 d^2 \right) x \left( e^{n \ln(x)} \right)^2}{1+2n} + \frac{e \left( a^2 e^2 + 6 a b d e + 3 b^2 d^2 \right) x \left( e^{n \ln(x)} \right)^3}{1+3n} + \frac{a d^2 \left( 3 a e + 2 b d \right) x e^{n \ln(x)}}{1+n} + \frac{b e^2 \left( 2 a e + 3 b d \right) x \left( e^{n \ln(x)} \right)^4}{1+4n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)^2\*(d+e\*x^n)^3,x)

[Out]  $a^2*d^3*x+b^2*e^3/(1+5*n)*x*\exp(n*\ln(x))^5+d*(3*a^2*e^2+6*a*b*d*e+b^2*d^2)/(1+2*n)*x*\exp(n*\ln(x))^2+e*(a^2*e^2+6*a*b*d*e+3*b^2*d^2)/(1+3*n)*x*\exp(n*\ln(x))^3+a*d^2*(3*a*e+2*b*d)/(1+n)*x*\exp(n*\ln(x))+b*e^2*(2*a*e+3*b*d)/(1+4*n)*x*\exp(n*\ln(x))^4$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^2\*(e\*x^n + d)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.253386, size = 900, normalized size = 5.7

$$(24 b^2 e^3 n^4 + 50 b^2 e^3 n^3 + 35 b^2 e^3 n^2 + 10 b^2 e^3 n + b^2 e^3) x x^{5 n} + (3 b^2 d e^2 + 2 a b e^3 + 30 (3 b^2 d e^2 + 2 a b e^3) n^4 + 61 (3 b^2 d e^2 + 2 a b e^3) n^3 + 41 (3 b^2 d e^2 + 2 a b e^3) n^2 + 11 (3 b^2 d e^2 + 2 a b e^3) n) x x^{4 n} + (3 b^2 d^2 e + 6 a b d^2 e^2 + a^2 e^3 + 40 (3 b^2 d^2 e + 6 a b d^2 e^2 + a^2 e^3) n^4 + 78 (3 b^2 d^2 e + 6 a b d^2 e^2 + a^2 e^3) n^3 + 49 (3 b^2 d^2 e + 6 a b d^2 e^2 + a^2 e^3) n^2 + 12 (3 b^2 d^2 e + 6 a b d^2 e^2 + a^2 e^3) n) x x^{3 n} + (b^2 d^3 + 6 a b d^2 e + 3 a^2 d^2 e^2 + 60 (b^2 d^3 + 6 a b d^2 e + 3 a^2 d^2 e^2) n^4 + 107 (b^2 d^3 + 6 a b d^2 e + 3 a^2 d^2 e^2) n^3 + 59 (b^2 d^3 + 6 a b d^2 e + 3 a^2 d^2 e^2) n^2 + 13 (b^2 d^3 + 6 a b d^2 e + 3 a^2 d^2 e^2) n) x x^{2 n} + (2 a b d^3 + 3 a^2 d^2 e + 120 (2 a b d^3 + 3 a^2 d^2 e) n^4 + 154 (2 a b d^3 + 3 a^2 d^2 e) n^3 + 71 (2 a b d^3 + 3 a^2 d^2 e) n^2 + 14 (2 a b d^3 + 3 a^2 d^2 e) n) x x^n + (120 a^2 d^3 n^5 + 274 a^2 d^3 n^4 + 225 a^2 d^3 n^3 + 85 a^2 d^3 n^2 + 15 a^2 d^3 n + a^2 d^3) x)/(120 n^5 + 274 n^4 + 225 n^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^2\*(e\*x^n + d)^3,x, algorithm="fricas")

[Out]  $((24*b^2*e^3*n^4 + 50*b^2*e^3*n^3 + 35*b^2*e^3*n^2 + 10*b^2*e^3*n + b^2*e^3)*x*x^{(5*n)} + (3*b^2*d^2*e^2 + 2*a*b*e^3 + 30*(3*b^2*d^2*e^2 + 2*a*b*e^3)*n^4 + 61*(3*b^2*d^2*e^2 + 2*a*b*e^3)*n^3 + 41*(3*b^2*d^2*e^2 + 2*a*b*e^3)*n^2 + 11*(3*b^2*d^2*e^2 + 2*a*b*e^3)*n)*x*x^{(4*n)} + (3*b^2*d^2*e + 6*a*b*d^2*e^2 + a^2*e^3 + 40*(3*b^2*d^2*e + 6*a*b*d^2*e^2 + a^2*e^3)*n^4 + 78*(3*b^2*d^2*e + 6*a*b*d^2*e^2 + a^2*e^3)*n^3 + 49*(3*b^2*d^2*e + 6*a*b*d^2*e^2 + a^2*e^3)*n^2 + 12*(3*b^2*d^2*e + 6*a*b*d^2*e^2 + a^2*e^3)*n)*x*x^{(3*n)} + (b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d^2*e^2 + 60*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d^2*e^2)*n^4 + 107*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d^2*e^2)*n^3 + 59*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d^2*e^2)*n^2 + 13*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d^2*e^2)*n)*x*x^{(2*n)} + (2*a*b*d^3 + 3*a^2*d^2*e + 120*(2*a*b*d^3 + 3*a^2*d^2*e)*n^4 + 154*(2*a*b*d^3 + 3*a^2*d^2*e)*n^3 + 71*(2*a*b*d^3 + 3*a^2*d^2*e)*n^2 + 14*(2*a*b*d^3 + 3*a^2*d^2*e)*n)*x*x^n + (120*a^2*d^3*n^5 + 274*a^2*d^3*n^4 + 225*a^2*d^3*n^3 + 85*a^2*d^3*n^2 + 15*a^2*d^3*n + a^2*d^3)*x)/(120*n^5 + 274*n^4 + 225*n^3$

+ 85\*n^2 + 15\*n + 1)

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*2\*(d+e\*x\*\*n)\*\*3,x)

[Out] Exception raised: TypeError

---

**GIAC/XCAS [A]** time = 0.226763, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^2\*(e\*x^n + d)^3,x, algorithm="giac")

[Out] Done

### 3.194 $\int (a + bx^n)^2 (d + ex^n)^2 dx$

**Optimal.** Leaf size=112

$$\frac{x^{2n+1}(a^2e^2 + 4abde + b^2d^2)}{2n+1} + a^2d^2x + \frac{2adx^{n+1}(ae + bd)}{n+1} + \frac{2bex^{3n+1}(ae + bd)}{3n+1} + \frac{b^2e^2x^{4n+1}}{4n+1}$$

[Out]  $a^2d^2x + (2ad(bd + ae)x^{n+1})/(n+1) + ((b^2d^2 + 4abde + a^2e^2)x^{2n+1})/(2n+1) + (2b^2e^2x^{4n+1})/(4n+1) + (2b^2e^2x^{4n+1})/(4n+1)$

**Rubi [A]** time = 0.187095, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{x^{2n+1}(a^2e^2 + 4abde + b^2d^2)}{2n+1} + a^2d^2x + \frac{2adx^{n+1}(ae + bd)}{n+1} + \frac{2bex^{3n+1}(ae + bd)}{3n+1} + \frac{b^2e^2x^{4n+1}}{4n+1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^2\*(d + e\*x^n)^2, x]

[Out]  $a^2d^2x + (2ad(bd + ae)x^{n+1})/(n+1) + ((b^2d^2 + 4abde + a^2e^2)x^{2n+1})/(2n+1) + (2b^2e^2x^{4n+1})/(4n+1) + (2b^2e^2x^{4n+1})/(4n+1)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2adx^{n+1}(ae + bd)}{n+1} + \frac{b^2e^2x^{4n+1}}{4n+1} + \frac{2bex^{3n+1}(ae + bd)}{3n+1} + d^2 \int a^2 dx + \frac{x^{2n+1}(a^2e^2 + bd(4ae + bd))}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*2\*(d+e\*x\*\*n)\*\*2, x)

[Out]  $2adx^{n+1}(ae + bd)/(n+1) + b^2e^2x^{4n+1}/(4n+1) + 2b^2e^2x^{4n+1}/(4n+1) + 2b^2e^2x^{4n+1}/(4n+1) + d^2 \int a^2 dx + x^{2n+1}(a^2e^2 + bd(4ae + bd))/(2n+1)$

**Mathematica [A]** time = 0.25921, size = 105, normalized size = 0.94

$$x \left( \frac{x^{2n}(a^2e^2 + 4abde + b^2d^2)}{2n+1} + a^2d^2 + \frac{2bex^{3n}(ae + bd)}{3n+1} + \frac{2adx^n(ae + bd)}{n+1} + \frac{b^2e^2x^{4n}}{4n+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^2\*(d + e\*x^n)^2, x]

[Out]  $x(a^2d^2 + (2ad(bd + ae)x^n)/(n+1) + ((b^2d^2 + 4abde + a^2e^2)x^{2n})/(2n+1) + (2b^2e^2x^{4n})/(4n+1) + (2b^2e^2x^{4n})/(4n+1))$

**Maple [A]** time = 0.017, size = 117, normalized size = 1.

$$a^2 d^2 x + \frac{(a^2 e^2 + 4 abde + b^2 d^2) x \left( e^{n \ln(x)} \right)^2}{1 + 2n} + \frac{b^2 e^2 x \left( e^{n \ln(x)} \right)^4}{1 + 4n} \\ + 2 \frac{ad(ae + bd) x e^{n \ln(x)}}{1 + n} + 2 \frac{be(ae + bd) x \left( e^{n \ln(x)} \right)^3}{1 + 3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)^2\*(d+e\*x^n)^2,x)

[Out]  $a^2 d^2 x + (a^2 e^2 + 4 a b d e + b^2 d^2) x \left( e^{n \ln(x)} \right)^2 / (1 + 2 n) * x * \exp(n * \ln(x))^{2 + b^2 e^2 / (1 + 4 n)} + 2 * a * d * (a e + b d) / (1 + n) * x * \exp(n * \ln(x))^{2 + b^2 e^2 / (1 + 4 n)} + 2 * b * e * (a e + b d) / (1 + 3 n) * x * \exp(n * \ln(x))^{2 + b^2 e^2 / (1 + 4 n)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^2\*(e\*x^n + d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.255879, size = 500, normalized size = 4.46

$$(6 b^2 e^2 n^3 + 11 b^2 e^2 n^2 + 6 b^2 e^2 n + b^2 e^2) x x^{4n} + 2 (b^2 d e + a b e^2 + 8 (b^2 d e + a b e^2) n^3 + 14 (b^2 d e + a b e^2) n^2 + 7 (b^2 d e + a b e^2) n)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^2\*(e\*x^n + d)^2,x, algorithm="fricas")

[Out]  $((6 b^2 e^2 n^3 + 11 b^2 e^2 n^2 + 6 b^2 e^2 n + b^2 e^2) * x * x^{4n} + 2 * (b^2 d e + a b e^2 + 8 * (b^2 d e + a b e^2) * n^3 + 14 * (b^2 d e + a b e^2) * n^2 + 7 * (b^2 d e + a b e^2) * n) * x^{4n} + 4 * a * b * d * e + a^2 * e^2 + 12 * (b^2 d^2 + 4 * a * b * d * e + a^2 * e^2) * n^3 + 19 * (b^2 d^2 + 4 * a * b * d * e + a^2 * e^2) * n^2 + 8 * (b^2 d^2 + 4 * a * b * d * e + a^2 * e^2) * n) * x * x^{2n} + 2 * (a * b * d^2 + a^2 * d * e + 24 * (a * b * d^2 + a^2 * d * e) * n^3 + 26 * (a * b * d^2 + a^2 * d * e) * n^2 + 9 * (a * b * d^2 + a^2 * d * e) * n) * x * x^n + (24 * a^2 * d^2 * n^4 + 50 * a^2 * d^2 * n^3 + 35 * a^2 * d^2 * n^2 + 10 * a^2 * d^2 * n + a^2 * d^2) * x) / (24 * n^4 + 50 * n^3 + 35 * n^2 + 10 * n + 1)$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*2\*(d+e\*x\*\*n)\*\*2,x)

[Out] Exception raised: TypeError

**GIAC/XCAS [A]** time = 0.227152, size = 782, normalized size = 6.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^2\*(e\*x^n + d)^2,x, algorithm="giac")

[Out]  $(24*a^2*d^2*n^4*x + 12*b^2*d^2*n^3*x*e^{(2*n*\ln(x))} + 48*a*b*d^2*n^3*x*e^{(n*\ln(x))} + 50*a^2*d^2*n^3*x + 19*b^2*d^2*n^2*x*e^{(2*n*\ln(x))} + 16*b^2*d^2*n^3*x*e^{(3*n*\ln(x) + 1)} + 48*a*b*d^2*n^3*x*e^{(2*n*\ln(x) + 1)} + 48*a^2*d^2*n^3*x*e^{(n*\ln(x) + 1)} + 52*a*b*d^2*n^2*x*e^{(n*\ln(x))} + 35*a^2*d^2*n^2*x + 8*b^2*d^2*n*x*e^{(2*n*\ln(x))} + 6*b^2*n^3*x*e^{(4*n*\ln(x) + 2)} + 16*a*b*n^3*x*e^{(3*n*\ln(x) + 2)} + 28*b^2*d^2*n^2*x*e^{(3*n*\ln(x) + 1)} + 12*a^2*n^3*x*e^{(2*n*\ln(x) + 2)} + 76*a*b*d^2*n^2*x*e^{(2*n*\ln(x) + 1)} + 52*a^2*d^2*n^2*x*e^{(n*\ln(x) + 1)} + 18*a*b*d^2*n*x*e^{(n*\ln(x))} + 10*a^2*d^2*n*x + b^2*d^2*x*e^{(2*n*\ln(x))} + 11*b^2*n^2*x*e^{(4*n*\ln(x) + 2)} + 28*a*b*n^2*x*e^{(3*n*\ln(x) + 2)} + 14*b^2*d^2*n*x*e^{(3*n*\ln(x) + 1)} + 19*a^2*n^2*x*e^{(2*n*\ln(x) + 2)} + 32*a*b*d^2*n*x*e^{(2*n*\ln(x) + 1)} + 18*a^2*d^2*n*x*e^{(n*\ln(x) + 1)} + 2*a*b*d^2*x*e^{(n*\ln(x))} + a^2*d^2*x + 6*b^2*n*x*e^{(4*n*\ln(x) + 2)} + 14*a*b*n*x*e^{(3*n*\ln(x) + 2)} + 2*b^2*d^2*x*e^{(3*n*\ln(x) + 1)} + 8*a^2*n*x*e^{(2*n*\ln(x) + 2)} + 4*a*b*d^2*x*e^{(2*n*\ln(x) + 1)} + 2*a^2*d^2*x*e^{(n*\ln(x) + 1)} + b^2*x*e^{(4*n*\ln(x) + 2)} + 2*a*b*x*e^{(3*n*\ln(x) + 2)} + a^2*x*e^{(2*n*\ln(x) + 2)})/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)$

### 3.195 $\int (a + bx^n)^2 (c + dx^n) dx$

**Optimal.** Leaf size=70

$$a^2cx + \frac{ax^{n+1}(ad + 2bc)}{n + 1} + \frac{bx^{2n+1}(2ad + bc)}{2n + 1} + \frac{b^2dx^{3n+1}}{3n + 1}$$

[Out]  $a^2c*x + (a*(2*b*c + a*d)*x^{(1 + n)})/(1 + n) + (b*(b*c + 2*a*d)*x^{(1 + 2*n)})/(1 + 2*n) + (b^2*d*x^{(1 + 3*n)})/(1 + 3*n)$

**Rubi [A]** time = 0.114034, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$a^2cx + \frac{ax^{n+1}(ad + 2bc)}{n + 1} + \frac{bx^{2n+1}(2ad + bc)}{2n + 1} + \frac{b^2dx^{3n+1}}{3n + 1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^2\*(c + d\*x^n), x]

[Out]  $a^2c*x + (a*(2*b*c + a*d)*x^{(1 + n)})/(1 + n) + (b*(b*c + 2*a*d)*x^{(1 + 2*n)})/(1 + 2*n) + (b^2*d*x^{(1 + 3*n)})/(1 + 3*n)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$a^2 \int c dx + \frac{ax^{n+1}(ad + 2bc)}{n + 1} + \frac{b^2dx^{3n+1}}{3n + 1} + \frac{bx^{2n+1}(2ad + bc)}{2n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*2\*(c+d\*x\*\*n), x)

[Out]  $a**2*Integral(c, x) + a*x**(n + 1)*(a*d + 2*b*c)/(n + 1) + b**2*d*x**(3*n + 1)/(3*n + 1) + b*x**(2*n + 1)*(2*a*d + b*c)/(2*n + 1)$

**Mathematica [A]** time = 0.110819, size = 65, normalized size = 0.93

$$x \left( a^2c + \frac{bx^{2n}(2ad + bc)}{2n + 1} + \frac{ax^n(ad + 2bc)}{n + 1} + \frac{b^2dx^{3n}}{3n + 1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^2\*(c + d\*x^n), x]

[Out]  $x*(a^2*c + (a*(2*b*c + a*d)*x^n)/(1 + n) + (b*(b*c + 2*a*d)*x^{(2*n)})/(1 + 2*n) + (b^2*d*x^{(3*n)})/(1 + 3*n))$

**Maple [A]** time = 0.014, size = 74, normalized size = 1.1

$$a^2cx + \frac{a(ad + 2bc)xe^{n \ln(x)}}{1 + n} + \frac{b(2ad + bc)x \left( e^{n \ln(x)} \right)^2}{1 + 2n} + \frac{b^2dx \left( e^{n \ln(x)} \right)^3}{1 + 3n}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((a+b*x^n)^2*(c+d*x^n),x)`

[Out]  $a^2c*x+a*(a*d+2*b*c)/(1+n)*x*\exp(n*\ln(x))+b*(2*a*d+b*c)/(1+2*n)*x*\exp(n*\ln(x))^2+b^2*d/(1+3*n)*x*\exp(n*\ln(x))^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^2*(d*x^n + c),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.254831, size = 236, normalized size = 3.37

$$\frac{(2b^2dn^2 + 3b^2dn + b^2d)xx^{3n} + (b^2c + 2abd + 3(b^2c + 2abd)n^2 + 4(b^2c + 2abd)n)xx^{2n} + (2abc + a^2d + 6(2abc + a^2d))}{6n^3 + 11n^2 + 6n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^2*(d*x^n + c),x, algorithm="fricas")`

[Out]  $((2*b^2*d*n^2 + 3*b^2*d*n + b^2*d)*x*x^{(3*n)} + (b^2*c + 2*a*b*d + 3*(b^2*c + 2*a*b*d)*n^2 + 4*(b^2*c + 2*a*b*d)*n)*x*x^{(2*n)} + (2*a*b*c + a^2*d + 6*(2*a*b*c + a^2*d)*n^2 + 5*(2*a*b*c + a^2*d)*n)*x*x^n + (6*a^2*c*n^3 + 11*a^2*c*n^2 + 6*a^2*c*n + a^2*c)*x)/(6*n^3 + 11*n^2 + 6*n + 1)$

**Sympy [A]** time = 2.77067, size = 726, normalized size = 10.37

$$\left\{ \begin{array}{l} a^2cx + a^2d \log(x) + 2abc \log(x) - \frac{2abd}{x} - \frac{b^2c}{x} - \frac{b^2d}{2x^2} \\ a^2cx + 2a^2d\sqrt{x} + 4abc\sqrt{x} + 2abd \log(x) + b^2c \log(x) - \frac{2b^2d}{\sqrt{x}} \\ a^2cx + \frac{3a^2dx^{\frac{3}{2}}}{2} + 3abcx^{\frac{3}{2}} + 6abd\sqrt[3]{x} + 3b^2c\sqrt[3]{x} + b^2d \log(x) \end{array} \right. \\ \frac{6a^2cn^3x}{6n^3+11n^2+6n+1} + \frac{11a^2cn^2x}{6n^3+11n^2+6n+1} + \frac{6a^2cnx}{6n^3+11n^2+6n+1} + \frac{a^2cx}{6n^3+11n^2+6n+1} + \frac{6a^2dn^2xx^n}{6n^3+11n^2+6n+1} + \frac{5a^2dnxx^n}{6n^3+11n^2+6n+1} + \frac{a^2dxx^n}{6n^3+11n^2+6n+1} + \frac{12abcn^2xx^n}{6n^3+11n^2+6n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n)**2*(c+d*x**n),x)`

[Out]  $\text{Piecewise}((a**2*c*x + a**2*d*\log(x) + 2*a*b*c*\log(x) - 2*a*b*d/x - b**2*c/x - b**2*d/(2*x**2), \text{Eq}(n, -1)), (a**2*c*x + 2*a**2*d*\text{sqrt}(x) + 4*a*b*c*\text{sqrt}(x) + 2*a*b*d*\log(x) + b**2*c*\log(x) - 2*b**2*d/\text{sqrt}(x), \text{Eq}(n, -1/2)), (a**2*c*x + 3*a**2*d*x**(2/3)/2 + 3*a*b*c*x**(2/3) + 6*a*b*d*x**(1/3) + 3*b**2*c*x**(1/3) + b**2*d*\log(x), \text{Eq}(n, -1/3)), (6*a**2*c*n**3*x/(6*n**3 + 11*n**2 + 6*n + 1) + 11*a**2*c*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a**2*c*n*x/(6*n**3 + 11*n**2 + 6*n + 1) + a**2*c*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a**2*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*a**2*d*n*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + a**2*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 12*a*b*c*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 2*a*b*c*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*b*d*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*a*b*d*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*a*b*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b**2*c*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*b**2*c*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b**2*c*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1))$

```

3 + 11*n**2 + 6*n + 1) + 2*b**2*d*n**2*x*x**(3*n)/(6*n**3 + 11*n*
**2 + 6*n + 1) + 3*b**2*d*n*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1
) + b**2*d*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1), True))

```

---

**GIAC/XCAS [A]** time = 0.219823, size = 342, normalized size = 4.89

$$6 a^2 c n^3 x + 2 b^2 d n^2 x e^{(3 n \ln(x))} + 3 b^2 c n^2 x e^{(2 n \ln(x))} + 6 a b d n^2 x e^{(2 n \ln(x))} + 12 a b c n^2 x e^{(n \ln(x))} + 6 a^2 d n^2 x e^{(n \ln(x))} + 11 a^2 c n^2 x + 3$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((b*x^n + a)^2*(d*x^n + c),x, algorithm="giac")

```

```

[Out] (6*a^2*c*n^3*x + 2*b^2*d*n^2*x*e^(3*n*ln(x)) + 3*b^2*c*n^2*x*e^(2
*n*ln(x)) + 6*a*b*d*n^2*x*e^(2*n*ln(x)) + 12*a*b*c*n^2*x*e^(n*ln(
x)) + 6*a^2*d*n^2*x*e^(n*ln(x)) + 11*a^2*c*n^2*x + 3*b^2*d*n*x*e^
(3*n*ln(x)) + 4*b^2*c*n*x*e^(2*n*ln(x)) + 8*a*b*d*n*x*e^(2*n*ln(x
)) + 10*a*b*c*n*x*e^(n*ln(x)) + 5*a^2*d*n*x*e^(n*ln(x)) + 6*a^2*c
*n*x + b^2*d*x*e^(3*n*ln(x)) + b^2*c*x*e^(2*n*ln(x)) + 2*a*b*d*x*
e^(2*n*ln(x)) + 2*a*b*c*x*e^(n*ln(x)) + a^2*d*x*e^(n*ln(x)) + a^2
*c*x)/(6*n^3 + 11*n^2 + 6*n + 1)

```

$$3.196 \quad \int \frac{(a+bx^n)^2}{c+dx^n} dx$$

**Optimal.** Leaf size=84

$$\frac{x(bc-ad)^2 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{cd^2} - \frac{bx(bc(n+1) - ad(2n+1))}{d^2(n+1)} + \frac{bx(a+bx^n)}{d(n+1)}$$

[Out]  $-\left(\frac{b^2c(1+n) - a^2d(1+2n)x}{d^2(1+n)}\right) + \frac{bx(a+bx^n)}{d(n+1)} + \frac{(b^2c - a^2d)^2 x \operatorname{Hypergeometric2F1}\left[1, n^{(-1)}, 1 + n^{(-1)}, -\left(\frac{dx^n}{c}\right)\right]}{c^2 d^2}$

**Rubi [A]** time = 0.21881, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{x(bc-ad)^2 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{cd^2} - \frac{bx(bc(n+1) - ad(2n+1))}{d^2(n+1)} + \frac{bx(a+bx^n)}{d(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^2/(c + d\*x^n), x]

[Out]  $-\left(\frac{b^2c(1+n) - a^2d(1+2n)x}{d^2(1+n)}\right) + \frac{bx(a+bx^n)}{d(n+1)} + \frac{(b^2c - a^2d)^2 x \operatorname{Hypergeometric2F1}\left[1, n^{(-1)}, 1 + n^{(-1)}, -\left(\frac{dx^n}{c}\right)\right]}{c^2 d^2}$

**Rubi in Sympy [A]** time = 22.8338, size = 70, normalized size = 0.83

$$\frac{bx(a+bx^n)}{d(n+1)} + \frac{bx(ad(2n+1) - bc(n+1))}{d^2(n+1)} + \frac{x(ad-bc)^2 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*2/(c+d\*x\*\*n), x)

[Out]  $\frac{bx(a+bx^n)}{d(n+1)} + \frac{bx(ad(2n+1) - bc(n+1))}{d^2(n+1)} + \frac{x(ad-bc)^2 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{cd^2}$

**Mathematica [A]** time = 0.069443, size = 82, normalized size = 0.98

$$\frac{a^2x}{c} + \frac{x(ad-bc)^2 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{cd^2} - \frac{x(bc-ad)^2}{cd^2} + \frac{b^2x^{n+1}}{d(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^2/(c + d\*x^n), x]

[Out]  $\frac{a^2x}{c} - \left(\frac{b^2c - a^2d}{d^2}\right) \frac{x}{c} + \frac{b^2x^{n+1}}{d(n+1)} + \frac{(-b^2c + a^2d)^2 x \operatorname{Hypergeometric2F1}\left[1, n^{(-1)}, 1 + n^{(-1)}, -\left(\frac{dx^n}{c}\right)\right]}{c^2 d^2}$

**Maple [F]** time = 0.062, size = 0, normalized size = 0.

$$\int \frac{(a + bx^n)^2}{c + dx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)^2/(c+d\*x^n), x)

[Out] int((a+b\*x^n)^2/(c+d\*x^n), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$(b^2c^2 - 2abcd + a^2d^2) \int \frac{1}{d^3x^n + cd^2} dx + \frac{b^2dxx^n - (b^2c(n+1) - 2abd(n+1))x}{d^2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^2/(d\*x^n + c), x, algorithm="maxima")

[Out] (b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*integrate(1/(d^3\*x^n + c\*d^2), x) + (b^2\*d\*x\*x^n - (b^2\*c\*(n+1) - 2\*a\*b\*d\*(n+1))\*x)/(d^2\*(n+1))

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^{2n} + 2abx^n + a^2}{dx^n + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^2/(d\*x^n + c), x, algorithm="fricas")

[Out] integral((b^2\*x^(2\*n) + 2\*a\*b\*x^n + a^2)/(d\*x^n + c), x)

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*2/(c+d\*x\*\*n), x)

[Out] Exception raised: TypeError

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^2}{dx^n + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^2/(d\*x^n + c), x, algorithm="giac")

[Out] integrate((b\*x^n + a)^2/(d\*x^n + c), x)

$$3.197 \quad \int \frac{(a+bx^n)^2}{(c+dx^n)^2} dx$$

**Optimal.** Leaf size=115

$$\frac{x(bc-ad)(ad(1-n)-bc(n+1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2 d^2 n} - \frac{bx(ad-bc(n+1))}{cd^2 n} - \frac{x(bc-ad)(a+bx^n)}{cdn(c+dx^n)}$$

[Out]  $-\left(\frac{b^*(a*d - b*c*(1+n))*x}{(c*d^{2*n})} - \frac{(b*c - a*d)*x*(a + b*x^n)}{(c*d^n*(c + d*x^n))} + \frac{(b*c - a*d)*(a*d*(1-n) - b*c*(1+n))}{c^2 d^2 n} \right) * x * \text{Hypergeometric2F1}\left[1, n^{(-1)}, 1 + n^{(-1)}, -\left(\frac{d*x^n}{c}\right)\right] / (c^2 d^2 n)$

**Rubi [A]** time = 0.244695, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{x(bc-ad)(ad(1-n)-bc(n+1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2 d^2 n} - \frac{bx(ad-bc(n+1))}{cd^2 n} - \frac{x(bc-ad)(a+bx^n)}{cdn(c+dx^n)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^2/(c + d\*x^n)^2, x]

[Out]  $-\left(\frac{b^*(a*d - b*c*(1+n))*x}{(c*d^{2*n})} - \frac{(b*c - a*d)*x*(a + b*x^n)}{(c*d^n*(c + d*x^n))} + \frac{(b*c - a*d)*(a*d*(1-n) - b*c*(1+n))}{c^2 d^2 n} \right) * x * \text{Hypergeometric2F1}\left[1, n^{(-1)}, 1 + n^{(-1)}, -\left(\frac{d*x^n}{c}\right)\right] / (c^2 d^2 n)$

**Rubi in Sympy [A]** time = 23.0974, size = 95, normalized size = 0.83

$$-\frac{bx(ad-bc(n+1))}{cd^2 n} + \frac{x(a+bx^n)(ad-bc)}{cdn(c+dx^n)} - \frac{x(ad-bc)(-adn+ad-bcn-bc) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2 d^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*2/(c+d\*x\*\*n)\*\*2, x)

[Out]  $-b*x*(a*d - b*c*(n+1))/(c*d^{2*n}) + x*(a + b*x^n)*(a*d - b*c)/(c*d^n*(c + d*x^n)) - x*(a*d - b*c)*(-a*d*n + a*d - b*c*n - b*c) * \text{hyper}((1, 1/n), (1 + 1/n), -d*x^n/c)/(c^2 d^2 n)$

**Mathematica [A]** time = 0.190573, size = 95, normalized size = 0.83

$$\frac{x\left(\frac{c(a^2 d^2 - 2abcd + b^2 c(cn + c + dnx^n))}{c + dx^n} - (bc - ad)(ad(n - 1) + bc(n + 1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)\right)}{c^2 d^2 n}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^2/(c + d\*x^n)^2, x]

[Out]  $(x*((c*(-2*a*b*c*d + a^2*d^2 + b^2*c*(c + c*n + d*n*x^n)))/(c + d*x^n) - (b*c - a*d)*(a*d*(-1 + n) + b*c*(1 + n)) * \text{Hypergeometric2F1}\left[1, n^{(-1)}, 1 + n^{(-1)}, -\left(\frac{d*x^n}{c}\right)\right]) / (c^2 d^2 n)$

---

**Maple [F]** time = 0.067, size = 0, normalized size = 0.

$$\int \frac{(a + bx^n)^2}{(c + dx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)^2/(c+d\*x^n)^2, x)

[Out] int((a+b\*x^n)^2/(c+d\*x^n)^2, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-(b^2c^2(n+1) - a^2d^2(n-1) - 2abcd) \int \frac{1}{cd^3nx^n + c^2d^2n} dx + \frac{b^2cdnxx^n + (b^2c^2(n+1) - 2abcd + a^2d^2)x}{cd^3nx^n + c^2d^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^2/(d\*x^n + c)^2, x, algorithm="maxima")

[Out] -(b^2\*c^2\*(n+1) - a^2\*d^2\*(n-1) - 2\*a\*b\*c\*d)\*integrate(1/(c\*d^3\*n\*x^n + c^2\*d^2\*n), x) + (b^2\*c\*d\*n\*x\*x^n + (b^2\*c^2\*(n+1) - 2\*a\*b\*c\*d + a^2\*d^2)\*x)/(c\*d^3\*n\*x^n + c^2\*d^2\*n)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^{2n} + 2abx^n + a^2}{d^2x^{2n} + 2cdx^n + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^2/(d\*x^n + c)^2, x, algorithm="fricas")

[Out] integral((b^2\*x^(2\*n) + 2\*a\*b\*x^n + a^2)/(d^2\*x^(2\*n) + 2\*c\*d\*x^n + c^2), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^n)^2}{(c + dx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*2/(c+d\*x\*\*n)\*\*2, x)

[Out] Integral((a + b\*x\*\*n)\*\*2/(c + d\*x\*\*n)\*\*2, x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^2}{(dx^n + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n + a)^2/(d*x^n + c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2/(d*x^n + c)^2, x)
```

$$3.198 \quad \int \frac{(a+bx^n)^2}{(c+dx^n)^3} dx$$

**Optimal.** Leaf size=160

$$\frac{x(-a^2d^2(2n^2-3n+1)+2abcd(1-n)-b^2c^2(n+1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3d^2n^2} + \frac{x(bc-ad)(ad(1-2n)-bc(n+1))}{2c^2d^2n^2(c+dx^n)} - \frac{x(bc-ad)(a+bx^n)}{2cdn(c+dx^n)^2}$$

[Out]  $-\frac{((b*c - a*d)*x*(a + b*x^n))/(2*c*d*n*(c + d*x^n)^2) + ((b*c - a*d)*(a*d*(1 - 2*n) - b*c*(1 + n))*x)/(2*c^2*d^2*n^2*(c + d*x^n)) - ((2*a*b*c*d*(1 - n) - b^2*c^2*(1 + n) - a^2*d^2*(1 - 3*n + 2*n^2)) * x * \text{Hypergeometric2F1}[1, n^(-1), 1 + n^(-1), -(d*x^n)/c])}{(2*c^3*d^2*n^2)}$

**Rubi [A]** time = 0.359242, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{x(-a^2d^2(2n^2-3n+1)+2abcd(1-n)-b^2c^2(n+1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3d^2n^2} + \frac{x(bc-ad)(ad(1-2n)-bc(n+1))}{2c^2d^2n^2(c+dx^n)} - \frac{x(bc-ad)(a+bx^n)}{2cdn(c+dx^n)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^2/(c + d\*x^n)^3, x]

[Out]  $-\frac{((b*c - a*d)*x*(a + b*x^n))/(2*c*d*n*(c + d*x^n)^2) + ((b*c - a*d)*(a*d*(1 - 2*n) - b*c*(1 + n))*x)/(2*c^2*d^2*n^2*(c + d*x^n)) - ((2*a*b*c*d*(1 - n) - b^2*c^2*(1 + n) - a^2*d^2*(1 - 3*n + 2*n^2)) * x * \text{Hypergeometric2F1}[1, n^(-1), 1 + n^(-1), -(d*x^n)/c])}{(2*c^3*d^2*n^2)}$

**Rubi in Sympy [A]** time = 27.9249, size = 141, normalized size = 0.88

$$\frac{x(a+bx^n)(ad-bc)}{2cdn(c+dx^n)^2} - \frac{x(ad-bc)(-2adn+ad-bcn-bc)}{2c^2d^2n^2(c+dx^n)} - \frac{x(-ad(-n+1)(-2adn+ad-bc)+bc(ad(-n+1)-bc(n+1))) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3d^2n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*2/(c+d\*x\*\*n)\*\*3, x)

[Out]  $x*(a + b*x**n)*(a*d - b*c)/(2*c*d*n*(c + d*x**n)**2) - x*(a*d - b*c)*(-2*a*d*n + a*d - b*c*n - b*c)/(2*c**2*d**2*n**2*(c + d*x**n)) - x*(-a*d*(-n + 1)*(-2*a*d*n + a*d - b*c) + b*c*(a*d*(-n + 1) - b*c*(n + 1)))*\text{hyper}((1, 1/n), (1 + 1/n, ), -d*x**n/c)/(2*c**3*d**2*n**2)$

**Mathematica [A]** time = 0.173422, size = 133, normalized size = 0.83

$$\frac{x\left((a^2d^2(2n^2-3n+1)+2abcd(n-1)+b^2c^2(n+1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{dx^n}{c}\right) + \frac{c^2n(bc-ad)^2}{(c+dx^n)^2} - \frac{c(bc-ad)(ad(2n-1)+b(2cn+c))}{c+dx^n}\right)}{2c^3d^2n^2}$$



Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^2/(c + d\*x^n)^3,x]

[Out] (x\*((c^2\*(b\*c - a\*d)^2\*n)/(c + d\*x^n)^2 - (c\*(b\*c - a\*d)\*(a\*d\*(-1 + 2\*n) + b\*(c + 2\*c\*n)))/(c + d\*x^n) + (2\*a\*b\*c\*d\*(-1 + n) + b^2\*c^2\*(1 + n) + a^2\*d^2\*(1 - 3\*n + 2\*n^2))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(d\*x^n)/c]))/(2\*c^3\*d^2\*n^2)

**Maple [F]** time = 0.079, size = 0, normalized size = 0.

$$\int \frac{(a + bx^n)^2}{(c + dx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)^2/(c+d\*x^n)^3,x)

[Out] int((a+b\*x^n)^2/(c+d\*x^n)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{((2n^2 - 3n + 1)a^2d^2 + b^2c^2(n + 1) + 2abcd(n - 1)) \int \frac{1}{2(c^2d^3n^2x^n + c^3d^2n^2)} dx - \frac{(b^2c^2d(2n + 1) - a^2d^3(2n - 1) - 2abcd^2)xx^n - (a^2cd^2(3n - 1) - b^2c^3(n + 1) - 2abc^2d(n - 1))x}{2(c^2d^4n^2x^{2n} + 2c^3d^3n^2x^n + c^4d^2n^2)}}{2(c^2d^4n^2x^{2n} + 2c^3d^3n^2x^n + c^4d^2n^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^2/(d\*x^n + c)^3,x, algorithm="maxima")

[Out] ((2\*n^2 - 3\*n + 1)\*a^2\*d^2 + b^2\*c^2\*(n + 1) + 2\*a\*b\*c\*d\*(n - 1)) \* integrate(1/2/(c^2\*d^3\*n^2\*x^n + c^3\*d^2\*n^2), x) - 1/2\*((b^2\*c^2\*d\*(2\*n + 1) - a^2\*d^3\*(2\*n - 1) - 2\*a\*b\*c\*d^2)\*x\*x^n - (a^2\*c\*d^2\*(3\*n - 1) - b^2\*c^3\*(n + 1) - 2\*a\*b\*c^2\*d\*(n - 1))\*x)/(c^2\*d^4\*n^2\*x^(2\*n) + 2\*c^3\*d^3\*n^2\*x^n + c^4\*d^2\*n^2)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^{2n} + 2abx^n + a^2}{d^3x^{3n} + 3cd^2x^{2n} + 3c^2dx^n + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^2/(d\*x^n + c)^3,x, algorithm="fricas")

[Out] integral((b^2\*x^(2\*n) + 2\*a\*b\*x^n + a^2)/(d^3\*x^(3\*n) + 3\*c\*d^2\*x^(2\*n) + 3\*c^2\*d\*x^n + c^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2/(c+d*x**n)**3,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^2}{(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n + a)^2/(d*x^n + c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2/(d*x^n + c)^3, x)
```

$$3.199 \quad \int \frac{(c+dx^n)^4}{a+bx^n} dx$$

**Optimal.** Leaf size=310

$$\frac{dx(c+dx^n)(-a^2d^2(6n^2+5n+1)+2abcd(3n+1)^2-b^2c^2(18n^2+7n+1))}{b^3(n+1)(2n+1)(3n+1)}$$

$$\frac{dx(a^3d^3(6n^3+11n^2+6n+1)-a^2bcd^2(24n^3+38n^2+19n+3)+ab^2c^2d(36n^3+45n^2+20n+3)-b^3c^3(24n^3+18n^2+6n+1))}{b^4(n+1)(2n+1)(3n+1)}$$

$$+\frac{x(bc-ad)^4 {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{ab^4} - \frac{dx(c+dx^n)^2(ad(3n+1)-b(6cn+c))}{b^2(6n^2+5n+1)} + \frac{dx(c+dx^n)^3}{b(3n+1)}$$

[Out]  $-\left(\left(d^3(a^3d^3(1+6n+11n^2+6n^3)-b^3c^3(1+7n+18n^2+24n^3))-a^2b^2cd^2(3+19n+38n^2+24n^3)+a^2b^2c^2d^2(3+20n+45n^2+36n^3)\right)x\right)/(b^4(1+n)^2(1+3n)) - \left(d^2(2abcd(1+3n)^2-a^2d^2(1+5n+6n^2)-b^2c^2(1+7n+18n^2))x^2(c+dx^n)\right)/(b^3(1+n)^2(1+3n)) - \left(d^2(a^2d(1+3n)-b^2(c+6cn))x^2(c+dx^n)^2\right)/(b^2(1+5n+6n^2)) + \left(d^2x^2(c+dx^n)^3\right)/(b(1+3n)) + \left((b^3c-a^2d)^4x^4\text{Hypergeometric2F1}[1, n^{-1}, 1+n^{-1}, -(bx^n/a)]\right)/(a^4b^4)$

**Rubi [A]** time = 1.12801, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{dx(c+dx^n)(-a^2d^2(6n^2+5n+1)+2abcd(3n+1)^2-b^2c^2(18n^2+7n+1))}{b^3(n+1)(2n+1)(3n+1)}$$

$$\frac{dx(a^3d^3(6n^3+11n^2+6n+1)-a^2bcd^2(24n^3+38n^2+19n+3)+ab^2c^2d(36n^3+45n^2+20n+3)-b^3c^3(24n^3+18n^2+6n+1))}{b^4(n+1)(2n+1)(3n+1)}$$

$$+\frac{x(bc-ad)^4 {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{ab^4} - \frac{dx(c+dx^n)^2(ad(3n+1)-b(6cn+c))}{b^2(6n^2+5n+1)} + \frac{dx(c+dx^n)^3}{b(3n+1)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^4/(a + b\*x^n), x]

[Out]  $-\left(\left(d^3(a^3d^3(1+6n+11n^2+6n^3)-b^3c^3(1+7n+18n^2+24n^3))-a^2b^2cd^2(3+19n+38n^2+24n^3)+a^2b^2c^2d^2(3+20n+45n^2+36n^3)\right)x\right)/(b^4(1+n)^2(1+3n)) - \left(d^2(2abcd(1+3n)^2-a^2d^2(1+5n+6n^2)-b^2c^2(1+7n+18n^2))x^2(c+dx^n)\right)/(b^3(1+n)^2(1+3n)) - \left(d^2(a^2d(1+3n)-b^2(c+6cn))x^2(c+dx^n)^2\right)/(b^2(1+5n+6n^2)) + \left(d^2x^2(c+dx^n)^3\right)/(b(1+3n)) + \left((b^3c-a^2d)^4x^4\text{Hypergeometric2F1}[1, n^{-1}, 1+n^{-1}, -(bx^n/a)]\right)/(a^4b^4)$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d\*x\*\*n)\*\*4/(a+b\*x\*\*n), x)

[Out] Timed out

**Mathematica [A]** time = 0.185418, size = 146, normalized size = 0.47

$$x \left( \frac{d^2 x^n (a^2 d^2 - 4abcd + 6b^2 c^2)}{b^3 (n+1)} + \frac{(bc - ad)^4 {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a} \right)}{ab^4} \right. \\ \left. - \frac{(bc - ad)^4}{ab^4} + \frac{d^3 x^{2n} (4bc - ad)}{b^2 (2n+1)} + \frac{c^4}{a} + \frac{d^4 x^{3n}}{3bn + b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^4/(a + b\*x^n), x]

[Out] x\*(c^4/a - (b\*c - a\*d)^4/(a\*b^4) + (d^2\*(6\*b^2\*c^2 - 4\*a\*b\*c\*d + a^2\*d^2)\*x^n)/(b^3\*(1 + n)) + (d^3\*(4\*b\*c - a\*d)\*x^(2\*n))/(b^2\*(1 + 2\*n)) + (d^4\*x^(3\*n))/(b + 3\*b\*n) + ((b\*c - a\*d)^4\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n)/a])/(a\*b^4))

**Maple [F]** time = 0.072, size = 0, normalized size = 0.

$$\int \frac{(c + dx^n)^4}{a + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d\*x^n)^4/(a+b\*x^n), x)

[Out] int((c+d\*x^n)^4/(a+b\*x^n), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$(b^4 c^4 - 4 ab^3 c^3 d + 6 a^2 b^2 c^2 d^2 - 4 a^3 b c d^3 + a^4 d^4) \int \frac{1}{b^5 x^n + ab^4} dx \\ + \frac{(2n^2 + 3n + 1)b^3 d^4 x x^{3n} + (4(3n^2 + 4n + 1)b^3 c d^3 - (3n^2 + 4n + 1)ab^2 d^4) x x^{2n} + (6(6n^2 + 5n + 1)b^3 c^2 d^2 - 4(6n^2 + 5n + 1)ab^2 c d + a^2 d^4) x x^n + a^4 d^4}{(6n^3 + 11n^2 + 6n + 1)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^4/(b\*x^n + a), x, algorithm="maxima")

[Out] (b^4\*c^4 - 4\*a\*b^3\*c^3\*d + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a^3\*b\*c\*d^3 + a^4\*d^4)\*integrate(1/(b^5\*x^n + a\*b^4), x) + ((2\*n^2 + 3\*n + 1)\*b^3\*d^4\*x\*x^(3\*n) + (4\*(3\*n^2 + 4\*n + 1)\*b^3\*c\*d^3 - (3\*n^2 + 4\*n + 1)\*a\*b^2\*d^4)\*x\*x^(2\*n) + (6\*(6\*n^2 + 5\*n + 1)\*b^3\*c^2\*d^2 - 4\*(6\*n^2 + 5\*n + 1)\*a\*b^2\*c\*d + a^2\*d^4)\*x\*x^n + (4\*(6\*n^3 + 11\*n^2 + 6\*n + 1)\*b^3\*c^3\*d - 6\*(6\*n^3 + 11\*n^2 + 6\*n + 1)\*a\*b^2\*c^2\*d^2 + 4\*(6\*n^3 + 11\*n^2 + 6\*n + 1)\*a^2\*b\*c\*d^3 - (6\*n^3 + 11\*n^2 + 6\*n + 1)\*a^3\*d^4)\*x)/((6\*n^3 + 11\*n^2 + 6\*n + 1)\*b^4)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{d^4 x^{4n} + 4cd^3 x^{3n} + 6c^2 d^2 x^{2n} + 4c^3 dx^n + c^4}{bx^n + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^4/(b\*x^n + a),x, algorithm="fricas")

[Out] integral((d^4\*x^(4\*n) + 4\*c\*d^3\*x^(3\*n) + 6\*c^2\*d^2\*x^(2\*n) + 4\*c^3\*d\*x^n + c^4)/(b\*x^n + a), x)

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*n)\*\*4/(a+b\*x\*\*n),x)

[Out] Exception raised: TypeError

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^4}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^4/(b\*x^n + a),x, algorithm="giac")

[Out] integrate((d\*x^n + c)^4/(b\*x^n + a), x)

$$3.200 \quad \int \frac{(c+dx^n)^3}{a+bx^n} dx$$

**Optimal.** Leaf size=173

$$\frac{dx (a^2 d^2 (2n^2 + 3n + 1) - abcd (6n^2 + 7n + 2) + b^2 c^2 (6n^2 + 4n + 1))}{b^3 (n + 1)(2n + 1)} + \frac{x(bc - ad)^3 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{ab^3} - \frac{dx(c + dx^n)(ad(2n + 1) - b(4cn + c))}{b^2(n + 1)(2n + 1)} + \frac{dx(c + dx^n)^2}{b(2n + 1)}$$

[Out]  $(d^*(a^2*d^2*(1 + 3*n + 2*n^2) + b^2*c^2*(1 + 4*n + 6*n^2) - a*b*c*d*(2 + 7*n + 6*n^2))*x)/(b^3*(1 + n)*(1 + 2*n)) - (d*(a*d*(1 + 2*n) - b*(c + 4*c*n))*x*(c + d*x^n))/(b^2*(1 + n)*(1 + 2*n)) + (d*x*(c + d*x^n)^2)/(b*(1 + 2*n)) + ((b*c - a*d)^3*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b*x^n)/a])/ (a*b^3)$

**Rubi [A]** time = 0.614258, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{dx (a^2 d^2 (2n^2 + 3n + 1) - abcd (6n^2 + 7n + 2) + b^2 c^2 (6n^2 + 4n + 1))}{b^3 (n + 1)(2n + 1)} + \frac{x(bc - ad)^3 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{ab^3} - \frac{dx(c + dx^n)(ad(2n + 1) - b(4cn + c))}{b^2(n + 1)(2n + 1)} + \frac{dx(c + dx^n)^2}{b(2n + 1)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^3/(a + b\*x^n), x]

[Out]  $(d*(a^2*d^2*(1 + 3*n + 2*n^2) + b^2*c^2*(1 + 4*n + 6*n^2) - a*b*c*d*(2 + 7*n + 6*n^2))*x)/(b^3*(1 + n)*(1 + 2*n)) - (d*(a*d*(1 + 2*n) - b*(c + 4*c*n))*x*(c + d*x^n))/(b^2*(1 + n)*(1 + 2*n)) + (d*x*(c + d*x^n)^2)/(b*(1 + 2*n)) + ((b*c - a*d)^3*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b*x^n)/a])/ (a*b^3)$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d\*x\*\*n)\*\*3/(a+b\*x\*\*n), x)

[Out] Timed out

**Mathematica [A]** time = 0.50718, size = 104, normalized size = 0.6

$$x \left( \frac{(bc - ad)^3 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{ab^3} + \frac{(ad - bc)^3}{ab^3} + \frac{d^2 x^n (3bc - ad)}{b^2(n + 1)} + \frac{c^3}{a} + \frac{d^3 x^{2n}}{2bn + b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^3/(a + b\*x^n), x]

[Out]  $x*(c^3/a + (-b*c) + a*d)^3/(a*b^3) + (d^2*(3*b*c - a*d)*x^n)/(b^2*(1 + n)) + (d^3*x^(2*n))/(b + 2*b*n) + ((b*c - a*d)^3*Hypergeom$

etric2F1[1, n^(-1), 1 + n^(-1), -((b\*x^n)/a)]/(a\*b^3))

**Maple [F]** time = 0.068, size = 0, normalized size = 0.

$$\int \frac{(c + dx^n)^3}{a + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d\*x^n)^3/(a+b\*x^n), x)

[Out] int((c+d\*x^n)^3/(a+b\*x^n), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3) \int \frac{1}{b^4x^n + ab^3} dx + \frac{b^2d^3(n+1)xx^{2n} + (3b^2cd^2(2n+1) - abd^3(2n+1))xx^n + (3(2n^2 + 3n + 1)b^2c^2d - 3(2n^2 + 3n + 1)abcd^2 + (2n^2 + 3n + 1)a^2d^3)}{(2n^2 + 3n + 1)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^3/(b\*x^n + a), x, algorithm="maxima")

[Out] (b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)\*integrate(1/(b^4\*x^n + a\*b^3), x) + (b^2\*d^3\*(n + 1)\*x\*x^(2\*n) + (3\*b^2\*c\*d^2\*(2\*n + 1) - a\*b\*d^3\*(2\*n + 1))\*x\*x^n + (3\*(2\*n^2 + 3\*n + 1)\*b^2\*c^2\*d - 3\*(2\*n^2 + 3\*n + 1)\*a\*b\*c\*d^2 + (2\*n^2 + 3\*n + 1)\*a^2\*d^3)\*x)/((2\*n^2 + 3\*n + 1)\*b^3)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^3x^{3n} + 3cd^2x^{2n} + 3c^2dx^n + c^3}{bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^3/(b\*x^n + a), x, algorithm="fricas")

[Out] integral((d^3\*x^(3\*n) + 3\*c\*d^2\*x^(2\*n) + 3\*c^2\*d\*x^n + c^3)/(b\*x^n + a), x)

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*n)\*\*3/(a+b\*x\*\*n), x)

[Out] Exception raised: TypeError

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^3}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^n + c)^3/(b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^3/(b*x^n + a), x)
```



$$3.201 \quad \int \frac{(c+dx^n)^2}{a+bx^n} dx$$

**Optimal.** Leaf size=84

$$\frac{x(bc-ad)^2 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{ab^2} - \frac{dx(ad(n+1) - b(2cn+c))}{b^2(n+1)} + \frac{dx(c+dx^n)}{b(n+1)}$$

[Out]  $-\left(\frac{d(a*d*(1+n) - b*(c + 2*c*n))*x}{(b^2*(1+n))} + \frac{d*x*(c + d*x^n)}{b*(1+n)} + \left(\frac{b*c - a*d}{a}\right)^2*x*\text{Hypergeometric2F1}\left[1, n^{(-1)}, 1 + n^{(-1)}, -\left(\frac{b*x^n}{a}\right)\right]\right)/(a*b^2)$

**Rubi [A]** time = 0.236371, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{x(bc-ad)^2 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{ab^2} - \frac{dx(ad(n+1) - b(2cn+c))}{b^2(n+1)} + \frac{dx(c+dx^n)}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^2/(a + b\*x^n), x]

[Out]  $-\left(\frac{d(a*d*(1+n) - b*(c + 2*c*n))*x}{(b^2*(1+n))} + \frac{d*x*(c + d*x^n)}{b*(1+n)} + \left(\frac{b*c - a*d}{a}\right)^2*x*\text{Hypergeometric2F1}\left[1, n^{(-1)}, 1 + n^{(-1)}, -\left(\frac{b*x^n}{a}\right)\right]\right)/(a*b^2)$

**Rubi in Sympy [A]** time = 22.1612, size = 70, normalized size = 0.83

$$\frac{dx(c+dx^n)}{b(n+1)} - \frac{dx(ad(n+1) - bc(2n+1))}{b^2(n+1)} + \frac{x(ad-bc)^2 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d\*x\*\*n)\*\*2/(a+b\*x\*\*n), x)

[Out]  $d*x*(c + d*x**n)/(b*(n + 1)) - d*x*(a*d*(n + 1) - b*c*(2*n + 1))/(b**2*(n + 1)) + x*(a*d - b*c)**2*hyper((1, 1/n), (1 + 1/n, ), -b*x**n/a)/(a*b**2)$

**Mathematica [A]** time = 0.0705425, size = 82, normalized size = 0.98

$$\frac{x(bc-ad)^2 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{ab^2} - \frac{x(ad-bc)^2}{ab^2} + \frac{c^2x}{a} + \frac{d^2x^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^2/(a + b\*x^n), x]

[Out]  $(c^2*x)/a - \left(\frac{-b*c + a*d}{a}\right)^2*x/(a*b^2) + \frac{d^2*x^{n+1}}{b*(1+n)} + \left(\frac{b*c - a*d}{a}\right)^2*x*\text{Hypergeometric2F1}\left[1, n^{(-1)}, 1 + n^{(-1)}, -\left(\frac{b*x^n}{a}\right)\right]\right)/(a*b^2)$

**Maple [F]** time = 0.062, size = 0, normalized size = 0.

$$\int \frac{(c + dx^n)^2}{a + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d\*x^n)^2/(a+b\*x^n), x)

[Out] int((c+d\*x^n)^2/(a+b\*x^n), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$(b^2c^2 - 2abcd + a^2d^2) \int \frac{1}{b^3x^n + ab^2} dx + \frac{bd^2xx^n + (2bcd(n+1) - ad^2(n+1))x}{b^2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^2/(b\*x^n + a), x, algorithm="maxima")

[Out] (b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*integrate(1/(b^3\*x^n + a\*b^2), x) + (b\*d^2\*x\*x^n + (2\*b\*c\*d\*(n+1) - a\*d^2\*(n+1))\*x)/(b^2\*(n+1))

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^2x^{2n} + 2cdx^n + c^2}{bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^2/(b\*x^n + a), x, algorithm="fricas")

[Out] integral((d^2\*x^(2\*n) + 2\*c\*d\*x^n + c^2)/(b\*x^n + a), x)

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*n)\*\*2/(a+b\*x\*\*n), x)

[Out] Exception raised: TypeError

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^2}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^2/(b\*x^n + a), x, algorithm="giac")

[Out] integrate((d\*x^n + c)^2/(b\*x^n + a), x)

$$3.202 \quad \int \frac{c+dx^n}{a+bx^n} dx$$

**Optimal.** Leaf size=42

$$\frac{x(bc-ad) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{ab} + \frac{dx}{b}$$

[Out] (d\*x)/b + ((b\*c - a\*d)\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n)/a])/(a\*b)

**Rubi [A]** time = 0.0525895, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x(bc-ad) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{ab} + \frac{dx}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)/(a + b\*x^n), x]

[Out] (d\*x)/b + ((b\*c - a\*d)\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n)/a])/(a\*b)

**Rubi in Sympy [A]** time = 6.35457, size = 31, normalized size = 0.74

$$\frac{dx}{b} - \frac{x(ad-bc) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d\*x\*\*n)/(a+b\*x\*\*n), x)

[Out] d\*x/b - x\*(a\*d - b\*c)\*hyper((1, 1/n), (1 + 1/n, ), -b\*x\*\*n/a)/(a\*b)

**Mathematica [A]** time = 0.0332002, size = 40, normalized size = 0.95

$$\frac{x\left((bc-ad) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) + ad\right)}{ab}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)/(a + b\*x^n), x]

[Out] (x\*(a\*d + (b\*c - a\*d)\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n)/a]))/(a\*b)

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int \frac{c + dx^n}{a + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c+d*x^n)/(a+b*x^n), x)`

[Out] `int((c+d*x^n)/(a+b*x^n), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$(bc - ad) \int \frac{1}{b^2 x^n + ab} dx + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)/(b*x^n + a), x, algorithm="maxima")`

[Out] `(b*c - a*d)*integrate(1/(b^2*x^n + a*b), x) + d*x/b`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^n + c}{bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)/(b*x^n + a), x, algorithm="fricas")`

[Out] `integral((d*x^n + c)/(b*x^n + a), x)`

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d*x**n)/(a+b*x**n), x)`

[Out] Exception raised: TypeError

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^n + c}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)/(b*x^n + a), x, algorithm="giac")`

[Out] `integrate((d*x^n + c)/(b*x^n + a), x)`

### 3.203 $\int \frac{1}{(a+bx^n)(c+dx^n)} dx$

**Optimal.** Leaf size=72

$$\frac{bx {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a(bc - ad)} - \frac{dx {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c(bc - ad)}$$

[Out] (b\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n)/a])/(a\*(b\*c - a\*d)) - (d\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(d\*x^n)/c])/(c\*(b\*c - a\*d))

**Rubi [A]** time = 0.0794435, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{bx {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a(bc - ad)} - \frac{dx {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^n)\*(c + d\*x^n)), x]

[Out] (b\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n)/a])/(a\*(b\*c - a\*d)) - (d\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(d\*x^n)/c])/(c\*(b\*c - a\*d))

**Rubi in Sympy [A]** time = 10.9658, size = 53, normalized size = 0.74

$$\frac{dx {}_2F_1\left(1, \frac{1}{n} \middle| -\frac{dx^n}{c}\right)}{c(ad - bc)} - \frac{bx {}_2F_1\left(1, \frac{1}{n} \middle| -\frac{bx^n}{a}\right)}{a(ad - bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a+b\*x\*\*n)/(c+d\*x\*\*n), x)

[Out] d\*x\*hyper((1, 1/n), (1 + 1/n, ), -d\*x\*\*n/c)/(c\*(a\*d - b\*c)) - b\*x\*hyper((1, 1/n), (1 + 1/n, ), -b\*x\*\*n/a)/(a\*(a\*d - b\*c))

**Mathematica [A]** time = 0.0545357, size = 64, normalized size = 0.89

$$\frac{x\left(ad {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right) - bc {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)\right)}{ac(ad - bc)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^n)\*(c + d\*x^n)), x]

[Out] (x\*(-(b\*c\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n)/a]) + a\*d\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(d\*x^n)/c]))/(a\*c\*(-(b\*c) + a\*d))

**Maple [F]** time = 0.091, size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^n)(c + dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*x^n)/(c+d\*x^n), x)

[Out] int(1/(a+b\*x^n)/(c+d\*x^n), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)(dx^n + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^n + a)\*(d\*x^n + c)), x, algorithm="maxima")

[Out] integrate(1/((b\*x^n + a)\*(d\*x^n + c)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bdx^{2n} + ac + (bc + ad)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^n + a)\*(d\*x^n + c)), x, algorithm="fricas")

[Out] integral(1/(b\*d\*x^(2\*n) + a\*c + (b\*c + a\*d)\*x^n), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^n)(c + dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x\*\*n)/(c+d\*x\*\*n), x)

[Out] Integral(1/((a + b\*x\*\*n)\*(c + d\*x\*\*n)), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)(dx^n + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^n + a)\*(d\*x^n + c)), x, algorithm="giac")

[Out] integrate(1/((b\*x^n + a)\*(d\*x^n + c)), x)

$$3.204 \quad \int \frac{1}{(a+bx^n)(c+dx^n)^2} dx$$

**Optimal.** Leaf size=123

$$\frac{b^2 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a(bc-ad)^2} + \frac{dx(bc(1-2n) - ad(1-n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2 n(bc-ad)^2} - \frac{dx}{cn(bc-ad)(c+dx^n)}$$

[Out]  $-\left(\frac{d*x}{c*(b*c - a*d)*n*(c + d*x^n)}\right) + (b^2*x*Hypergeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, -((b*x^n)/a)]/(a*(b*c - a*d)^2) + (d*(b*c*(1 - 2*n) - a*d*(1 - n))*x*Hypergeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, -((d*x^n)/c)]/(c^2*(b*c - a*d)^2*n)$

**Rubi [A]** time = 0.359577, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{b^2 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a(bc-ad)^2} - \frac{dx(ad(1-n) - b(c-2cn)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2 n(bc-ad)^2} - \frac{dx}{cn(bc-ad)(c+dx^n)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^n)\*(c + d\*x^n)^2), x]

[Out]  $-\left(\frac{d*x}{c*(b*c - a*d)*n*(c + d*x^n)}\right) + (b^2*x*Hypergeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, -((b*x^n)/a)]/(a*(b*c - a*d)^2) - (d*(a*d*(1 - n) - b*(c - 2*c*n))*x*Hypergeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, -((d*x^n)/c)]/(c^2*(b*c - a*d)^2*n)$

**Rubi in Sympy [A]** time = 48.4865, size = 100, normalized size = 0.81

$$\frac{dx}{cn(c+dx^n)(ad-bc)} - \frac{dx(-adn + ad + 2bcn - bc) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2 n(ad-bc)^2} + \frac{b^2 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a+b\*x\*\*n)/(c+d\*x\*\*n)\*\*2, x)

[Out]  $d*x/(c*n*(c + d*x**n)*(a*d - b*c)) - d*x*(-a*d*n + a*d + 2*b*c*n - b*c)*hyper((1, 1/n), (1 + 1/n), -d*x**n/c)/(c**2*n*(a*d - b*c)**2) + b**2*x*hyper((1, 1/n), (1 + 1/n), -b*x**n/a)/(a*(a*d - b*c)**2)$

**Mathematica [A]** time = 0.217176, size = 121, normalized size = 0.98

$$\frac{x \left( b^2 c^2 n (c + dx^n) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) + ad \left( (c + dx^n)(ad(n-1) + b(c-2cn)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right) + c(ad-bc) \right) \right)}{ac^2 n(bc-ad)^2 (c+dx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^n)\*(c + d\*x^n)^2), x]

[Out]  $(x*(b^2*c^2*n*(c + d*x^n)*Hypergeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, -((b*x^n)/a)] + a*d*(c*(-b*c) + a*d) + (a*d*(-1 + n) + b*(c - 2*c*n))*(c + d*x^n)*Hypergeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, -((d*x^n)/c)])/(c^2*n*(bc - ad)^2*(c + dx^n))$

$(x^n/c)])))/(a*c^2*(b*c - a*d)^{2*n}*(c + d*x^n))$

**Maple [F]** time = 0.165, size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^n)(c + dx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*x^n)/(c+d\*x^n)^2,x)

[Out] int(1/(a+b\*x^n)/(c+d\*x^n)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$b^2 \int \frac{1}{ab^2c^2 - 2a^2bcd + a^3d^2 + (b^3c^2 - 2ab^2cd + a^2bd^2)x^n} dx$$

$$- (bcd(2n - 1) - ad^2(n - 1)) \int \frac{1}{b^2c^4n - 2abc^3dn + a^2c^2d^2n + (b^2c^3dn - 2abc^2d^2n + a^2cd^3n)x^n} dx$$

$$- \frac{dx}{bc^3n - ac^2dn + (bc^2dn - acd^2n)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^n + a)\*(d\*x^n + c)^2),x, algorithm="maxima")

[Out] b^2\*integrate(1/(a\*b^2\*c^2 - 2\*a^2\*b\*c\*d + a^3\*d^2 + (b^3\*c^2 - 2\*a\*b^2\*c\*d + a^2\*b\*d^2)\*x^n), x) - (b\*c\*d\*(2\*n - 1) - a\*d^2\*(n - 1))\*integrate(1/(b^2\*c^4\*n - 2\*a\*b\*c^3\*d\*n + a^2\*c^2\*d^2\*n + (b^2\*c^3\*d\*n - 2\*a\*b\*c^2\*d^2\*n + a^2\*c\*d^3\*n)\*x^n), x) - d\*x/(b\*c^3\*n - a\*c^2\*d\*n + (b\*c^2\*d\*n - a\*c\*d^2\*n)\*x^n)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bd^2x^{3n} + ac^2 + (2bcd + ad^2)x^{2n} + (bc^2 + 2acd)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^n + a)\*(d\*x^n + c)^2),x, algorithm="fricas")

[Out] integral(1/(b\*d^2\*x^(3\*n) + a\*c^2 + (2\*b\*c\*d + a\*d^2)\*x^(2\*n) + (b\*c^2 + 2\*a\*c\*d)\*x^n), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x\*\*n)/(c+d\*x\*\*n)\*\*2,x)

[Out] Timed out



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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)(dx^n + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^n + a)*(d*x^n + c)^2),x, algorithm="giac")`

[Out] `integrate(1/((b*x^n + a)*(d*x^n + c)^2), x)`

$$3.205 \quad \int \frac{1}{(a+bx^n)(c+dx^n)^3} dx$$

**Optimal.** Leaf size=210

$$\frac{dx (a^2 d^2 (2n^2 - 3n + 1) - 2abcd (3n^2 - 4n + 1) + b^2 c^2 (6n^2 - 5n + 1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3 n^2 (bc - ad)^3} + \frac{b^3 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a(bc - ad)^3} - \frac{dx(ad(1 - 2n) - b(c - 4cn))}{2c^2 n^2 (bc - ad)^2 (c + dx^n)} - \frac{dx}{2cn(bc - ad)(c + dx^n)^2}$$

[Out]  $-(d*x)/(2*c*(b*c - a*d)*n*(c + d*x^n)^2) - (d*(a*d*(1 - 2*n) - b*(c - 4*c*n))*x)/(2*c^2*(b*c - a*d)^2*n^2*(c + d*x^n)) + (b^3*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b*x^n)/a])/(a*(b*c - a*d)^3) - (d*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(d*x^n)/c])/(2*c^3*(b*c - a*d)^3*n^2)$

**Rubi [A]** time = 0.818794, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{dx (a^2 d^2 (2n^2 - 3n + 1) - 2abcd (3n^2 - 4n + 1) + b^2 c^2 (6n^2 - 5n + 1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3 n^2 (bc - ad)^3} + \frac{b^3 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a(bc - ad)^3} + \frac{dx(bc(1 - 4n) - ad(1 - 2n))}{2c^2 n^2 (bc - ad)^2 (c + dx^n)} - \frac{dx}{2cn(bc - ad)(c + dx^n)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^n)\*(c + d\*x^n)^3), x]

[Out]  $-(d*x)/(2*c*(b*c - a*d)*n*(c + d*x^n)^2) + (d*(b*c*(1 - 4*n) - a*d*(1 - 2*n))*x)/(2*c^2*(b*c - a*d)^2*n^2*(c + d*x^n)) + (b^3*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b*x^n)/a])/(a*(b*c - a*d)^3) - (d*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(d*x^n)/c])/(2*c^3*(b*c - a*d)^3*n^2)$

**Rubi in Sympy [A]** time = 112.827, size = 218, normalized size = 1.04

$$\frac{dx}{2cn(c + dx^n)^2(ad - bc)} - \frac{dx(-2adn + ad + 4bcn - bc)}{2c^2 n^2 (c + dx^n)(ad - bc)^2} + \frac{dx(ad(-2adn + ad + 2bcn - bc(-2n + 1)) - bc(-n + 1)(-2adn + ad + 4bcn - bc) - n(ad - bc)(-2adn + ad + 2bcn)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3 n^2 (ad - bc)^3} - \frac{b^3 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a(ad - bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a+b\*x\*\*n)/(c+d\*x\*\*n)\*\*3, x)

[Out]  $d*x/(2*c*n*(c + d*x**n)**2*(a*d - b*c)) - d*x*(-2*a*d*n + a*d + 4*b*c*n - b*c)/(2*c**2*n**2*(c + d*x**n)*(a*d - b*c)**2) + d*x*(a*d*(-2*a*d*n + a*d + 2*b*c*n - b*c*(-2*n + 1)) - b*c*(-n + 1)*(-2*a*d*n + a*d + 4*b*c*n - b*c) - n*(a*d - b*c)*(-2*a*d*n + a*d + 2*b*c*n))*hyper((1, 1/n), (1 + 1/n, ), -d*x**n/c)/(2*c**3*n**2*(a*d$

$-b^3c)^{**3}) - b^{**3}x^*hyper((1, 1/n), (1 + 1/n, ), -b^*x^{**n}/a)/(a^*(a^*d - b^*c)^{**3})$

**Mathematica [A]** time = 0.333671, size = 210, normalized size = 1.

$$\frac{x \left( -ad(c + dx^n)^2 (a^2d^2 (2n^2 - 3n + 1) - 2abcd (3n^2 - 4n + 1) + b^2c^2 (6n^2 - 5n + 1)) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c} \right) + 2b^3c^3n^2 (c + dx^n) \right)}{2ac^3n^2(bc - ad)^3 (c + dx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^n)\*(c + d\*x^n)^3), x]

[Out]  $(x^* (- (a^*c^2*d^*(b^*c - a^*d)^2*n) + a^*c*d^*(b^*c - a^*d)*(a^*d^*(-1 + 2*n) + b^*(c - 4*c^n))*(c + d*x^n) + 2*b^3*c^3*n^2*(c + d*x^n)^2*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)] - a^*d^*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d^*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))*(c + d*x^n)^2*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(2*a*c^3*(b^*c - a^*d)^3*n^2*(c + d*x^n)^2)$

**Maple [F]** time = 0.175, size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^n)(c + dx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*x^n)/(c+d\*x^n)^3, x)

[Out] int(1/(a+b\*x^n)/(c+d\*x^n)^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-b^3 \int \frac{1}{ab^3c^3 - 3a^2b^2c^2d + 3a^3bcd^2 - a^4d^3 + (b^4c^3 - 3ab^3c^2d + 3a^2b^2cd^2 - a^3bd^3)x^n} dx + ((6n^2 - 5n + 1)b^2c^2d - 2(3n^2 - 4n + 1)abcd^2 + (2n^2 - 3n + 1)a^2d^3) \int \frac{1}{\frac{2(b^3c^6n^2 - 3ab^2c^5dn^2 + 3a^2bc^4d^2n^2 - a^3c^3d^3n^2 + (b^3c^5dn^2 - 3ab^2c^4d^2n^2 + 3a^2bc^3d^3n^2 - a^3c^2d^4n^2)x^n)}{(bcd^2(4n - 1) - ad^3(2n - 1))xx^n + (bc^2d(5n - 1) - acd^2(3n - 1))x} + \frac{2(b^2c^6n^2 - 2abc^5dn^2 + a^2c^4d^2n^2 + (b^2c^4d^2n^2 - 2abc^3d^3n^2 + a^2c^2d^4n^2)x^{2n} + 2(b^2c^5dn^2 - 2abc^4d^2n^2 + a^2c^3d^3n^2)x^n)}}{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^n + a)\*(d\*x^n + c)^3), x, algorithm="maxima")

[Out]  $-b^3*integrate(-1/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3 + (b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^n), x) + ((6*n^2 - 5*n + 1)*b^2*c^2*d - 2*(3*n^2 - 4*n + 1)*a*b*c*d^2 + (2*n^2 - 3*n + 1)*a^2*d^3)*integrate(-1/2/(b^3*c^6*n^2 - 3*a*b^2*c^5*d*n^2 + 3*a^2*b*c^4*d^2*n^2 - a^3*c^3*d^3*n^2 + (b^3*c^5*d*n^2 - 3*a*b^2*c^4*d^2*n^2 + 3*a^2*b*c^3*d^3*n^2 - a^3*c^2*d^4*n^2)*x^n), x) - 1/2*((b^3*c^5*d^2*(4*n - 1) - a^3*d^3*(2*n - 1))*x*x^n + (b^2*c^4*d^2*(5*n - 1) - a^2*c^3*d^2*(3*n - 1))*x)/(b^2*c^6*n^2 - 2*a*b*c^5*d*n^2 + a^2*c^4*d^2*n^2 + (b^2*c^4*d^2*n^2 - 2*a*b*c^3*d^3*n^2 + a^2*c^2*d^4*n^2)*x^(2*n) + 2*(b^2*c^5*d*n^2 - 2*a*b*c^4*d^2*n^2 + a^2*c^3*d^3*n^2)*x^n)$

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bd^3x^{4n} + ac^3 + (3bcd^2 + ad^3)x^{3n} + 3(bc^2d + acd^2)x^{2n} + (bc^3 + 3ac^2d)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^n + a)*(d*x^n + c)^3), x, algorithm="fricas")`

[Out] `integral(1/(b*d^3*x^(4*n) + a*c^3 + (3*b*c*d^2 + a*d^3)*x^(3*n) + 3*(b*c^2*d + a*c*d^2)*x^(2*n) + (b*c^3 + 3*a*c^2*d)*x^n), x)`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*x**n)/(c+d*x**n)**3, x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^n + a)*(d*x^n + c)^3), x, algorithm="giac")`

[Out] `integrate(1/((b*x^n + a)*(d*x^n + c)^3), x)`

$$3.206 \quad \int \frac{(c+dx^n)^4}{(a+bx^n)^2} dx$$

**Optimal.** Leaf size=341

$$\frac{x(bc-ad)^3(bc(1-n)-ad(3n+1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2b^4n} \\ - \frac{dx(c+dx^n)(a^2d^2(6n^2+5n+1)-2abcd(5n^2+4n+1)+b^2c^2(2n^2+3n+1))}{ab^3n(n+1)(2n+1)} \\ - \frac{dx(-a^3d^3(6n^3+11n^2+6n+1)+a^2bcd^2(16n^3+26n^2+15n+3)-ab^2c^2d(12n^3+17n^2+12n+3)+b^3c^3(2n^2+3n+1))}{ab^4n(n+1)(2n+1)} \\ + \frac{dx(c+dx^n)^2(ad(3n+1)-b(2cn+c))}{ab^2n(2n+1)} + \frac{x(bc-ad)(c+dx^n)^3}{abn(a+bx^n)}$$

[Out]  $-\left(\left(d^*(b^3*c^3*(1+3*n+2*n^2)-a^3*d^3*(1+6*n+11*n^2+6*n^3)-a*b^2*c^2*d*(3+12*n+17*n^2+12*n^3)+a^2*b*c*d^2*(3+15*n+26*n^2+16*n^3))*x\right)/(a*b^4*n*(1+n)*(1+2*n))\right) - \left(d^*(b^2*c^2*(1+3*n+2*n^2)-2*a*b*c*d*(1+4*n+5*n^2)+a^2*d^2*(1+5*n+6*n^2))*x*(c+d*x^n)\right)/(a*b^3*n*(1+n)*(1+2*n)) + \left(d^*(a*d*(1+3*n)-b*(c+2*c*n))*x*(c+d*x^n)^2\right)/(a*b^2*n*(1+2*n)) + \left((b*c-a*d)*x*(c+d*x^n)^3\right)/(a*b*n*(a+b*x^n)) - \left((b*c-a*d)^3*(b*c*(1-n)-a*d*(1+3*n))*x*Hypergeometric2F1[1, n^(-1), 1+n^(-1), -(b*x^n)/a]\right)/(a^2*b^4*n)$

**Rubi [A]** time = 1.2872, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{x(bc-ad)^3(bc(1-n)-ad(3n+1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2b^4n} \\ - \frac{dx(c+dx^n)(a^2d^2(6n^2+5n+1)-2abcd(5n^2+4n+1)+b^2c^2(2n^2+3n+1))}{ab^3n(n+1)(2n+1)} \\ - \frac{dx(-a^3d^3(6n^3+11n^2+6n+1)+a^2bcd^2(16n^3+26n^2+15n+3)-ab^2c^2d(12n^3+17n^2+12n+3)+b^3c^3(2n^2+3n+1))}{ab^4n(n+1)(2n+1)} \\ + \frac{dx(c+dx^n)^2(ad(3n+1)-b(2cn+c))}{ab^2n(2n+1)} + \frac{x(bc-ad)(c+dx^n)^3}{abn(a+bx^n)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^4/(a + b\*x^n)^2, x]

[Out]  $-\left(\left(d^*(b^3*c^3*(1+3*n+2*n^2)-a^3*d^3*(1+6*n+11*n^2+6*n^3)-a*b^2*c^2*d*(3+12*n+17*n^2+12*n^3)+a^2*b*c*d^2*(3+15*n+26*n^2+16*n^3))*x\right)/(a*b^4*n*(1+n)*(1+2*n))\right) - \left(d^*(b^2*c^2*(1+3*n+2*n^2)-2*a*b*c*d*(1+4*n+5*n^2)+a^2*d^2*(1+5*n+6*n^2))*x*(c+d*x^n)\right)/(a*b^3*n*(1+n)*(1+2*n)) + \left(d^*(a*d*(1+3*n)-b*(c+2*c*n))*x*(c+d*x^n)^2\right)/(a*b^2*n*(1+2*n)) + \left((b*c-a*d)*x*(c+d*x^n)^3\right)/(a*b*n*(a+b*x^n)) - \left((b*c-a*d)^3*(b*c*(1-n)-a*d*(1+3*n))*x*Hypergeometric2F1[1, n^(-1), 1+n^(-1), -(b*x^n)/a]\right)/(a^2*b^4*n)$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d\*x\*\*n)\*\*4/(a+b\*x\*\*n)\*\*2, x)

[Out] Timed out

**Mathematica [A]** time = 0.337356, size = 217, normalized size = 0.64

$$x \left( \frac{(bc-ad)^3(ad(3n+1)+bc(n-1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2n} + \frac{(ad-bc)^3(ad(3n+1)+bc(n-1))}{a^2n} + \frac{-a^4d^4+4a^3bcd^3-6a^2b^2c^2d^2+4ab^3c^3d+b^4c^4(n-1)}{a^2n} + \frac{2bd^3x^n(2n+1)}{n+1} \right) / b^4$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^4/(a + b\*x^n)^2, x]

[Out] (x\*((4\*a\*b^3\*c^3\*d - 6\*a^2\*b^2\*c^2\*d^2 + 4\*a^3\*b\*c\*d^3 - a^4\*d^4 + b^4\*c^4\*(-1 + n))/(a^2\*n) + ((-(b\*c) + a\*d)^3\*(b\*c\*(-1 + n) + a\*d\*(1 + 3\*n)))/(a^2\*n) + (2\*b\*d^3\*(2\*b\*c - a\*d)\*x^n)/(1 + n) + (b^2\*d^4\*x^(2\*n))/(1 + 2\*n) + (b\*c - a\*d)^4/(a\*n\*(a + b\*x^n)) + ((b\*c - a\*d)^3\*(b\*c\*(-1 + n) + a\*d\*(1 + 3\*n))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n)/a])/(a^2\*n)))/b^4

**Maple [F]** time = 0.078, size = 0, normalized size = 0.

$$\int \frac{(c + dx^n)^4}{(a + bx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d\*x^n)^4/(a+b\*x^n)^2, x)

[Out] int((c+d\*x^n)^4/(a+b\*x^n)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-(a^4d^4(3n+1) - 4a^3bcd^3(2n+1) + 6a^2b^2c^2d^2(n+1) - b^4c^4(n-1) - 4ab^3c^3d) \int \frac{1}{ab^5nx^n + a^2b^4n} dx + \frac{(n^2+n)ab^3d^4xx^{3n} + (4(2n^2+n)ab^3cd^3 - (3n^2+n)a^2b^2d^4)xx^{2n} + (6(2n^3+3n^2+n)ab^3c^2d^2 - 4(4n^3+4n^2+n)a^2b^2d^4)}{ab^5nx^n + a^2b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^4/(b\*x^n + a)^2, x, algorithm="maxima")

[Out] -(a^4\*d^4\*(3\*n + 1) - 4\*a^3\*b\*c\*d^3\*(2\*n + 1) + 6\*a^2\*b^2\*c^2\*d^2\*(n + 1) - b^4\*c^4\*(n - 1) - 4\*a\*b^3\*c^3\*d)\*integrate(1/(a\*b^5\*n\*x^n + a^2\*b^4\*n), x) + ((n^2 + n)\*a\*b^3\*d^4\*x\*x^(3\*n) + (4\*(2\*n^2 + n)\*a\*b^3\*c\*d^3 - (3\*n^2 + n)\*a^2\*b^2\*d^4)\*x\*x^(2\*n) + (6\*(2\*n^3 + 3\*n^2 + n)\*a\*b^3\*c^2\*d^2 - 4\*(4\*n^3 + 4\*n^2 + n)\*a^2\*b^2\*c\*d^3 + (6\*n^3 + 5\*n^2 + n)\*a^3\*b\*d^4)\*x\*x^n + ((2\*n^2 + 3\*n + 1)\*b^4\*c^4 - 4\*(2\*n^2 + 3\*n + 1)\*a\*b^3\*c^3\*d + 6\*(2\*n^3 + 5\*n^2 + 4\*n + 1)\*a^2\*b^2\*c^2\*d^2 - 4\*(4\*n^3 + 8\*n^2 + 5\*n + 1)\*a^3\*b\*c\*d^3 + (6\*n^3 + 11\*n^2 + 6\*n + 1)\*a^4\*d^4)\*x)/((2\*n^3 + 3\*n^2 + n)\*a\*b^5\*x^n + (2\*n^3 + 3\*n^2 + n)\*a^2\*b^4)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^4x^{4n} + 4cd^3x^{3n} + 6c^2d^2x^{2n} + 4c^3dx^n + c^4}{b^2x^{2n} + 2abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^4/(b*x^n + a)^2,x, algorithm="fricas")`

[Out] `integral((d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4*c^3*d*x^n + c^4)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d*x**n)**4/(a+b*x**n)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^4}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^4/(b*x^n + a)^2,x, algorithm="giac")`

[Out] `integrate((d*x^n + c)^4/(b*x^n + a)^2, x)`

$$3.207 \quad \int \frac{(c+dx^n)^3}{(a+bx^n)^2} dx$$

**Optimal.** Leaf size=200

$$\frac{x(bc-ad)^2(bc(1-n)-ad(2n+1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2b^3n} - \frac{dx(a^2d^2(2n^2+3n+1)-abcd(3n^2+4n+2)+b^2c^2(n+1))}{ab^3n(n+1)} - \frac{dx(c+dx^n)(bc(n+1)-ad(2n+1))}{ab^2n(n+1)} + \frac{x(bc-ad)(c+dx^n)^2}{abn(a+bx^n)}$$

[Out]  $-\left(\frac{d^2(b^2c^2(1+n)+a^2d^2(1+3n+2n^2)-ab^2cd^2(2+4n+3n^2))x}{a^2b^3n(1+n)} - \frac{d^2(bc(1+n)-ad(1+2n))x^2(c+dx^n)}{ab^3n(n+1)} + \frac{(bc-a^2d)x^2(c+dx^n)^2}{ab^2n(n+1)(a+bx^n)} - \frac{(bc-a^2d)^2(bc(1-n)-ad(1+2n))x^2 \operatorname{Hypergeometric2F1}\left[1, n^{-1}, 1+n^{-1}, -(bx^n/a)\right]}{a^2b^3n}\right)$

**Rubi [A]** time = 0.609973, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{x(bc-ad)^2(bc(1-n)-ad(2n+1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2b^3n} - \frac{dx(a^2d^2(2n^2+3n+1)-abcd(3n^2+4n+2)+b^2c^2(n+1))}{ab^3n(n+1)} - \frac{dx(c+dx^n)(bc(n+1)-ad(2n+1))}{ab^2n(n+1)} + \frac{x(bc-ad)(c+dx^n)^2}{abn(a+bx^n)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^3/(a + b\*x^n)^2, x]

[Out]  $-\left(\frac{d^2(b^2c^2(1+n)+a^2d^2(1+3n+2n^2)-ab^2cd^2(2+4n+3n^2))x}{a^2b^3n(1+n)} - \frac{d^2(bc(1+n)-ad(1+2n))x^2(c+dx^n)}{ab^3n(n+1)} + \frac{(bc-a^2d)x^2(c+dx^n)^2}{ab^2n(n+1)(a+bx^n)} - \frac{(bc-a^2d)^2(bc(1-n)-ad(1+2n))x^2 \operatorname{Hypergeometric2F1}\left[1, n^{-1}, 1+n^{-1}, -(bx^n/a)\right]}{a^2b^3n}\right)$

**Rubi in Sympy [A]** time = 65.6107, size = 172, normalized size = 0.86

$$\frac{x(c+dx^n)^2(ad-bc)}{abn(a+bx^n)} + \frac{dx(c(ad-bc(-n+1))+dx^n(ad(2n+1)-bc(n+1)))}{ab^2n(n+1)} - \frac{dx(adn+ad-bc)(2adn+ad-3bcn-bc)}{ab^3n(n+1)} + \frac{x(ad-bc)^2(2adn+ad+bcn-bc) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d\*x\*\*n)\*\*3/(a+b\*x\*\*n)\*\*2, x)

[Out]  $-x^2(c+d*x^n)^2(a*d-b*c)/(a*b*n*(a+b*x^n)) + d*x^2(c*(a*d-b*c*(-n+1))+d*x^n*(a*d*(2*n+1)-b*c*(n+1)))/(a*b^2*n*(n+1)) - d*x^2(a*d*n+a*d-b*c)*(2*a*d*n+a*d-3*b*c*n-b*c)/(a*b^3*n*(n+1)) + x^2(a*d-b*c)^2*(2*a*d*n+a*d+b*c*n$



- b\*c)\*hyper((1, 1/n), (1 + 1/n), -b\*x\*\*n/a)/(a\*\*2\*b\*\*3\*n)

**Mathematica [A]** time = 0.457563, size = 167, normalized size = 0.84

$$x \left( \frac{a(-a^3 d^3 (2n^2 + 3n + 1) + a^2 b d^2 (3c(n+1)^2 - dn(2n+1)x^n) + ab^2 d(-3c^2(n+1) + 3cdn(n+1)x^n + d^2 n x^{2n}) + b^3 c^3 (n+1))}{(n+1)(a+bx^n)} + (bc - ad)^2 (ad(2n+1) + bc(n-1)) \right) / a^2 b^3 n$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^3/(a + b\*x^n)^2, x]

[Out] (x\*((a\*(b^3\*c^3\*(1+n) - a^3\*d^3\*(1+3\*n+2\*n^2) + a^2\*b\*d^2\*(3\*c\*(1+n)^2 - d\*n\*(1+2\*n)\*x^n) + a\*b^2\*d\*(-3\*c^2\*(1+n) + 3\*c\*d\*n\*(1+n)\*x^n + d^2\*n\*x^(2\*n))))/((1+n)\*(a+b\*x^n)) + (b\*c - a\*d)^2\*(b\*c\*(-1+n) + a\*d\*(1+2\*n))\*Hypergeometric2F1[1, n^(-1), 1+n^(-1), -(b\*x^n)/a])/((a^2\*b^3\*n))

**Maple [F]** time = 0.073, size = 0, normalized size = 0.

$$\int \frac{(c + dx^n)^3}{(a + bx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d\*x^n)^3/(a+b\*x^n)^2, x)

[Out] int((c+d\*x^n)^3/(a+b\*x^n)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(a^3 d^3 (2n+1) - 3 a^2 b c d^2 (n+1) + b^3 c^3 (n-1) + 3 a b^2 c^2 d) \int \frac{1}{a b^4 n x^n + a^2 b^3 n} dx + \frac{a b^2 d^3 n x x^{2n} + (3(n^2 + n) a b^2 c d^2 - (2n^2 + n) a^2 b d^3) x x^n + (3(n^2 + 2n + 1) a^2 b c d^2 - (2n^2 + 3n + 1) a^3 d^3 + b^3 c^3 (n+1) - 3 a^2 b^2 c^2 d) x}{(n^2 + n) a b^4 x^n + (n^2 + n) a^2 b^3}}{(n^2 + n) a b^4 x^n + (n^2 + n) a^2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^3/(b\*x^n + a)^2, x, algorithm="maxima")

[Out] (a^3\*d^3\*(2\*n+1) - 3\*a^2\*b\*c\*d^2\*(n+1) + b^3\*c^3\*(n-1) + 3\*a\*b^2\*c^2\*d)\*integrate(1/(a\*b^4\*n\*x^n + a^2\*b^3\*n), x) + (a\*b^2\*d^3\*n\*x\*x^(2\*n) + (3\*(n^2+n)\*a\*b^2\*c\*d^2 - (2\*n^2+n)\*a^2\*b\*d^3)\*x\*x^n + (3\*(n^2+2\*n+1)\*a^2\*b\*c\*d^2 - (2\*n^2+3\*n+1)\*a^3\*d^3 + b^3\*c^3\*(n+1) - 3\*a\*b^2\*c^2\*d\*(n+1))\*x)/((n^2+n)\*a\*b^4\*x^n + (n^2+n)\*a^2\*b^3)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{d^3 x^{3n} + 3 c d^2 x^{2n} + 3 c^2 d x^n + c^3}{b^2 x^{2n} + 2 a b x^n + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^3/(b\*x^n + a)^2, x, algorithm="fricas")

[Out] `integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d*x**n)**3/(a+b*x**n)**2,x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^3}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^3/(b*x^n + a)^2,x, algorithm="giac")`

[Out] `integrate((d*x^n + c)^3/(b*x^n + a)^2, x)`

$$3.208 \quad \int \frac{(c+dx^n)^2}{(a+bx^n)^2} dx$$

**Optimal.** Leaf size=115

$$\frac{x(bc-ad)(bc(1-n)-ad(n+1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2b^2n} - \frac{dx(bc-ad(n+1))}{ab^2n} + \frac{x(bc-ad)(c+dx^n)}{abn(a+bx^n)}$$

[Out]  $-\left(\frac{d(b^*c - a^*d(1+n))^*x}{(a^*b^{2*n})} + \frac{(b^*c - a^*d)^*x^*(c + d^*x^n)}{(a^*b^n(a + b^*x^n))} - \frac{(b^*c - a^*d)^*(b^*c(1-n) - a^*d(1+n))^*x^* \text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, -(b^*x^n/a)]}{(a^{2*n} b^{2*n})}\right)$

**Rubi [A]** time = 0.236724, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{x(bc-ad)(bc(1-n)-ad(n+1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2b^2n} - \frac{dx(bc-ad(n+1))}{ab^2n} + \frac{x(bc-ad)(c+dx^n)}{abn(a+bx^n)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^2/(a + b\*x^n)^2, x]

[Out]  $-\left(\frac{d(b^*c - a^*d(1+n))^*x}{(a^*b^{2*n})} + \frac{(b^*c - a^*d)^*x^*(c + d^*x^n)}{(a^*b^n(a + b^*x^n))} - \frac{(b^*c - a^*d)^*(b^*c(1-n) - a^*d(1+n))^*x^* \text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, -(b^*x^n/a)]}{(a^{2*n} b^{2*n})}\right)$

**Rubi in Sympy [A]** time = 22.5983, size = 97, normalized size = 0.84

$$\frac{x(c+dx^n)(ad-bc)}{abn(a+bx^n)} - \frac{dx(-ad(n+1)+bc)}{ab^2n} - \frac{x(ad-bc)(adn+ad+bcn-bc) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d\*x\*\*n)\*\*2/(a+b\*x\*\*n)\*\*2, x)

[Out]  $-x^*(c + d^*x^{*n})^*(a^*d - b^*c)/(a^*b^n(a + b^*x^{*n})) - d^*x^*(-a^*d(n + 1) + b^*c)/(a^*b^{2*n}) - x^*(a^*d - b^*c)^*(a^*d^n + a^*d + b^*c^n - b^*c) \text{hyper}((1, 1/n), (1 + 1/n), -b^*x^{*n}/a)/(a^{2*n} b^{2*n})$

**Mathematica [A]** time = 0.199505, size = 96, normalized size = 0.83

$$\frac{x\left(\frac{a(a^2d^2(n+1)+abd(dnx^n-2c)+b^2c^2)}{a+bx^n} + (bc-ad)(ad(n+1)+bc(n-1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)\right)}{a^2b^2n}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^2/(a + b\*x^n)^2, x]

[Out]  $(x^*((a^*(b^{2*n}c^2 + a^{2*n}d^{2*n}(1+n) + a^*b^*d^*(-2*c + d^n*x^n)))/(a + b^*x^n) + (b^*c - a^*d)^*(b^*c^{(-1+n)} + a^*d^*(1+n)) \text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, -(b^*x^n/a)]))/(a^{2*n} b^{2*n})$

---

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int \frac{(c + dx^n)^2}{(a + bx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d\*x^n)^2/(a+b\*x^n)^2, x)

[Out] int((c+d\*x^n)^2/(a+b\*x^n)^2, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-(a^2d^2(n+1) - b^2c^2(n-1) - 2abcd) \int \frac{1}{ab^3nx^n + a^2b^2n} dx + \frac{abd^2nxx^n + (a^2d^2(n+1) + b^2c^2 - 2abcd)x}{ab^3nx^n + a^2b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^2/(b\*x^n + a)^2, x, algorithm="maxima")

[Out]  $-(a^2d^2(n+1) - b^2c^2(n-1) - 2ab^2cd) \int \frac{1}{(ab^3n^2x^{2n} + a^2b^2n^2)}, x) + (ab^2d^2n^2x^{2n} + (a^2d^2(n+1) + b^2c^2 - 2ab^2cd)x) / (ab^3n^2x^{2n} + a^2b^2n^2)$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^2x^{2n} + 2cdx^n + c^2}{b^2x^{2n} + 2abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^2/(b\*x^n + a)^2, x, algorithm="fricas")

[Out] integral((d^2\*x^(2\*n) + 2\*c\*d\*x^n + c^2)/(b^2\*x^(2\*n) + 2\*a\*b\*x^n + a^2), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c + dx^n)^2}{(a + bx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*n)\*\*2/(a+b\*x\*\*n)\*\*2, x)

[Out] Integral((c + d\*x\*\*n)\*\*2/(a + b\*x\*\*n)\*\*2, x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^2}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^n + c)^2/(b*x^n + a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^2/(b*x^n + a)^2, x)
```

$$3.209 \quad \int \frac{c+dx^n}{(a+bx^n)^2} dx$$

**Optimal.** Leaf size=72

$$\frac{x(ad - bc(1 - n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2bn} + \frac{x(bc - ad)}{abn(a + bx^n)}$$

[Out] ((b\*c - a\*d)\*x)/(a\*b\*n\*(a + b\*x^n)) + ((a\*d - b\*c\*(1 - n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n)/a])/(a^2\*b\*n)

**Rubi [A]** time = 0.0844173, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x(ad - bc(1 - n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2bn} + \frac{x(bc - ad)}{abn(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)/(a + b\*x^n)^2, x]

[Out] ((b\*c - a\*d)\*x)/(a\*b\*n\*(a + b\*x^n)) + ((a\*d - b\*c\*(1 - n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n)/a])/(a^2\*b\*n)

**Rubi in Sympy [A]** time = 9.0002, size = 53, normalized size = 0.74

$$-\frac{x(ad - bc)}{abn(a + bx^n)} + \frac{x(ad - bc(-n + 1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d\*x\*\*n)/(a+b\*x\*\*n)\*\*2, x)

[Out] -x\*(a\*d - b\*c)/(a\*b\*n\*(a + b\*x\*\*n)) + x\*(a\*d - b\*c\*(-n + 1))\*hyper((1, 1/n), (1 + 1/n, ), -b\*x\*\*n/a)/(a\*\*2\*b\*n)

**Mathematica [A]** time = 0.0768954, size = 68, normalized size = 0.94

$$\frac{x\left((a + bx^n)(ad + bc(n - 1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) + a(bc - ad)\right)}{a^2bn(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)/(a + b\*x^n)^2, x]

[Out] (x\*(a\*(b\*c - a\*d) + (a\*d + b\*c\*(-1 + n))\*(a + b\*x^n)\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n)/a])/(a^2\*b\*n\*(a + b\*x^n))

**Maple [F]** time = 0.065, size = 0, normalized size = 0.

$$\int \frac{c + dx^n}{(a + bx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c+d*x^n)/(a+b*x^n)^2,x)`

[Out] `int((c+d*x^n)/(a+b*x^n)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$(bc(n-1) + ad) \int \frac{1}{ab^2nx^n + a^2bn} dx + \frac{(bc - ad)x}{ab^2nx^n + a^2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)/(b*x^n + a)^2,x, algorithm="maxima")`

[Out] `(b*c*(n - 1) + a*d)*integrate(1/(a*b^2*n*x^n + a^2*b*n), x) + (b*c - a*d)*x/(a*b^2*n*x^n + a^2*b*n)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^n + c}{b^2x^{2n} + 2abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)/(b*x^n + a)^2,x, algorithm="fricas")`

[Out] `integral((d*x^n + c)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)`

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d*x**n)/(a+b*x**n)**2,x)`

[Out] Exception raised: TypeError

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^n + c}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)/(b*x^n + a)^2,x, algorithm="giac")`

[Out] `integrate((d*x^n + c)/(b*x^n + a)^2, x)`

$$3.210 \quad \int \frac{1}{(a+bx^n)^2(c+dx^n)} dx$$

**Optimal.** Leaf size=122

$$\frac{bx(ad(1-2n)-bc(1-n)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2} + \frac{d^2x {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{dx^n}{c}\right)}{c(bc-ad)^2} + \frac{bx}{an(bc-ad)(a+bx^n)}$$

[Out] (b\*x)/(a\*(b\*c - a\*d)\*n\*(a + b\*x^n)) + (b\*(a\*d\*(1 - 2\*n) - b\*c\*(1 - n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b\*x^n)/a)])/(a^2\*(b\*c - a\*d)^2\*n) + (d^2\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d\*x^n)/c)])/(c\*(b\*c - a\*d)^2)

**Rubi [A]** time = 0.343957, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{bx(ad(1-2n)-bc(1-n)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2} + \frac{d^2x {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{dx^n}{c}\right)}{c(bc-ad)^2} + \frac{bx}{an(bc-ad)(a+bx^n)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^n)^2\*(c + d\*x^n)), x]

[Out] (b\*x)/(a\*(b\*c - a\*d)\*n\*(a + b\*x^n)) + (b\*(a\*d\*(1 - 2\*n) - b\*c\*(1 - n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b\*x^n)/a)])/(a^2\*(b\*c - a\*d)^2\*n) + (d^2\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d\*x^n)/c)])/(c\*(b\*c - a\*d)^2)

**Rubi in Sympy [A]** time = 48.6244, size = 100, normalized size = 0.82

$$\frac{d^2x {}_2F_1\left(1, \frac{1}{n} \middle| -\frac{dx^n}{c}\right)}{c(ad-bc)^2} - \frac{bx}{an(a+bx^n)(ad-bc)} + \frac{bx(-2adn+ad+bcn-bc) {}_2F_1\left(1, \frac{1}{n} \middle| -\frac{bx^n}{a}\right)}{a^2n(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a+b\*x\*\*n)\*\*2/(c+d\*x\*\*n), x)

[Out] d\*\*2\*x\*hyper((1, 1/n), (1 + 1/n, ), -d\*x\*\*n/c)/(c\*(a\*d - b\*c)\*\*2) - b\*x/(a\*n\*(a + b\*x\*\*n)\*(a\*d - b\*c)) + b\*x\*(-2\*a\*d\*n + a\*d + b\*c\*n - b\*c)\*hyper((1, 1/n), (1 + 1/n, ), -b\*x\*\*n/a)/(a\*\*2\*n\*(a\*d - b\*c)\*\*2)

**Mathematica [A]** time = 0.254896, size = 108, normalized size = 0.89

$$\frac{x \left( \frac{b^2c-abd}{a^2n+abnx^n} + \frac{b(ad(1-2n)+bc(n-1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2n} + \frac{d^2 {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{dx^n}{c}\right)}{c} \right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^n)^2\*(c + d\*x^n)), x]

[Out] (x\*((b^2\*c - a\*b\*d)/(a^2\*n + a\*b\*n\*x^n) + (b\*(a\*d\*(1 - 2\*n) + b\*c\*(-1 + n))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b\*x^n)/a)]



$)/(a^2 \cdot n) + (d^2 \cdot \text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, -(d \cdot x^n)/c])/c)/(b \cdot c - a \cdot d)^2$

**Maple [F]** time = 0.154, size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^n)^2 (c + dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*x^n)^2/(c+d\*x^n), x)

[Out] int(1/(a+b\*x^n)^2/(c+d\*x^n), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$d^2 \int \frac{1}{b^2 c^3 - 2 abc^2 d + a^2 cd^2 + (b^2 c^2 d - 2 abcd^2 + a^2 d^3) x^n} dx - (abd(2n - 1) - b^2 c(n - 1)) \int \frac{1}{a^2 b^2 c^2 n - 2 a^3 bcdn + a^4 d^2 n + (ab^3 c^2 n - 2 a^2 b^2 c d n + a^3 b d^2 n) x^n} dx + \frac{bx}{a^2 bcn - a^3 dn + (ab^2 cn - a^2 bdn) x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^n + a)^2\*(d\*x^n + c)), x, algorithm="maxima")

[Out] d^2\*integrate(1/(b^2\*c^3 - 2\*a\*b\*c^2\*d + a^2\*c\*d^2 + (b^2\*c^2\*d - 2\*a\*b\*c\*d^2 + a^2\*d^3)\*x^n), x) - (a\*b\*d\*(2\*n - 1) - b^2\*c\*(n - 1))\*integrate(1/(a^2\*b^2\*c^2\*n - 2\*a^3\*b\*c\*d\*n + a^4\*d^2\*n + (a\*b^3\*c^2\*n - 2\*a^2\*b^2\*c\*d\*n + a^3\*b\*d^2\*n)\*x^n), x) + b\*x/(a^2\*b\*c\*n - a^3\*d\*n + (a\*b^2\*c\*n - a^2\*b\*d\*n)\*x^n)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2 dx^{3n} + a^2 c + (b^2 c + 2 abd)x^{2n} + (2 abc + a^2 d)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^n + a)^2\*(d\*x^n + c)), x, algorithm="fricas")

[Out] integral(1/(b^2\*d\*x^(3\*n) + a^2\*c + (b^2\*c + 2\*a\*b\*d)\*x^(2\*n) + (2\*a\*b\*c + a^2\*d)\*x^n), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x\*\*n)\*\*2/(c+d\*x\*\*n), x)

[Out] Timed out

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^n + a)^2\*(d\*x^n + c)),x, algorithm="giac")

[Out] integrate(1/((b\*x^n + a)^2\*(d\*x^n + c)), x)

$$3.211 \quad \int \frac{1}{(a+bx^n)^2(c+dx^n)^2} dx$$

**Optimal.** Leaf size=193

$$\begin{aligned} & \frac{b^2x(ad(1-3n) - b(c-cn)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2n(bc-ad)^3} \\ & - \frac{d^2x(bc(1-3n) - ad(1-n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2n(bc-ad)^3} \\ & + \frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)} + \frac{dx(ad+bc)}{acn(bc-ad)^2(c+dx^n)} \end{aligned}$$

[Out]  $(d*(b*c + a*d)*x)/(a*c*(b*c - a*d)^{2*n}*(c + d*x^n)) + (b*x)/(a*(b*c - a*d)^n*(a + b*x^n)*(c + d*x^n)) + (b^2*(a*d*(1 - 3*n) - b*(c - c*n))*x*Hypergeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, -(b*x^n)/a])/ (a^2*(b*c - a*d)^{3*n} - (d^2*(b*c*(1 - 3*n) - a*d*(1 - n))*x*Hypergeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, -(d*x^n)/c])/(c^2*(b*c - a*d)^{3*n})$

**Rubi [A]** time = 0.694641, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\begin{aligned} & \frac{b^2x(ad(1-3n) - b(c-cn)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2n(bc-ad)^3} \\ & + \frac{d^2x(ad(1-n) - b(c-3cn)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2n(bc-ad)^3} \\ & + \frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)} + \frac{dx(ad+bc)}{acn(bc-ad)^2(c+dx^n)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^n)^2\*(c + d\*x^n)^2), x]

[Out]  $(d*(b*c + a*d)*x)/(a*c*(b*c - a*d)^{2*n}*(c + d*x^n)) + (b*x)/(a*(b*c - a*d)^n*(a + b*x^n)*(c + d*x^n)) + (b^2*(a*d*(1 - 3*n) - b*(c - c*n))*x*Hypergeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, -(b*x^n)/a])/ (a^2*(b*c - a*d)^{3*n} + (d^2*(a*d*(1 - n) - b*(c - 3*c*n))*x*Hypergeometric2F1[1, n^{(-1)}, 1 + n^{(-1)}, -(d*x^n)/c])/(c^2*(b*c - a*d)^{3*n})$

**Rubi in Sympy [A]** time = 105.36, size = 162, normalized size = 0.84

$$\begin{aligned} & \frac{dx}{cn(a+bx^n)(c+dx^n)(ad-bc)} - \frac{d^2x(-adn + ad + 3bcn - bc) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2n(ad-bc)^3} \\ & + \frac{bx(ad+bc)}{acn(a+bx^n)(ad-bc)^2} - \frac{b^2x(-3adn + ad + bcn - bc) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2n(ad-bc)^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a+b\*x\*\*n)\*\*2/(c+d\*x\*\*n)\*\*2, x)

[Out]  $d*x/(c*n*(a + b*x^n)*(c + d*x^n)*(a*d - b*c)) - d**2*x*(-a*d*n + a*d + 3*b*c*n - b*c)*hyper((1, 1/n), (1 + 1/n), -d*x^n/c)/(c**2*n*(a*d - b*c)**3) + b*x*(a*d + b*c)/(a*c*n*(a + b*x^n)*(a*d -$

$$b^*c)^{**2}) - b^{**2}x^*(-3*a*d^n + a*d + b^*c^n - b^*c)^*hyper((1, 1/n), (1 + 1/n, ), -b^*x^{**n}/a)/(a^{**2}n^*(a*d - b^*c)^{**3})$$

**Mathematica [A]** time = 0.344232, size = 147, normalized size = 0.76

$$x \left( \frac{b^2(ad(1-3n)+bc(n-1)){}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2} + \frac{b^2(bc-ad)}{a(ax^n)} + \frac{d^2(bc(3n-1)-ad(n-1)){}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2} + \frac{d^2(bc-ad)}{c(c+dx^n)} \right) / n(bc-ad)^3$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^n)^2\*(c + d\*x^n)^2), x]

[Out] (x\*((b^2\*(b\*c - a\*d))/(a\*(a + b\*x^n)) + (d^2\*(b\*c - a\*d))/(c\*(c + d\*x^n)) + (b^2\*(a\*d\*(1 - 3\*n) + b\*c\*(-1 + n))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n)/a])/a^2 + (d^2\*(-a\*d\*(-1 + n) + b\*c\*(-1 + 3\*n))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(d\*x^n)/c])/c^2)/(b\*c - a\*d)^3\*n

**Maple [F]** time = 0.153, size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^n)^2 (c + dx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*x^n)^2/(c+d\*x^n)^2, x)

[Out] int(1/(a+b\*x^n)^2/(c+d\*x^n)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(ab^2d(3n-1) - b^3c(n-1)) \int \frac{1}{a^2b^3c^3n - 3a^3b^2c^2dn + 3a^4bcd^2n - a^5d^3n + (ab^4c^3n - 3a^2b^3c^2dn + 3a^3b^2cd^2n - a^4bd^3n)x^n} dx - (bcd^2(3n-1) - ad^3(n-1)) \int \frac{1}{b^3c^5n - 3ab^2c^4dn + 3a^2bc^3d^2n - a^3c^2d^3n + (b^3c^4dn - 3ab^2c^3d^2n + 3a^2bc^2d^3n - a^3cd^4n)x^n} dx}{a^2b^2c^4n - 2a^3bc^3dn + a^4c^2d^2n + (ab^3c^3dn - 2a^2b^2c^2d^2n + a^3bcd^3n)x^{2n} + (ab^3c^4n - a^2b^2c^3dn - a^3bc^2d^2n + a^4cd^3n)x^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^n + a)^2\*(d\*x^n + c)^2), x, algorithm="maxima")

[Out] (a\*b^2\*d\*(3\*n - 1) - b^3\*c\*(n - 1))\*integrate(-1/(a^2\*b^3\*c^3\*n - 3\*a^3\*b^2\*c^2\*d\*n + 3\*a^4\*b\*c\*d^2\*n - a^5\*d^3\*n + (a\*b^4\*c^3\*n - 3\*a^2\*b^3\*c^2\*d\*n + 3\*a^3\*b^2\*c\*d^2\*n - a^4\*b\*d^3\*n)\*x^n), x) - (b\*c\*d^2\*(3\*n - 1) - a\*d^3\*(n - 1))\*integrate(-1/(b^3\*c^5\*n - 3\*a\*b^2\*c^4\*d\*n + 3\*a^2\*b\*c^3\*d^2\*n - a^3\*c^2\*d^3\*n + (b^3\*c^4\*d\*n - 3\*a\*b^2\*c^3\*d^2\*n + 3\*a^2\*b\*c^2\*d^3\*n - a^3\*c\*d^4\*n)\*x^n), x) + ((b^2\*c\*d + a\*b\*d^2)\*x\*x^n + (b^2\*c^2 + a^2\*d^2)\*x)/(a^2\*b^2\*c^4\*n - 2\*a^3\*b\*c^3\*d\*n + a^4\*c^2\*d^2\*n + (a\*b^3\*c^3\*d\*n - 2\*a^2\*b^2\*c^2\*d^2\*n + a^3\*b\*c\*d^3\*n)\*x^(2\*n) + (a\*b^3\*c^4\*n - a^2\*b^2\*c^3\*d\*n - a^3\*b\*c^2\*d^2\*n + a^4\*c\*d^3\*n)\*x^n)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2 d^2 x^{4n} + a^2 c^2 + 2(b^2 c d + a b d^2) x^{3n} + (b^2 c^2 + 4 a b c d + a^2 d^2) x^{2n} + 2(a b c^2 + a^2 c d) x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^n + a)^2\*(d\*x^n + c)^2),x, algorithm="fricas")

[Out] integral(1/(b^2\*d^2\*x^(4\*n) + a^2\*c^2 + 2\*(b^2\*c\*d + a\*b\*d^2)\*x^(3\*n) + (b^2\*c^2 + 4\*a\*b\*c\*d + a^2\*d^2)\*x^(2\*n) + 2\*(a\*b\*c^2 + a^2\*c\*d)\*x^n), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x\*\*n)\*\*2/(c+d\*x\*\*n)\*\*2,x)

[Out] Timed out

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^n + a)^2\*(d\*x^n + c)^2),x, algorithm="giac")

[Out] integrate(1/((b\*x^n + a)^2\*(d\*x^n + c)^2), x)

$$3.212 \quad \int \frac{1}{(a+bx^n)^2(c+dx^n)^3} dx$$

**Optimal.** Leaf size=299

$$\begin{aligned} & \frac{b^3x(ad(1-4n)-bc(1-n)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2n(bc-ad)^4} - \frac{dx(-a^2d^2(1-2n)+abcd(1-6n)-2b^2c^2n)}{2ac^2n^2(bc-ad)^3(c+dx^n)} \\ & + \frac{d^2x(a^2d^2(2n^2-3n+1)-2abcd(4n^2-5n+1)+b^2c^2(12n^2-7n+1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3n^2(bc-ad)^4} \\ & + \frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)^2} + \frac{dx(ad+2bc)}{2acn(bc-ad)^2(c+dx^n)^2} \end{aligned}$$

[Out] (d\*(2\*b\*c + a\*d)\*x)/(2\*a\*c\*(b\*c - a\*d)^2\*n\*(c + d\*x^n)^2) + (b\*x)/(a\*(b\*c - a\*d)\*n\*(a + b\*x^n)\*(c + d\*x^n)^2) - (d\*(a\*b\*c\*d\*(1 - 6\*n) - a^2\*d^2\*(1 - 2\*n) - 2\*b^2\*c^2\*n)\*x)/(2\*a\*c^2\*(b\*c - a\*d)^3\*n^2\*(c + d\*x^n)) + (b^3\*(a\*d\*(1 - 4\*n) - b\*c\*(1 - n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n/a)])/ (a^2\*(b\*c - a\*d)^4\*n) + (d^2\*(a^2\*d^2\*(1 - 3\*n + 2\*n^2) - 2\*a\*b\*c\*d\*(1 - 5\*n + 4\*n^2) + b^2\*c^2\*(1 - 7\*n + 12\*n^2))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(d\*x^n/c)])/ (2\*c^3\*(b\*c - a\*d)^4\*n^2)

**Rubi [A]** time = 1.30478, antiderivative size = 299, normalized size of antiderivative = 1., number of rules used = 6, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\begin{aligned} & \frac{b^3x(ad(1-4n)-bc(1-n)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2n(bc-ad)^4} - \frac{dx(-a^2d^2(1-2n)+abcd(1-6n)-2b^2c^2n)}{2ac^2n^2(bc-ad)^3(c+dx^n)} \\ & + \frac{d^2x(a^2d^2(2n^2-3n+1)-2abcd(4n^2-5n+1)+b^2c^2(12n^2-7n+1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3n^2(bc-ad)^4} \\ & + \frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)^2} + \frac{dx(ad+2bc)}{2acn(bc-ad)^2(c+dx^n)^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^n)^2\*(c + d\*x^n)^3), x]

[Out] (d\*(2\*b\*c + a\*d)\*x)/(2\*a\*c\*(b\*c - a\*d)^2\*n\*(c + d\*x^n)^2) + (b\*x)/(a\*(b\*c - a\*d)\*n\*(a + b\*x^n)\*(c + d\*x^n)^2) - (d\*(a\*b\*c\*d\*(1 - 6\*n) - a^2\*d^2\*(1 - 2\*n) - 2\*b^2\*c^2\*n)\*x)/(2\*a\*c^2\*(b\*c - a\*d)^3\*n^2\*(c + d\*x^n)) + (b^3\*(a\*d\*(1 - 4\*n) - b\*c\*(1 - n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n/a)])/ (a^2\*(b\*c - a\*d)^4\*n) + (d^2\*(a^2\*d^2\*(1 - 3\*n + 2\*n^2) - 2\*a\*b\*c\*d\*(1 - 5\*n + 4\*n^2) + b^2\*c^2\*(1 - 7\*n + 12\*n^2))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(d\*x^n/c)])/ (2\*c^3\*(b\*c - a\*d)^4\*n^2)

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a+b\*x\*\*n)\*\*2/(c+d\*x\*\*n)\*\*3, x)

[Out] Timed out

**Mathematica [A]** time = 0.535841, size = 233, normalized size = 0.78

$$x \left( \frac{2b^3n(ad(1-4n)+bc(n-1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2} + \frac{d^2(a^2d^2(2n^2-3n+1)-2abcd(4n^2-5n+1)+b^2c^2(12n^2-7n+1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{dx^n}{c}\right)}{c^3} + \frac{2b^3n(bc-ad)}{a(a+bx^n)} \right) \frac{1}{2n^2(bc-ad)^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^n)^2\*(c + d\*x^n)^3), x]

[Out] (x\*((2\*b^3\*(b\*c - a\*d)^n)/(a\*(a + b\*x^n)) + (d^2\*(b\*c - a\*d)^2\*n)/(c\*(c + d\*x^n)^2 + (d^2\*(-(b\*c) + a\*d)\*(a\*d\*(-1 + 2\*n) + b\*(c - 6\*c\*n)))/(c^2\*(c + d\*x^n)) + (2\*b^3\*(a\*d\*(1 - 4\*n) + b\*c\*(-1 + n)))\*n\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b\*x^n)/a)]/a^2 + (d^2\*(a^2\*d^2\*(1 - 3\*n + 2\*n^2) - 2\*a\*b\*c\*d\*(1 - 5\*n + 4\*n^2) + b^2\*c^2\*(1 - 7\*n + 12\*n^2))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d\*x^n)/c)]/c^3))/(2\*(b\*c - a\*d)^4\*n^2)

**Maple [F]** time = 0.187, size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^n)^2 (c + dx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*x^n)^2/(c+d\*x^n)^3, x)

[Out] int(1/(a+b\*x^n)^2/(c+d\*x^n)^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^n + a)^2\*(d\*x^n + c)^3), x, algorithm="maxima")

[Out] ((12\*n^2 - 7\*n + 1)\*b^2\*c^2\*d^2 - 2\*(4\*n^2 - 5\*n + 1)\*a\*b\*c\*d^3 + (2\*n^2 - 3\*n + 1)\*a^2\*d^4)\*integrate(1/2/(b^4\*c^7\*n^2 - 4\*a\*b^3\*c^6\*d\*n^2 + 6\*a^2\*b^2\*c^5\*d^2\*n^2 - 4\*a^3\*b\*c^4\*d^3\*n^2 + a^4\*c^3\*d^4\*n^2 + (b^4\*c^6\*d\*n^2 - 4\*a\*b^3\*c^5\*d^2\*n^2 + 6\*a^2\*b^2\*c^4\*d^3\*n^2 - 4\*a^3\*b\*c^3\*d^4\*n^2 + a^4\*c^2\*d^5\*n^2)\*x^n), x) - (a\*b^3\*d\*(4\*n - 1) - b^4\*c\*(n - 1))\*integrate(1/(a^2\*b^4\*c^4\*n - 4\*a^3\*b^3\*c^3\*d\*n + 6\*a^4\*b^2\*c^2\*d^2\*n - 4\*a^5\*b\*c\*d^3\*n + a^6\*d^4\*n + (a\*b^5\*c^4\*n - 4\*a^2\*b^4\*c^3\*d\*n + 6\*a^3\*b^3\*c^2\*d^2\*n - 4\*a^4\*b^2\*c\*d^3\*n + a^5\*b\*d^4\*n)\*x^n), x) + 1/2\*((a\*b^2\*c\*d^3\*(6\*n - 1) - a^2\*b\*d^4\*(2\*n - 1) + 2\*b^3\*c^2\*d^2\*n)\*x^(2\*n) + (a\*b^2\*c^2\*d^2\*(7\*n - 1) - a^3\*d^4\*(2\*n - 1) + 4\*b^3\*c^3\*d\*n + 3\*a^2\*b\*c\*d^3\*n)\*x\*x^n + (a^2\*b\*c^2\*d^2\*(7\*n - 1) - a^3\*c\*d^3\*(3\*n - 1) + 2\*b^3\*c^4\*n)\*x)/(a^2\*b^3\*c^7\*n^2 - 3\*a^3\*b^2\*c^6\*d\*n^2 + 3\*a^4\*b\*c^5\*d^2\*n^2 - a^5\*c^4\*d^3\*n^2 + (a\*b^4\*c^5\*d^2\*n^2 - 3\*a^2\*b^3\*c^4\*d^3\*n^2 + 3\*a^3\*b^2\*c^3\*d^4\*n^2 - a^4\*b\*c^2\*d^5\*n^2)\*x^(3\*n) + (2\*a\*b^4\*c^6\*d\*n^2 - 5\*a^2\*b^3\*c^5\*d^2\*n^2 + 3\*a^3\*b^2\*c^4\*d^3\*n^2 + a^4\*b\*c^3\*d^4\*n^2 - a^5\*c^2\*d^5\*n^2)\*x^(2\*n) + (a\*b^4\*c^7\*n^2 - a^2\*b^3\*c^6\*d\*n^2 - 3\*a^3\*b^2\*c^5\*d^2\*n^2 + 5\*a^4\*b\*c^4\*d^3\*n^2 - 2\*a^5\*c^3\*d^4\*n^2)\*x^n)

**Ericas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{b^2 d^3 x^{5n} + a^2 c^3 + (3 b^2 c d^2 + 2 a b d^3) x^{4n} + (3 b^2 c^2 d + 6 a b c d^2 + a^2 d^3) x^{3n} + (b^2 c^3 + 6 a b c^2 d + 3 a^2 c d^2) x^{2n} + (2 a b c^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^n + a)^2*(d*x^n + c)^3),x, algorithm="fricas")`

[Out] `integral(1/(b^2*d^3*x^(5*n) + a^2*c^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*x^(4*n) + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(3*n) + (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(2*n) + (2*a*b*c^3 + 3*a^2*c^2*d)*x^n), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*x**n)**2/(c+d*x**n)**3,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^n + a)^2*(d*x^n + c)^3),x, algorithm="giac")`

[Out] `integrate(1/((b*x^n + a)^2*(d*x^n + c)^3), x)`



### 3.213 $\int (a + bx^n)^p (c + dx^n)^q dx$

**Optimal.** Leaf size=81

$$x (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} (c + dx^n)^q \left(\frac{dx^n}{c} + 1\right)^{-q} F_1\left(\frac{1}{n}; -p, -q; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)$$

[Out]  $(x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^{(-1)}, -p, -q, 1 + n^{(-1)}, -((b*x^n)/a), -((d*x^n)/c)]/((1 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)$

**Rubi [A]** time = 0.128784, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$x (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} (c + dx^n)^q \left(\frac{dx^n}{c} + 1\right)^{-q} F_1\left(\frac{1}{n}; -p, -q; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p\*(c + d\*x^n)^q,x]

[Out]  $(x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^{(-1)}, -p, -q, 1 + n^{(-1)}, -((b*x^n)/a), -((d*x^n)/c)]/((1 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)$

**Rubi in Sympy [A]** time = 21.6673, size = 63, normalized size = 0.78

$$x \left(1 + \frac{bx^n}{a}\right)^{-p} \left(1 + \frac{dx^n}{c}\right)^{-q} (a + bx^n)^p (c + dx^n)^q \text{appellf}_1\left(\frac{1}{n}, -p, -q, 1 + \frac{1}{n}, -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*p\*(c+d\*x\*\*n)\*\*q,x)

[Out]  $x*(1 + b*x**n/a)**(-p)*(1 + d*x**n/c)**(-q)*(a + b*x**n)**p*(c + d*x**n)**q*appellf1(1/n, -p, -q, 1 + 1/n, -b*x**n/a, -d*x**n/c)$

**Mathematica [B]** time = 0.661915, size = 190, normalized size = 2.35

$$\frac{ac(n+1)x(a+bx^n)^p(c+dx^n)^q F_1\left(\frac{1}{n}; -p, -q; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{bcnpx^n F_1\left(1 + \frac{1}{n}; 1 - p, -q; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) + adnqx^n F_1\left(1 + \frac{1}{n}; -p, 1 - q; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) + ac(n+1)F_1\left(\frac{1}{n}; -p, -q; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^n)^p\*(c + d\*x^n)^q,x]

[Out]  $(a*c*(1+n)*x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^{(-1)}, -p, -q, 1 + n^{(-1)}, -((b*x^n)/a), -((d*x^n)/c)]/(b*c*n^p*x^n*AppellF1[1 + n^{(-1)}, 1 - p, -q, 2 + n^{(-1)}, -((b*x^n)/a), -((d*x^n)/c)] + a*d*n^q*x^n*AppellF1[1 + n^{(-1)}, -p, 1 - q, 2 + n^{(-1)}, -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1+n)*AppellF1[n^{(-1)}, -p, -q, 1 + n^{(-1)}, -((b*x^n)/a), -((d*x^n)/c)])$

**Maple [F]** time = 0.207, size = 0, normalized size = 0.

$$\int (a + bx^n)^p (c + dx^n)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*x^n)^p*(c+d*x^n)^q,x)`

[Out] `int((a+b*x^n)^p*(c+d*x^n)^q,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (dx^n + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^p*(d*x^n + c)^q,x, algorithm="maxima")`

[Out] `integrate((b*x^n + a)^p*(d*x^n + c)^q, x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((bx^n + a)^p(dx^n + c)^q, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^p*(d*x^n + c)^q,x, algorithm="fricas")`

[Out] `integral((b*x^n + a)^p*(d*x^n + c)^q, x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n)**p*(c+d*x**n)**q,x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (dx^n + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^p*(d*x^n + c)^q,x, algorithm="giac")`

[Out] `integrate((b*x^n + a)^p*(d*x^n + c)^q, x)`

### 3.214 $\int (a + bx^n)^p (c + dx^n)^3 dx$

**Optimal.** Leaf size=402

$$\frac{dx (a + bx^n)^{p+1} (a^2 d^2 (2n^2 + 3n + 1) - abcd (n^2(p + 7) + n(2p + 9) + 2) + b^2 c^2 (n^2 (p^2 + 6p + 11) + 2n(p + 3) + 1))}{b^3 (np + n + 1)(n(p + 2) + 1)(n(p + 3) + 1)}$$

$$\frac{x (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} (a^3 d^3 (2n^2 + 3n + 1) - 3a^2 bcd^2 (n + 1)(n(p + 3) + 1) + 3ab^2 c^2 d (n^2 (p^2 + 5p + 6) + n(2p + 5) + 1))}{b^3 (np + n + 1)(n(p + 2) + 1)(n(p + 3) + 1)}$$

$$- \frac{dx (c + dx^n) (a + bx^n)^{p+1} (ad(2n + 1) - bc(n(p + 5) + 1))}{b^2 (n(p + 2) + 1)(n(p + 3) + 1)} + \frac{dx (c + dx^n)^2 (a + bx^n)^{p+1}}{b(np + 3n + 1)}$$

[Out]  $(d^*(a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + n^2*(7 + p) + n*(9 + 2*p)) + b^2*c^2*(1 + 2*n*(3 + p) + n^2*(11 + 6*p + p^2))) * x^*(a + b*x^n)^{(1 + p)} / (b^3*(1 + n + n*p)^*(1 + n*(2 + p))^*(1 + n*(3 + p))) - (d^*(a*d*(1 + 2*n) - b*c*(1 + n*(5 + p))) * x^*(a + b*x^n)^{(1 + p)} * (c + d*x^n) / (b^2*(1 + n*(2 + p))^*(1 + n*(3 + p))) + (d*x^*(a + b*x^n)^{(1 + p)} * (c + d*x^n)^2) / (b*(1 + 3*n + n*p)) - ((a^3*d^3*(1 + 3*n + 2*n^2) - 3*a^2*b*c*d^2*(1 + n)*(1 + n*(3 + p)) + 3*a*b^2*c^2*d*(1 + n*(5 + 2*p) + n^2*(6 + 5*p + p^2)) - b^3*c^3*(1 + 3*n*(2 + p) + n^2*(11 + 12*p + 3*p^2) + n^3*(6 + 11*p + 6*p^2 + p^3))) * x^*(a + b*x^n)^p * Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -(b*x^n/a)] / (b^3*(1 + n + n*p)^*(1 + n*(2 + p))^*(1 + n*(3 + p))^*(1 + (b*x^n/a)^p)$

**Rubi [A]** time = 1.28482, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{dx (a + bx^n)^{p+1} (a^2 d^2 (2n^2 + 3n + 1) - abcd (n^2(p + 7) + n(2p + 9) + 2) + b^2 c^2 (n^2 (p^2 + 6p + 11) + 2n(p + 3) + 1))}{b^3 (np + n + 1)(n(p + 2) + 1)(n(p + 3) + 1)}$$

$$\frac{x (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} (a^3 d^3 (2n^2 + 3n + 1) - 3a^2 bcd^2 (n + 1)(n(p + 3) + 1) + 3ab^2 c^2 d (n^2 (p^2 + 5p + 6) + n(2p + 5) + 1))}{b^3 (np + n + 1)(n(p + 2) + 1)(n(p + 3) + 1)}$$

$$- \frac{dx (c + dx^n) (a + bx^n)^{p+1} (ad(2n + 1) - b(cn(p + 5) + c))}{b^2 (n(p + 2) + 1)(n(p + 3) + 1)} + \frac{dx (c + dx^n)^2 (a + bx^n)^{p+1}}{b(n(p + 3) + 1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p\*(c + d\*x^n)^3, x]

[Out]  $(d^*(a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + n^2*(7 + p) + n*(9 + 2*p)) + b^2*c^2*(1 + 2*n*(3 + p) + n^2*(11 + 6*p + p^2))) * x^*(a + b*x^n)^{(1 + p)} / (b^3*(1 + n + n*p)^*(1 + n*(2 + p))^*(1 + n*(3 + p))) - (d^*(a*d*(1 + 2*n) - b*(c + c*n*(5 + p))) * x^*(a + b*x^n)^{(1 + p)} * (c + d*x^n) / (b^2*(1 + n*(2 + p))^*(1 + n*(3 + p))) + (d*x^*(a + b*x^n)^{(1 + p)} * (c + d*x^n)^2) / (b*(1 + n*(3 + p))) - ((a^3*d^3*(1 + 3*n + 2*n^2) - 3*a^2*b*c*d^2*(1 + n)*(1 + n*(3 + p)) + 3*a*b^2*c^2*d*(1 + n*(5 + 2*p) + n^2*(6 + 5*p + p^2)) - b^3*c^3*(1 + 3*n*(2 + p) + n^2*(11 + 12*p + 3*p^2) + n^3*(6 + 11*p + 6*p^2 + p^3))) * x^*(a + b*x^n)^p * Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -(b*x^n/a)] / (b^3*(1 + n + n*p)^*(1 + n*(2 + p))^*(1 + n*(3 + p))^*(1 + (b*x^n/a)^p)$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*p\*(c+d\*x\*\*n)\*\*3, x)

[Out] Timed out

**Mathematica [A]** time = 0.444014, size = 168, normalized size = 0.42

$$x(a + bx^n)^p \left( \frac{bx^n}{a} + 1 \right)^{-p} \left( c^3 {}_2F_1 \left( \frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a} \right) + \frac{3c^2 dx^n {}_2F_1 \left( 1 + \frac{1}{n}, -p; 2 + \frac{1}{n}; -\frac{bx^n}{a} \right)}{n + 1} \right. \\ \left. + \frac{3cd^2 x^{2n} {}_2F_1 \left( 2 + \frac{1}{n}, -p; 3 + \frac{1}{n}; -\frac{bx^n}{a} \right)}{2n + 1} + \frac{d^3 x^{3n} {}_2F_1 \left( 3 + \frac{1}{n}, -p; 4 + \frac{1}{n}; -\frac{bx^n}{a} \right)}{3n + 1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^p\*(c + d\*x^n)^3,x]

[Out] (x\*(a + b\*x^n)^p\*((3\*c^2\*d\*x^n\*Hypergeometric2F1[1 + n^(-1), -p, 2 + n^(-1), -(b\*x^n)/a])/(1 + n) + (3\*c\*d^2\*x^(2\*n)\*Hypergeometric2F1[2 + n^(-1), -p, 3 + n^(-1), -(b\*x^n)/a])/(1 + 2\*n) + (d^3\*x^(3\*n)\*Hypergeometric2F1[3 + n^(-1), -p, 4 + n^(-1), -(b\*x^n)/a])/(1 + 3\*n) + c^3\*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -(b\*x^n)/a]))/(1 + (b\*x^n)/a)^p

**Maple [F]** time = 0.159, size = 0, normalized size = 0.

$$\int (a + bx^n)^p (c + dx^n)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)^p\*(c+d\*x^n)^3,x)

[Out] int((a+b\*x^n)^p\*(c+d\*x^n)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^n + c)^3 (bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^3\*(b\*x^n + a)^p,x, algorithm="maxima")

[Out] integrate((d\*x^n + c)^3\*(b\*x^n + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((d^3 x^{3n} + 3cd^2 x^{2n} + 3c^2 dx^n + c^3)(bx^n + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^3\*(b\*x^n + a)^p,x, algorithm="fricas")

[Out] integral((d^3\*x^(3\*n) + 3\*c\*d^2\*x^(2\*n) + 3\*c^2\*d\*x^n + c^3)\*(b\*x^n + a)^p, x)

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n)**p*(c+d*x**n)**3,x)`

[Out] Exception raised: TypeError

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^3*(b*x^n + a)^p,x, algorithm="giac")`

[Out] Exception raised: TypeError

### 3.215 $\int (a + bx^n)^p (c + dx^n)^2 dx$

**Optimal.** Leaf size=202

$$\frac{x(a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} (bc(np + n + 1)(ad - bc(n(p + 2) + 1)) - ad(ad(n + 1) - bc(n(p + 3) + 1))) {}_2F_1\left(\frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{b^2(np + n + 1)(n(p + 2) + 1)} - \frac{dx(a + bx^n)^{p+1} (ad(n + 1) - bc(n(p + 3) + 1))}{b^2(np + n + 1)(n(p + 2) + 1)} + \frac{dx(c + dx^n)(a + bx^n)^{p+1}}{b(np + 2n + 1)}$$

[Out]  $-\left(\left(d^*(a*d*(1+n) - b*c*(1+n*(3+p)))\right)*x^*(a + b*x^n)^{(1+p)}\right) / \left(b^{2*(1+n+n*p)}*(1+n*(2+p))\right) + \left(d*x^*(a + b*x^n)^{(1+p)}*(c + d*x^n)\right) / \left(b*(1+2*n+n*p)\right) - \left(\left(b*c*(1+n+n*p)^*(a*d - b*c*(1+n*(2+p))) - a*d*(a*d*(1+n) - b*c*(1+n*(3+p)))\right)*x^*(a + b*x^n)^p * \text{Hypergeometric2F1}[n^{(-1)}, -p, 1 + n^{(-1)}, -(b*x^n)/a]\right) / \left(b^{2*(1+n+n*p)}*(1+n*(2+p))\right)^*(1 + (b*x^n)/a)^p$

**Rubi [A]** time = 0.624457, antiderivative size = 197, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{dx(a + bx^n)^{p+1} (ad(n + 1) - b(cn(p + 3) + c))}{b^2(np + n + 1)(n(p + 2) + 1)} - \frac{x(a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \left(c(ad - b(cn(p + 2) + c)) - \frac{ad(ad(n+1)-b(cn(p+3)+c))}{b(np+n+1)}\right) {}_2F_1\left(\frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{b(n(p + 2) + 1)} + \frac{dx(c + dx^n)(a + bx^n)^{p+1}}{b(n(p + 2) + 1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p\*(c + d\*x^n)^2, x]

[Out]  $-\left(\left(d^*(a*d*(1+n) - b*(c + c*n*(3+p)))\right)*x^*(a + b*x^n)^{(1+p)}\right) / \left(b^{2*(1+n+n*p)}*(1+n*(2+p))\right) + \left(d*x^*(a + b*x^n)^{(1+p)}*(c + d*x^n)\right) / \left(b*(1+n*(2+p))\right) - \left(\left(c*(a*d - b*(c + c*n*(2+p))) - (a*d*(a*d*(1+n) - b*(c + c*n*(3+p)))\right) / \left(b*(1+n+n*p)\right)\right)*x^*(a + b*x^n)^p * \text{Hypergeometric2F1}[n^{(-1)}, -p, 1 + n^{(-1)}, -(b*x^n)/a] / \left(b*(1+n*(2+p))\right)^*(1 + (b*x^n)/a)^p$

**Rubi in Sympy [A]** time = 36.2797, size = 173, normalized size = 0.86

$$\frac{dx(a + bx^n)^{p+1} (c + dx^n)}{b(n(p + 2) + 1)} - \frac{dx(a + bx^n)^{p+1} (ad(n + 1) - bc(n(p + 3) + 1))}{b^2(n(p + 1) + 1)(n(p + 2) + 1)} + \frac{x\left(1 + \frac{bx^n}{a}\right)^{-p} (a + bx^n)^p (ad(ad(n + 1) - bc(n(p + 3) + 1)) - bc(ad - bc(n(p + 2) + 1))(n(p + 1) + 1)) {}_2F_1\left(-p, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{b^2(n(p + 1) + 1)(n(p + 2) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*p\*(c+d\*x\*\*n)\*\*2,x)

[Out]  $d*x^*(a + b*x**n)**(p + 1)*(c + d*x**n)/(b*(n*(p + 2) + 1)) - d*x^*(a + b*x**n)**(p + 1)*(a*d*(n + 1) - b*c*(n*(p + 3) + 1))/(b**2*(n*(p + 1) + 1)*(n*(p + 2) + 1)) + x*(1 + b*x**n/a)**(-p)*(a + b*x**n)**p*(a*d*(a*d*(n + 1) - b*c*(n*(p + 3) + 1)) - b*c*(a*d - b*c*(n*(p + 2) + 1))*(n*(p + 1) + 1)*hyper((-p, 1/n), (1 + 1/n), -b*x**n/a)/(b**2*(n*(p + 1) + 1)*(n*(p + 2) + 1))$

**Mathematica [A]** time = 0.24786, size = 140, normalized size = 0.69

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \left( (n+1) \left( c^2(2n+1) {}_2F_1\left(\frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) + d^2 x^{2n} {}_2F_1\left(2 + \frac{1}{n}, -p; 3 + \frac{1}{n}; -\frac{bx^n}{a}\right) \right) + 2cd(2n+1) \right)}{(n+1)(2n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^p\*(c + d\*x^n)^2,x]

[Out] (x\*(a + b\*x^n)^p\*(2\*c\*d\*(1 + 2\*n)\*x^n\*Hypergeometric2F1[1 + n^(-1), -p, 2 + n^(-1), -(b\*x^n)/a]) + (1 + n)\*(d^2\*x^(2\*n)\*Hypergeometric2F1[2 + n^(-1), -p, 3 + n^(-1), -(b\*x^n)/a]) + c^2\*(1 + 2\*n)\*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -(b\*x^n)/a]))/(1 + n)\*(1 + 2\*n)\*(1 + (b\*x^n)/a)^p

**Maple [F]** time = 0.137, size = 0, normalized size = 0.

$$\int (a + bx^n)^p (c + dx^n)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)^p\*(c+d\*x^n)^2,x)

[Out] int((a+b\*x^n)^p\*(c+d\*x^n)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^n + c)^2 (bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^2\*(b\*x^n + a)^p,x, algorithm="maxima")

[Out] integrate((d\*x^n + c)^2\*(b\*x^n + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((d^2 x^{2n} + 2cdx^n + c^2)(bx^n + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^2\*(b\*x^n + a)^p,x, algorithm="fricas")

[Out] integral((d^2\*x^(2\*n) + 2\*c\*d\*x^n + c^2)\*(b\*x^n + a)^p, x)

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**2,x)
```

```
[Out] Exception raised: TypeError
```

---

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^n + c)^2*(b*x^n + a)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```



### 3.216 $\int (a + bx^n)^p (c + dx^n) dx$

**Optimal.** Leaf size=98

$$\frac{dx(a + bx^n)^{p+1}}{b(np + n + 1)} - \frac{x(a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} (ad - bc(np + n + 1)) {}_2F_1\left(\frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{b(np + n + 1)}$$

[Out]  $(d*x*(a + b*x^n)^{(1 + p)})/(b*(1 + n + n*p)) - ((a*d - b*c*(1 + n + n*p))*x*(a + b*x^n)^p*Hypergeometric2F1[n^{(-1)}, -p, 1 + n^{(-1)}, -((b*x^n)/a)])/(b*(1 + n + n*p)*(1 + (b*x^n)/a)^p)$

**Rubi [A]** time = 0.119854, antiderivative size = 89, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$x(a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \left(c - \frac{ad}{bnp + bn + b}\right) {}_2F_1\left(\frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) + \frac{dx(a + bx^n)^{p+1}}{b(np + n + 1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p\*(c + d\*x^n), x]

[Out]  $(d*x*(a + b*x^n)^{(1 + p)})/(b*(1 + n + n*p)) + ((c - (a*d)/(b + b*n + b*n*p))*x*(a + b*x^n)^p*Hypergeometric2F1[n^{(-1)}, -p, 1 + n^{(-1)}, -((b*x^n)/a)])/(1 + (b*x^n)/a)^p$

**Rubi in Sympy [A]** time = 11.0367, size = 80, normalized size = 0.82

$$\frac{dx(a + bx^n)^{p+1}}{b(np + n + 1)} - \frac{x \left(1 + \frac{bx^n}{a}\right)^{-p} (a + bx^n)^p (ad - bc(n(p + 1) + 1)) {}_2F_1\left(-p, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{b(n(p + 1) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*p\*(c+d\*x\*\*n), x)

[Out]  $d*x*(a + b*x**n)**(p + 1)/(b*(n*p + n + 1)) - x*(1 + b*x**n/a)**(-p)*(a + b*x**n)**p*(a*d - b*c*(n*(p + 1) + 1))*hyper((-p, 1/n), (1 + 1/n, ), -b*x**n/a)/(b*(n*(p + 1) + 1))$

**Mathematica [A]** time = 0.102702, size = 85, normalized size = 0.87

$$\frac{x(a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \left(c(n + 1) {}_2F_1\left(\frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) + dx^n {}_2F_1\left(1 + \frac{1}{n}, -p; 2 + \frac{1}{n}; -\frac{bx^n}{a}\right)\right)}{n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^p\*(c + d\*x^n), x]

[Out]  $(x*(a + b*x^n)^p*(d*x^n*Hypergeometric2F1[1 + n^{(-1)}, -p, 2 + n^{(-1)}, -((b*x^n)/a)]) + c*(1 + n)*Hypergeometric2F1[n^{(-1)}, -p, 1 + n^{(-1)}, -((b*x^n)/a)])/((1 + n)*(1 + (b*x^n)/a)^p)$

**Maple [F]** time = 0.135, size = 0, normalized size = 0.

$$\int (a + bx^n)^p (c + dx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*x^n)^p*(c+d*x^n),x)`

[Out] `int((a+b*x^n)^p*(c+d*x^n),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^n + c)(bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)*(b*x^n + a)^p,x, algorithm="maxima")`

[Out] `integrate((d*x^n + c)*(b*x^n + a)^p, x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx^n + c)(bx^n + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)*(b*x^n + a)^p,x, algorithm="fricas")`

[Out] `integral((d*x^n + c)*(b*x^n + a)^p, x)`

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n)**p*(c+d*x**n),x)`

[Out] Exception raised: TypeError

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)*(b*x^n + a)^p,x, algorithm="giac")`

[Out] Exception raised: TypeError

### 3.217 $\int (a + bx^n)^p dx$

**Optimal.** Leaf size=46

$$x (a + bx^n)^p \left( \frac{bx^n}{a} + 1 \right)^{-p} {}_2F_1 \left( \frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a} \right)$$

[Out] (x\*(a + b\*x^n)^p\*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -(b\*x^n)/a])/(1 + (b\*x^n)/a)^p

**Rubi [A]** time = 0.0316681, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$x (a + bx^n)^p \left( \frac{bx^n}{a} + 1 \right)^{-p} {}_2F_1 \left( \frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a} \right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p, x]

[Out] (x\*(a + b\*x^n)^p\*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -(b\*x^n)/a])/(1 + (b\*x^n)/a)^p

**Rubi in Sympy [A]** time = 3.70442, size = 36, normalized size = 0.78

$$x \left( 1 + \frac{bx^n}{a} \right)^{-p} (a + bx^n)^p {}_2F_1 \left( -p, \frac{1}{n} \middle| -\frac{bx^n}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*p, x)

[Out] x\*(1 + b\*x\*\*n/a)\*\*(-p)\*(a + b\*x\*\*n)\*\*p\*hyper((-p, 1/n), (1 + 1/n), -b\*x\*\*n/a)

**Mathematica [A]** time = 0.0267131, size = 46, normalized size = 1.

$$x (a + bx^n)^p \left( \frac{bx^n}{a} + 1 \right)^{-p} {}_2F_1 \left( \frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^p, x]

[Out] (x\*(a + b\*x^n)^p\*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -(b\*x^n)/a])/(1 + (b\*x^n)/a)^p

**Maple [F]** time = 0.002, size = 0, normalized size = 0.

$$\int (a + bx^n)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*x^n)^p, x)`

[Out] `int((a+b*x^n)^p, x)`

---

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^p, x, algorithm="maxima")`

[Out] `integrate((b*x^n + a)^p, x)`

---

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((bx^n + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^p, x, algorithm="fricas")`

[Out] `integral((b*x^n + a)^p, x)`

---

**Sympy** [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n)**p, x)`

[Out] Exception raised: TypeError

---

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^p, x, algorithm="giac")`

[Out] `integrate((b*x^n + a)^p, x)`

$$3.218 \quad \int \frac{(a+bx^n)^p}{c+dx^n} dx$$

**Optimal.** Leaf size=59

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} F_1\left(\frac{1}{n}; -p, 1; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{c}$$

[Out] (x\*(a + b\*x^n)^p\*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -(b\*x^n)/a, -((d\*x^n)/c)])/(c\*(1 + (b\*x^n)/a)^p)

**Rubi [A]** time = 0.084008, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} F_1\left(\frac{1}{n}; -p, 1; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p/(c + d\*x^n), x]

[Out] (x\*(a + b\*x^n)^p\*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -(b\*x^n)/a, -((d\*x^n)/c)])/(c\*(1 + (b\*x^n)/a)^p)

**Rubi in Sympy [A]** time = 21.4334, size = 44, normalized size = 0.75

$$\frac{x \left(1 + \frac{bx^n}{a}\right)^{-p} (a + bx^n)^p \text{appellf1}\left(\frac{1}{n}, 1, -p, 1 + \frac{1}{n}, -\frac{dx^n}{c}, -\frac{bx^n}{a}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*p/(c+d\*x\*\*n), x)

[Out] x\*(1 + b\*x\*\*n/a)\*\*(-p)\*(a + b\*x\*\*n)\*\*p\*appellf1(1/n, 1, -p, 1 + 1/n, -d\*x\*\*n/c, -b\*x\*\*n/a)/c

**Mathematica [B]** time = 0.418044, size = 180, normalized size = 3.05

$$\frac{ac(n+1)x(a+bx^n)^p F_1\left(\frac{1}{n}; -p, 1; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{(c+dx^n)\left(bcnp^n F_1\left(1 + \frac{1}{n}; 1 - p, 1; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) - adnx^n F_1\left(1 + \frac{1}{n}; -p, 2; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) + ac(n+1)F_1\left(\frac{1}{n}; -\right)}{c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^n)^p/(c + d\*x^n), x]

[Out] (a\*c\*(1 + n)\*x\*(a + b\*x^n)^p\*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -(b\*x^n)/a, -((d\*x^n)/c)]/((c + d\*x^n)\*(b\*c\*n^p\*x^n\*AppellF1[1 + n^(-1), 1 - p, 1, 2 + n^(-1), -(b\*x^n)/a, -((d\*x^n)/c)] - a\*d\*n\*x^n\*AppellF1[1 + n^(-1), -p, 2, 2 + n^(-1), -(b\*x^n)/a, -((d\*x^n)/c)] + a\*c\*(1 + n)\*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -(b\*x^n)/a, -((d\*x^n)/c)]))

**Maple [F]** time = 0.122, size = 0, normalized size = 0.

$$\int \frac{(a + bx^n)^p}{c + dx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)^p/(c+d\*x^n), x)

[Out] int((a+b\*x^n)^p/(c+d\*x^n), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p}{dx^n + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^p/(d\*x^n + c), x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^p/(d\*x^n + c), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^n + a)^p}{dx^n + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^p/(d\*x^n + c), x, algorithm="fricas")

[Out] integral((b\*x^n + a)^p/(d\*x^n + c), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^n)^p}{c + dx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*p/(c+d\*x\*\*n), x)

[Out] Integral((a + b\*x\*\*n)\*\*p/(c + d\*x\*\*n), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p}{dx^n + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^p/(d\*x^n + c), x, algorithm="giac")

[Out] integrate((b\*x^n + a)^p/(d\*x^n + c), x)

$$3.219 \quad \int \frac{(a+bx^n)^p}{(c+dx^n)^2} dx$$

**Optimal.** Leaf size=59

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} F_1\left(\frac{1}{n}; -p, 2; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{c^2}$$

[Out]  $(x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])/(c^2*(1 + (b*x^n)/a)^p)$

**Rubi [A]** time = 0.0831748, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} F_1\left(\frac{1}{n}; -p, 2; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p/(c + d\*x^n)^2, x]

[Out]  $(x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])/(c^2*(1 + (b*x^n)/a)^p)$

**Rubi in Sympy [A]** time = 19.9456, size = 46, normalized size = 0.78

$$\frac{x \left(1 + \frac{bx^n}{a}\right)^{-p} (a + bx^n)^p \text{appellf1}\left(\frac{1}{n}, 2, -p, 1 + \frac{1}{n}, -\frac{dx^n}{c}, -\frac{bx^n}{a}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*p/(c+d\*x\*\*n)\*\*2, x)

[Out]  $x*(1 + b*x**n/a)**(-p)*(a + b*x**n)**p*appellf1(1/n, 2, -p, 1 + 1/n, -d*x**n/c, -b*x**n/a)/c**2$

**Mathematica [B]** time = 0.473617, size = 180, normalized size = 3.05

$$\frac{ac(n+1)x(a+bx^n)^p F_1\left(\frac{1}{n}; -p, 2; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{(c+dx^n)^2 \left( bcnp x^n F_1\left(1 + \frac{1}{n}; 1 - p, 2; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) - 2adnx^n F_1\left(1 + \frac{1}{n}; -p, 3; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) + ac(n+1)F_1\left(\frac{1}{n}; \right. \right.$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^n)^p/(c + d\*x^n)^2, x]

[Out]  $(a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]/((c + d*x^n)^2*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 2*a*d*n*x^n*AppellF1[1 + n^(-1), -p, 3, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))$

**Maple [F]** time = 0.103, size = 0, normalized size = 0.

$$\int \frac{(a + bx^n)^p}{(c + dx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)^p/(c+d\*x^n)^2,x)

[Out] int((a+b\*x^n)^p/(c+d\*x^n)^2,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^p/(d\*x^n + c)^2,x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^p/(d\*x^n + c)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^n + a)^p}{d^2x^{2n} + 2cdx^n + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^p/(d\*x^n + c)^2,x, algorithm="fricas")

[Out] integral((b\*x^n + a)^p/(d^2\*x^(2\*n) + 2\*c\*d\*x^n + c^2), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^n)^p}{(c + dx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*p/(c+d\*x\*\*n)\*\*2,x)

[Out] Integral((a + b\*x\*\*n)\*\*p/(c + d\*x\*\*n)\*\*2, x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^p/(d\*x^n + c)^2,x, algorithm="giac")

[Out] integrate((b\*x^n + a)^p/(d\*x^n + c)^2, x)



$$3.220 \quad \int \frac{(a+bx^n)^p}{(c+dx^n)^3} dx$$

**Optimal.** Leaf size=59

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} F_1\left(\frac{1}{n}; -p, 3; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{c^3}$$

[Out]  $(x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])/(c^3*(1 + (b*x^n)/a)^p)$

**Rubi [A]** time = 0.084313, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} F_1\left(\frac{1}{n}; -p, 3; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p/(c + d\*x^n)^3, x]

[Out]  $(x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])/(c^3*(1 + (b*x^n)/a)^p)$

**Rubi in Sympy [A]** time = 20.1158, size = 46, normalized size = 0.78

$$\frac{x \left(1 + \frac{bx^n}{a}\right)^{-p} (a + bx^n)^p \text{appellf1}\left(\frac{1}{n}, 3, -p, 1 + \frac{1}{n}, -\frac{dx^n}{c}, -\frac{bx^n}{a}\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*p/(c+d\*x\*\*n)\*\*3, x)

[Out]  $x*(1 + b*x**n/a)**(-p)*(a + b*x**n)**p*appellf1(1/n, 3, -p, 1 + 1/n, -d*x**n/c, -b*x**n/a)/c**3$

**Mathematica [B]** time = 0.692187, size = 180, normalized size = 3.05

$$\frac{ac(n+1)x(a+bx^n)^p F_1\left(\frac{1}{n}; -p, 3; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{(c+dx^n)^3 \left( bcnp x^n F_1\left(1 + \frac{1}{n}; 1 - p, 3; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) - 3adnx^n F_1\left(1 + \frac{1}{n}; -p, 4; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) + ac(n+1)F_1\left(\frac{1}{n}; \right. \right.$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^n)^p/(c + d\*x^n)^3, x]

[Out]  $(a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]/((c + d*x^n)^3*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p, 3, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 3*a*d*n*x^n*AppellF1[1 + n^(-1), -p, 4, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))$

**Maple [F]** time = 0.112, size = 0, normalized size = 0.

$$\int \frac{(a + bx^n)^p}{(c + dx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)^p/(c+d\*x^n)^3,x)

[Out] int((a+b\*x^n)^p/(c+d\*x^n)^3,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^p/(d\*x^n + c)^3,x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^p/(d\*x^n + c)^3, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^n + a)^p}{d^3x^{3n} + 3cd^2x^{2n} + 3c^2dx^n + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^p/(d\*x^n + c)^3,x, algorithm="fricas")

[Out] integral((b\*x^n + a)^p/(d^3\*x^(3\*n) + 3\*c\*d^2\*x^(2\*n) + 3\*c^2\*d\*x^n + c^3), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*p/(c+d\*x\*\*n)\*\*3,x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^p/(d\*x^n + c)^3,x, algorithm="giac")

[Out] integrate((b\*x^n + a)^p/(d\*x^n + c)^3, x)

$$3.221 \quad \int (a + bx^n)^p (c + dx^n)^{-1 - \frac{1}{n} - p} dx$$

**Optimal.** Leaf size=93

$$\frac{x(a + bx^n)^p (c + dx^n)^{-\frac{1}{n} - p} \left( \frac{c(ax^n + b)}{a(c + dx^n)} \right)^{-p} {}_2F_1 \left( \frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{(bc - ad)x^n}{a(dx^n + c)} \right)}{c}$$

[Out] (x\*(a + b\*x^n)^p\*(c + d\*x^n)^(-n^(-1) - p)\*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])/(c\*((c + b\*x^n))/(a\*(c + d\*x^n)))^p

**Rubi [A]** time = 0.0726765, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$\frac{x(a + bx^n)^p (c + dx^n)^{-\frac{1}{n} - p} \left( \frac{c(ax^n + b)}{a(c + dx^n)} \right)^{-p} {}_2F_1 \left( \frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{(bc - ad)x^n}{a(dx^n + c)} \right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p\*(c + d\*x^n)^(-1 - n^(-1) - p), x]

[Out] (x\*(a + b\*x^n)^p\*(c + d\*x^n)^(-n^(-1) - p)\*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])/(c\*((c + b\*x^n))/(a\*(c + d\*x^n)))^p

**Rubi in Sympy [A]** time = 8.19064, size = 83, normalized size = 0.89

$$\frac{x \left( \frac{a(c + dx^n)}{c(a + bx^n)} \right)^{p+1+\frac{1}{n}} (a + bx^n)^{p+1} (c + dx^n)^{-p-1-\frac{1}{n}} {}_2F_1 \left( \frac{1}{n}, p+1+\frac{1}{n}; 1+\frac{1}{n}; -\frac{x^n(ad-bc)}{c(a+bx^n)} \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*p\*(c+d\*x\*\*n)\*\*(-1-1/n-p), x)

[Out] x\*(a\*(c + d\*x\*\*n)/(c\*(a + b\*x\*\*n)))\*\* (p + 1 + 1/n)\* (a + b\*x\*\*n)\*\* (p + 1)\*(c + d\*x\*\*n)\*\* (-p - 1 - 1/n)\*hyper((1/n, p + 1 + 1/n), (1 + 1/n, ), -x\*\*n\*(a\*d - b\*c)/(c\*(a + b\*x\*\*n)))/a

**Mathematica [A]** time = 0.558127, size = 93, normalized size = 1.

$$\frac{x(a + bx^n)^p (c + dx^n)^{-\frac{np+1}{n}} \left( \frac{c(ax^n + b)}{a(c + dx^n)} \right)^{-p} {}_2F_1 \left( \frac{1}{n}, -p; 1 + \frac{1}{n}; \frac{(ad - bc)x^n}{a(dx^n + c)} \right)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^p\*(c + d\*x^n)^(-1 - n^(-1) - p), x]

[Out] (x\*(a + b\*x^n)^p\*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), ((-b\*c) + a\*d)\*x^n)/(a\*(c + d\*x^n))])/(c\*((c\*(a + b\*x^n))/(a\*(c + d\*x^n)))^p\*(c + d\*x^n)^((1 + n\*p)/n))

**Maple [F]** time = 0.249, size = 0, normalized size = 0.

$$\int (a + bx^n)^p (c + dx^n)^{-1-n^{-1}-p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)^p\*(c+d\*x^n)^(-1-1/n-p), x)

[Out] int((a+b\*x^n)^p\*(c+d\*x^n)^(-1-1/n-p), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (dx^n + c)^{-p-\frac{1}{n}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^p\*(d\*x^n + c)^(-p - 1/n - 1), x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^p\*(d\*x^n + c)^(-p - 1/n - 1), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^n + a)^p}{(dx^n + c)^{\frac{np+n+1}{n}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^p\*(d\*x^n + c)^(-p - 1/n - 1), x, algorithm="fricas")

[Out] integral((b\*x^n + a)^p/(d\*x^n + c)^((n\*p + n + 1)/n), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*p\*(c+d\*x\*\*n)\*\*(-1-1/n-p), x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (dx^n + c)^{-p-\frac{1}{n}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^p\*(d\*x^n + c)^(-p - 1/n - 1), x, algorithm="giac")

[Out] integrate((b\*x^n + a)^p\*(d\*x^n + c)^(-p - 1/n - 1), x)

$$3.222 \quad \int (a + bx^n)^3 (c + dx^n)^{-4-\frac{1}{n}} dx$$

**Optimal.** Leaf size=178

$$\frac{6a^3n^3x(c+dx^n)^{-1/n}}{c^4(n+1)(2n+1)(3n+1)} + \frac{6a^2n^2x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^3(n+1)(2n+1)(3n+1)} \\ + \frac{3anx(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c^2(6n^2+5n+1)} + \frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}$$

[Out]  $(x^*(a + b*x^n)^3*(c + d*x^n)^{-3 - n^(-1)})/(c*(1 + 3*n)) + (3*a*n*x*(a + b*x^n)^2*(c + d*x^n)^{-2 - n^(-1)})/(c^2*(1 + 5*n + 6*n^2)) + (6*a^2*n^2*x*(a + b*x^n)*(c + d*x^n)^{-1 - n^(-1)})/(c^3*(1 + n)*(1 + 2*n)*(1 + 3*n)) + (6*a^3*n^3*x)/(c^4*(1 + n)*(1 + 2*n)*(1 + 3*n)*(c + d*x^n)^n^(-1))$

**Rubi [A]** time = 0.240627, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{6a^3n^3x(c+dx^n)^{-1/n}}{c^4(n+1)(2n+1)(3n+1)} + \frac{6a^2n^2x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^3(n+1)(2n+1)(3n+1)} \\ + \frac{3anx(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c^2(6n^2+5n+1)} + \frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^3\*(c + d\*x^n)^(-4 - n^(-1)), x]

[Out]  $(x^*(a + b*x^n)^3*(c + d*x^n)^{-3 - n^(-1)})/(c*(1 + 3*n)) + (3*a*n*x*(a + b*x^n)^2*(c + d*x^n)^{-2 - n^(-1)})/(c^2*(1 + 5*n + 6*n^2)) + (6*a^2*n^2*x*(a + b*x^n)*(c + d*x^n)^{-1 - n^(-1)})/(c^3*(1 + n)*(1 + 2*n)*(1 + 3*n)) + (6*a^3*n^3*x)/(c^4*(1 + n)*(1 + 2*n)*(1 + 3*n)*(c + d*x^n)^n^(-1))$

**Rubi in Sympy [A]** time = 35.5955, size = 156, normalized size = 0.88

$$\frac{6a^3n^3x(c+dx^n)^{-\frac{1}{n}}}{c^4(n+1)(2n+1)(3n+1)} + \frac{6a^2n^2x(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c^3(n+1)(2n+1)(3n+1)} \\ + \frac{3anx(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c^2(2n+1)(3n+1)} + \frac{x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{c(3n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*3\*(c+d\*x\*\*n)\*\*(-4-1/n), x)

[Out]  $6*a**3*n**3*x*(c + d*x**n)**(-1/n)/(c**4*(n + 1)*(2*n + 1)*(3*n + 1)) + 6*a**2*n**2*x*(a + b*x**n)*(c + d*x**n)**(-1 - 1/n)/(c**3*(n + 1)*(2*n + 1)*(3*n + 1)) + 3*a*n*x*(a + b*x**n)**2*(c + d*x**n)**(-2 - 1/n)/(c**2*(2*n + 1)*(3*n + 1)) + x*(a + b*x**n)**3*(c + d*x**n)**(-3 - 1/n)/(c*(3*n + 1))$

**Mathematica [C]** time = 0.607878, size = 198, normalized size = 1.11

$$x(c+dx^n)^{-1/n} \left( a^3 \left( \frac{dx^n}{c} + 1 \right)^{\frac{1}{n}} {}_2F_1 \left( 4 + \frac{1}{n}, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c} \right) + \frac{3a^2bx^n \left( \frac{dx^n}{c} + 1 \right)^{\frac{1}{n}} {}_2F_1 \left( 1 + \frac{1}{n}, 4 + \frac{1}{n}; 2 + \frac{1}{n}; -\frac{dx^n}{c} \right)}{n+1} + \frac{3ab^2x^{2n} \left( \frac{dx^n}{c} + 1 \right)^{\frac{1}{n}} {}_2F_1 \left( 2n + 1, \frac{1}{n}; 2n + 1; -\frac{dx^n}{c} \right)}{2n+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^3\*(c + d\*x^n)^(-4 - n^(-1)),x]

[Out] (x\*((b^3\*c^3\*x^(3\*n))/((1 + 3\*n)\*(c + d\*x^n)^3) + (3\*a^2\*b\*x^n\*(1 + (d\*x^n)/c))^n^(-1)\*Hypergeometric2F1[1 + n^(-1), 4 + n^(-1), 2 + n^(-1), -(d\*x^n)/c])/(1 + n) + (3\*a\*b^2\*x^(2\*n)\*(1 + (d\*x^n)/c))^n^(-1)\*Hypergeometric2F1[2 + n^(-1), 4 + n^(-1), 3 + n^(-1), -(d\*x^n)/c])/(1 + 2\*n) + a^3\*(1 + (d\*x^n)/c))^n^(-1)\*Hypergeometric2F1[4 + n^(-1), n^(-1), 1 + n^(-1), -(d\*x^n)/c])/(c^4\*(c + d\*x^n)^n^(-1))

**Maple [F]** time = 0.203, size = 0, normalized size = 0.

$$\int (a + bx^n)^3 (c + dx^n)^{-4-n^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)^3\*(c+d\*x^n)^(-4-1/n),x)

[Out] int((a+b\*x^n)^3\*(c+d\*x^n)^(-4-1/n),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^3(dx^n + c)^{-\frac{1}{n}-4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^3\*(d\*x^n + c)^(-1/n - 4),x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^3\*(d\*x^n + c)^(-1/n - 4), x)

**Fricas [A]** time = 0.255308, size = 645, normalized size = 3.62

$$(6a^3d^4n^3 + b^3c^3d + (2b^3c^3d + 3ab^2c^2d^2 + 6a^2bcd^3)n^2 + 3(b^3c^3d + ab^2c^2d^2)n)xx^{4n} + (24a^3cd^3n^3 + b^3c^4 + 3ab^2c^3d + 2(b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^3\*(d\*x^n + c)^(-1/n - 4),x, algorithm="fricas")

[Out] ((6\*a^3\*d^4\*n^3 + b^3\*c^3\*d + (2\*b^3\*c^3\*d + 3\*a\*b^2\*c^2\*d^2 + 6\*a^2\*b\*c\*d^3)\*n^2 + 3\*(b^3\*c^3\*d + a\*b^2\*c^2\*d^2)\*n)\*x\*x^(4\*n) + (24\*a^3\*c\*d^3\*n^3 + b^3\*c^4 + 3\*a\*b^2\*c^3\*d + 2\*(b^3\*c^4 + 6\*a\*b^2\*c^3\*d + 12\*a^2\*b\*c^2\*d^2 + 3\*a^3\*c\*d^3)\*n^2 + 3\*(b^3\*c^4 + 5\*a\*b^2\*c^3\*d + 2\*a^2\*b\*c^2\*d^2)\*n)\*x\*x^(3\*n) + 3\*(12\*a^3\*c^2\*d^2\*n^3 + a\*b^2\*c^4 + a^2\*b\*c^3\*d + (3\*a\*b^2\*c^4 + 12\*a^2\*b\*c^3\*d + 7\*a^3\*c^2\*d^2)\*n^2 + (4\*a\*b^2\*c^4 + 7\*a^2\*b\*c^3\*d + a^3\*c^2\*d^2)\*n)\*x\*x^(2\*n) + (24\*a^3\*c^3\*d\*n^3 + 3\*a^2\*b\*c^4 + a^3\*c^3\*d + 2\*(9\*a^2\*b\*c^4 + 13\*a^3\*c^3\*d)\*n^2 + 3\*(5\*a^2\*b\*c^4 + 3\*a^3\*c^3\*d)\*n)\*x\*x^n + (6\*a^3\*c^4\*n^3 + 11\*a^3\*c^4\*n^2 + 6\*a^3\*c^4\*n + a^3\*c^4)\*x)/((6\*c^4\*n^3 + 11\*c^4\*n^2 + 6\*c^4\*n + c^4)\*(d\*x^n + c)^((4\*n + 1)/n))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n)**3*(c+d*x**n)**(-4-1/n),x)`

[Out] Timed out

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^3*(d*x^n + c)^(-1/n - 4),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.223 \quad \int (a + bx^n)^2 (c + dx^n)^{-3-\frac{1}{n}} dx$$

**Optimal.** Leaf size=116

$$\frac{2a^2n^2x(c+dx^n)^{-1/n}}{c^3(n+1)(2n+1)} + \frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^2(n+1)(2n+1)} + \frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

[Out]  $(x^*(a + b*x^n)^2*(c + d*x^n)^{(-2 - n^(-1))})/(c*(1 + 2*n)) + (2*a*n*x*(a + b*x^n)*(c + d*x^n)^{(-1 - n^(-1))})/(c^2*(1 + n)*(1 + 2*n)) + (2*a^2*n^2*x)/(c^3*(1 + n)*(1 + 2*n)*(c + d*x^n)^{n^(-1)})$

**Rubi [A]** time = 0.112751, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{2a^2n^2x(c+dx^n)^{-1/n}}{c^3(n+1)(2n+1)} + \frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^2(n+1)(2n+1)} + \frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^2\*(c + d\*x^n)^(-3 - n^(-1)), x]

[Out]  $(x^*(a + b*x^n)^2*(c + d*x^n)^{(-2 - n^(-1))})/(c*(1 + 2*n)) + (2*a*n*x*(a + b*x^n)*(c + d*x^n)^{(-1 - n^(-1))})/(c^2*(1 + n)*(1 + 2*n)) + (2*a^2*n^2*x)/(c^3*(1 + n)*(1 + 2*n)*(c + d*x^n)^{n^(-1)})$

**Rubi in Sympy [A]** time = 18.1007, size = 100, normalized size = 0.86

$$\frac{2a^2n^2x(c+dx^n)^{-\frac{1}{n}}}{c^3(n+1)(2n+1)} + \frac{2anx(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c^2(n+1)(2n+1)} + \frac{x(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c(2n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*2\*(c+d\*x\*\*n)\*\*(-3-1/n), x)

[Out]  $2*a**2*n**2*x*(c + d*x**n)**(-1/n)/(c**3*(n + 1)*(2*n + 1)) + 2*a*n*x*(a + b*x**n)*(c + d*x**n)**(-1 - 1/n)/(c**2*(n + 1)*(2*n + 1)) + x*(a + b*x**n)**2*(c + d*x**n)**(-2 - 1/n)/(c*(2*n + 1))$

**Mathematica [C]** time = 0.313694, size = 139, normalized size = 1.2

$$\frac{x(c+dx^n)^{-1/n} \left( a^2 \left( \frac{dx^n}{c} + 1 \right)^{\frac{1}{n}} {}_2F_1 \left( 3 + \frac{1}{n}, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c} \right) + \frac{2abx^n \left( \frac{dx^n}{c} + 1 \right)^{\frac{1}{n}} {}_2F_1 \left( 1 + \frac{1}{n}, 3 + \frac{1}{n}; 2 + \frac{1}{n}; -\frac{dx^n}{c} \right)}{c^3} + \frac{b^2c^2x^{2n}}{(2n+1)(c+dx^n)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^2\*(c + d\*x^n)^(-3 - n^(-1)), x]

[Out]  $(x^*((b^2*c^2*x^(2*n))/((1 + 2*n)*(c + d*x^n)^2) + (2*a*b*x^n*(1 + (d*x^n)/c)^{n^(-1)}*Hypergeometric2F1[1 + n^(-1), 3 + n^(-1), 2 + n^(-1), -(d*x^n)/c]))/(1 + n) + a^2*(1 + (d*x^n)/c)^{n^(-1)}*Hypergeometric2F1[3 + n^(-1), n^(-1), 1 + n^(-1), -(d*x^n)/c]))/(c^3*(c + d*x^n)^{n^(-1)})$



**Maple [F]** time = 0.177, size = 0, normalized size = 0.

$$\int (a + bx^n)^2 (c + dx^n)^{-3-n^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)^2\*(c+d\*x^n)^(-3-1/n), x)

[Out] int((a+b\*x^n)^2\*(c+d\*x^n)^(-3-1/n), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^2 (dx^n + c)^{-\frac{1}{n}-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^2\*(d\*x^n + c)^(-1/n - 3), x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^2\*(d\*x^n + c)^(-1/n - 3), x)

**Fricas [A]** time = 0.256654, size = 312, normalized size = 2.69

$$\frac{(2a^2d^3n^2 + b^2c^2d + (b^2c^2d + 2abcd^2)n)xx^{3n} + (6a^2cd^2n^2 + b^2c^3 + 2abc^2d + (b^2c^3 + 6abc^2d + 2a^2cd^2)n)xx^{2n} + (6a^2c^2d^2 + 3a^2c^2d + (b^2c^2d + 2abcd^2)n)xx^{n} + (2c^3n^2 + 3c^3n + c^3)(dx^n + c)^{\frac{3n+1}{n}}}{(2c^3n^2 + 3c^3n + c^3)(dx^n + c)^{\frac{3n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^2\*(d\*x^n + c)^(-1/n - 3), x, algorithm="fricas")

[Out] ((2\*a^2\*d^3\*n^2 + b^2\*c^2\*d + (b^2\*c^2\*d + 2\*a\*b\*c\*d^2)\*n)\*x\*x^(3\*n) + (6\*a^2\*c\*d^2\*n^2 + b^2\*c^3 + 2\*a\*b\*c^2\*d + (b^2\*c^3 + 6\*a\*b\*c^2\*d + 2\*a^2\*c\*d^2)\*n)\*x\*x^(2\*n) + (6\*a^2\*c^2\*d\*n^2 + 2\*a\*b\*c^3 + a^2\*c^2\*d + (4\*a\*b\*c^3 + 5\*a^2\*c^2\*d)\*n)\*x\*x^n + (2\*a^2\*c^3\*n^2 + 3\*a^2\*c^3\*n + a^2\*c^3)\*x)/((2\*c^3\*n^2 + 3\*c^3\*n + c^3)\*(d\*x^n + c)^((3\*n + 1)/n))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*2\*(c+d\*x\*\*n)\*\*(-3-1/n), x)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.224 \quad \int (a + bx^n)(c + dx^n)^{-2-\frac{1}{n}} dx$$

**Optimal.** Leaf size=58

$$\frac{x(a + bx^n)(c + dx^n)^{-\frac{1}{n}-1}}{c(n+1)} + \frac{anx(c + dx^n)^{-1/n}}{c^2(n+1)}$$

[Out]  $(x*(a + b*x^n)*(c + d*x^n)^{(-1 - n^(-1))})/(c*(1 + n)) + (a*n*x)/(c^2*(1 + n)*(c + d*x^n)^{n^(-1)})$

**Rubi [A]** time = 0.0483245, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{x(a + bx^n)(c + dx^n)^{-\frac{1}{n}-1}}{c(n+1)} + \frac{anx(c + dx^n)^{-1/n}}{c^2(n+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^n)*(c + d*x^n)^{(-2 - n^(-1))}, x]$

[Out]  $(x*(a + b*x^n)*(c + d*x^n)^{(-1 - n^(-1))})/(c*(1 + n)) + (a*n*x)/(c^2*(1 + n)*(c + d*x^n)^{n^(-1)})$

**Rubi in Sympy [A]** time = 6.74866, size = 48, normalized size = 0.83

$$\frac{anx(c + dx^n)^{-\frac{1}{n}}}{c^2(n+1)} + \frac{x(a + bx^n)(c + dx^n)^{-1-\frac{1}{n}}}{c(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a+b*x**n)*(c+d*x**n)**(-2-1/n), x)$

[Out]  $a*n*x*(c + d*x**n)**(-1/n)/(c**2*(n + 1)) + x*(a + b*x**n)*(c + d*x**n)**(-1 - 1/n)/(c*(n + 1))$

**Mathematica [C]** time = 0.169288, size = 82, normalized size = 1.41

$$\frac{x(c + dx^n)^{-\frac{n+1}{n}} \left( a(n+1)(c + dx^n) \left( \frac{dx^n}{c} + 1 \right)^{\frac{1}{n}} {}_2F_1 \left( 2 + \frac{1}{n}, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c} \right) + bcx^n \right)}{c^2(n+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*x^n)*(c + d*x^n)^{(-2 - n^(-1))}, x]$

[Out]  $(x*(b*c*x^n + a*(1 + n)*(c + d*x^n)*(1 + (d*x^n)/c)^{n^(-1)} \text{Hypergeometric2F1}[2 + n^(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*(1 + n)*(c + d*x^n)^{(1 + n)/n})$

**Maple [F]** time = 0.165, size = 0, normalized size = 0.

$$\int (a + bx^n)(c + dx^n)^{-2-n^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*x^n)*(c+d*x^n)^(-2-1/n),x)`

[Out] `int((a+b*x^n)*(c+d*x^n)^(-2-1/n),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)(dx^n + c)^{-\frac{1}{n}-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 2),x, algorithm="maxima")`

[Out] `integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 2), x)`

**Fricas** [A] time = 0.267403, size = 115, normalized size = 1.98

$$\frac{(ad^2n + bcd)xx^{2n} + (2acdn + bc^2 + acd)xx^n + (ac^2n + ac^2)x}{(c^2n + c^2)(dx^n + c)^{\frac{2n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 2),x, algorithm="fricas")`

[Out] `((a*d^2*n + b*c*d)*x*x^(2*n) + (2*a*c*d*n + b*c^2 + a*c*d)*x*x^n + (a*c^2*n + a*c^2)*x)/((c^2*n + c^2)*(d*x^n + c)^((2*n + 1)/n))`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n)*(c+d*x**n)**(-2-1/n),x)`

[Out] Timed out

**GIAC/XCAS** [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 2),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.225 \quad \int (c + dx^n)^{-1-\frac{1}{n}} dx$$

**Optimal.** Leaf size=18

$$\frac{x(c + dx^n)^{-1/n}}{c}$$

[Out] x/(c\*(c + d\*x^n)^n^(-1))

**Rubi [A]** time = 0.0116419, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{x(c + dx^n)^{-1/n}}{c}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^(-1 - n^(-1)), x]

[Out] x/(c\*(c + d\*x^n)^n^(-1))

**Rubi in Sympy [A]** time = 1.39143, size = 12, normalized size = 0.67

$$\frac{x(c + dx^n)^{-\frac{1}{n}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d\*x\*\*n)\*\*(-1-1/n), x)

[Out] x\*(c + d\*x\*\*n)\*\*(-1/n)/c

**Mathematica [A]** time = 0.0399012, size = 18, normalized size = 1.

$$\frac{x(c + dx^n)^{-1/n}}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^(-1 - n^(-1)), x]

[Out] x/(c\*(c + d\*x^n)^n^(-1))

**Maple [B]** time = 0.036, size = 53, normalized size = 2.9

$$xe^{(-1-n^{-1})\ln(c+de^{n\ln(x)})} + \frac{dx e^{n\ln(x)}}{c} e^{(-1-n^{-1})\ln(c+de^{n\ln(x)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d\*x^n)^(-1-1/n), x)

[Out] x\*exp((-1-1/n)\*ln(c+d\*exp(n\*ln(x))))+d/c\*x\*exp(n\*ln(x))\*exp((-1-1/n)\*ln(c+d\*exp(n\*ln(x))))

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^n + c)^{-\frac{1}{n}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^(-1/n - 1), x, algorithm="maxima")`

[Out] `integrate((d*x^n + c)^(-1/n - 1), x)`

---

**Fricas [A]** time = 0.248533, size = 42, normalized size = 2.33

$$\frac{dx^n + cx}{(dx^n + c)^{\frac{n+1}{n}} c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^(-1/n - 1), x, algorithm="fricas")`

[Out] `(d*x*x^n + c*x)/((d*x^n + c)^((n + 1)/n)*c)`

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d*x**n)**(-1-1/n), x)`

[Out] Exception raised: RecursionError

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^n + c)^{-\frac{1}{n}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^(-1/n - 1), x, algorithm="giac")`

[Out] `integrate((d*x^n + c)^(-1/n - 1), x)`

$$3.226 \quad \int \frac{(c+dx^n)^{-1/n}}{a+bx^n} dx$$

**Optimal.** Leaf size=53

$$\frac{x(c+dx^n)^{-1/n} {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a}$$

[Out] (x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])/ (a\*(c + d\*x^n)^n^(-1))

**Rubi [A]** time = 0.0478586, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{x(c+dx^n)^{-1/n} {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^n)\*(c + d\*x^n)^n^(-1)), x]

[Out] (x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])/ (a\*(c + d\*x^n)^n^(-1))

**Rubi in Sympy [A]** time = 6.25931, size = 39, normalized size = 0.74

$$\frac{x(c+dx^n)^{-\frac{1}{n}} {}_2F_1\left(\frac{1}{n}, 1 \mid -\frac{x^n(-ad+bc)}{a(c+dx^n)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a+b\*x\*\*n)/((c+d\*x\*\*n)\*\*(1/n)), x)

[Out] x\*(c + d\*x\*\*n)\*\*(-1/n)\*hyper((1/n, 1), (1 + 1/n, ), -x\*\*n\*(-a\*d + b\*c)/(a\*(c + d\*x\*\*n)))/a

**Mathematica [A]** time = 0.300655, size = 79, normalized size = 1.49

$$\frac{x(c+dx^n)^{-1/n} \left(\frac{a(c+dx^n)}{c(a+bx^n)}\right)^{\frac{1}{n}} {}_2F_1\left(\frac{1}{n}, \frac{1}{n}; 1 + \frac{1}{n}; \frac{(bc-ad)x^n}{c(bx^n+a)}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^n)\*(c + d\*x^n)^n^(-1)), x]

[Out] (x\*((a\*(c + d\*x^n))/(c\*(a + b\*x^n)))^n^(-1)\*Hypergeometric2F1[n^(-1), n^(-1), 1 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))])/ (a\*(c + d\*x^n)^n^(-1))

**Maple [F]** time = 0.12, size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^n) \sqrt[n]{c + dx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x)`

[Out] `int(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}}}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)),x, algorithm="maxima")`

[Out] `integrate((d*x^n + c)^(-1/n)/(b*x^n + a), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^n + a)(dx^n + c)^{\frac{1}{n}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)),x, algorithm="fricas")`

[Out] `integral(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)`

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c + dx^n)^{-\frac{1}{n}}}{a + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*x**n)/((c+d*x**n)**(1/n)),x)`

[Out] `Integral((c + d*x**n)**(-1/n)/(a + b*x**n), x)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)(dx^n + c)^{\frac{1}{n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)`



$$3.227 \quad \int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} dx$$

**Optimal.** Leaf size=54

$$\frac{cx(c+dx^n)^{-1/n} {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2}$$

[Out] (c\*x\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n)))])/(a^2\*(c + d\*x^n)^n^(-1))

**Rubi [A]** time = 0.0502847, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{cx(c+dx^n)^{-1/n} {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^(1 - n^(-1))/(a + b\*x^n)^2, x]

[Out] (c\*x\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n)))])/(a^2\*(c + d\*x^n)^n^(-1))

**Rubi in Sympy [A]** time = 6.1553, size = 42, normalized size = 0.78

$$\frac{cx(c+dx^n)^{-\frac{1}{n}} {}_2F_1\left(\frac{1}{n}, 2 \mid \frac{x^n(-ad+bc)}{a(c+dx^n)}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d\*x\*\*n)\*\*(1-1/n)/(a+b\*x\*\*n)\*\*2, x)

[Out] c\*x\*(c + d\*x\*\*n)\*\*(-1/n)\*hyper((1/n, 2), (1 + 1/n, ), -x\*\*n\*(-a\*d + b\*c)/(a\*(c + d\*x\*\*n)))/a\*\*2

**Mathematica [A]** time = 0.134758, size = 53, normalized size = 0.98

$$\frac{cx(c+dx^n)^{-1/n} {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; \frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^(1 - n^(-1))/(a + b\*x^n)^2, x]

[Out] (c\*x\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), ((-(b\*c) + a\*d)\*x^n)/(a\*(c + d\*x^n)))]/(a^2\*(c + d\*x^n)^n^(-1))

**Maple [F]** time = 0.112, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)^2} (c+dx^n)^{1-n^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x)`

[Out] `int((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+1}}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2,x, algorithm="maxima")`

[Out] `integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^n + c)^{\frac{n-1}{n}}}{b^2x^{2n} + 2abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2,x, algorithm="fricas")`

[Out] `integral((d*x^n + c)^((n - 1)/n)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+1}}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2,x, algorithm="giac")`

[Out] `integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2, x)`

$$3.228 \quad \int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3} dx$$

**Optimal.** Leaf size=56

$$\frac{c^2 x (c + dx^n)^{-1/n} {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^3}$$

[Out] (c^2\*x\*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])/a^3\*(c + d\*x^n)^n^(-1))

**Rubi [A]** time = 0.0518859, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{c^2 x (c + dx^n)^{-1/n} {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^(2 - n^(-1))/(a + b\*x^n)^3, x]

[Out] (c^2\*x\*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])/a^3\*(c + d\*x^n)^n^(-1))

**Rubi in Sympy [A]** time = 6.46959, size = 44, normalized size = 0.79

$$\frac{c^2 x (c + dx^n)^{-\frac{1}{n}} {}_2F_1\left(\frac{1}{n}, 3; 1 + \frac{1}{n}; -\frac{x^n(-ad+bc)}{a(c+dx^n)}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d\*x\*\*n)\*\*(2-1/n)/(a+b\*x\*\*n)\*\*3, x)

[Out] c\*\*2\*x\*(c + d\*x\*\*n)\*\*(-1/n)\*hyper((1/n, 3), (1 + 1/n, ), -x\*\*n\*(-a\*d + b\*c)/(a\*(c + d\*x\*\*n)))/a\*\*3

**Mathematica [F]** time = 179.999, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x^n)^(2 - n^(-1))/(a + b\*x^n)^3, x]

[Out] \$Aborted

**Maple [F]** time = 0.115, size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^n)^3} (c + dx^n)^{2-n^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x)`

[Out] `int((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+2}}{(bx^n + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3,x, algorithm="maxima")`

[Out] `integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^n + c)^{\frac{2n-1}{n}}}{b^3x^{3n} + 3ab^2x^{2n} + 3a^2bx^n + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3,x, algorithm="fricas")`

[Out] `integral((d*x^n + c)^((2*n - 1)/n)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d*x**n)**(2-1/n)/(a+b*x**n)**3,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+2}}{(bx^n + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3,x, algorithm="giac")`

[Out] `integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3, x)`

$$3.229 \quad \int (a + bx^n)^p (c + dx^n)^{-2 - \frac{1}{n} - p} dx$$

**Optimal.** Leaf size=193

$$\frac{x(a + bx^n)^{p+1} (c + dx^n)^{-\frac{1}{n} - p - 1} (n(p+1)(bc - ad) + bc) \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p-1} {}_2F_1\left(\frac{1}{n}, -p-1; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{acn(p+1)(bc - ad)} - \frac{bx(a + bx^n)^{p+1} (c + dx^n)^{-\frac{1}{n} - p - 1}}{an(p+1)(bc - ad)}$$

[Out]  $-\left((b^*x^*(a + b^*x^n)^{(1+p)}*(c + d^*x^n)^{(-1 - n^{(-1)} - p)})/(a^*(b^*c - a^*d)^*n^*(1+p))\right) + \left((b^*c + (b^*c - a^*d)^*n^*(1+p))*x^*(a + b^*x^n)^{(1+p)}*((c^*(a + b^*x^n))/(a^*(c + d^*x^n)))^{(-1-p)}*(c + d^*x^n)^{(-1 - n^{(-1)} - p)}*\text{Hypergeometric2F1}[n^{(-1)}, -1 - p, 1 + n^{(-1)}, -((b^*c - a^*d)^*x^n)/(a^*(c + d^*x^n))]\right)/(a^*c^*(b^*c - a^*d)^*n^*(1+p))$

**Rubi [A]** time = 0.206221, antiderivative size = 179, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{x(a + bx^n)^{p+1} (c + dx^n)^{-\frac{1}{n} - p - 1} \left(\frac{b}{n(p+1)(bc-ad)} + \frac{1}{c}\right) \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p-1} {}_2F_1\left(\frac{1}{n}, -p-1; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a} - \frac{bx(a + bx^n)^{p+1} (c + dx^n)^{-\frac{1}{n} - p - 1}}{an(p+1)(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p\*(c + d\*x^n)^(-2 - n^(-1) - p), x]

[Out]  $-\left((b^*x^*(a + b^*x^n)^{(1+p)}*(c + d^*x^n)^{(-1 - n^{(-1)} - p)})/(a^*(b^*c - a^*d)^*n^*(1+p))\right) + \left((c^{(-1)} + b/((b^*c - a^*d)^*n^*(1+p)))*x^*(a + b^*x^n)^{(1+p)}*((c^*(a + b^*x^n))/(a^*(c + d^*x^n)))^{(-1-p)}*(c + d^*x^n)^{(-1 - n^{(-1)} - p)}*\text{Hypergeometric2F1}[n^{(-1)}, -1 - p, 1 + n^{(-1)}, -((b^*c - a^*d)^*x^n)/(a^*(c + d^*x^n))]\right)/a$

**Rubi in Sympy [A]** time = 25.2427, size = 155, normalized size = 0.8

$$\frac{bx(a + bx^n)^{p+1} (c + dx^n)^{-p-1-\frac{1}{n}}}{an(p+1)(ad - bc)} - \frac{x\left(\frac{a(c+dx^n)}{c(a+bx^n)}\right)^{p+2+\frac{1}{n}} (a + bx^n)^{p+2} (c + dx^n)^{-p-2-\frac{1}{n}} (bc - n(p+1)(ad - bc)) {}_2F_1\left(\frac{1}{n}, p+2 + \frac{1}{n}; 1 + \frac{1}{n}; -\frac{x^n(ad-bc)}{c(a+bx^n)}\right)}{a^2n(p+1)(ad - bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*p\*(c+d\*x\*\*n)\*\*(-2-1/n-p), x)

[Out]  $b^*x^*(a + b^*x^n)^{(p+1)}*(c + d^*x^n)^{(-p - 1 - 1/n)}/(a^*n^*(p + 1)^*(a^*d - b^*c)) - x^*(a^*(c + d^*x^n)/(c^*(a + b^*x^n)))^{(p + 2 + 1/n)}*(a + b^*x^n)^{(p + 2)}*(c + d^*x^n)^{(-p - 2 - 1/n)}*(b^*c - n^*(p + 1)^*(a^*d - b^*c))*\text{hyper}((1/n, p + 2 + 1/n), (1 + 1/n), -x^*n^*(a^*d - b^*c)/(c^*(a + b^*x^n)))/(a^*2^n^*(p + 1)^*(a^*d - b^*c))$

**Mathematica [B]** time = 53.2575, size = 1414, normalized size = 7.33

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^n)^p\*(c + d\*x^n)^(-2 - n^(-1) - p), x]

[Out] (c^4\*(1 + n)\*(1 + 2\*n)\*(1 + 3\*n)\*x\*(a + b\*x^n)^(3 + p)\*(c + d\*x^n)^(-2 - n^(-1) - p)\*(1 + (d\*x^n)/c)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*(Hypergeometric2F1[1, -p, 1 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + (d\*n\*x^n\*((c\*Hypergeometric2F1[1, -p, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))])/(1 + n) + ((b\*c - a\*d)\*x^n\*Gamma[1 + n^(-1)]\*Gamma[1 - p]\*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))])/(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]))/c^2)/(-(c\*d\*(1 + 3\*n)\*(1 + n + n\*p)\*x^n\*(a + b\*x^n)^2\*(c^2\*(1 + n)\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[1, -p, 1 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + d\*n\*x^n\*(c\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[1, -p, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + (b\*c - a\*d)\*(1 + n)\*x^n\*Gamma[1 + n^(-1)]\*Gamma[1 - p]\*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))])) + b\*c\*n\*(1 + 3\*n)\*p\*x^n\*(a + b\*x^n)\*(c + d\*x^n)\*(c^2\*(1 + n)\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[1, -p, 1 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + d\*n\*x^n\*(c\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[1, -p, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + (b\*c - a\*d)\*(1 + n)\*x^n\*Gamma[1 + n^(-1)]\*Gamma[1 - p]\*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))])) + c\*(1 + 3\*n)\*(a + b\*x^n)^2\*(c + d\*x^n)\*(c^2\*(1 + n)\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[1, -p, 1 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + d\*n\*x^n\*(c\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[1, -p, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + (b\*c - a\*d)\*(1 + n)\*x^n\*Gamma[1 + n^(-1)]\*Gamma[1 - p]\*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))])) + n^2\*x^n\*(c + d\*x^n)\*(a\*c^2\*(-(b\*c) + a\*d)\*(1 + 2\*n)\*(1 + 3\*n)\*p\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[2, 1 - p, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + c\*d\*(1 + 3\*n)\*(a + b\*x^n)^2\*(c\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[1, -p, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + (b\*c - a\*d)\*(1 + n)\*x^n\*Gamma[1 + n^(-1)]\*Gamma[1 - p]\*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) - d\*(b\*c - a\*d)\*x^n\*(b\*c\*(1 + n)\*(1 + 3\*n)\*x^n\*(a + b\*x^n)\*Gamma[1 + n^(-1)]\*Gamma[1 - p]\*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) - c\*(1 + n)\*(1 + 3\*n)\*(a + b\*x^n)^2\*Gamma[1 + n^(-1)]\*Gamma[1 - p]\*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + a\*c\*n\*(1 + 3\*n)\*p\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) - 2\*a\*(-(b\*c) + a\*d)\*n\*(1 + n)\*(-1 + p)\*x^n\*Gamma[1 + n^(-1)]\*Gamma[1 - p]\*Hypergeometric2F1[3, 2 - p, 4 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]))

**Maple [F]** time = 0.264, size = 0, normalized size = 0.

$$\int (a + bx^n)^p (c + dx^n)^{-2-n^{-1}-p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)^p\*(c+d\*x^n)^(-2-1/n-p), x)

[Out] int((a+b\*x^n)^p\*(c+d\*x^n)^(-2-1/n-p), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (dx^n + c)^{-p-\frac{1}{n}-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2),x, algorithm="maxima")`

[Out] `integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^n + a)^p}{(dx^n + c)^{\frac{np+2n+1}{n}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2),x, algorithm="fricas")`

[Out] `integral((b*x^n + a)^p/(d*x^n + c)^((n*p + 2*n + 1)/n), x)`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n)**p*(c+d*x**n)**(-2-1/n-p), x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (dx^n + c)^{-p - \frac{1}{n} - 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2),x, algorithm="giac")`

[Out] `integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2), x)`

$$3.230 \quad \int (a + bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c + dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} dx$$

**Optimal.** Leaf size=57

$$\frac{x(a + bx^n)^{-\frac{bc}{n(bc-ad)}} (c + dx^n)^{\frac{ad}{n(bc-ad)}}}{ac}$$

[Out]  $(x^*(c + d*x^n)^{((a*d)/((b*c - a*d)*n))})/(a*c*(a + b*x^n)^{((b*c)/((b*c - a*d)*n))})$

**Rubi [A]** time = 0.0845283, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 69,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.014$

$$\frac{x(a + bx^n)^{-\frac{bc}{n(bc-ad)}} (c + dx^n)^{\frac{ad}{n(bc-ad)}}}{ac}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^((a\*d\*n - b\*c\*(1 + n)))/((b\*c - a\*d)\*n)]\*(c + d\*x^n)^((a\*d - b\*c\*n)/n)

[Out]  $(x^*(c + d*x^n)^{((a*d)/((b*c - a*d)*n))})/(a*c*(a + b*x^n)^{((b*c)/((b*c - a*d)*n))})$

**Rubi in Sympy [A]** time = 13.5218, size = 41, normalized size = 0.72

$$\frac{x(a + bx^n)^{\frac{bc}{n(ad-bc)}} (c + dx^n)^{-\frac{ad}{n(ad-bc)}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*\*((a\*d\*n-b\*c\*(1+n)))/(-a\*d+b\*c)/n\*(c+d\*x\*\*n)\*\*((a\*d-b\*c\*n+a\*d)/(-a\*d\*n+b\*c\*n)),x)

[Out]  $x^*(a + b*x**n)**(b*c/(n*(a*d - b*c)))*(c + d*x**n)**(-a*d/(n*(a*d - b*c)))/(a*c)$

**Mathematica [C]** time = 1.93871, size = 461, normalized size = 8.09

$$ac(n+1)x(ad-bc)(a+bx^n)^{\frac{adn-bc(n+1)}{n(bc-ad)}}(c+dx^n)^{\frac{adn+ad-bcn}{bcn-adn}}$$

$$bcx^n(bc(n+1)-adn)F_1\left(1+\frac{1}{n};\frac{bc+2bnc-2adn}{bcn-adn},\frac{bcn-ad(n+1)}{(bc-ad)n};2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-a\left(dx^n(ad(n+1)-bcn)F_1\left(1+\frac{1}{n};\frac{bc+bnc-2adn}{bcn-adn},\frac{bcn-ad(n+1)}{(bc-ad)n};2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^n)^((a\*d\*n - b\*c\*(1 + n)))/((b\*c - a\*d)\*n)]\*(c + d\*x^n)^((a\*d - b\*c\*n)/n)

[Out]  $(a*c*(-(b*c) + a*d)*(1 + n)*x*(a + b*x^n)^{((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))})/(a*c*(a + b*x^n)^{((b*c)/((b*c - a*d)*n))})$   
 $\text{AppellF1}[n^{(-1)}, (b*c + b*c*n - a*d*n)/(b*c*n - a*d*n), (b*c*n - a*d*(1 + n))/((b*c - a*d)*n), 1 + n^{(-1)}, -((b*x^n)/a), -((d*x^n)/c)]/(b*c*(-(a*d*n) + b*c*(1 + n))*x^n*\text{AppellF1}[1 + n^{(-1)}, (b*c + 2*b*c*n - 2*a*d*n)/(b*c*n - a*d*n), (b*c*n - a*d*(1 + n))/((b*c - a*d)*n), 2 + n^{(-1)}, -((b*x^n)/a), -((d*x^n)/c)] - a*(d*(-(b*c*n) + a*d*(1 + n))*x^n*\text{AppellF1}[1 + n^{(-1)}, (b*c + b*c*n - a*d*n)/(b*c*n - a*d*n), -((a*d - 2*b*c*n + 2*a*d*n)/(b*c*n - a*d*n)), 2 + n^{(-1)}, -((b*x^n)/a), -((d*x^n)/c)] + c*(b*c - a*d)*(1 + n)*\text{AppellF1}[n^{(-1)}, (b*c + b*c*n - a*d*n)/(b*c*n - a*d*n), (b*c*n - a*d*(1 + n))/((b*c - a*d)*n), 1 + n^{(-1)}, -((b*x^n)/a), -((d*x^n)/c)]$



- a\*d\*(1 + n))/((b\*c - a\*d)^n), 1 + n^(-1), -((b\*x^n)/a), -((d\*x^n)/c))

**Maple [F]** time = 0.306, size = 0, normalized size = 0.

$$\int (a + bx^n)^{\frac{adn-bc(1+n)}{(-ad+bc)n}} (c + dx^n)^{\frac{adn-bcn+ad}{-adn+bcn}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)^((a\*d\*n-b\*c\*(1+n))/(-a\*d+b\*c)/n)\*(c+d\*x^n)^((a\*d\*n-b\*c\*n+a\*d)/(-a\*d\*n+b\*c\*n)), x)

[Out] int((a+b\*x^n)^((a\*d\*n-b\*c\*(1+n))/(-a\*d+b\*c)/n)\*(c+d\*x^n)^((a\*d\*n-b\*c\*n+a\*d)/(-a\*d\*n+b\*c\*n)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^{-\frac{bc(n+1)-adn}{(bc-ad)n}} (dx^n + c)^{-\frac{bcn-adn-ad}{bcn-adn}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^(-(b\*c\*(n + 1) - a\*d\*n)/((b\*c - a\*d)^n))\*(d\*x^n + c)^(-(b\*c\*n - a\*d\*n - a\*d)/(b\*c\*n - a\*d\*n)), x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^(-(b\*c\*(n + 1) - a\*d\*n)/((b\*c - a\*d)^n))\*(d\*x^n + c)^(-(b\*c\*n - a\*d\*n - a\*d)/(b\*c\*n - a\*d\*n)), x)

**Fricas [A]** time = 0.264862, size = 146, normalized size = 2.56

$$\frac{(bdxx^{2n} + acx + (bc + ad)xx^n)(dx^n + c)^{\frac{ad-(bc-ad)n}{(bc-ad)n}}}{(bx^n + a)^{\frac{bc+(bc-ad)n}{(bc-ad)n}} ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^n + a)^((b\*c\*(n + 1) - a\*d\*n)/((b\*c - a\*d)^n))\*(d\*x^n + c)^((b\*c\*n - a\*d\*n - a\*d)/(b\*c\*n - a\*d\*n)), x)

[Out] (b\*d\*x\*x^(2\*n) + a\*c\*x + (b\*c + a\*d)\*x\*x^n)\*(d\*x^n + c)^((a\*d - (b\*c - a\*d)^n)/((b\*c - a\*d)^n))/((b\*x^n + a)^((b\*c + (b\*c - a\*d)^n)/((b\*c - a\*d)^n))\*a\*c)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*((a\*d\*n-b\*c\*(1+n))/(-a\*d+b\*c)/n)\*(c+d\*x\*\*n)\*\*((a\*d\*n-b\*c\*n+a\*d)/(-a\*d\*n+b\*c\*n)), x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)^{\frac{bc(n+1)-adn}{(bc-ad)n}} (dx^n + c)^{\frac{bcn-adn-ad}{bcn-adn}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^n + a)^((b\*c\*(n + 1) - a\*d\*n)/((b\*c - a\*d)\*n)) \* (d\*x^n + c)^((b\*

[Out] integrate(1/((b\*x^n + a)^((b\*c\*(n + 1) - a\*d\*n)/((b\*c - a\*d)\*n)) \* (d\*x^n + c)^((b\*c\*n - a\*d\*n - a\*d)/(b\*c\*n - a\*d\*n))), x)

$$3.231 \quad \int (a + bx^n)^2 (c + dx^n)^{-4-\frac{1}{n}} dx$$

**Optimal.** Leaf size=327

$$\begin{aligned} & \frac{2a^2n^2x(c+dx^n)^{-1/n}(3adn-b(3cn+c))}{c^4(n+1)(2n+1)(3n+1)(bc-ad)} - \frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}(3adn-b(3cn+c))}{c^3(n+1)(2n+1)(3n+1)(bc-ad)} \\ & - \frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}(3adn-b(3cn+c))}{c^2(6n^2+5n+1)(bc-ad)} \\ & - \frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}(3adn-b(3cn+c))}{3acn(3n+1)(bc-ad)} - \frac{bx(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{3an(bc-ad)} \end{aligned}$$

[Out]  $-(b*x*(a+b*x^n)^3*(c+d*x^n)^{-3-n*(-1)})/(3*a*(b*c-a*d)*n) - ((3*a*d*n-b*(c+3*c*n))*x*(a+b*x^n)^3*(c+d*x^n)^{-3-n*(-1)})/(3*a*c*(b*c-a*d)*n*(1+3*n)) - ((3*a*d*n-b*(c+3*c*n))*x*(a+b*x^n)^2*(c+d*x^n)^{-2-n*(-1)})/(c^2*(b*c-a*d)*(1+5*n+6*n^2)) - (2*a*n*(3*a*d*n-b*(c+3*c*n))*x*(a+b*x^n)*(c+d*x^n)^{-1-n*(-1)})/(c^3*(b*c-a*d)*(1+n)*(1+2*n)*(1+3*n)) - (2*a^2*n^2*(3*a*d*n-b*(c+3*c*n))*x)/(c^4*(b*c-a*d)*(1+n)*(1+2*n)*(1+3*n)*(c+d*x^n)^n*(-1))$

**Rubi [A]** time = 0.548162, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\begin{aligned} & \frac{2a^2n^2x(c+dx^n)^{-1/n}(3adn-b(3cn+c))}{c^4(n+1)(2n+1)(3n+1)(bc-ad)} - \frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}(3adn-b(3cn+c))}{c^3(n+1)(2n+1)(3n+1)(bc-ad)} \\ & - \frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}(3adn-b(3cn+c))}{c^2(6n^2+5n+1)(bc-ad)} \\ & - \frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}(3adn-b(3cn+c))}{3acn(3n+1)(bc-ad)} - \frac{bx(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{3an(bc-ad)} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a+b*x^n)^2*(c+d*x^n)^{-4-n*(-1)},x]$

[Out]  $-(b*x*(a+b*x^n)^3*(c+d*x^n)^{-3-n*(-1)})/(3*a*(b*c-a*d)*n) - ((3*a*d*n-b*(c+3*c*n))*x*(a+b*x^n)^3*(c+d*x^n)^{-3-n*(-1)})/(3*a*c*(b*c-a*d)*n*(1+3*n)) - ((3*a*d*n-b*(c+3*c*n))*x*(a+b*x^n)^2*(c+d*x^n)^{-2-n*(-1)})/(c^2*(b*c-a*d)*(1+5*n+6*n^2)) - (2*a*n*(3*a*d*n-b*(c+3*c*n))*x*(a+b*x^n)*(c+d*x^n)^{-1-n*(-1)})/(c^3*(b*c-a*d)*(1+n)*(1+2*n)*(1+3*n)) - (2*a^2*n^2*(3*a*d*n-b*(c+3*c*n))*x)/(c^4*(b*c-a*d)*(1+n)*(1+2*n)*(1+3*n)*(c+d*x^n)^n*(-1))$

**Rubi in Sympy [A]** time = 81.8544, size = 286, normalized size = 0.87

$$\begin{aligned} & \frac{2a^2n^2x(c+dx^n)^{-\frac{1}{n}}(-3adn+3bcn+bc)}{c^4(n+1)(2n+1)(3n+1)(ad-bc)} - \frac{2anx(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}(-3adn+3bcn+bc)}{c^3(n+1)(2n+1)(3n+1)(ad-bc)} \\ & - \frac{x(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}(-3adn+3bcn+bc)}{c^2(2n+1)(3n+1)(ad-bc)} + \frac{bx(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3an(ad-bc)} \\ & - \frac{x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}(-3adn+3bcn+bc)}{3acn(3n+1)(ad-bc)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a+b*x**n)**2*(c+d*x**n)**(-4-1/n),x)$

[Out]  $-2*a**2*n**2*x*(c+d*x**n)**(-1/n)*(-3*a*d*n+3*b*c*n+b*c)/(c**4*(n+1)*(2*n+1)*(3*n+1)*(a*d-b*c)) - 2*a*n*x*(a+b*x**$

$$\frac{n(c + dx^n)^{-1/n} (-1 - 1/n)^{-3} (-3ad^n + 3b^2c^n + b^3c) / (c^3(n+1)(2n+1)(3n+1)(ad - b^2c)) - x^n (a + bx^n)^{-2} (c + dx^n)^{-2} (-2 - 1/n)^{-3} (-3ad^n + 3b^2c^n + b^3c) / (c^2(2n+1)(3n+1)(ad - b^2c)) + b^2x^{2n} (a + bx^n)^{-3} (c + dx^n)^{-3} (-3 - 1/n) / (3a^n(ad - b^2c)) - x^n (a + bx^n)^{-3} (c + dx^n)^{-3} (-3 - 1/n)^{-3} (-3ad^n + 3b^2c^n + b^3c) / (3a^2c^n(3n+1)(ad - b^2c))}{c^4(n+1)(2n+1)}$$

**Mathematica [C]** time = 0.304528, size = 153, normalized size = 0.47

$$x(c + dx^n)^{-1/n} \left(\frac{dx^n}{c} + 1\right)^{\frac{1}{n}} \left((n+1) \left(a^2(2n+1) {}_2F_1\left(4 + \frac{1}{n}, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right) + b^2x^{2n} {}_2F_1\left(2 + \frac{1}{n}, 4 + \frac{1}{n}; 3 + \frac{1}{n}; -\frac{dx^n}{c}\right)\right) + 2ab\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^2\*(c + d\*x^n)^(-4 - n^(-1)), x]

[Out] (x\*(1 + (d\*x^n)/c)^n^(-1)\*(2\*a\*b\*(1 + 2\*n)\*x^n\*Hypergeometric2F1[1 + n^(-1), 4 + n^(-1), 2 + n^(-1), -((d\*x^n)/c)] + (1 + n)\*(b^2\*x^(2\*n)\*Hypergeometric2F1[2 + n^(-1), 4 + n^(-1), 3 + n^(-1), -((d\*x^n)/c)] + a^2\*(1 + 2\*n)\*Hypergeometric2F1[4 + n^(-1), n^(-1), 1 + n^(-1), -((d\*x^n)/c)]))/c^4\*(1 + n)\*(1 + 2\*n)\*(c + d\*x^n)^n^(-1))

**Maple [F]** time = 0.21, size = 0, normalized size = 0.

$$\int (a + bx^n)^2 (c + dx^n)^{-4-n^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^n)^2\*(c+d\*x^n)^(-4-1/n), x)

[Out] int((a+b\*x^n)^2\*(c+d\*x^n)^(-4-1/n), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^2 (dx^n + c)^{-\frac{1}{n}-4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^2\*(d\*x^n + c)^(-1/n - 4), x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^2\*(d\*x^n + c)^(-1/n - 4), x)

**Fricas [A]** time = 0.260355, size = 540, normalized size = 1.65

$$(6a^2d^4n^3 + b^2c^2d^2n + (b^2c^2d^2 + 4abcd^3)n^2)xx^{4n} + (24a^2cd^3n^3 + b^2c^3d + 2(2b^2c^3d + 8abc^2d^2 + 3a^2cd^3)n^2 + (5b^2c^3d + 4abcd^3)n) x^{4n+1} + (4abcd^3n^2 + b^2c^3d + 2(2b^2c^3d + 8abc^2d^2 + 3a^2cd^3)n^2 + (5b^2c^3d + 4abcd^3)n) x^{4n+2} + (4abcd^3n^2 + b^2c^3d + 2(2b^2c^3d + 8abc^2d^2 + 3a^2cd^3)n^2 + (5b^2c^3d + 4abcd^3)n) x^{4n+3} + (4abcd^3n^2 + b^2c^3d + 2(2b^2c^3d + 8abc^2d^2 + 3a^2cd^3)n^2 + (5b^2c^3d + 4abcd^3)n) x^{4n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^n + a)^2\*(d\*x^n + c)^(-1/n - 4), x, algorithm="fricas")

```
[Out] ((6*a^2*d^4*n^3 + b^2*c^2*d^2*n + (b^2*c^2*d^2 + 4*a*b*c*d^3)*n^2)
*x*x^(4*n) + (24*a^2*c*d^3*n^3 + b^2*c^3*d + 2*(2*b^2*c^3*d + 8*
a*b*c^2*d^2 + 3*a^2*c*d^3)*n^2 + (5*b^2*c^3*d + 4*a*b*c^2*d^2)*n)
*x*x^(3*n) + (36*a^2*c^2*d^2*n^3 + b^2*c^4 + 2*a*b*c^3*d + 3*(b^2
*c^4 + 8*a*b*c^3*d + 7*a^2*c^2*d^2)*n^2 + (4*b^2*c^4 + 14*a*b*c^3
*d + 3*a^2*c^2*d^2)*n)*x*x^(2*n) + (24*a^2*c^3*d*n^3 + 2*a*b*c^4
+ a^2*c^3*d + 2*(6*a*b*c^4 + 13*a^2*c^3*d)*n^2 + (10*a*b*c^4 + 9*
a^2*c^3*d)*n)*x*x^n + (6*a^2*c^4*n^3 + 11*a^2*c^4*n^2 + 6*a^2*c^4
*n + a^2*c^4)*x)/((6*c^4*n^3 + 11*c^4*n^2 + 6*c^4*n + c^4)*(d*x^n
+ c)^((4*n + 1)/n))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(c+d*x**n)**(-4-1/n),x)
```

```
[Out] Timed out
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.232 \quad \int (a + bx^n)(c + dx^n)^{-3-\frac{1}{n}} dx$$

**Optimal.** Leaf size=127

$$\frac{nx(c + dx^n)^{-1/n}(2adn + bc)}{c^3d(n+1)(2n+1)} + \frac{x(c + dx^n)^{-\frac{1}{n}-1}(2adn + bc)}{c^2d(n+1)(2n+1)} - \frac{x(bc - ad)(c + dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

[Out] -(((b\*c - a\*d)\*x\*(c + d\*x^n)^(-2 - n^(-1)))/(c\*d\*(1 + 2\*n))) + ((b\*c + 2\*a\*d\*n)\*x\*(c + d\*x^n)^(-1 - n^(-1)))/(c^2\*d\*(1 + n)\*(1 + 2\*n)) + (n\*(b\*c + 2\*a\*d\*n)\*x)/(c^3\*d\*(1 + n)\*(1 + 2\*n)\*(c + d\*x^n)^n^(-1))

**Rubi [A]** time = 0.172598, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$\frac{nx(c + dx^n)^{-1/n}(2adn + bc)}{c^3d(n+1)(2n+1)} + \frac{x(c + dx^n)^{-\frac{1}{n}-1}(2adn + bc)}{c^2d(n+1)(2n+1)} - \frac{x(bc - ad)(c + dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)\*(c + d\*x^n)^(-3 - n^(-1)), x]

[Out] -(((b\*c - a\*d)\*x\*(c + d\*x^n)^(-2 - n^(-1)))/(c\*d\*(1 + 2\*n))) + ((b\*c + 2\*a\*d\*n)\*x\*(c + d\*x^n)^(-1 - n^(-1)))/(c^2\*d\*(1 + n)\*(1 + 2\*n)) + (n\*(b\*c + 2\*a\*d\*n)\*x)/(c^3\*d\*(1 + n)\*(1 + 2\*n)\*(c + d\*x^n)^n^(-1))

**Rubi in Sympy [A]** time = 17.8297, size = 105, normalized size = 0.83

$$\frac{x(c + dx^n)^{-2-\frac{1}{n}}(ad - bc)}{cd(2n+1)} + \frac{x(c + dx^n)^{-1-\frac{1}{n}}(2adn + bc)}{c^2d(n+1)(2n+1)} + \frac{nx(c + dx^n)^{-\frac{1}{n}}(2adn + bc)}{c^3d(n+1)(2n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a+b\*x\*\*n)\*(c+d\*x\*\*n)\*\*(-3-1/n), x)

[Out] x\*(c + d\*x\*\*n)\*\*(-2 - 1/n)\*(a\*d - b\*c)/(c\*d\*(2\*n + 1)) + x\*(c + d\*x\*\*n)\*\*(-1 - 1/n)\*(2\*a\*d\*n + b\*c)/(c\*\*2\*d\*(n + 1)\*(2\*n + 1)) + n\*x\*(c + d\*x\*\*n)\*\*(-1/n)\*(2\*a\*d\*n + b\*c)/(c\*\*3\*d\*(n + 1)\*(2\*n + 1))

**Mathematica [C]** time = 0.139311, size = 96, normalized size = 0.76

$$\frac{x(c + dx^n)^{-1/n} \left( \frac{dx^n}{c} + 1 \right)^{\frac{1}{n}} \left( a(n+1) {}_2F_1 \left( 3 + \frac{1}{n}, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c} \right) + bx^n {}_2F_1 \left( 1 + \frac{1}{n}, 3 + \frac{1}{n}; 2 + \frac{1}{n}; -\frac{dx^n}{c} \right) \right)}{c^3(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)\*(c + d\*x^n)^(-3 - n^(-1)), x]

[Out] (x\*(1 + (d\*x^n)/c)^n^(-1)\*(b\*x^n\*Hypergeometric2F1[1 + n^(-1), 3 + n^(-1), 2 + n^(-1), -((d\*x^n)/c)] + a\*(1 + n)\*Hypergeometric2F1[3 + n^(-1), n^(-1), 1 + n^(-1), -((d\*x^n)/c)]))/(c^3\*(1 + n)\*(c + d\*x^n)^n^(-1))

---

**Maple [F]** time = 0.18, size = 0, normalized size = 0.

$$\int (a + bx^n)(c + dx^n)^{-3-n^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*x^n)*(c+d*x^n)^(-3-1/n),x)`

[Out] `int((a+b*x^n)*(c+d*x^n)^(-3-1/n),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)(dx^n + c)^{-\frac{1}{n}-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 3),x, algorithm="maxima")`

[Out] `integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 3), x)`

---

**Fricas [A]** time = 0.260778, size = 234, normalized size = 1.84

$$\frac{(2ad^3n^2 + bcd^2n)xx^{3n} + (6acd^2n^2 + bc^2d + (3bc^2d + 2acd^2)n)xx^{2n} + (6ac^2dn^2 + bc^3 + ac^2d + (2bc^3 + 5ac^2d)n)xx^n + (2c^3n^2 + 3c^3n + c^3)(dx^n + c)^{\frac{3n+1}{n}}}{(2c^3n^2 + 3c^3n + c^3)(dx^n + c)^{\frac{3n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 3),x, algorithm="fricas")`

[Out] `((2*a*d^3*n^2 + b*c*d^2*n)*x*x^(3*n) + (6*a*c*d^2*n^2 + b*c^2*d + (3*b*c^2*d + 2*a*c*d^2)*n)*x*x^(2*n) + (6*a*c^2*d*n^2 + b*c^3 + a*c^2*d + (2*b*c^3 + 5*a*c^2*d)*n)*x*x^n + (2*a*c^3*n^2 + 3*a*c^3*n + a*c^3)*x)/((2*c^3*n^2 + 3*c^3*n + c^3)*(d*x^n + c)^((3*n + 1)/n))`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n)*(c+d*x**n)**(-3-1/n),x)`

[Out] Timed out

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```



### 3.233 $\int (c + dx^n)^{-2-\frac{1}{n}} dx$

**Optimal.** Leaf size=50

$$\frac{nx(c+dx^n)^{-1/n}}{c^2(n+1)} + \frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}$$

[Out]  $(x*(c+d*x^n)^{(-1-n^{-1})})/(c*(1+n)) + (n*x)/(c^2*(1+n)*(c+d*x^n)^{n^{-1}})$

**Rubi [A]** time = 0.0396865, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{nx(c+dx^n)^{-1/n}}{c^2(n+1)} + \frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^(-2 - n^(-1)), x]

[Out]  $(x*(c+d*x^n)^{(-1-n^{-1})})/(c*(1+n)) + (n*x)/(c^2*(1+n)*(c+d*x^n)^{n^{-1}})$

**Rubi in Sympy [A]** time = 3.68181, size = 39, normalized size = 0.78

$$\frac{x(c+dx^n)^{-1-\frac{1}{n}}}{c(n+1)} + \frac{nx(c+dx^n)^{-\frac{1}{n}}}{c^2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d\*x\*\*n)\*\*(-2-1/n), x)

[Out]  $x*(c+d*x**n)**(-1-1/n)/(c*(n+1)) + n*x*(c+d*x**n)**(-1/n)/(c**2*(n+1))$

**Mathematica [C]** time = 0.0370937, size = 55, normalized size = 1.1

$$\frac{x(c+dx^n)^{-1/n} \left(\frac{dx^n}{c} + 1\right)^{\frac{1}{n}} {}_2F_1\left(2 + \frac{1}{n}, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^(-2 - n^(-1)), x]

[Out]  $(x*(1+(d*x^n)/c)^{n^{-1}}*Hypergeometric2F1[2+n^{-1}, n^{-1}, 1+n^{-1}, -((d*x^n)/c)])/(c^2*(c+d*x^n)^{n^{-1}})$

**Maple [F]** time = 0.131, size = 0, normalized size = 0.

$$\int (c + dx^n)^{-2-n^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c+d*x^n)^(-2-1/n),x)`

[Out] `int((c+d*x^n)^(-2-1/n),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^n + c)^{-\frac{1}{n}-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^(-1/n - 2),x, algorithm="maxima")`

[Out] `integrate((d*x^n + c)^(-1/n - 2), x)`

**Fricas [A]** time = 0.256259, size = 92, normalized size = 1.84

$$\frac{d^2 n x x^{2n} + (2 c d n + c d) x x^n + (c^2 n + c^2) x}{(c^2 n + c^2) (d x^n + c)^{\frac{2n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^(-1/n - 2),x, algorithm="fricas")`

[Out] `(d^2*n*x*x^(2*n) + (2*c*d*n + c*d)*x*x^n + (c^2*n + c^2)*x)/((c^2*n + c^2)*(d*x^n + c)^((2*n + 1)/n))`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d*x**n)**(-2-1/n),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^n + c)^{-\frac{1}{n}-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^(-1/n - 2),x, algorithm="giac")`

[Out] `integrate((d*x^n + c)^(-1/n - 2), x)`

$$3.234 \quad \int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n} dx$$

**Optimal.** Leaf size=95

$$\frac{bx(c+dx^n)^{-1/n} {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a(bc-ad)} - \frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}$$

[Out]  $-\left(\frac{d*x}{c*(b*c - a*d)*(c + d*x^n)^{n*(-1)}}\right) + (b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*c - a*d)*x^n)/(a*(c + d*x^n))]) / (a*(b*c - a*d)*(c + d*x^n)^{n*(-1)})$

**Rubi [A]** time = 0.105637, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{bx(c+dx^n)^{-1/n} {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a(bc-ad)} - \frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^(-1 - n^(-1))/(a + b\*x^n), x]

[Out]  $-\left(\frac{d*x}{c*(b*c - a*d)*(c + d*x^n)^{n*(-1)}}\right) + (b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*c - a*d)*x^n)/(a*(c + d*x^n))]) / (a*(b*c - a*d)*(c + d*x^n)^{n*(-1)})$

**Rubi in Sympy [A]** time = 13.6199, size = 70, normalized size = 0.74

$$\frac{dx(c+dx^n)^{-\frac{1}{n}}}{c(ad-bc)} - \frac{bx(c+dx^n)^{-\frac{1}{n}} {}_2F_1\left(\frac{1}{n}, 1; 1 + \frac{1}{n}; -\frac{x^n(-ad+bc)}{a(c+dx^n)}\right)}{a(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d\*x\*\*n)\*\*(-1-1/n)/(a+b\*x\*\*n), x)

[Out]  $d*x*(c + d*x**n)**(-1/n)/(c*(a*d - b*c)) - b*x*(c + d*x**n)**(-1/n)*hyper((1/n, 1), (1 + 1/n, ), -x**n*(-a*d + b*c)/(a*(c + d*x**n)))/(a*(a*d - b*c))$

**Mathematica [C]** time = 2.23558, size = 153, normalized size = 1.61

$$\frac{x(c+dx^n)^{-\frac{n+1}{n}} \left( \frac{bnx^{2n}(ad-bc) {}_2F_1\left(2, 2+\frac{1}{n}; 3+\frac{1}{n}; \frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{a^2(2n+1)(c+dx^n)} + \frac{bx^n \left(\frac{(ad-bc)x^n}{a(dx^n+c)}, 1, 1+\frac{1}{n}\right)}{a} + \frac{a(c+dx^n)}{c(a+bx^n)} \right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^(-1 - n^(-1))/(a + b\*x^n), x]

[Out]  $(x*((a*(c + d*x^n))/(c*(a + b*x^n)) + (b*x^n*HurwitzLerchPhi[(-(b*c) + a*d)*x^n)/(a*(c + d*x^n)], 1, 1 + n^(-1)))/a + (b*(-(b*c) + a*d)*n*x^(2*n)*Hypergeometric2F1[2, 2 + n^(-1), 3 + n^(-1), ((-b*c) + a*d)*x^n]/(a*(c + d*x^n)))/(a^2*(1 + 2*n)*(c + d*x^n)) / (a*(c + d*x^n)^((1 + n)/n))$

---

**Maple [F]** time = 0.126, size = 0, normalized size = 0.

$$\int \frac{1}{a + bx^n} (c + dx^n)^{-1-n^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c+d*x^n)^(-1-1/n)/(a+b*x^n),x)`

[Out] `int((c+d*x^n)^(-1-1/n)/(a+b*x^n),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}-1}}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a),x, algorithm="maxima")`

[Out] `integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^n + a)(dx^n + c)^{\frac{n+1}{n}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a),x, algorithm="fricas")`

[Out] `integral(1/((b*x^n + a)*(d*x^n + c)^((n + 1)/n)), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d*x**n)**(-1-1/n)/(a+b*x**n),x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}-1}}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a),x, algorithm="giac")`

[Out] `integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a), x)`

$$3.235 \quad \int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2} dx$$

**Optimal.** Leaf size=127

$$\frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{an(bc-ad)(a+bx^n)} - \frac{x(c+dx^n)^{-1/n}(adn+bc(1-n)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2n(bc-ad)}$$

[Out] (b\*x)/(a\*(b\*c - a\*d)\*n\*(a + b\*x^n)\*(c + d\*x^n)^((1 - n)/n)) - ((b\*c\*(1 - n) + a\*d\*n)\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -( (b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])/ (a^2\*(b\*c - a\*d)\*n\*(c + d\*x^n)^n^(-1))

**Rubi [A]** time = 0.151078, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{an(bc-ad)(a+bx^n)} - \frac{x(c+dx^n)^{-1/n}(adn+b(c-cn)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2n(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^n)^2\*(c + d\*x^n)^n^(-1)), x]

[Out] (b\*x)/(a\*(b\*c - a\*d)\*n\*(a + b\*x^n)\*(c + d\*x^n)^((1 - n)/n)) - ((a\*d\*n + b\*(c - c\*n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -( (b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])/ (a^2\*(b\*c - a\*d)\*n\*(c + d\*x^n)^n^(-1))

**Rubi in Sympy [A]** time = 19.2463, size = 95, normalized size = 0.75

$$-\frac{bx(c+dx^n)^{\frac{n-1}{n}}}{an(a+bx^n)(ad-bc)} + \frac{x(c+dx^n)^{-\frac{1}{n}}(adn-bcn+bc) {}_2F_1\left(\frac{1}{n}, 1 \mid -\frac{x^n(-ad+bc)}{a(c+dx^n)}\right)}{a^2n(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a+b\*x\*\*n)\*\*2/((c+d\*x\*\*n)\*\*(1/n)), x)

[Out] -b\*x\*(c + d\*x\*\*n)\*\*((n - 1)/n)/(a\*n\*(a + b\*x\*\*n)\*(a\*d - b\*c)) + x\*(c + d\*x\*\*n)\*\*(-1/n)\*(a\*d\*n - b\*c\*n + b\*c)\*hyper((1/n, 1), (1 + 1/n, ), -x\*\*n\*(-a\*d + b\*c)/(a\*(c + d\*x\*\*n)))/(a\*\*2\*n\*(a\*d - b\*c))

**Mathematica [B]** time = 62.1905, size = 1070, normalized size = 8.43

$$-cd(1-n)(2n+1)(3n+1)(bx^n+a)^2 \left( 2(bc-ad)n(dx^n+c) \left( 2+\frac{1}{n} \right) {}_2F_1\left( 2, 3; 3+\frac{1}{n}; \frac{(bc-ad)x^n}{c(bx^n+a)} \right) x^n + c(bx^n+a)(dnx^n+c) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^n)^2\*(c + d\*x^n)^n^(-1)), x]

[Out] (c^2\*(1 + 2\*n)\*(1 + 3\*n)\*x\*(a + b\*x^n)\*(1 + (d\*x^n)/c)\*Gamma[2 + n^(-1)]\*Gamma[3 + n^(-1)]\*((c\*(c + c\*n + d\*n\*x^n)\*Hypergeometric2

$$\begin{aligned}
&F1[1, 2, 2 + n^{(-1)}, ((b^*c - a^*d)^*x^n)/(c^*(a + b^*x^n))]/Gamma[2 \\
&+ n^{(-1)}] + (2^*(b^*c - a^*d)^*n^*x^n^*(c + d^*x^n)^*Hypergeometric2F1[2, \\
&3, 3 + n^{(-1)}, ((b^*c - a^*d)^*x^n)/(c^*(a + b^*x^n))]/((a + b^*x^n)^* \\
&Gamma[3 + n^{(-1)}]))/((c + d^*x^n)^n^{(-1)}*(-(c^*d^*(1 - n)^*(1 + 2^*n) \\
&*(1 + 3^*n)^*x^n^*(a + b^*x^n)^2^*(c^*(a + b^*x^n)^*(c + c^*n + d^*n^*x^n)^*G \\
&amma[3 + n^{(-1)}]^*Hypergeometric2F1[1, 2, 2 + n^{(-1)}, ((b^*c - a^*d) \\
&^*x^n)/(c^*(a + b^*x^n))] + 2^*(b^*c - a^*d)^*n^*x^n^*(c + d^*x^n)^*Gamma[2 \\
&+ n^{(-1)}]^*Hypergeometric2F1[2, 3, 3 + n^{(-1)}, ((b^*c - a^*d)^*x^n)/(c \\
&^*(a + b^*x^n))])) - 2^*b^*c^*n^*(1 + 2^*n)^*(1 + 3^*n)^*x^n^*(a + b^*x^n)^*( \\
&c + d^*x^n)^*(c^*(a + b^*x^n)^*(c + c^*n + d^*n^*x^n)^*Gamma[3 + n^{(-1)}]^*H \\
&ypergeometric2F1[1, 2, 2 + n^{(-1)}, ((b^*c - a^*d)^*x^n)/(c^*(a + b^*x^ \\
&n))] + 2^*(b^*c - a^*d)^*n^*x^n^*(c + d^*x^n)^*Gamma[2 + n^{(-1)}]^*Hypergeo \\
&metric2F1[2, 3, 3 + n^{(-1)}, ((b^*c - a^*d)^*x^n)/(c^*(a + b^*x^n))] + \\
&c^*(1 + 2^*n)^*(1 + 3^*n)^*(a + b^*x^n)^2^*(c + d^*x^n)^*(c^*(a + b^*x^n)^*( \\
&c + c^*n + d^*n^*x^n)^*Gamma[3 + n^{(-1)}]^*Hypergeometric2F1[1, 2, 2 + \\
&n^{(-1)}, ((b^*c - a^*d)^*x^n)/(c^*(a + b^*x^n))] + 2^*(b^*c - a^*d)^*n^*x^n^* \\
&(c + d^*x^n)^*Gamma[2 + n^{(-1)}]^*Hypergeometric2F1[2, 3, 3 + n^{(-1)}, \\
&((b^*c - a^*d)^*x^n)/(c^*(a + b^*x^n))] + n^2^*x^n^*(c + d^*x^n)^*(c^2^*d \\
&^*(1 + 2^*n)^*(1 + 3^*n)^*(a + b^*x^n)^3^*Gamma[3 + n^{(-1)}]^*Hypergeometr \\
&ic2F1[1, 2, 2 + n^{(-1)}, ((b^*c - a^*d)^*x^n)/(c^*(a + b^*x^n))] + 2^*c^* \\
&d^*(b^*c - a^*d)^*(1 + 2^*n)^*(1 + 3^*n)^*x^n^*(a + b^*x^n)^2^*Gamma[2 + n^{( \\
&-1)}]^*Hypergeometric2F1[2, 3, 3 + n^{(-1)}, ((b^*c - a^*d)^*x^n)/(c^*(a \\
&+ b^*x^n))] - 2^*b^*c^*(b^*c - a^*d)^*(1 + 2^*n)^*(1 + 3^*n)^*x^n^*(a + b^*x^n \\
&)^*(c + d^*x^n)^*Gamma[2 + n^{(-1)}]^*Hypergeometric2F1[2, 3, 3 + n^{(-1) \\
&)}, ((b^*c - a^*d)^*x^n)/(c^*(a + b^*x^n))] + 2^*c^*(b^*c - a^*d)^*(1 + 2^*n) \\
&^*(1 + 3^*n)^*(a + b^*x^n)^2^*(c + d^*x^n)^*Gamma[2 + n^{(-1)}]^*Hypergeome \\
&tric2F1[2, 3, 3 + n^{(-1)}, ((b^*c - a^*d)^*x^n)/(c^*(a + b^*x^n))] + 2^* \\
&a^*c^*(b^*c - a^*d)^*(1 + 3^*n)^*(a + b^*x^n)^*(c + c^*n + d^*n^*x^n)^*Gamma[3 \\
&+ n^{(-1)}]^*Hypergeometric2F1[2, 3, 3 + n^{(-1)}, ((b^*c - a^*d)^*x^n)/ \\
&(c^*(a + b^*x^n))] + 12^*a^*(b^*c - a^*d)^2^*n^*(1 + 2^*n)^*x^n^*(c + d^*x^n) \\
&^*Gamma[2 + n^{(-1)}]^*Hypergeometric2F1[3, 4, 4 + n^{(-1)}, ((b^*c - a^* \\
&d)^*x^n)/(c^*(a + b^*x^n))]))
\end{aligned}$$

**Maple [F]** time = 0.116, size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^n)^2 \sqrt[n]{c + dx^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*x^n)^2/((c+d\*x^n)^(1/n)),x)

[Out] int(1/(a+b\*x^n)^2/((c+d\*x^n)^(1/n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}}}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^n + a)^2\*(d\*x^n + c)^(1/n)),x, algorithm="maxima")

[Out] integrate((d\*x^n + c)^(-1/n)/(b\*x^n + a)^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(b^2x^{2n} + 2abx^n + a^2)(dx^n + c)^{\left(\frac{1}{n}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)),x, algorithm="fricas")`

[Out] `integral(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)*(d*x^n + c)^(1/n)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*x**n)**2/((c+d*x**n)**(1/n)),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)^{\frac{1}{n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)), x)`

$$3.236 \quad \int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3} dx$$

**Optimal.** Leaf size=131

$$\frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2an(bc-ad)(a+bx^n)^2} - \frac{cx(c+dx^n)^{-1/n}(2adn+bc(1-2n)) {}_2F_1\left(2, \frac{1}{n}; 1+\frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{2a^3n(bc-ad)}$$

[Out] (b\*x\*(c+d\*x^n)^(2-n^(-1)))/(2\*a\*(b\*c-a\*d)\*n\*(a+b\*x^n)^2) - (c\*(b\*c\*(1-2\*n)+2\*a\*d\*n)\*x\*Hypergeometric2F1[2, n^(-1), 1+n^(-1), -((b\*c-a\*d)\*x^n)/(a\*(c+d\*x^n))])/(2\*a^3\*(b\*c-a\*d)\*n\*(c+d\*x^n)^n^(-1))

**Rubi [A]** time = 0.16193, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2an(bc-ad)(a+bx^n)^2} - \frac{cx(c+dx^n)^{-1/n}(2adn+b(c-2cn)) {}_2F_1\left(2, \frac{1}{n}; 1+\frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{2a^3n(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(c+d\*x^n)^(1-n^(-1))/(a+b\*x^n)^3, x]

[Out] (b\*x\*(c+d\*x^n)^(2-n^(-1)))/(2\*a\*(b\*c-a\*d)\*n\*(a+b\*x^n)^2) - (c\*(2\*a\*d\*n+b\*(c-2\*c\*n))\*x\*Hypergeometric2F1[2, n^(-1), 1+n^(-1), -((b\*c-a\*d)\*x^n)/(a\*(c+d\*x^n))])/(2\*a^3\*(b\*c-a\*d)\*n\*(c+d\*x^n)^n^(-1))

**Rubi in Sympy [A]** time = 19.6972, size = 105, normalized size = 0.8

$$-\frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2an(a+bx^n)^2(ad-bc)} + \frac{cx(c+dx^n)^{-\frac{1}{n}}(2adn-2bcn+bc) {}_2F_1\left(\frac{1}{n}, 2; 1+\frac{1}{n}; -\frac{x^n(-ad+bc)}{a(c+dx^n)}\right)}{2a^3n(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d\*x\*\*n)\*\*(1-1/n)/(a+b\*x\*\*n)\*\*3, x)

[Out] -b\*x\*(c+d\*x\*\*n)\*\*(2-1/n)/(2\*a\*n\*(a+b\*x\*\*n)\*\*2\*(a\*d-b\*c)) + c\*x\*(c+d\*x\*\*n)\*\*(-1/n)\*(2\*a\*d\*n-2\*b\*c\*n+b\*c)\*hyper((1/n, 2), (1+1/n, ), -x\*\*n\*(-a\*d+b\*c)/(a\*(c+d\*x\*\*n)))/(2\*a\*\*3\*n\*(a\*d-b\*c))

**Mathematica [B]** time = 43.7676, size = 1241, normalized size = 9.47

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(c+d\*x^n)^(1-n^(-1))/(a+b\*x^n)^3, x]

[Out] -((c^3\*(1+n)\*(1+2\*n)\*(1+3\*n)\*x\*(c+d\*x^n)^(2-n^(-1)))\*Gamma[2+n^(-1)]\*(Hypergeometric2F1[1, 3, 1+n^(-1), ((b\*c-a\*d)\*x^n)/(c\*(a+b\*x^n))]) + (d\*n\*x^n\*((c\*Hypergeometric2F1[1, 3, 2+n^(-1), ((b\*c-a\*d)\*x^n)/(c\*(a+b\*x^n))])/(1+n) + (3\*(b\*c-a



```

*d)*x^n*Gamma[1 + n^(-1)]*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b
*c - a*d)*x^n)/(c*(a + b*x^n))]/((1 + 2*n)*(a + b*x^n)*Gamma[2 +
n^(-1)]))/c^2))/c*d*(1 - 2*n)*(1 + 3*n)*x^n*(a + b*x^n)^2*(c^2
*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F
1[1, 3, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + d*n*x^n*
(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3
, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + 3*(b*c - a*d)*
(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeometric2F1[2, 4, 3 + n^(-1),
((b*c - a*d)*x^n)/(c*(a + b*x^n))])) + 3*b*c*n*(1 + 3*n)*x^n*(a
+ b*x^n)*(c + d*x^n)*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 +
n^(-1)]*Hypergeometric2F1[1, 3, 1 + n^(-1), ((b*c - a*d)*x^n)/(c
*(a + b*x^n))] + d*n*x^n*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1
)]*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a +
b*x^n))] + 3*(b*c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeome
tric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])) -
c*(1 + 3*n)*(a + b*x^n)^2*(c + d*x^n)*(c^2*(1 + n)*(1 + 2*n)*(a +
b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3, 1 + n^(-1), ((b
*c - a*d)*x^n)/(c*(a + b*x^n))] + d*n*x^n*(c*(1 + 2*n)*(a + b*x^n
)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b*c - a
*d)*x^n)/(c*(a + b*x^n))] + 3*(b*c - a*d)*(1 + n)*x^n*Gamma[1 + n
^(-1)]*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(
a + b*x^n))])) + n^2*x^n*(c + d*x^n)*(3*a*c^2*(-(b*c) + a*d)*(1 +
2*n)*(1 + 3*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[2
, 4, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - c*d*(1 + 3*
n)*(a + b*x^n)^2*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hyper
geometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]
+ 3*(b*c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeometric2F1[2
, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])) + 3*d*(b*c
- a*d)*x^n*(b*c*(1 + n)*(1 + 3*n)*x^n*(a + b*x^n)*Gamma[1 + n^(-1
)]*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a +
b*x^n))] - c*(1 + n)*(1 + 3*n)*(a + b*x^n)^2*Gamma[1 + n^(-1)]*Hy
pergeometric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n
))]) - a*c*n*(1 + 3*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hypergeometri
c2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + 8*a*(
-(b*c) + a*d)*n*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeometric2F1[3
, 5, 4 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]))))

```

**Maple [F]** time = 0.113, size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^n)^3} (c + dx^n)^{1-n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d\*x^n)^(1-1/n)/(a+b\*x^n)^3,x)

[Out] int((c+d\*x^n)^(1-1/n)/(a+b\*x^n)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+1}}{(bx^n + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^(-1/n + 1)/(b\*x^n + a)^3,x, algorithm="maxima")

[Out] integrate((d\*x^n + c)^(-1/n + 1)/(b\*x^n + a)^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(dx^n + c)^{\frac{n-1}{n}}}{b^3 x^{3n} + 3ab^2 x^{2n} + 3a^2 b x^n + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3, x, algorithm="fricas")`

[Out] `integral((d*x^n + c)^((n - 1)/n)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**3, x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+1}}{(bx^n + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3, x, algorithm="giac")`

[Out] `integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3, x)`

$$3.237 \quad \int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4} dx$$

**Optimal.** Leaf size=133

$$\frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3an(bc-ad)(a+bx^n)^3} - \frac{c^2x(c+dx^n)^{-1/n}(3adn+bc(1-3n)) {}_2F_1\left(3, \frac{1}{n}; 1+\frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{3a^4n(bc-ad)}$$

[Out] (b\*x\*(c+d\*x^n)^(3-n^(-1)))/(3\*a\*(b\*c-a\*d)\*n\*(a+b\*x^n)^3) - (c^2\*(b\*c\*(1-3\*n)+3\*a\*d\*n)\*x\*Hypergeometric2F1[3, n^(-1), 1+n^(-1), -((b\*c-a\*d)\*x^n)/(a\*(c+d\*x^n))])/(3\*a^4\*(b\*c-a\*d)\*n\*(c+d\*x^n)^n^(-1))

**Rubi [A]** time = 0.171492, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3an(bc-ad)(a+bx^n)^3} - \frac{c^2x(c+dx^n)^{-1/n}(3adn+b(c-3cn)) {}_2F_1\left(3, \frac{1}{n}; 1+\frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{3a^4n(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(c+d\*x^n)^(2-n^(-1))/(a+b\*x^n)^4, x]

[Out] (b\*x\*(c+d\*x^n)^(3-n^(-1)))/(3\*a\*(b\*c-a\*d)\*n\*(a+b\*x^n)^3) - (c^2\*(3\*a\*d\*n+b\*(c-3\*c\*n))\*x\*Hypergeometric2F1[3, n^(-1), 1+n^(-1), -((b\*c-a\*d)\*x^n)/(a\*(c+d\*x^n))])/(3\*a^4\*(b\*c-a\*d)\*n\*(c+d\*x^n)^n^(-1))

**Rubi in Sympy [A]** time = 20.6569, size = 107, normalized size = 0.8

$$-\frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3an(a+bx^n)^3(ad-bc)} + \frac{c^2x(c+dx^n)^{-\frac{1}{n}}(3adn-3bcn+bc) {}_2F_1\left(\frac{1}{n}, 3 \left| -\frac{x^n(-ad+bc)}{a(c+dx^n)} \right. \right)}{3a^4n(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d\*x\*\*n)\*\*(2-1/n)/(a+b\*x\*\*n)\*\*4, x)

[Out] -b\*x\*(c+d\*x\*\*n)\*\*(3-1/n)/(3\*a\*n\*(a+b\*x\*\*n)\*\*3\*(a\*d-b\*c)) + c\*\*2\*x\*(c+d\*x\*\*n)\*\*(-1/n)\*(3\*a\*d\*n-3\*b\*c\*n+b\*c)\*hyper((1/n, 3), (1+1/n, ), -x\*\*n\*(-a\*d+b\*c)/(a\*(c+d\*x\*\*n)))/(3\*a\*\*4\*n\*(a\*d-b\*c))

**Mathematica [F]** time = 179.999, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(c+d\*x^n)^(2-n^(-1))/(a+b\*x^n)^4, x]

[Out] \$Aborted

**Maple [F]** time = 0.137, size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^n)^4} (c + dx^n)^{2-n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d\*x^n)^(2-1/n)/(a+b\*x^n)^4,x)

[Out] int((c+d\*x^n)^(2-1/n)/(a+b\*x^n)^4,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+2}}{(bx^n + a)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^(-1/n + 2)/(b\*x^n + a)^4,x, algorithm="maxima")

[Out] integrate((d\*x^n + c)^(-1/n + 2)/(b\*x^n + a)^4, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^n + c)^{\frac{2n-1}{n}}}{b^4x^{4n} + 4ab^3x^{3n} + 6a^2b^2x^{2n} + 4a^3bx^n + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^(-1/n + 2)/(b\*x^n + a)^4,x, algorithm="fricas")

[Out] integral((d\*x^n + c)^((2\*n - 1)/n)/(b^4\*x^(4\*n) + 4\*a\*b^3\*x^(3\*n) + 6\*a^2\*b^2\*x^(2\*n) + 4\*a^3\*b\*x^n + a^4), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*n)\*\*(2-1/n)/(a+b\*x\*\*n)\*\*4,x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+2}}{(bx^n + a)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^n + c)^(-1/n + 2)/(b\*x^n + a)^4,x, algorithm="giac")

[Out] integrate((d\*x^n + c)^(-1/n + 2)/(b\*x^n + a)^4, x)

$$3.238 \quad \int x^5 \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx$$

**Optimal.** Leaf size=152

$$\frac{(dx - c)^{7/2}(c + dx)^{7/2} (ad^2 + 3bc^2)}{7d^8} + \frac{c^2(dx - c)^{5/2}(c + dx)^{5/2} (2ad^2 + 3bc^2)}{5d^8} \\ + \frac{c^4(dx - c)^{3/2}(c + dx)^{3/2} (ad^2 + bc^2)}{3d^8} + \frac{b(dx - c)^{9/2}(c + dx)^{9/2}}{9d^8}$$

[Out]  $(c^4*(b*c^2 + a*d^2)*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(3*d^8) + (c^2*(3*b*c^2 + 2*a*d^2)*(-c + d*x)^{(5/2)}*(c + d*x)^{(5/2)})/(5*d^8) + ((3*b*c^2 + a*d^2)*(-c + d*x)^{(7/2)}*(c + d*x)^{(7/2)})/(7*d^8) + (b*(-c + d*x)^{(9/2)}*(c + d*x)^{(9/2)})/(9*d^8)$

**Rubi [A]** time = 0.406873, antiderivative size = 164, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$\frac{4c^2x^2(dx - c)^{3/2}(c + dx)^{3/2} (3ad^2 + 2bc^2)}{105d^6} + \frac{x^4(dx - c)^{3/2}(c + dx)^{3/2} (3ad^2 + 2bc^2)}{21d^4} \\ + \frac{8c^4(dx - c)^{3/2}(c + dx)^{3/2} (3ad^2 + 2bc^2)}{315d^8} + \frac{bx^6(dx - c)^{3/2}(c + dx)^{3/2}}{9d^2}$$

Antiderivative was successfully verified.

[In] Int[x^5\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2), x]

[Out]  $(8*c^4*(2*b*c^2 + 3*a*d^2)*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(315*d^8) + (4*c^2*(2*b*c^2 + 3*a*d^2)*x^2*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(105*d^6) + ((2*b*c^2 + 3*a*d^2)*x^4*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(21*d^4) + (b*x^6*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(9*d^2)$

**Rubi in Sympy [A]** time = 25.8529, size = 150, normalized size = 0.99

$$\frac{bx^6(-c + dx)^{\frac{3}{2}}(c + dx)^{\frac{3}{2}}}{9d^2} + \frac{8c^4(-c + dx)^{\frac{3}{2}}(c + dx)^{\frac{3}{2}}(3ad^2 + 2bc^2)}{315d^8} \\ + \frac{4c^2x^2(-c + dx)^{\frac{3}{2}}(c + dx)^{\frac{3}{2}}(3ad^2 + 2bc^2)}{105d^6} + \frac{x^4(-c + dx)^{\frac{3}{2}}(c + dx)^{\frac{3}{2}}(3ad^2 + 2bc^2)}{21d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2), x)

[Out]  $b*x**6*(-c + d*x)**(3/2)*(c + d*x)**(3/2)/(9*d**2) + 8*c**4*(-c + d*x)**(3/2)*(c + d*x)**(3/2)*(3*a*d**2 + 2*b*c**2)/(315*d**8) + 4*c**2*x**2*(-c + d*x)**(3/2)*(c + d*x)**(3/2)*(3*a*d**2 + 2*b*c**2)/(105*d**6) + x**4*(-c + d*x)**(3/2)*(c + d*x)**(3/2)*(3*a*d**2 + 2*b*c**2)/(21*d**4)$

**Mathematica [A]** time = 0.094186, size = 109, normalized size = 0.72

$$\frac{\sqrt{dx - c}\sqrt{c + dx} (c^2 - d^2x^2) (3ad^2 (8c^4 + 12c^2d^2x^2 + 15d^4x^4) + b (16c^6 + 24c^4d^2x^2 + 30c^2d^4x^4 + 35d^6x^6))}{315d^8}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2), x]

[Out]  $-(\text{Sqrt}[-c + d*x] * \text{Sqrt}[c + d*x] * (c^2 - d^2*x^2) * (3*a*d^2*(8*c^4 + 12*c^2*d^2*x^2 + 15*d^4*x^4) + b*(16*c^6 + 24*c^4*d^2*x^2 + 30*c^2*d^4*x^4 + 35*d^6*x^6)))/(315*d^8)$

**Maple [A]** time = 0.01, size = 92, normalized size = 0.6

$$\frac{35bx^6d^6 + 45ad^6x^4 + 30bc^2d^4x^4 + 36ac^2d^4x^2 + 24bc^4d^2x^2 + 24ac^4d^2 + 16bc^6}{315d^8} (dx + c)^{\frac{3}{2}} (dx - c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)`

[Out]  $1/315*(d*x+c)^{3/2}*(35*b*d^6*x^6+45*a*d^6*x^4+30*b*c^2*d^4*x^4+36*a*c^2*d^4*x^2+24*b*c^4*d^2*x^2+24*a*c^4*d^2+16*b*c^6)*(d*x-c)^{3/2}/d^8$

**Maxima [A]** time = 1.39231, size = 240, normalized size = 1.58

$$\begin{aligned} & \frac{(d^2x^2 - c^2)^{\frac{3}{2}}bx^6}{9d^2} + \frac{2(d^2x^2 - c^2)^{\frac{3}{2}}bc^2x^4}{21d^4} + \frac{(d^2x^2 - c^2)^{\frac{3}{2}}ax^4}{7d^2} + \frac{8(d^2x^2 - c^2)^{\frac{3}{2}}bc^4x^2}{105d^6} \\ & + \frac{4(d^2x^2 - c^2)^{\frac{3}{2}}ac^2x^2}{35d^4} + \frac{16(d^2x^2 - c^2)^{\frac{3}{2}}bc^6}{315d^8} + \frac{8(d^2x^2 - c^2)^{\frac{3}{2}}ac^4}{105d^6} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^5,x, algorithm="maxima")`

[Out]  $1/9*(d^2*x^2 - c^2)^{3/2}*b*x^6/d^2 + 2/21*(d^2*x^2 - c^2)^{3/2}*b*c^2*x^4/d^4 + 1/7*(d^2*x^2 - c^2)^{3/2}*a*x^4/d^2 + 8/105*(d^2*x^2 - c^2)^{3/2}*b*c^4*x^2/d^6 + 4/35*(d^2*x^2 - c^2)^{3/2}*a*c^2*x^2/d^4 + 16/315*(d^2*x^2 - c^2)^{3/2}*b*c^6/d^8 + 8/105*(d^2*x^2 - c^2)^{3/2}*a*c^4/d^6$

**Fricas [A]** time = 0.743305, size = 741, normalized size = 4.88

$$\frac{8960bd^{18}x^{18} + 16bc^{18} + 24ac^{16}d^2 - 2880(9bc^2d^{16} - 4ad^{18})x^{16} + 144(181bc^4d^{14} - 236ac^2d^{16})x^{14} - 24(461bc^6d^{12} - 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^5,x, algorithm="fricas")`

[Out]  $-1/315*(8960*b*d^{18}*x^{18} + 16*b*c^{18} + 24*a*c^{16}*d^2 - 2880*(9*b*c^2*d^{16} - 4*a*d^{18})*x^{16} + 144*(181*b*c^4*d^{14} - 236*a*c^2*d^{16})*x^{14} - 24*(461*b*c^6*d^{12} - 1426*a*c^4*d^{14})*x^{12} + 9*(27*b*c^8*d^{10} - 1832*a*c^6*d^{12})*x^{10} + 45*(160*b*c^{10}*d^8 + 289*a*c^8*d^{10})*x^8 - 21*(429*b*c^{12}*d^6 + 646*a*c^{10}*d^8)*x^6 + 2079*(2*b*c^{14}*d^4 + 3*a*c^{12}*d^6)*x^4 - 324*(2*b*c^{16}*d^2 + 3*a*c^{14}*d^4)*x^2 - (8960*b*d^{17}*x^{17} - 320*(67*b*c^2*d^{15} - 36*a*d^{17})*x^{15} + 235*2*(7*b*c^4*d^{13} - 12*a*c^2*d^{15})*x^{13} - 8*(619*b*c^6*d^{11} - 2694*a*c^4*d^{13})*x^{11} - 5*(233*b*c^8*d^9 + 1704*a*c^6*d^{11})*x^9 + 45*(143*b*c^{10}*d^7 + 225*a*c^8*d^9)*x^7 - 3003*(2*b*c^{12}*d^5 + 3*a*c^{10}*d^7)*x^5 + 924*(2*b*c^{14}*d^3 + 3*a*c^{12}*d^5)*x^3 - 72*(2*b*c^{16}*d + 3*a*c^{14}*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c)/(256*d^{17}*x^9 - 576*c^2*d^{15}*x^7 + 432*c^4*d^{13}*x^5 - 120*c^6*d^{11}*x^3 + 9*c^8*d^9*x - (256*d^{16}*x^8 - 448*c^2*d^{14}*x^6 + 240*c^4*d^{12}*x^4 - 40*c^6*d^{10}*x^2 + c^8*d^8)*sqrt(d*x + c)*sqrt(d*x - c))$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^5 (a + bx^2) \sqrt{-c + dx} \sqrt{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2),x)

[Out] Integral(x\*\*5\*(a + b\*x\*\*2)\*sqrt(-c + d\*x)\*sqrt(c + d\*x), x)

---

**GIAC/XCAS [A]** time = 0.257139, size = 309, normalized size = 2.03

$$3 \left( \left( 3 \left( (dx + c) \left( 5(dx + c) \left( \frac{dx+c}{d^5} - \frac{6c}{d^5} \right) + \frac{74c^2}{d^5} \right) - \frac{96c^3}{d^5} \right) (dx + c) + \frac{203c^4}{d^5} \right) (dx + c) - \frac{70c^5}{d^5} \right) (dx + c)^{\frac{3}{2}} \sqrt{dx - ca} + \left( \left( \left( 5(dx + c) \right) \right) \right)$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*sqrt(d\*x + c)\*sqrt(d\*x - c)\*x^5,x, algorithm="giac")

[Out] 1/315\*(3\*((3\*((d\*x + c)\*(5\*(d\*x + c))\*((d\*x + c)/d^5 - 6\*c/d^5) + 74\*c^2/d^5) - 96\*c^3/d^5)\*(d\*x + c) + 203\*c^4/d^5)\*(d\*x + c) - 70\*c^5/d^5)\*(d\*x + c)^(3/2)\*sqrt(d\*x - c)\*a + (((5\*((d\*x + c)\*(7\*(d\*x + c))\*((d\*x + c)/d^7 - 8\*c/d^7) + 195\*c^2/d^7) - 386\*c^3/d^7)\*(d\*x + c) + 2369\*c^4/d^7)\*(d\*x + c) - 1836\*c^5/d^7)\*(d\*x + c) + 861\*c^6/d^7)\*(d\*x + c) - 210\*c^7/d^7)\*(d\*x + c)^(3/2)\*sqrt(d\*x - c)\*b)/d

### 3.239 $\int x^3 \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx$

**Optimal.** Leaf size=109

$$\frac{(dx - c)^{5/2}(c + dx)^{5/2} (ad^2 + 2bc^2)}{5d^6} + \frac{c^2(dx - c)^{3/2}(c + dx)^{3/2} (ad^2 + bc^2)}{3d^6} + \frac{b(dx - c)^{7/2}(c + dx)^{7/2}}{7d^6}$$

[Out]  $(c^2(b^2c^2 + a^2d^2) * (-c + d*x)^{(3/2)} * (c + d*x)^{(3/2)}) / (3*d^6) + ((2*b^2c^2 + a^2d^2) * (-c + d*x)^{(5/2)} * (c + d*x)^{(5/2)}) / (5*d^6) + (b * (-c + d*x)^{(7/2)} * (c + d*x)^{(7/2)}) / (7*d^6)$

**Rubi [A]** time = 0.291181, antiderivative size = 118, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$\frac{2c^2(dx - c)^{3/2}(c + dx)^{3/2} (7ad^2 + 4bc^2)}{105d^6} + \frac{x^2(dx - c)^{3/2}(c + dx)^{3/2} (7ad^2 + 4bc^2)}{35d^4} + \frac{bx^4(dx - c)^{3/2}(c + dx)^{3/2}}{7d^2}$$

Antiderivative was successfully verified.

[In] `Int[x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2), x]`

[Out]  $(2*c^2*(4*b^2c^2 + 7*a^2d^2) * (-c + d*x)^{(3/2)} * (c + d*x)^{(3/2)}) / (105*d^6) + ((4*b^2c^2 + 7*a^2d^2) * x^2 * (-c + d*x)^{(3/2)} * (c + d*x)^{(3/2)}) / (35*d^4) + (b*x^4 * (-c + d*x)^{(3/2)} * (c + d*x)^{(3/2)}) / (7*d^2)$

**Rubi in Sympy [A]** time = 17.5402, size = 105, normalized size = 0.96

$$\frac{bx^4(-c + dx)^{\frac{3}{2}}(c + dx)^{\frac{3}{2}}}{7d^2} + \frac{2c^2(-c + dx)^{\frac{3}{2}}(c + dx)^{\frac{3}{2}}(7ad^2 + 4bc^2)}{105d^6} + \frac{x^2(-c + dx)^{\frac{3}{2}}(c + dx)^{\frac{3}{2}}(7ad^2 + 4bc^2)}{35d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2), x)`

[Out]  $b*x**4*(-c + d*x)**(3/2)*(c + d*x)**(3/2)/(7*d**2) + 2*c**2*(-c + d*x)**(3/2)*(c + d*x)**(3/2)*(7*a*d**2 + 4*b*c**2)/(105*d**6) + x**2*(-c + d*x)**(3/2)*(c + d*x)**(3/2)*(7*a*d**2 + 4*b*c**2)/(35*d**4)$

**Mathematica [A]** time = 0.0750872, size = 88, normalized size = 0.81

$$\frac{\sqrt{dx - c}\sqrt{c + dx} (d^2x^2 - c^2) (7ad^2 (2c^2 + 3d^2x^2) + b (8c^4 + 12c^2d^2x^2 + 15d^4x^4))}{105d^6}$$

Antiderivative was successfully verified.

[In] `Integrate[x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2), x]`

[Out]  $(\text{Sqrt}[-c + d*x] * \text{Sqrt}[c + d*x] * (-c^2 + d^2*x^2) * (7*a*d^2 * (2*c^2 + 3*d^2*x^2) + b * (8*c^4 + 12*c^2*d^2*x^2 + 15*d^4*x^4))) / (105*d^6)$

**Maple [A]** time = 0.009, size = 68, normalized size = 0.6

$$\frac{15bd^4x^4 + 21ad^4x^2 + 12bc^2d^2x^2 + 14ac^2d^2 + 8bc^4}{105d^6} (dx + c)^{\frac{3}{2}} (dx - c)^{\frac{3}{2}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)`

[Out]  $\frac{1}{105} (d^2x^2 - c^2)^{3/2} (15b^2d^4x^4 + 21a^2d^4x^2 + 12b^2c^2d^2x^2 + 4a^2c^2d^2 + 8b^2c^4) (d^2x^2 - c^2)^{3/2} / d^6$

**Maxima [A]** time = 1.38088, size = 167, normalized size = 1.53

$$\frac{(d^2x^2 - c^2)^{3/2}bx^4}{7d^2} + \frac{4(d^2x^2 - c^2)^{3/2}bc^2x^2}{35d^4} + \frac{(d^2x^2 - c^2)^{3/2}ax^2}{5d^2} + \frac{8(d^2x^2 - c^2)^{3/2}bc^4}{105d^6} + \frac{2(d^2x^2 - c^2)^{3/2}ac^2}{15d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^3,x, algorithm="maxima")`

[Out]  $\frac{1}{7} (d^2x^2 - c^2)^{3/2} b^2x^4/d^2 + \frac{4}{35} (d^2x^2 - c^2)^{3/2} b^2c^2x^2/d^4 + \frac{1}{5} (d^2x^2 - c^2)^{3/2} a^2x^2/d^2 + \frac{8}{105} (d^2x^2 - c^2)^{3/2} b^2c^4/d^6 + \frac{2}{15} (d^2x^2 - c^2)^{3/2} a^2c^2/d^4$

**Fricas [A]** time = 0.413169, size = 582, normalized size = 5.34

$$\frac{960bd^{14}x^{14} - 8bc^{14} - 14ac^{12}d^2 - 336(7bc^2d^{12} - 4ad^{14})x^{12} + 1736(bc^4d^{10} - 2ac^2d^{12})x^{10} - 7(89bc^6d^8 - 328ac^4d^{10})x^8 - 7(89b^3c^6d^8 - 328a^3c^4d^{10})x^6 - 105(7b^2c^4d^8 + 12a^2c^8d^6)x^4 + 49(4b^2c^4d^8 + 7a^2c^8d^6)x^2 - (960b^2d^{13}x^{13} - 48(39b^2c^2d^{11} - 28a^2d^{13})x^{11} + 40(23b^2c^4d^9 - 70a^2c^2d^{11})x^9 - (337b^2c^6d^7 - 1064a^2c^4d^9)x^7 + 21(33b^2c^8d^5 + 49a^2c^6d^7)x^5 - 105(4b^2c^{10}d^3 + 7a^2c^8d^5)x^3 + 14(4b^2c^{12}d + 7a^2c^{10}d^3)x}{(64d^{13}x^7 - 112c^2d^{11}x^5 + 56c^4d^9x^3 - 7c^6d^7x - (64d^{12}x^6 - 80c^2d^{10}x^4 + 24c^4d^8x^2 - c^6d^6))\sqrt{d^2x^2 - c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^3,x, algorithm="fricas")`

[Out]  $\frac{-1/105*(960*b*d^{14}*x^{14} - 8*b*c^{14} - 14*a*c^{12}*d^2 - 336*(7*b*c^2*d^{12} - 4*a*d^{14})*x^{12} + 1736*(b*c^4*d^{10} - 2*a*c^2*d^{12})*x^{10} - 7*(89*b^3*c^6*d^8 - 328*a^3*c^4*d^{10})*x^8 + 7*(118*b^2*c^8*d^6 + 109*a^2*c^6*d^8)*x^6 - 105*(7*b^2*c^4*d^8 + 12*a^2*c^8*d^6)*x^4 + 49*(4*b^2*c^4*d^8 + 7*a^2*c^8*d^6)*x^2 - (960*b^2*d^{13}*x^{13} - 48*(39*b^2*c^2*d^{11} - 28*a^2*d^{13})*x^{11} + 40*(23*b^2*c^4*d^9 - 70*a^2*c^2*d^{11})*x^9 - (337*b^2*c^6*d^7 - 1064*a^2*c^4*d^9)*x^7 + 21*(33*b^2*c^8*d^5 + 49*a^2*c^6*d^7)*x^5 - 105*(4*b^2*c^{10}d^3 + 7*a^2*c^8*d^5)*x^3 + 14*(4*b^2*c^{12}d + 7*a^2*c^{10}d^3)*x}{(64*d^{13}*x^7 - 112*c^2*d^{11}*x^5 + 56*c^4*d^9*x^3 - 7*c^6*d^7*x - (64*d^{12}*x^6 - 80*c^2*d^{10}*x^4 + 24*c^4*d^8*x^2 - c^6*d^6))*sqrt(d^2*x^2 - c^2)}$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 (a + bx^2) \sqrt{-c + dx} \sqrt{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)`

[Out] `Integral(x**3*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)`

**GIAC/XCAS [A]** time = 0.230614, size = 227, normalized size = 2.08

$$\frac{7 \left( (dx + c) \left( 3(dx + c) \left( \frac{dx+c}{d^3} - \frac{4c}{d^3} \right) + \frac{17c^2}{d^3} \right) - \frac{10c^3}{d^3} \right) (dx + c)^{\frac{3}{2}} \sqrt{dx - ca} + \left( \left( 3 \left( (dx + c) \left( 5(dx + c) \left( \frac{dx+c}{d^5} - \frac{6c}{d^5} \right) + \frac{74c^2}{d^5} \right) - \frac{96c^3}{d^5} \right) \right) \sqrt{dx - ca} \right)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^3,x, algorithm="giac")
```

```
[Out] 1/105*(7*((d*x + c)*(3*(d*x + c)*((d*x + c)/d^3 - 4*c/d^3) + 17*c
^2/d^3) - 10*c^3/d^3)*(d*x + c)^(3/2)*sqrt(d*x - c)*a + ((3*((d*x
+ c)*(5*(d*x + c)*((d*x + c)/d^5 - 6*c/d^5) + 74*c^2/d^5) - 96*c
^3/d^5)*(d*x + c) + 203*c^4/d^5)*(d*x + c) - 70*c^5/d^5)*(d*x + c
)^(3/2)*sqrt(d*x - c)*b)/d
```

### 3.240 $\int x\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2) dx$

**Optimal.** Leaf size=67

$$\frac{(dx-c)^{3/2}(c+dx)^{3/2}(ad^2+bc^2)}{3d^4} + \frac{b(dx-c)^{5/2}(c+dx)^{5/2}}{5d^4}$$

[Out]  $((b*c^2 + a*d^2)*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(3*d^4) + (b*(-c + d*x)^{(5/2)}*(c + d*x)^{(5/2)})/(5*d^4)$

**Rubi [A]** time = 0.151357, antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{(dx-c)^{3/2}(c+dx)^{3/2}(5ad^2+2bc^2)}{15d^4} + \frac{bx^2(dx-c)^{3/2}(c+dx)^{3/2}}{5d^2}$$

Antiderivative was successfully verified.

[In] `Int[x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2), x]`

[Out]  $((2*b*c^2 + 5*a*d^2)*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(15*d^4) + (b*x^2*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(5*d^2)$

**Rubi in Sympy [A]** time = 9.89443, size = 61, normalized size = 0.91

$$\frac{bx^2(-c+dx)^{\frac{3}{2}}(c+dx)^{\frac{3}{2}}}{5d^2} + \frac{(-c+dx)^{\frac{3}{2}}(c+dx)^{\frac{3}{2}}(5ad^2+2bc^2)}{15d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2), x)`

[Out]  $b*x^2*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)}/(5*d^2) + (-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)}*(5*a*d^2 + 2*b*c^2)/(15*d^4)$

**Mathematica [A]** time = 0.0582542, size = 62, normalized size = 0.93

$$\frac{\sqrt{dx-c}\sqrt{c+dx}(d^2x^2-c^2)(5ad^2+2bc^2+3bd^2x^2)}{15d^4}$$

Antiderivative was successfully verified.

[In] `Integrate[x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2), x]`

[Out]  $(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]*(-c^2 + d^2*x^2)*(2*b*c^2 + 5*a*d^2 + 3*b*d^2*x^2))/(15*d^4)$

**Maple [A]** time = 0.006, size = 44, normalized size = 0.7

$$\frac{3bd^2x^2 + 5ad^2 + 2bc^2}{15d^4}(dx+c)^{\frac{3}{2}}(dx-c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)`

[Out]  $\frac{1}{15} \cdot (d \cdot x + c)^{3/2} \cdot (3 \cdot b \cdot d^2 \cdot x^2 + 5 \cdot a \cdot d^2 + 2 \cdot b \cdot c^2) \cdot (d \cdot x - c)^{3/2} / d^4$

**Maxima [A]** time = 1.37831, size = 95, normalized size = 1.42

$$\frac{(d^2 x^2 - c^2)^{\frac{3}{2}} b x^2}{5 d^2} + \frac{2 (d^2 x^2 - c^2)^{\frac{3}{2}} b c^2}{15 d^4} + \frac{(d^2 x^2 - c^2)^{\frac{3}{2}} a}{3 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x,x, algorithm="maxima")`

[Out]  $\frac{1}{5} \cdot (d^2 \cdot x^2 - c^2)^{3/2} \cdot b \cdot x^2 / d^2 + \frac{2}{15} \cdot (d^2 \cdot x^2 - c^2)^{3/2} \cdot b \cdot c^2 / d^4 + \frac{1}{3} \cdot (d^2 \cdot x^2 - c^2)^{3/2} \cdot a / d^2$

**Fricas [A]** time = 0.279759, size = 421, normalized size = 6.28

$$\frac{48 b d^{10} x^{10} + 2 b c^{10} + 5 a c^8 d^2 - 20 (5 b c^2 d^8 - 4 a d^{10}) x^8 + 5 (7 b c^4 d^6 - 44 a c^2 d^8) x^6 + 5 (8 b c^6 d^4 + 41 a c^4 d^6) x^4 - 5 (5 b c^8 d^2 + 41 a c^6 d^4) x^2 - 5 (5 b c^{10} d^0 + 41 a c^8 d^2) x^0}{15 (16 d^9 x^5 - 20 c^2 d^7 x^3 + 5 c^4 d^5 x - 5 c^6 d^3 + 5 a c^4 d^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x,x, algorithm="fricas")`

[Out]  $-\frac{1}{15} \cdot (48 \cdot b \cdot d^{10} \cdot x^{10} + 2 \cdot b \cdot c^{10} + 5 \cdot a \cdot c^8 \cdot d^2 - 20 \cdot (5 \cdot b \cdot c^2 \cdot d^8 - 4 \cdot a \cdot d^{10}) \cdot x^8 + 5 \cdot (7 \cdot b \cdot c^4 \cdot d^6 - 44 \cdot a \cdot c^2 \cdot d^8) \cdot x^6 + 5 \cdot (8 \cdot b \cdot c^6 \cdot d^4 + 41 \cdot a \cdot c^4 \cdot d^6) \cdot x^4 - 5 \cdot (5 \cdot b \cdot c^8 \cdot d^2 + 14 \cdot a \cdot c^6 \cdot d^4) \cdot x^2 - (48 \cdot b \cdot d^9 \cdot x^9 - 4 \cdot (19 \cdot b \cdot c^2 \cdot d^7 - 20 \cdot a \cdot d^9) \cdot x^7 + 3 \cdot (b \cdot c^4 \cdot d^5 - 6 \cdot a \cdot c^2 \cdot d^7) \cdot x^5 + 5 \cdot (7 \cdot b \cdot c^6 \cdot d^3 + 25 \cdot a \cdot c^4 \cdot d^5) \cdot x^3 - 5 \cdot (2 \cdot b \cdot c^8 \cdot d + 5 \cdot a \cdot c^6 \cdot d^3) \cdot x) \cdot \sqrt{d \cdot x + c} \cdot \sqrt{d \cdot x - c}) / (16 \cdot d^9 \cdot x^5 - 20 \cdot c^2 \cdot d^7 \cdot x^3 + 5 \cdot c^4 \cdot d^5 \cdot x - (16 \cdot d^8 \cdot x^4 - 12 \cdot c^2 \cdot d^6 \cdot x^2 + c^4 \cdot d^4) \cdot \sqrt{d \cdot x + c} \cdot \sqrt{d \cdot x - c})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x (a + b x^2) \sqrt{-c + d x} \sqrt{c + d x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)`

[Out] `Integral(x*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)`

**GIAC/XCAS [A]** time = 0.217173, size = 126, normalized size = 1.88

$$\frac{\left( (d x + c) \left( 3 (d x + c) \left( \frac{d x + c}{d^3} - \frac{4 c}{d^3} \right) + \frac{17 c^2}{d^3} \right) - \frac{10 c^3}{d^3} \right) (d x + c)^{\frac{3}{2}} \sqrt{d x - c} b + \frac{5 (d x + c)^{\frac{3}{2}} (d x - c)^{\frac{3}{2}} a}{d}}{15 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x,x, algorithm="giac")`

[Out]  $\frac{1}{15} \left( ((d^*x + c)^3 (d^*x + c) / d^3 - 4^*c/d^3) + 17^*c^2/d^3 - 10^*c^3/d^3 \right) (d^*x + c)^{3/2} \sqrt{d^*x - c} b + 5^*(d^*x + c)^{3/2} (d^*x - c)^{3/2} a/d / d$

$$3.241 \quad \int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x} dx$$

**Optimal.** Leaf size=80

$$a\sqrt{dx-c}\sqrt{c+dx} - ac \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right) + \frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

[Out] a\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x] + (b\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2))/(3\*d^2) - a\*c\*ArcTan[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/c]

**Rubi [A]** time = 0.286411, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$

$$a\sqrt{dx-c}\sqrt{c+dx} - ac \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right) + \frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2))/x, x]

[Out] a\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x] + (b\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2))/(3\*d^2) - a\*c\*ArcTan[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/c]

**Rubi in Sympy [A]** time = 17.3838, size = 65, normalized size = 0.81

$$-ac \operatorname{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right) + a\sqrt{-c+dx}\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2)/x, x)

[Out] -a\*c\*atan(sqrt(-c + d\*x)\*sqrt(c + d\*x)/c) + a\*sqrt(-c + d\*x)\*sqrt(c + d\*x) + b\*(-c + d\*x)\*\*(3/2)\*(c + d\*x)\*\*(3/2)/(3\*d\*\*2)

**Mathematica [A]** time = 0.142888, size = 75, normalized size = 0.94

$$\frac{\sqrt{dx-c}\sqrt{c+dx}(3ad^2 - bc^2 + bd^2x^2)}{3d^2} + ac \tan^{-1}\left(\frac{c}{\sqrt{dx-c}\sqrt{c+dx}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2))/x, x]

[Out] (Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(-(b\*c^2) + 3\*a\*d^2 + b\*d^2\*x^2))/(3\*d^2) + a\*c\*ArcTan[c/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])]

**Maple [B]** time = 0.02, size = 174, normalized size = 2.2

$$\frac{1}{3d^2}\sqrt{dx-c}\sqrt{c+dx}\left(x^2bd^2\sqrt{-c^2\sqrt{d^2x^2-c^2}}+3ac^2\ln\left(-2\frac{c^2-\sqrt{-c^2\sqrt{d^2x^2-c^2}}}{x}\right)d^2+3a\sqrt{d^2x^2-c^2}d^2\sqrt{-c^2}-bc^2\sqrt{-c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x)`

[Out]  $\frac{1}{3} (d^2 x - c)^{1/2} (d^2 x + c)^{1/2} (x^2 b d^2 (-c^2)^{1/2} (d^2 x^2 - c^2)^{1/2} + 3 a^2 c^2 \ln(-2 (c^2 - (-c^2)^{1/2} (d^2 x^2 - c^2)^{1/2})) / x) d^2 + 3 a (d^2 x^2 - c^2)^{1/2} d^2 (-c^2)^{1/2} - b c^2 (-c^2)^{1/2} (d^2 x^2 - c^2)^{1/2} / (d^2 x^2 - c^2)^{1/2} / d^2 / (-c^2)^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.250421, size = 383, normalized size = 4.79

$$\frac{4 b d^6 x^6 - b c^6 + 3 a c^4 d^2 - 3 (3 b c^2 d^4 - 4 a d^6) x^4 + 3 (2 b c^4 d^2 - 5 a c^2 d^4) x^2 - (4 b d^5 x^5 - (7 b c^2 d^3 - 12 a d^5) x^3 + 3 (b c^4 d - 3 a c^2 d^3) x) \sqrt{d x + c} \sqrt{d x - c}}{3 (4 d^5 x^3 - 3 c^2 d^3 x - (4 d^4 x^2 - 3 c^2 d^2) \sqrt{d x + c} \sqrt{d x - c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x,x, algorithm="fricas")`

[Out]  $-1/3 (4 b^2 d^6 x^6 - b^2 c^6 + 3 a^2 c^4 d^2 - 3 (3 b^2 c^2 d^4 - 4 a^2 d^6) x^4 + 3 (2 b^2 c^4 d^2 - 5 a^2 c^2 d^4) x^2 - (4 b^2 d^5 x^5 - (7 b^2 c^2 d^3 - 12 a^2 d^5) x^3 + 3 (b^2 c^4 d - 3 a^2 c^2 d^3) x) \sqrt{d x + c} \sqrt{d x - c} + 6 (4 a^2 c^2 d^5 x^3 - 3 a^2 c^3 d^3 x - (4 a^2 c^2 d^4 x^2 - a^2 c^3 d^2) \sqrt{d x + c} \sqrt{d x - c})) \arctan(- (d x - \sqrt{d x + c} \sqrt{d x - c}) / c) / (4 d^5 x^3 - 3 c^2 d^3 x - (4 d^4 x^2 - 3 c^2 d^2) \sqrt{d x + c} \sqrt{d x - c})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b x^2) \sqrt{-c + d x} \sqrt{c + d x}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x,x)`

[Out] `Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x, x)`

**GIAC/XCAS [A]** time = 0.224712, size = 109, normalized size = 1.36

$$2 a c \arctan\left(\frac{(\sqrt{d x + c} - \sqrt{d x - c})^2}{2 c}\right) + \frac{1}{1920} (3 a d^6 + ((d x + c) b d^4 - 2 b c d^4) (d x + c)) \sqrt{d x + c} \sqrt{d x - c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c) + 1/1920*(3  
*a*d^6 + ((d*x + c)*b*d^4 - 2*b*c*d^4)*(d*x + c))*sqrt(d*x + c)*s  
qrt(d*x - c)
```



$$3.242 \quad \int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3} dx$$

**Optimal.** Leaf size=96

$$-\frac{(2bc^2 - ad^2) \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c} - \frac{a\sqrt{dx-c}\sqrt{c+dx}}{2x^2} + b\sqrt{dx-c}\sqrt{c+dx}$$

[Out] b\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x] - (a\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]) / (2\*x^2) - ((2\*b\*c^2 - a\*d^2)\*ArcTan[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/c]) / (2\*c)

**Rubi [A]** time = 0.319887, antiderivative size = 114, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$

$$\frac{1}{2}\sqrt{dx-c}\sqrt{c+dx}\left(2b - \frac{ad^2}{c^2}\right) - \frac{(2bc^2 - ad^2) \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c} + \frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{2c^2x^2}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2))/x^3, x]

[Out] ((2\*b - (a\*d^2)/c^2)\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/2 + (a\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2))/(2\*c^2\*x^2) - ((2\*b\*c^2 - a\*d^2)\*ArcTan[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/c]) / (2\*c)

**Rubi in Sympy [A]** time = 18.789, size = 95, normalized size = 0.99

$$\frac{a(-c+dx)^{\frac{3}{2}}(c+dx)^{\frac{3}{2}}}{2c^2x^2} + \frac{(ad^2 - 2bc^2) \operatorname{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c} - \frac{\sqrt{-c+dx}\sqrt{c+dx}(ad^2 - 2bc^2)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2)/x\*\*3, x)

[Out] a\*(-c + d\*x)\*\*(3/2)\*(c + d\*x)\*\*(3/2)/(2\*c\*\*2\*x\*\*2) + (a\*d\*\*2 - 2\*b\*c\*\*2)\*atan(sqrt(-c + d\*x)\*sqrt(c + d\*x)/c)/(2\*c) - sqrt(-c + d\*x)\*sqrt(c + d\*x)\*(a\*d\*\*2 - 2\*b\*c\*\*2)/(2\*c\*\*2)

**Mathematica [C]** time = 0.19657, size = 105, normalized size = 1.09

$$\frac{1}{2} \left( \frac{(2bx^2 - a) \sqrt{dx-c}\sqrt{c+dx}}{x^2} + \left( 2ibc - \frac{iad^2}{c} \right) \log \left( \frac{-4\sqrt{dx-c}\sqrt{c+dx} + 4ic}{2bc^2x - ad^2x} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2))/x^3, x]

[Out] ((Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(-a + 2\*b\*x^2))/x^2 + ((2\*I)\*b\*c - (I\*a\*d^2)/c)\*Log[((4\*I)\*c - 4\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(2\*b\*c^2\*x - a\*d^2\*x)]/2

**Maple [B]** time = 0.02, size = 182, normalized size = 1.9

$$-\frac{1}{2x^2}\sqrt{dx-c}\sqrt{dx+c}\left(\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^2ad^2-2\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^2bc^2-2x^2b\sqrt{-c^2}\sqrt{d^2x^2-c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/x^3,x)

[Out] -1/2\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)\*(ln(-2\*(c^2-(-c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2))/x)\*x^2\*a\*d^2-2\*ln(-2\*(c^2-(-c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2))/x)\*x^2\*b\*c^2-2\*x^2\*b\*sqrt(-c^2)\*sqrt(d^2\*x^2-c^2)+a\*(d^2\*x^2-c^2)^(1/2)\*(-c^2)^(1/2))/(d^2\*x^2-c^2)^(1/2)/x^2/(-c^2)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*sqrt(d\*x + c)\*sqrt(d\*x - c)/x^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.25108, size = 419, normalized size = 4.36

$$\frac{8bcd^4x^6 - ac^5 - 2(5bc^3d^2 + 2acd^4)x^4 + (2bc^5 + 5ac^3d^2)x^2 - (8bcd^3x^5 + 3ac^3dx - 2(3bc^3d + 2acd^3)x^3)\sqrt{dx+c}\sqrt{dx-c}}{2(4cd^3x^5 - 3c^3a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*sqrt(d\*x + c)\*sqrt(d\*x - c)/x^3,x, algorithm="fricas")

[Out] -1/2\*(8\*b\*c\*d^4\*x^6 - a\*c^5 - 2\*(5\*b\*c^3\*d^2 + 2\*a\*c\*d^4)\*x^4 + (2\*b\*c^5 + 5\*a\*c^3\*d^2)\*x^2 - (8\*b\*c\*d^3\*x^5 + 3\*a\*c^3\*d\*x - 2\*(3\*b\*c^3\*d + 2\*a\*c\*d^3)\*x^3)\*sqrt(d\*x + c)\*sqrt(d\*x - c) + 2\*(4\*(2\*b\*c^2\*d^3 - a\*d^5)\*x^5 - 3\*(2\*b\*c^4\*d - a\*c^2\*d^3)\*x^3 - (4\*(2\*b\*c^2\*d^2 - a\*d^4)\*x^4 - (2\*b\*c^4 - a\*c^2\*d^2)\*x^2)\*sqrt(d\*x + c)\*sqrt(d\*x - c)\*arctan(-(d\*x - sqrt(d\*x + c)\*sqrt(d\*x - c))/c)/(4\*c\*d^3\*x^5 - 3\*c^3\*d\*x^3 - (4\*c\*d^2\*x^4 - c^3\*x^2)\*sqrt(d\*x + c)\*sqrt(d\*x - c))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)\sqrt{-c + dx}\sqrt{c + dx}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2)/x\*\*3,x)

[Out] Integral((a + b\*x\*\*2)\*sqrt(-c + d\*x)\*sqrt(c + d\*x)/x\*\*3, x)

**GIAC/XCAS [A]** time = 0.239757, size = 212, normalized size = 2.21

$$\frac{\sqrt{dx+c}\sqrt{dx-c}bd + \frac{(2bc^2d-ad^3) \arctan\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2c}\right)}{c} + \frac{2\left(ad^3(\sqrt{dx+c}-\sqrt{dx-c})^6 - 4ac^2d^3(\sqrt{dx+c}-\sqrt{dx-c})^2\right)}{\left((\sqrt{dx+c}-\sqrt{dx-c})^4 + 4c^2\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*sqrt(d\*x + c)\*sqrt(d\*x - c)/x^3,x, algorithm="giac")

[Out] (sqrt(d\*x + c)\*sqrt(d\*x - c)\*b\*d + (2\*b\*c^2\*d - a\*d^3)\*arctan(1/2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2/c)/c + 2\*(a\*d^3\*(sqrt(d\*x + c) - sqrt(d\*x - c))^6 - 4\*a\*c^2\*d^3\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2)/((sqrt(d\*x + c) - sqrt(d\*x - c))^4 + 4\*c^2)^2/d

$$3.243 \quad \int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5} dx$$

**Optimal.** Leaf size=121

$$-\frac{\sqrt{dx-c}\sqrt{c+dx}(ad^2+4bc^2)}{8c^2x^2} + \frac{d^2(ad^2+4bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^3} + \frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{4c^2x^4}$$

[Out]  $-\left((4*b*c^2 + a*d^2)*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]\right)/(8*c^2*x^2) + (a*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(4*c^2*x^4) + (d^2*(4*b*c^2 + a*d^2)*\text{ArcTan}[(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/c])/(8*c^3)$

**Rubi [A]** time = 0.398807, antiderivative size = 164, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$-\frac{\sqrt{dx-c}(c+dx)^{3/2}(ad^2+4bc^2)}{8c^3x^2} + \frac{d\sqrt{dx-c}\sqrt{c+dx}(ad^2+4bc^2)}{8c^3x} + \frac{d^2(ad^2+4bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^3} + \frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{4c^2x^4}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2))/x^5, x]

[Out]  $(d*(4*b*c^2 + a*d^2)*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/(8*c^3*x) - ((4*b*c^2 + a*d^2)*\text{Sqrt}[-c + d*x]*(c + d*x)^{(3/2)})/(8*c^3*x^2) + (a*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(4*c^2*x^4) + (d^2*(4*b*c^2 + a*d^2)*\text{ArcTan}[(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/c])/(8*c^3)$

**Rubi in Sympy [A]** time = 22.6634, size = 141, normalized size = 1.17

$$\frac{a(-c+dx)^{\frac{3}{2}}(c+dx)^{\frac{3}{2}}}{4c^2x^4} + \frac{d^2(ad^2+4bc^2)\text{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{8c^3} + \frac{d\sqrt{-c+dx}\sqrt{c+dx}(ad^2+4bc^2)}{8c^3x} - \frac{\sqrt{-c+dx}(c+dx)^{\frac{3}{2}}(ad^2+4bc^2)}{8c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2)/x\*\*5, x)

[Out]  $a*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)}/(4*c^2*x^4) + d^2*(a*d^2 + 4*b*c^2)*\text{atan}(\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)/c)/(8*c^3) + d*\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)*(a*d^2 + 4*b*c^2)/(8*c^3*x) - \text{sqrt}(-c + d*x)*(c + d*x)^{(3/2)}*(a*d^2 + 4*b*c^2)/(8*c^3*x^2)$

**Mathematica [C]** time = 0.179346, size = 132, normalized size = 1.09

$$\frac{c\sqrt{dx-c}\sqrt{c+dx}(-2ac^2+ad^2x^2-4bc^2x^2)-id^2x^4(ad^2+4bc^2)\log\left(\frac{16c^2(\sqrt{dx-c}\sqrt{c+dx}-ic)}{d^2x(ad^2+4bc^2)}\right)}{8c^3x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2))/x^5, x]



[In] integrate((b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2)/x\*\*5,x)

[Out] Exception raised: MellinTransformStripError

**GIAC/XCAS [A]** time = 0.244135, size = 437, normalized size = 3.61

$$\frac{(4bc^2d^3+ad^5) \arctan\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2c}\right)}{c^3} - \frac{2\left(4bc^2d^3(\sqrt{dx+c}-\sqrt{dx-c})^{14} - ad^5(\sqrt{dx+c}-\sqrt{dx-c})^{14} + 16bc^4d^3(\sqrt{dx+c}-\sqrt{dx-c})^{10} + 28ac^2d^5(\sqrt{dx+c}-\sqrt{dx-c})^6\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*sqrt(d\*x + c)\*sqrt(d\*x - c)/x^5,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/4*((4*b*c^2*d^3 + a*d^5)*\arctan(1/2*(\sqrt{d*x + c} - \sqrt{d*x - c}))^2/c)/c^3 - 2*(4*b*c^2*d^3*(\sqrt{d*x + c} - \sqrt{d*x - c})^{14} \\ & - a*d^5*(\sqrt{d*x + c} - \sqrt{d*x - c})^{14} + 16*b*c^4*d^3*(\sqrt{d*x + c} - \sqrt{d*x - c})^{10} + 28*a*c^2*d^5*(\sqrt{d*x + c} - \sqrt{d*x - c})^6 - \\ & 112*a*c^4*d^5*(\sqrt{d*x + c} - \sqrt{d*x - c})^6 - 256*b*c^8*d^3*(\sqrt{d*x + c} - \sqrt{d*x - c})^2 + 64*a*c^6*d^5*(\sqrt{d*x + c} - \sqrt{d*x - c})^2)/((\sqrt{d*x + c} - \sqrt{d*x - c})^4 + 4*c^2)^4 \\ & *c^2)/d \end{aligned}$$

### 3.244 $\int x^4 \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx$

**Optimal.** Leaf size=208

$$\frac{c^2 x (dx - c)^{3/2} (c + dx)^{3/2} (8ad^2 + 5bc^2)}{64d^6} + \frac{x^3 (dx - c)^{3/2} (c + dx)^{3/2} (8ad^2 + 5bc^2)}{48d^4} - \frac{c^6 (8ad^2 + 5bc^2) \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{64d^7} + \frac{c^4 x \sqrt{dx-c} \sqrt{c+dx} (8ad^2 + 5bc^2)}{128d^6} + \frac{bx^5 (dx - c)^{3/2} (c + dx)^{3/2}}{8d^2}$$

[Out] (c^4\*(5\*b\*c^2 + 8\*a\*d^2)\*x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(128\*d^6) + (c^2\*(5\*b\*c^2 + 8\*a\*d^2)\*x\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2))/(64\*d^6) + ((5\*b\*c^2 + 8\*a\*d^2)\*x^3\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2))/(48\*d^4) + (b\*x^5\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2))/(8\*d^2) - (c^6\*(5\*b\*c^2 + 8\*a\*d^2)\*ArcTanh[Sqrt[-c + d\*x]/Sqrt[c + d\*x]])/(64\*d^7)

**Rubi [A]** time = 0.476242, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$

$$\frac{c^2 x (dx - c)^{3/2} (c + dx)^{3/2} (8ad^2 + 5bc^2)}{64d^6} + \frac{x^3 (dx - c)^{3/2} (c + dx)^{3/2} (8ad^2 + 5bc^2)}{48d^4} - \frac{c^6 (8ad^2 + 5bc^2) \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{64d^7} + \frac{c^4 x \sqrt{dx-c} \sqrt{c+dx} (8ad^2 + 5bc^2)}{128d^6} + \frac{bx^5 (dx - c)^{3/2} (c + dx)^{3/2}}{8d^2}$$

Antiderivative was successfully verified.

[In] Int[x^4\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2), x]

[Out] (c^4\*(5\*b\*c^2 + 8\*a\*d^2)\*x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(128\*d^6) + (c^2\*(5\*b\*c^2 + 8\*a\*d^2)\*x\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2))/(64\*d^6) + ((5\*b\*c^2 + 8\*a\*d^2)\*x^3\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2))/(48\*d^4) + (b\*x^5\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2))/(8\*d^2) - (c^6\*(5\*b\*c^2 + 8\*a\*d^2)\*ArcTanh[Sqrt[-c + d\*x]/Sqrt[c + d\*x]])/(64\*d^7)

**Rubi in Sympy [A]** time = 34.3333, size = 187, normalized size = 0.9

$$\frac{bx^5(-c + dx)^{\frac{3}{2}}(c + dx)^{\frac{3}{2}}}{8d^2} - \frac{c^6(8ad^2 + 5bc^2) \operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{64d^7} + \frac{c^4 x \sqrt{-c + dx} \sqrt{c + dx} (8ad^2 + 5bc^2)}{128d^6} + \frac{c^2 x (-c + dx)^{\frac{3}{2}} (c + dx)^{\frac{3}{2}} (8ad^2 + 5bc^2)}{64d^6} + \frac{x^3 (-c + dx)^{\frac{3}{2}} (c + dx)^{\frac{3}{2}} (8ad^2 + 5bc^2)}{48d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2), x)

[Out] b\*x\*\*5\*(-c + d\*x)\*\*(3/2)\*(c + d\*x)\*\*(3/2)/(8\*d\*\*2) - c\*\*6\*(8\*a\*d\*\*2 + 5\*b\*c\*\*2)\*atanh(sqrt(c + d\*x)/sqrt(-c + d\*x))/(64\*d\*\*7) + c\*\*4\*x\*sqrt(-c + d\*x)\*sqrt(c + d\*x)\*(8\*a\*d\*\*2 + 5\*b\*c\*\*2)/(128\*d\*\*6) + c\*\*2\*x\*(-c + d\*x)\*\*(3/2)\*(c + d\*x)\*\*(3/2)\*(8\*a\*d\*\*2 + 5\*b\*c\*\*2)/(64\*d\*\*6) + x\*\*3\*(-c + d\*x)\*\*(3/2)\*(c + d\*x)\*\*(3/2)\*(8\*a\*d\*\*2 + 5\*b\*c\*\*2)/(48\*d\*\*4)

**Mathematica [A]** time = 0.177939, size = 146, normalized size = 0.7

$$\frac{dx \sqrt{dx - c} \sqrt{c + dx} (8ad^2 (-3c^4 - 2c^2 d^2 x^2 + 8d^4 x^4) - b (15c^6 + 10c^4 d^2 x^2 + 8c^2 d^4 x^4 - 48d^6 x^6)) - 3 (8ac^6 d^2 + 5bc^8) \log\left(\sqrt{\frac{dx-c}{c+dx}}\right)}{384d^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*sqrt[-c + d\*x]\*sqrt[c + d\*x]\*(a + b\*x^2),x]

[Out] (d\*x\*sqrt[-c + d\*x]\*sqrt[c + d\*x]\*(8\*a\*d^2\*(-3\*c^4 - 2\*c^2\*d^2\*x^2 + 8\*d^4\*x^4) - b\*(15\*c^6 + 10\*c^4\*d^2\*x^2 + 8\*c^2\*d^4\*x^4 - 48\*d^6\*x^6)) - 3\*(5\*b\*c^8 + 8\*a\*c^6\*d^2)\*Log[d\*x + sqrt[-c + d\*x]\*sqrt[c + d\*x]])/(384\*d^7)

**Maple [C]** time = 0.032, size = 298, normalized size = 1.4

$$\frac{\operatorname{csgn}(d)}{384 d^7} \sqrt{dx - c} \sqrt{dx + c} \left( 48 \operatorname{csgn}(d) x^7 b d^7 \sqrt{d^2 x^2 - c^2} + 64 \operatorname{csgn}(d) x^5 a d^7 \sqrt{d^2 x^2 - c^2} - 8 \operatorname{csgn}(d) x^5 b c^2 d^5 \sqrt{d^2 x^2 - c^2} - 16 \operatorname{csgn}(d) x^3 a c^2 d^5 \sqrt{d^2 x^2 - c^2} - 16 \operatorname{csgn}(d) x^3 b c^4 d^5 \sqrt{d^2 x^2 - c^2} - 16 \operatorname{csgn}(d) x^3 a c^2 d^5 \sqrt{d^2 x^2 - c^2} - 16 \operatorname{csgn}(d) x^3 b c^4 d^5 \sqrt{d^2 x^2 - c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2),x)

[Out] 1/384\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)\*(48\*csgn(d)\*x^7\*b\*d^7\*(d^2\*x^2-c^2)^(1/2)+64\*csgn(d)\*x^5\*a\*d^7\*(d^2\*x^2-c^2)^(1/2)-8\*csgn(d)\*x^5\*b\*c^2\*d^5\*(d^2\*x^2-c^2)^(1/2)-16\*csgn(d)\*x^3\*a\*c^2\*d^5\*(d^2\*x^2-c^2)^(1/2)-10\*csgn(d)\*x^3\*b\*c^4\*d^5\*(d^2\*x^2-c^2)^(1/2)-24\*a\*c^4\*x\*(d^2\*x^2-c^2)^(1/2)\*d^3\*csgn(d)-15\*b\*c^6\*x\*(d^2\*x^2-c^2)^(1/2)\*csgn(d)\*d-24\*a\*c^6\*ln((csgn(d)\*(d^2\*x^2-c^2)^(1/2)+d\*x)\*csgn(d))\*d^2-15\*b\*c^8\*ln((csgn(d)\*(d^2\*x^2-c^2)^(1/2)+d\*x)\*csgn(d))\*csgn(d)/(d^2\*x^2-c^2)^(1/2)/d^7

**Maxima [A]** time = 1.41082, size = 356, normalized size = 1.71

$$\begin{aligned} & \frac{(d^2 x^2 - c^2)^{\frac{3}{2}} b x^5}{8 d^2} + \frac{5 (d^2 x^2 - c^2)^{\frac{3}{2}} b c^2 x^3}{48 d^4} + \frac{(d^2 x^2 - c^2)^{\frac{3}{2}} a x^3}{6 d^2} \\ & - \frac{5 b c^8 \log\left(2 d^2 x + 2 \sqrt{d^2 x^2 - c^2} \sqrt{d^2}\right)}{128 \sqrt{d^2} d^6} - \frac{a c^6 \log\left(2 d^2 x + 2 \sqrt{d^2 x^2 - c^2} \sqrt{d^2}\right)}{16 \sqrt{d^2} d^4} \\ & + \frac{5 \sqrt{d^2 x^2 - c^2} b c^6 x}{128 d^6} + \frac{\sqrt{d^2 x^2 - c^2} a c^4 x}{16 d^4} + \frac{5 (d^2 x^2 - c^2)^{\frac{3}{2}} b c^4 x}{64 d^6} + \frac{(d^2 x^2 - c^2)^{\frac{3}{2}} a c^2 x}{8 d^4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*sqrt(d\*x + c)\*sqrt(d\*x - c)\*x^4,x, algorithm="maxima")

[Out] 1/8\*(d^2\*x^2 - c^2)^(3/2)\*b\*x^5/d^2 + 5/48\*(d^2\*x^2 - c^2)^(3/2)\*b\*c^2\*x^3/d^4 + 1/6\*(d^2\*x^2 - c^2)^(3/2)\*a\*x^3/d^2 - 5/128\*b\*c^8\*log(2\*d^2\*x + 2\*sqrt(d^2\*x^2 - c^2)\*sqrt(d^2))/(sqrt(d^2)\*d^6) - 1/16\*a\*c^6\*log(2\*d^2\*x + 2\*sqrt(d^2\*x^2 - c^2)\*sqrt(d^2))/(sqrt(d^2)\*d^4) + 5/128\*sqrt(d^2\*x^2 - c^2)\*b\*c^6\*x/d^6 + 1/16\*sqrt(d^2\*x^2 - c^2)\*a\*c^4\*x/d^4 + 5/64\*(d^2\*x^2 - c^2)^(3/2)\*b\*c^4\*x/d^6 + 1/8\*(d^2\*x^2 - c^2)^(3/2)\*a\*c^2\*x/d^4

**Fricas [A]** time = 0.651568, size = 976, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*sqrt(d\*x + c)\*sqrt(d\*x - c)\*x^4,x, algorithm="fricas")

[Out] -1/384\*(6144\*b\*d^16\*x^16 - 8192\*(2\*b\*c^2\*d^14 - a\*d^16)\*x^14 + 2048\*(7\*b\*c^4\*d^12 - 11\*a\*c^2\*d^14)\*x^12 - 1024\*(5\*b\*c^6\*d^10 - 19\*



$$a^4 c^4 d^{12} x^{10} + 288 (11 b^4 c^8 d^8 - 8 a^4 c^6 d^{10}) x^8 - 192 (17 b^4 c^{10} d^6 + 24 a^4 c^8 d^8) x^6 + 248 (5 b^4 c^{12} d^4 + 8 a^4 c^{10} d^6) x^4 - 24 (5 b^4 c^{14} d^2 + 8 a^4 c^{12} d^4) x^2 - (6144 b^4 d^{15} x^15 - 1024 (13 b^4 c^2 d^{13} - 8 a^4 d^{15}) x^{13} + 768 (11 b^4 c^4 d^{11} - 24 a^4 c^2 d^{13}) x^{11} - 128 (17 b^4 c^6 d^9 - 88 a^4 c^4 d^{11}) x^9 + 48 (53 b^4 c^8 d^7 + 32 a^4 c^6 d^9) x^7 - 24 (87 b^4 c^{10} d^5 + 136 a^4 c^8 d^7) x^5 + 94 (5 b^4 c^{12} d^3 + 8 a^4 c^{10} d^5) x^3 - 3 (5 b^4 c^{14} d + 8 a^4 c^{12} d^3) x) \sqrt{d x + c} \sqrt{d x - c} - 3 (5 b^4 c^{16} + 8 a^4 c^{14} d^2 + 128 (5 b^4 c^8 d^8 + 8 a^4 c^6 d^{10}) x^8 - 256 (5 b^4 c^{10} d^6 + 8 a^4 c^8 d^8) x^6 + 160 (5 b^4 c^{12} d^4 + 8 a^4 c^{10} d^6) x^4 - 32 (5 b^4 c^{14} d^2 + 8 a^4 c^{12} d^4) x^2 - 8 (16 (5 b^4 c^8 d^7 + 8 a^4 c^6 d^9) x^7 - 24 (5 b^4 c^{10} d^5 + 8 a^4 c^8 d^7) x^5 + 10 (5 b^4 c^{12} d^3 + 8 a^4 c^{10} d^5) x^3 - (5 b^4 c^{14} d + 8 a^4 c^{12} d^3) x) \sqrt{d x + c} \sqrt{d x - c}) \log(-d x + \sqrt{d x + c} \sqrt{d x - c})) / (128 d^{15} x^8 - 256 c^2 d^{13} x^6 + 160 c^4 d^{11} x^4 - 32 c^6 d^9 x^2 + c^8 d^7 - 8 (16 d^{14} x^7 - 24 c^2 d^{12} x^5 + 10 c^4 d^{10} x^3 - c^6 d^8 x) \sqrt{d x + c} \sqrt{d x - c}))$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 (a + b x^2) \sqrt{-c + d x} \sqrt{c + d x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2),x)

[Out] Integral(x\*\*4\*(a + b\*x\*\*2)\*sqrt(-c + d\*x)\*sqrt(c + d\*x), x)

**GIAC/XCAS [A]** time = 0.306101, size = 394, normalized size = 1.89

$$8 \left( \frac{6 c^6 \ln\left(\frac{-\sqrt{d x+c}+\sqrt{d x-c}}{d^4}\right)}{d^4} + \left( \left( 2 \left( (d x+c) \left( 4 (d x+c) \left( \frac{d x+c}{d^4} - \frac{5 c}{d^4} \right) + \frac{39 c^2}{d^4} \right) - \frac{37 c^3}{d^4} \right) (d x+c) + \frac{31 c^4}{d^4} \right) (d x+c) - \frac{3 c^5}{d^4} \right) \sqrt{d x+c} \sqrt{d x-c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*sqrt(d\*x + c)\*sqrt(d\*x - c)\*x^4,x, algorithm="giac")

[Out] 1/384\*(8\*(6\*c^6\*ln(abs(-sqrt(d\*x + c) + sqrt(d\*x - c)))/d^4 + ((2\*(d\*x + c)\*(4\*(d\*x + c)\*((d\*x + c)/d^4 - 5\*c/d^4) + 39\*c^2/d^4) - 37\*c^3/d^4)\*(d\*x + c) + 31\*c^4/d^4)\*(d\*x + c) - 3\*c^5/d^4)\*sqrt(d\*x + c)\*sqrt(d\*x - c))\*a + (30\*c^8\*ln(abs(-sqrt(d\*x + c) + sqrt(d\*x - c)))/d^6 + ((2\*((4\*((d\*x + c)\*(6\*(d\*x + c)\*((d\*x + c)/d^6 - 7\*c/d^6) + 125\*c^2/d^6) - 205\*c^3/d^6)\*(d\*x + c) + 795\*c^4/d^6)\*(d\*x + c) - 449\*c^5/d^6)\*(d\*x + c) + 251\*c^6/d^6)\*(d\*x + c) - 15\*c^7/d^6)\*sqrt(d\*x + c)\*sqrt(d\*x - c))\*b)/d

$$3.245 \quad \int x^2 \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx$$

**Optimal.** Leaf size=159

$$\frac{c^2 x \sqrt{dx - c} \sqrt{c + dx} (2ad^2 + bc^2)}{16d^4} + \frac{x(dx - c)^{3/2} (c + dx)^{3/2} (2ad^2 + bc^2)}{8d^4} - \frac{c^4 (2ad^2 + bc^2) \tanh^{-1} \left( \frac{\sqrt{dx - c}}{\sqrt{c + dx}} \right)}{8d^5} + \frac{bx^3 (dx - c)^{3/2} (c + dx)^{3/2}}{6d^2}$$

[Out]  $(c^2 x \sqrt{dx - c} \sqrt{c + dx} (2ad^2 + bc^2)) / (16d^4) + ((b^2 c^2 + 2a^2 d^2) x^2 (-c + dx)^{3/2} (c + dx)^{3/2}) / (8d^4) + (b^2 x^3 (-c + dx)^{3/2} (c + dx)^{3/2}) / (6d^2) - (c^4 (b^2 c^2 + 2a^2 d^2) \operatorname{ArcTanh}[\sqrt{dx - c} / \sqrt{c + dx}]) / (8d^5)$

**Rubi [A]** time = 0.377282, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$

$$\frac{c^2 x \sqrt{dx - c} \sqrt{c + dx} (2ad^2 + bc^2)}{16d^4} + \frac{x(dx - c)^{3/2} (c + dx)^{3/2} (2ad^2 + bc^2)}{8d^4} - \frac{c^4 (2ad^2 + bc^2) \tanh^{-1} \left( \frac{\sqrt{dx - c}}{\sqrt{c + dx}} \right)}{8d^5} + \frac{bx^3 (dx - c)^{3/2} (c + dx)^{3/2}}{6d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2 \sqrt{-c + dx} \sqrt{c + dx} (a + b x^2), x]$

[Out]  $(c^2 x \sqrt{dx - c} \sqrt{c + dx} (2ad^2 + bc^2)) / (16d^4) + ((b^2 c^2 + 2a^2 d^2) x^2 (-c + dx)^{3/2} (c + dx)^{3/2}) / (8d^4) + (b^2 x^3 (-c + dx)^{3/2} (c + dx)^{3/2}) / (6d^2) - (c^4 (b^2 c^2 + 2a^2 d^2) \operatorname{ArcTanh}[\sqrt{dx - c} / \sqrt{c + dx}]) / (8d^5)$

**Rubi in Sympy [A]** time = 23.9385, size = 139, normalized size = 0.87

$$\frac{bx^3 (-c + dx)^{3/2} (c + dx)^{3/2}}{6d^2} - \frac{c^4 (2ad^2 + bc^2) \operatorname{atanh} \left( \frac{\sqrt{c + dx}}{\sqrt{-c + dx}} \right)}{8d^5} + \frac{c^2 x \sqrt{-c + dx} \sqrt{c + dx} (2ad^2 + bc^2)}{16d^4} + \frac{x (-c + dx)^{3/2} (c + dx)^{3/2} (2ad^2 + bc^2)}{8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^2 (b x^2 + a) (d x - c)^{1/2} (d x + c)^{1/2}, x)$

[Out]  $b x^3 (-c + dx)^{3/2} (c + dx)^{3/2} / (6d^2) - c^4 (2a^2 d^2 + b^2 c^2) \operatorname{atanh}(\sqrt{c + dx} / \sqrt{-c + dx}) / (8d^5) + c^2 x \sqrt{-c + dx} \sqrt{c + dx} (2ad^2 + bc^2) / (16d^4) + x^2 (-c + dx)^{3/2} (c + dx)^{3/2} (2a^2 d^2 + b^2 c^2) / (8d^4)$

**Mathematica [A]** time = 0.122253, size = 120, normalized size = 0.75

$$\frac{dx \sqrt{dx - c} \sqrt{c + dx} (b (-3c^4 - 2c^2 d^2 x^2 + 8d^4 x^4) - 6ad^2 (c^2 - 2d^2 x^2)) - 3 (2ac^4 d^2 + bc^6) \log(\sqrt{dx - c} \sqrt{c + dx} + dx)}{48d^5}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^2 \sqrt{-c + dx} \sqrt{c + dx} (a + b x^2), x]$

[Out]  $(d^2 x^2 - c^2)^{3/2} b x^3 - \frac{bc^6 \log(2d^2 x + 2\sqrt{d^2 x^2 - c^2} \sqrt{d^2})}{6d^2} - \frac{ac^4 \log(2d^2 x + 2\sqrt{d^2 x^2 - c^2} \sqrt{d^2})}{8\sqrt{d^2} d^2} + \frac{\sqrt{d^2 x^2 - c^2} bc^4 x}{16d^4} + \frac{\sqrt{d^2 x^2 - c^2} ac^2 x}{8d^2} + \frac{(d^2 x^2 - c^2)^{3/2} bc^2 x}{8d^4} + \frac{(d^2 x^2 - c^2)^{3/2} ax}{4d^2}$

**Maple [C]** time = 0.018, size = 240, normalized size = 1.5

$$\frac{\text{csgn}(d)}{48d^5} \sqrt{dx-c} \sqrt{dx+c} \left( 8 \text{csgn}(d) x^5 b d^5 \sqrt{d^2 x^2 - c^2} + 12 \text{csgn}(d) x^3 a d^5 \sqrt{d^2 x^2 - c^2} - 2 \text{csgn}(d) x^3 b c^2 d^3 \sqrt{d^2 x^2 - c^2} - 6 a c^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)`

[Out]  $\frac{1}{48} (d^2 x^2 - c^2)^{3/2} b x^3 - \frac{bc^6 \log(2d^2 x + 2\sqrt{d^2 x^2 - c^2} \sqrt{d^2})}{6d^2} - \frac{ac^4 \log(2d^2 x + 2\sqrt{d^2 x^2 - c^2} \sqrt{d^2})}{8\sqrt{d^2} d^2} + \frac{\sqrt{d^2 x^2 - c^2} bc^4 x}{16d^4} + \frac{\sqrt{d^2 x^2 - c^2} ac^2 x}{8d^2} + \frac{(d^2 x^2 - c^2)^{3/2} bc^2 x}{8d^4} + \frac{(d^2 x^2 - c^2)^{3/2} ax}{4d^2}$

**Maxima [A]** time = 1.3891, size = 284, normalized size = 1.79

$$\frac{(d^2 x^2 - c^2)^{3/2} b x^3}{6d^2} - \frac{bc^6 \log(2d^2 x + 2\sqrt{d^2 x^2 - c^2} \sqrt{d^2})}{16\sqrt{d^2} d^4} - \frac{ac^4 \log(2d^2 x + 2\sqrt{d^2 x^2 - c^2} \sqrt{d^2})}{8\sqrt{d^2} d^2} + \frac{\sqrt{d^2 x^2 - c^2} bc^4 x}{16d^4} + \frac{\sqrt{d^2 x^2 - c^2} ac^2 x}{8d^2} + \frac{(d^2 x^2 - c^2)^{3/2} bc^2 x}{8d^4} + \frac{(d^2 x^2 - c^2)^{3/2} ax}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^2,x, algorithm="maxima")`

[Out]  $\frac{1}{6} (d^2 x^2 - c^2)^{3/2} b x^3 / d^2 - \frac{1}{16} b c^6 \log(2d^2 x + 2\sqrt{d^2 x^2 - c^2} \sqrt{d^2}) / (\sqrt{d^2} d^4) - \frac{1}{8} a c^4 \log(2d^2 x + 2\sqrt{d^2 x^2 - c^2} \sqrt{d^2}) / (\sqrt{d^2} d^2) + \frac{1}{16} \sqrt{d^2 x^2 - c^2} b c^4 x / d^4 + \frac{1}{8} \sqrt{d^2 x^2 - c^2} a c^2 x / d^2 + \frac{1}{4} (d^2 x^2 - c^2)^{3/2} b c^2 x / d^4 + \frac{1}{4} (d^2 x^2 - c^2)^{3/2} a x / d^2$

**Fricas [A]** time = 0.345833, size = 741, normalized size = 4.66

$$256 b d^{12} x^{12} - 192 (3 b c^2 d^{10} - 2 a d^{12}) x^{10} + 48 (7 b c^4 d^8 - 20 a c^2 d^{10}) x^8 + 4 (17 b c^6 d^6 + 210 a c^4 d^8) x^6 - 6 (17 b c^8 d^4 + 50 a c^6 d^6)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^2,x, algorithm="fricas")`

[Out]  $-1/48 (256 b d^{12} x^{12} - 192 (3 b c^2 d^{10} - 2 a d^{12}) x^{10} + 48 (7 b c^4 d^8 - 20 a c^2 d^{10}) x^8 + 4 (17 b c^6 d^6 + 210 a c^4 d^8) x^6 - 6 (17 b c^8 d^4 + 50 a c^6 d^6) x^4 - 6 (17 b c^{10} d^2 + 2 a c^8 d^4) x^2 - (256 b d^{11} x^{11} - 64 (7 b c^2 d^9 - 6 a d^{11}) x^9 + 48 (3 b c^4 d^7 - 16 a c^2 d^9) x^7 + 4 (25 b c^6 d^5 + 12 6 a c^4 d^7) x^5 - 4 (13 b c^8 d^3 + 30 a c^6 d^5) x^3 + 3 (b c^{10} d + 2 a c^8 d^3) x) \sqrt{d x + c} \sqrt{d x - c} + 3 (b c^{12} + 2 a c^{10} d^2 - 32 (b c^6 d^6 + 2 a c^4 d^8) x^6 + 48 (b c^8 d^4 + 2 a c^6 d^6) x^4 - 18 (b c^{10} d^2 + 2 a c^8 d^4) x^2 + 2 (16 (b c^6 d^5 + 2 a c^4 d^7) x^5 - 16 (b c^8 d^3 + 2 a c^6 d^5) x^3 + 3$



### 3.246 $\int \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx$

**Optimal.** Leaf size=114

$$\frac{x\sqrt{dx-c}\sqrt{c+dx}(4ad^2+bc^2)}{8d^2} - \frac{c^2(4ad^2+bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^3} + \frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

[Out]  $((b*c^2 + 4*a*d^2)*x*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/(8*d^2) + (b*x*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(4*d^2) - (c^2*(b*c^2 + 4*a*d^2)*\text{ArcTanh}[\text{Sqrt}[-c + d*x]/\text{Sqrt}[c + d*x]])/(4*d^3)$

**Rubi [A]** time = 0.161227, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{1}{8}x\sqrt{dx-c}\sqrt{c+dx}\left(4a + \frac{bc^2}{d^2}\right) - \frac{c^2(4ad^2+bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^3} + \frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]*(a + b*x^2), x]$

[Out]  $((4*a + (b*c^2)/d^2)*x*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/8 + (b*x*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(4*d^2) - (c^2*(b*c^2 + 4*a*d^2)*\text{ArcTanh}[\text{Sqrt}[-c + d*x]/\text{Sqrt}[c + d*x]])/(4*d^3)$

**Rubi in Sympy [A]** time = 19.3635, size = 129, normalized size = 1.13

$$-\frac{ac^2 \operatorname{atanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d} + \frac{ax\sqrt{-c+dx}\sqrt{c+dx}}{2} - \frac{bc^4 \operatorname{atanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^3} + \frac{bc^2x\sqrt{-c+dx}\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{\frac{3}{2}}(c+dx)^{\frac{3}{2}}}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2), x)$

[Out]  $-a*c**2*\operatorname{atanh}(\text{sqrt}(-c + d*x)/\text{sqrt}(c + d*x))/d + a*x*\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)/2 - b*c**4*\operatorname{atanh}(\text{sqrt}(-c + d*x)/\text{sqrt}(c + d*x))/(4*d**3) + b*c**2*x*\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)/(8*d**2) + b*x*(-c + d*x)**(3/2)*(c + d*x)**(3/2)/(4*d**2)$

**Mathematica [A]** time = 0.0825454, size = 96, normalized size = 0.84

$$\frac{dx\sqrt{dx-c}\sqrt{c+dx}(4ad^2 - bc^2 + 2bd^2x^2) - (4ac^2d^2 + bc^4) \log\left(\sqrt{dx-c}\sqrt{c+dx} + dx\right)}{8d^3}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]*(a + b*x^2), x]$

[Out]  $(d*x*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]*(-(b*c^2) + 4*a*d^2 + 2*b*d^2*x^2) - (b*c^4 + 4*a*c^2*d^2)*\text{Log}[d*x + \text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]])/(8*d^3)$

**Maple [C]** time = 0.016, size = 182, normalized size = 1.6

$$\frac{\operatorname{csgn}(d)}{8d^3} \sqrt{dx-c} \sqrt{dx+c} \left( 2 \operatorname{csgn}(d) x^3 b d^3 \sqrt{d^2 x^2 - c^2} + 4 a x \sqrt{d^2 x^2 - c^2} d^3 \operatorname{csgn}(d) - b c^2 x \sqrt{d^2 x^2 - c^2} \operatorname{csgn}(d) d - 4 a c^2 \ln \left( \left( \left( \frac{d^2 x^2 - c^2}{d^2} \right)^{1/2} + d x \right) \operatorname{csgn}(d) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)`

[Out]  $1/8*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}*(2*\operatorname{csgn}(d)*x^3*b*d^3*(d^2*x^2-c^2)^{(1/2)}+4*a*x*(d^2*x^2-c^2)^{(1/2)}*d^3*\operatorname{csgn}(d)-b*c^2*x*(d^2*x^2-c^2)^{(1/2)}*\operatorname{csgn}(d)*d-4*a*c^2*\ln((\operatorname{csgn}(d)*(d^2*x^2-c^2)^{(1/2)}+d*x)*\operatorname{csgn}(d)))*\operatorname{csgn}(d)/(d^2*x^2-c^2)^{(1/2)}/d^3$

**Maxima [A]** time = 1.43454, size = 205, normalized size = 1.8

$$\frac{ac^2 \log\left(2d^2x + 2\sqrt{d^2x^2 - c^2}\sqrt{d^2}\right)}{2\sqrt{d^2}} - \frac{bc^4 \log\left(2d^2x + 2\sqrt{d^2x^2 - c^2}\sqrt{d^2}\right)}{8\sqrt{d^2}d^2} + \frac{1}{2}\sqrt{d^2x^2 - c^2}ax + \frac{\sqrt{d^2x^2 - c^2}bc^2x}{8d^2} + \frac{(d^2x^2 - c^2)^{\frac{3}{2}}bx}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c),x, algorithm="maxima")`

[Out]  $-1/2*a*c^2*\log(2*d^2*x + 2*\sqrt{d^2*x^2 - c^2}*\sqrt{d^2})/\sqrt{d^2} - 1/8*b*c^4*\log(2*d^2*x + 2*\sqrt{d^2*x^2 - c^2}*\sqrt{d^2})/(\sqrt{d^2}*d^2) + 1/2*\sqrt{d^2*x^2 - c^2}*a*x + 1/8*\sqrt{d^2*x^2 - c^2}*b*c^2*x/d^2 + 1/4*(d^2*x^2 - c^2)^{(3/2)}*b*x/d^2$

**Fricas [A]** time = 0.259909, size = 517, normalized size = 4.54

$$16bd^8x^8 - 32(bc^2d^6 - ad^8)x^6 + 4(5bc^4d^4 - 12ac^2d^6)x^4 - 4(bc^6d^2 - 4ac^4d^4)x^2 - (16bd^7x^7 - 8(3bc^2d^5 - 4ad^7)x^5 + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c),x, algorithm="fricas")`

[Out]  $-1/8*(16*b*d^8*x^8 - 32*(b*c^2*d^6 - a*d^8)*x^6 + 4*(5*b*c^4*d^4 - 12*a*c^2*d^6)*x^4 - 4*(b*c^6*d^2 - 4*a*c^4*d^4)*x^2 - (16*b*d^7*x^7 - 8*(3*b*c^2*d^5 - 4*a*d^7)*x^5 + 2*(5*b*c^4*d^3 - 16*a*c^2*d^5)*x^3 - (b*c^6*d - 4*a*c^4*d^3)*x)*\sqrt{d*x + c}*\sqrt{d*x - c} - (b*c^8 + 4*a*c^6*d^2 + 8*(b*c^4*d^4 + 4*a*c^2*d^6)*x^4 - 8*(b*c^6*d^2 + 4*a*c^4*d^4)*x^2 - 4*(2*(b*c^4*d^3 + 4*a*c^2*d^5)*x^3 - (b*c^6*d + 4*a*c^4*d^3)*x)*\sqrt{d*x + c}*\sqrt{d*x - c})*\log(-d*x + \sqrt{d*x + c}*\sqrt{d*x - c}))/ (8*d^7*x^4 - 8*c^2*d^5*x^2 + c^4*d^3 - 4*(2*d^6*x^3 - c^2*d^4*x)*\sqrt{d*x + c}*\sqrt{d*x - c})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + bx^2) \sqrt{-c + dx} \sqrt{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2),x)

[Out] Integral((a + b\*x\*\*2)\*sqrt(-c + d\*x)\*sqrt(c + d\*x), x)

---

**GIAC/XCAS [A]** time = 0.279106, size = 204, normalized size = 1.79

$$\frac{4\left(\sqrt{dx+c}\sqrt{dx-c}cdx + 2c^2\ln\left(\left|-\sqrt{dx+c} + \sqrt{dx-c}\right|\right)\right)a + \left(\frac{2c^4\ln\left(\left|-\sqrt{dx+c} + \sqrt{dx-c}\right|\right)}{d^2} + \left((dx+c)\left(2(dx+c)\left(\frac{dx+c}{d^2} - \frac{3c}{d^2}\right) + \frac{5c}{d^2}\right)\right)\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*sqrt(d\*x + c)\*sqrt(d\*x - c),x, algorithm="giac")

[Out] 1/8\*(4\*(sqrt(d\*x + c)\*sqrt(d\*x - c)\*d\*x + 2\*c^2\*ln(abs(-sqrt(d\*x + c) + sqrt(d\*x - c))))\*a + (2\*c^4\*ln(abs(-sqrt(d\*x + c) + sqrt(d\*x - c)))/d^2 + ((d\*x + c)\*(2\*(d\*x + c)\*((d\*x + c)/d^2 - 3\*c/d^2) + 5\*c^2/d^2) - c^3/d^2)\*sqrt(d\*x + c)\*sqrt(d\*x - c))\*b)/d

$$3.247 \quad \int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2} dx$$

**Optimal.** Leaf size=104

$$\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}\left(b-\frac{2ad^2}{c^2}\right) - \frac{(bc^2-2ad^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d} + \frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

[Out]  $((b - (2*a*d^2)/c^2)*x*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/2 + (a*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(c^2*x) - ((b*c^2 - 2*a*d^2)*\text{ArcTanh}[\text{Sqrt}[-c + d*x]/\text{Sqrt}[c + d*x]])/d$

**Rubi [A]** time = 0.255858, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}\left(b-\frac{2ad^2}{c^2}\right) - \frac{(bc^2-2ad^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d} + \frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2))/x^2, x]

[Out]  $((b - (2*a*d^2)/c^2)*x*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/2 + (a*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(c^2*x) - ((b*c^2 - 2*a*d^2)*\text{ArcTanh}[\text{Sqrt}[-c + d*x]/\text{Sqrt}[c + d*x]])/d$

**Rubi in Sympy [A]** time = 16.7444, size = 90, normalized size = 0.87

$$\frac{a(-c+dx)^{\frac{3}{2}}(c+dx)^{\frac{3}{2}}}{c^2x} + \frac{(2ad^2-bc^2)\text{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{d} - \frac{x\sqrt{-c+dx}\sqrt{c+dx}(2ad^2-bc^2)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2)/x\*\*2, x)

[Out]  $a*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)}/(c^2*x) + (2*a*d^2 - b*c^2)*\text{atanh}(\text{sqrt}(c + d*x)/\text{sqrt}(-c + d*x))/d - x*\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)*(2*a*d^2 - b*c^2)/(2*c^2)$

**Mathematica [A]** time = 0.107133, size = 81, normalized size = 0.78

$$\frac{(2ad^2-bc^2)\log\left(\sqrt{dx-c}\sqrt{c+dx}+dx\right)}{2d} + \left(\frac{bx}{2} - \frac{a}{x}\right)\sqrt{dx-c}\sqrt{c+dx}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2))/x^2, x]

[Out]  $((-a/x) + (b*x)/2)*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x] + (((-b*c^2) + 2*a*d^2)*\text{Log}[d*x + \text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]])/(2*d)$

**Maple [C]** time = 0.021, size = 153, normalized size = 1.5

$$\frac{\text{csgn}(d)}{2dx}\sqrt{dx-c}\sqrt{c+dx} + \left(\text{csgn}(d)x^2bd\sqrt{d^2x^2-c^2} + 2\ln\left(\left(\text{csgn}(d)\sqrt{d^2x^2-c^2}+dx\right)\text{csgn}(d)\right)\right)xad^2 - \ln\left(\left(\text{csgn}(d)\sqrt{d^2x^2-c^2}\right.\right.$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x)`

[Out]  $\frac{1}{2} (d^2 x^2 - c^2)^{1/2} (d x + c)^{1/2} (c \operatorname{sgn}(d) x^2 b d (d^2 x^2 - c^2)^{1/2} + 2 \ln((c \operatorname{sgn}(d) (d^2 x^2 - c^2)^{1/2} + d x) \operatorname{sgn}(d)) x^2 a d^2 - \ln((c \operatorname{sgn}(d) (d^2 x^2 - c^2)^{1/2} + d x) \operatorname{sgn}(d)) x^2 b c^2 - 2 a (d^2 x^2 - c^2)^{1/2} \operatorname{sgn}(d) d) \operatorname{sgn}(d) / (d^2 x^2 - c^2)^{1/2} / d / x$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.241695, size = 379, normalized size = 3.64

$$\frac{4 b d^5 x^6 - 5 b c^2 d^3 x^4 - 2 a c^4 d + (b c^4 d + 4 a c^2 d^3) x^2 - (4 b d^4 x^5 - 3 b c^2 d^2 x^3 + 4 a c^2 d^2 x) \sqrt{d x + c} \sqrt{d x - c} - (4 (b c^2 d^3 - 2 a c^2 d^2) \sqrt{d x + c} \sqrt{d x - c} - 2 (4 d^4 x^4 - 3 c^2 d^2 x^2 - (4 d^3 x^3 - 3 c^2 d x) \sqrt{d x + c} \sqrt{d x - c}))}{2 (4 d^4 x^4 - 3 c^2 d^2 x^2 - (4 d^3 x^3 - 3 c^2 d x) \sqrt{d x + c} \sqrt{d x - c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x^2,x, algorithm="fricas")`

[Out]  $-\frac{1}{2} (4 b^2 d^5 x^6 - 5 b^2 c^2 d^3 x^4 - 2 a^2 c^4 d + (b^2 c^4 d + 4 a^2 c^2 d^3) x^2 - (4 b^2 d^4 x^5 - 3 b^2 c^2 d^2 x^3 + 4 a^2 c^2 d^2 x) \operatorname{sqrt}(d x + c) \operatorname{sqrt}(d x - c) - (4 (b^2 c^2 d^3 - 2 a^2 d^5) x^4 - 3 (b^2 c^4 d - 2 a^2 c^2 d^3) x^2 - (4 (b^2 c^2 d^2 - 2 a^2 d^4) x^3 - (b^2 c^4 - 2 a^2 c^2 d^2) x) \operatorname{sqrt}(d x + c) \operatorname{sqrt}(d x - c))) \log(-d x + \operatorname{sqrt}(d x + c) \operatorname{sqrt}(d x - c)) / (4 d^4 x^4 - 3 c^2 d^2 x^2 - (4 d^3 x^3 - 3 c^2 d x) \operatorname{sqrt}(d x + c) \operatorname{sqrt}(d x - c))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b x^2) \sqrt{-c + d x} \sqrt{c + d x}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**2,x)`

[Out] `Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x**2, x)`

**GIAC/XCAS [A]** time = 0.230448, size = 149, normalized size = 1.43

$$\frac{\frac{6144 a c^2 d^2}{(\sqrt{d x + c} - \sqrt{d x - c})^4 + 4 c^2} - 2 ((d x + c) b - b c) \sqrt{d x + c} \sqrt{d x - c} - (b c^2 - 2 a d^2) \ln\left(\left(\sqrt{d x + c} - \sqrt{d x - c}\right)^4\right)}{768 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x^2,x, algorithm="giac")
```

```
[Out] -1/768*(6144*a*c^2*d^2/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2) - 2*((d*x + c)*b - b*c)*sqrt(d*x + c)*sqrt(d*x - c) - (b*c^2 - 2*a*d^2)*ln((sqrt(d*x + c) - sqrt(d*x - c))^4))/d
```

$$3.248 \quad \int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4} dx$$

Optimal. Leaf size=84

$$\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3} - \frac{b\sqrt{dx-c}\sqrt{c+dx}}{x} + 2bd \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)$$

[Out]  $-\left(\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x}\right) + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + 2bd \operatorname{ArcTanh}\left[\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right]$

Rubi [A] time = 0.259594, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$

$$\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3} - \frac{b\sqrt{dx-c}\sqrt{c+dx}}{x} + 2bd \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\left(\sqrt{-c+dx}\sqrt{c+dx}\right)(a+bx^2)/x^4, x\right]$

[Out]  $-\left(\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x}\right) + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + 2bd \operatorname{ArcTanh}\left[\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right]$

Rubi in Sympy [A] time = 15.5248, size = 70, normalized size = 0.83

$$\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + 2bd \operatorname{atanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right) - \frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}\left(\left(bx^2+a\right)\left(dx-c\right)^{1/2}\left(dx+c\right)^{1/2}/x^4, x\right)$

[Out]  $a(-c+dx)^{3/2}(c+dx)^{3/2}/(3c^2x^3) + 2bd \operatorname{atanh}\left(\sqrt{-c+dx}/\sqrt{c+dx}\right) - b\sqrt{-c+dx}\sqrt{c+dx}/x$

Mathematica [A] time = 0.162109, size = 78, normalized size = 0.93

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(a\left(\frac{d^2x^2}{c^2}-1\right)-3bx^2\right)}{3x^3} + bd \log\left(\sqrt{dx-c}\sqrt{c+dx}+dx\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}\left[\left(\sqrt{-c+dx}\sqrt{c+dx}\right)(a+bx^2)/x^4, x\right]$

[Out]  $\left(\sqrt{-c+dx}\sqrt{c+dx}\right)\left(-3bx^2+a\left(-1+\frac{d^2x^2}{c^2}\right)\right)/(3x^3) + bd \operatorname{Log}\left[d\sqrt{-c+dx}\sqrt{c+dx}\right]$

Maple [C] time = 0.022, size = 153, normalized size = 1.8

$$\frac{\operatorname{csgn}(d)\sqrt{dx-c}\sqrt{dx+c}}{3c^2x^3}\left(3 \ln\left(\left(\operatorname{csgn}(d)\sqrt{d^2x^2-c^2}+dx\right)\operatorname{csgn}(d)\right)x^3bc^2d + \operatorname{csgn}(d)x^2ad^2\sqrt{d^2x^2-c^2} - 3 \operatorname{csgn}(d)x^2bc^2\sqrt{d^2x^2-c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x^2+a)*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/x^4, x)$

[Out]  $\frac{1}{3}*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}*(3*\ln((\text{csgn}(d)*(d^2*x^2-c^2))^{(1/2)}+d*x)*\text{csgn}(d))*x^3*b*c^2*d+\text{csgn}(d)*x^2*a*d^2*(d^2*x^2-c^2)^{(1/2)}-3*\text{csgn}(d)*x^2*b*c^2*(d^2*x^2-c^2)^{(1/2)}-\text{csgn}(d)*a*c^2*(d^2*x^2-c^2)^{(1/2)})*\text{csgn}(d)/(d^2*x^2-c^2)^{(1/2)}/c^2/x^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((b*x^2+a)*\text{sqrt}(d*x+c)*\text{sqrt}(d*x-c)/x^4, x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.237663, size = 313, normalized size = 3.73

$$\frac{ac^4 - 6(bc^2d^2 - ad^4)x^4 + 3(bc^4 - 2ac^2d^2)x^2 + 3(ac^2dx + 2(bc^2d - ad^3)x^3)\sqrt{dx+c}\sqrt{dx-c} - 3(4bd^4x^6 - 3bc^2d^2x^4 - 3(4d^3x^6 - 3c^2dx^4 - (4d^2x^5 - c^2x^3)\sqrt{dx+c}\sqrt{dx-c}))}{3(4d^3x^6 - 3c^2dx^4 - (4d^2x^5 - c^2x^3)\sqrt{dx+c}\sqrt{dx-c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((b*x^2+a)*\text{sqrt}(d*x+c)*\text{sqrt}(d*x-c)/x^4, x, \text{algorithm}="fricas")$

[Out]  $\frac{1}{3}*(a*c^4 - 6*(b*c^2*d^2 - a*d^4)*x^4 + 3*(b*c^4 - 2*a*c^2*d^2)*x^2 + 3*(a*c^2*d*x + 2*(b*c^2*d - a*d^3)*x^3)*\text{sqrt}(d*x+c)*\text{sqrt}(d*x-c) - 3*(4*b*d^4*x^6 - 3*b*c^2*d^2*x^4 - (4*b*d^3*x^5 - b*c^2*d*x^3)*\text{sqrt}(d*x+c)*\text{sqrt}(d*x-c))*\log(-d*x + \text{sqrt}(d*x+c)*\text{sqrt}(d*x-c)))/(4*d^3*x^6 - 3*c^2*d*x^4 - (4*d^2*x^5 - c^2*x^3)*\text{sqrt}(d*x+c)*\text{sqrt}(d*x-c))$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: MellinTransformStripError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**4, x)$

[Out] Exception raised: MellinTransformStripError

**GIAC/XCAS [A]** time = 0.23387, size = 231, normalized size = 2.75

$$3bd^2\ln\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right) + \frac{16\left(3bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8 - 3ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8 + 24bc^4d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4 + 48bc^6d^2 - 16ac^4d^4\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4 + 4c^2\right)^3}$$

6 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*sqrt(d\*x + c)\*sqrt(d\*x - c)/x^4,x, algorithm="giac")

[Out] 
$$\frac{-1/6*(3*b*d^2*\ln((\sqrt{d*x + c}) - \sqrt{d*x - c}))^4 + 16*(3*b*c^2*d^2*(\sqrt{d*x + c}) - \sqrt{d*x - c})^8 - 3*a*d^4*(\sqrt{d*x + c}) - \sqrt{d*x - c})^8 + 24*b*c^4*d^2*(\sqrt{d*x + c}) - \sqrt{d*x - c})^4 + 48*b*c^6*d^2 - 16*a*c^4*d^4)/((\sqrt{d*x + c}) - \sqrt{d*x - c})^4 + 4*c^2)^3}{d}$$

$$3.249 \quad \int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}} dx$$

**Optimal.** Leaf size=125

$$\frac{(6ac^2 + 5b) \cosh^{-1}(cx)}{16c^7} + \frac{x\sqrt{cx-1}\sqrt{cx+1}(6ac^2 + 5b)}{16c^6} + \frac{x^3\sqrt{cx-1}\sqrt{cx+1}(6ac^2 + 5b)}{24c^4} + \frac{bx^5\sqrt{cx-1}\sqrt{cx+1}}{6c^2}$$

[Out]  $((5*b + 6*a*c^2)*x*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x])/(16*c^6) + ((5*b + 6*a*c^2)*x^3*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x])/(24*c^4) + (b*x^5*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x])/(6*c^2) + ((5*b + 6*a*c^2)*\text{ArcCosh}[c*x])/(16*c^7)$

**Rubi [A]** time = 0.33463, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{(6ac^2 + 5b) \cosh^{-1}(cx)}{16c^7} + \frac{x\sqrt{cx-1}\sqrt{cx+1}(6ac^2 + 5b)}{16c^6} + \frac{x^3\sqrt{cx-1}\sqrt{cx+1}(6ac^2 + 5b)}{24c^4} + \frac{bx^5\sqrt{cx-1}\sqrt{cx+1}}{6c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(a + b\*x^2))/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out]  $((5*b + 6*a*c^2)*x*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x])/(16*c^6) + ((5*b + 6*a*c^2)*x^3*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x])/(24*c^4) + (b*x^5*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x])/(6*c^2) + ((5*b + 6*a*c^2)*\text{ArcCosh}[c*x])/(16*c^7)$

**Rubi in Sympy [A]** time = 17.229, size = 116, normalized size = 0.93

$$\frac{bx^5\sqrt{cx-1}\sqrt{cx+1}}{6c^2} + \frac{x^3(6ac^2 + 5b)\sqrt{cx-1}\sqrt{cx+1}}{24c^4} + \frac{x(6ac^2 + 5b)\sqrt{cx-1}\sqrt{cx+1}}{16c^6} + \frac{(6ac^2 + 5b)\text{acosh}(cx)}{16c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(b\*x\*\*2+a)/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2), x)

[Out]  $b*x^5*\text{sqrt}(c*x - 1)*\text{sqrt}(c*x + 1)/(6*c^2) + x^3*(6*a*c^2 + 5*b)*\text{sqrt}(c*x - 1)*\text{sqrt}(c*x + 1)/(24*c^4) + x*(6*a*c^2 + 5*b)*\text{sqrt}(c*x - 1)*\text{sqrt}(c*x + 1)/(16*c^6) + (6*a*c^2 + 5*b)*\text{acosh}(c*x)/(16*c^7)$

**Mathematica [A]** time = 0.13668, size = 102, normalized size = 0.82

$$\frac{3(6ac^2 + 5b) \log\left(cx + \sqrt{cx-1}\sqrt{cx+1}\right) + cx\sqrt{cx-1}\sqrt{cx+1}(6ac^2(2c^2x^2 + 3) + b(8c^4x^4 + 10c^2x^2 + 15))}{48c^7}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*(a + b\*x^2))/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out]  $(c*x*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x]*(6*a*c^2*(3 + 2*c^2*x^2) + b*(15 + 10*c^2*x^2 + 8*c^4*x^4)) + 3*(5*b + 6*a*c^2)*\text{Log}[c*x + \text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x]])/(48*c^7)$

**Maple [C]** time = 0.05, size = 191, normalized size = 1.5

$$\frac{\text{csgn}(c)}{48c^7} \sqrt{cx-1}\sqrt{cx+1} \left( 8 \text{csgn}(c) x^5 b c^5 \sqrt{c^2 x^2 - 1} + 12 \text{csgn}(c) x^3 a c^5 \sqrt{c^2 x^2 - 1} + 10 b x^3 \sqrt{c^2 x^2 - 1} c^3 \text{csgn}(c) + 18 a x \sqrt{c^2 x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^4*(b*x^2+a)/(c*x-1)^{(1/2)/(c*x+1)^{(1/2)}, x)$

[Out]  $1/48*(c*x-1)^{(1/2)*(c*x+1)^{(1/2)*(8*\text{csgn}(c)*x^5*b*c^5*(c^2*x^2-1)^{(1/2)+12*\text{csgn}(c)*x^3*a*c^5*(c^2*x^2-1)^{(1/2)+10*b*x^3*(c^2*x^2-1)^{(1/2)*c^3*\text{csgn}(c)+18*a*x*(c^2*x^2-1)^{(1/2)*c^3*\text{csgn}(c)+15*b*x*(c^2*x^2-1)^{(1/2)*\text{csgn}(c)*c+18*a*\ln((\text{csgn}(c)*(c^2*x^2-1)^{(1/2)+c*x})*\text{csgn}(c))*c^2+15*b*\ln((\text{csgn}(c)*(c^2*x^2-1)^{(1/2)+c*x})*\text{csgn}(c)))*\text{csgn}(c)/(c^2*x^2-1)^{(1/2)/c^7}$

**Maxima [A]** time = 1.48597, size = 231, normalized size = 1.85

$$\frac{\sqrt{c^2x^2-1}bx^5}{6c^2} + \frac{\sqrt{c^2x^2-1}ax^3}{4c^2} + \frac{5\sqrt{c^2x^2-1}bx^3}{24c^4} + \frac{3\sqrt{c^2x^2-1}ax}{8c^4} + \frac{3a \log\left(2c^2x + 2\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{8\sqrt{c^2}c^4} + \frac{5\sqrt{c^2x^2-1}bx}{16c^6} + \frac{5b \log\left(2c^2x + 2\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{16\sqrt{c^2}c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((b*x^2 + a)*x^4/(\text{sqrt}(c*x + 1)*\text{sqrt}(c*x - 1)), x, \text{algorithm}="maxima")$

[Out]  $1/6*\text{sqrt}(c^2*x^2 - 1)*b*x^5/c^2 + 1/4*\text{sqrt}(c^2*x^2 - 1)*a*x^3/c^2 + 5/24*\text{sqrt}(c^2*x^2 - 1)*b*x^3/c^4 + 3/8*\text{sqrt}(c^2*x^2 - 1)*a*x/c^4 + 3/8*a*\log(2*c^2*x + 2*\text{sqrt}(c^2*x^2 - 1)*\text{sqrt}(c^2))/(\text{sqrt}(c^2)*c^4) + 5/16*\text{sqrt}(c^2*x^2 - 1)*b*x/c^6 + 5/16*b*\log(2*c^2*x + 2*\text{sqrt}(c^2*x^2 - 1)*\text{sqrt}(c^2))/(\text{sqrt}(c^2)*c^6)$

**Fricas [A]** time = 0.270148, size = 581, normalized size = 4.65

$$256bc^{12}x^{12} + 192(2ac^{12} - bc^{10})x^{10} - 48(4ac^{10} - 3bc^8)x^8 - 4(174ac^8 + 157bc^6)x^6 + 102(6ac^6 + 5bc^4)x^4 - 18(6ac^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((b*x^2 + a)*x^4/(\text{sqrt}(c*x + 1)*\text{sqrt}(c*x - 1)), x, \text{algorithm}="fricas")$

[Out]  $-1/48*(256*b*c^12*x^12 + 192*(2*a*c^12 - b*c^10)*x^10 - 48*(4*a*c^10 - 3*b*c^8)*x^8 - 4*(174*a*c^8 + 157*b*c^6)*x^6 + 102*(6*a*c^6 + 5*b*c^4)*x^4 - 18*(6*a*c^4 + 5*b*c^2)*x^2 - (256*b*c^11*x^11 + 144*b*c^9*x^9 + 64*(6*a*c^11 - b*c^9)*x^9 - 4*(162*a*c^7 + 137*b*c^5)*x^5 + 52*(6*a*c^5 + 5*b*c^3)*x^3 - 3*(6*a*c^3 + 5*b*c)*x)*\text{sqrt}(c*x + 1)*\text{sqrt}(c*x - 1) + 3*(32*(6*a*c^8 + 5*b*c^6)*x^6 - 48*(6*a*c^6 + 5*b*c^4)*x^4 - 6*a*c^2 + 18*(6*a*c^4 + 5*b*c^2)*x^2 - 2*(16*(6*a*c^7 + 5*b*c^5)*x^5 - 16*(6*a*c^5 + 5*b*c^3)*x^3 + 3*(6*a*c^3 + 5*b*c)*x)*\text{sqrt}(c*x + 1)*\text{sqrt}(c*x - 1) - 5*b)*\log(-c*x + \text{sqrt}(c*x + 1)*\text{sqrt}(c*x - 1)))/(32*c^13*x^6 - 48*c^11*x^4 + 18*c^9*x^2 - c^7 - 2*(16*c^12*x^5 - 16*c^10*x^3 + 3*c^8*x)*\text{sqrt}(c*x + 1)$

\*sqrt(c\*x - 1))

**Sympy [A]** time = 134.074, size = 216, normalized size = 1.73

$$\frac{aG_{6,6}^{6,2} \left( \begin{matrix} -\frac{7}{4}, -\frac{5}{4} & -\frac{3}{2}, -\frac{3}{2}, -1, 1 \\ -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 0 \end{matrix} \middle| \frac{1}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^5} - \frac{iaG_{6,6}^{2,6} \left( \begin{matrix} -\frac{5}{2}, -\frac{9}{4}, -2, -\frac{7}{4}, -\frac{3}{2}, 1 \\ -\frac{9}{4}, -\frac{7}{4} & -\frac{5}{2}, -2, -2, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^5} + \frac{bG_{6,6}^{6,2} \left( \begin{matrix} -\frac{11}{4}, -\frac{9}{4} & -\frac{5}{2}, -\frac{5}{2}, -2, 1 \\ -3, -\frac{11}{4}, -\frac{5}{2}, -\frac{9}{4}, -2, 0 \end{matrix} \middle| \frac{1}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^7} - \frac{ibG_{6,6}^{2,6} \left( \begin{matrix} -\frac{7}{2}, -\frac{13}{4}, -3, -\frac{11}{4}, -\frac{5}{2}, 1 \\ -\frac{13}{4}, -\frac{11}{4} & -\frac{7}{2}, -3, -3, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(b\*x\*\*2+a)/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2),x)

[Out] a\*meijerg((( -7/4, -5/4), (-3/2, -3/2, -1, 1)), ((-2, -7/4, -3/2, -5/4, -1, 0), ()), 1/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*5) - I\*a\*meijerg((( -5/2, -9/4, -2, -7/4, -3/2, 1), ()), ((-9/4, -7/4), (-5/2, -2, -2, 0)), exp\_polar(2\*I\*pi)/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*5) + b\*meijerg((( -11/4, -9/4), (-5/2, -5/2, -2, 1)), ((-3, -11/4, -5/2, -9/4, -2, 0), ()), 1/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*7) - I\*b\*meijerg((( -7/2, -13/4, -3, -11/4, -5/2, 1), ()), ((-13/4, -11/4), (-7/2, -3, -3, 0)), exp\_polar(2\*I\*pi)/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*7)

**GIAC/XCAS [A]** time = 0.246501, size = 205, normalized size = 1.64

$$\frac{(30 ac^{38} + 33 bc^{36} - (54 ac^{38} + 85 bc^{36} - 2(18 ac^{38} + 55 bc^{36} - (6 ac^{38} + 45 bc^{36} + 4((cx + 1)bc^{36} - 5 bc^{36})(cx + 1))(cx + 1)))(cx + 1)}{34603008 c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^4/(sqrt(c\*x + 1)\*sqrt(c\*x - 1)),x, algorithm="giac")

[Out] -1/34603008\*((30\*a\*c^38 + 33\*b\*c^36 - (54\*a\*c^38 + 85\*b\*c^36 - 2\*(18\*a\*c^38 + 55\*b\*c^36 - (6\*a\*c^38 + 45\*b\*c^36 + 4\*((c\*x + 1)\*b\*c^36 - 5\*b\*c^36)\*(c\*x + 1))\*(c\*x + 1))\*(c\*x + 1))\*sqrt(c\*x + 1)\*sqrt(c\*x - 1) + 6\*(6\*a\*c^38 + 5\*b\*c^36)\*ln(abs(-sqrt(c\*x + 1) + sqrt(c\*x - 1))))/c



$$3.250 \quad \int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}} dx$$

**Optimal.** Leaf size=103

$$\frac{2\sqrt{cx-1}\sqrt{cx+1}(5ac^2+4b)}{15c^6} + \frac{x^2\sqrt{cx-1}\sqrt{cx+1}(5ac^2+4b)}{15c^4} + \frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}$$

[Out] (2\*(4\*b + 5\*a\*c^2)\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(15\*c^6) + ((4\*b + 5\*a\*c^2)\*x^2\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(15\*c^4) + (b\*x^4\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(5\*c^2)

**Rubi [A]** time = 0.26647, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\frac{2\sqrt{cx-1}\sqrt{cx+1}(5ac^2+4b)}{15c^6} + \frac{x^2\sqrt{cx-1}\sqrt{cx+1}(5ac^2+4b)}{15c^4} + \frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*x^2))/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] (2\*(4\*b + 5\*a\*c^2)\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(15\*c^6) + ((4\*b + 5\*a\*c^2)\*x^2\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(15\*c^4) + (b\*x^4\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(5\*c^2)

**Rubi in Sympy [A]** time = 13.3902, size = 95, normalized size = 0.92

$$\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2} + \frac{x^2(5ac^2+4b)\sqrt{cx-1}\sqrt{cx+1}}{15c^4} + \frac{2(5ac^2+4b)\sqrt{cx-1}\sqrt{cx+1}}{15c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(b\*x\*\*2+a)/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2), x)

[Out] b\*x\*\*4\*sqrt(c\*x - 1)\*sqrt(c\*x + 1)/(5\*c\*\*2) + x\*\*2\*(5\*a\*c\*\*2 + 4\*b)\*sqrt(c\*x - 1)\*sqrt(c\*x + 1)/(15\*c\*\*4) + 2\*(5\*a\*c\*\*2 + 4\*b)\*sqrt(c\*x - 1)\*sqrt(c\*x + 1)/(15\*c\*\*6)

**Mathematica [A]** time = 0.0631832, size = 61, normalized size = 0.59

$$\frac{\sqrt{cx-1}\sqrt{cx+1}(5ac^2(c^2x^2+2)+b(3c^4x^4+4c^2x^2+8))}{15c^6}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*x^2))/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] (Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]\*(5\*a\*c^2\*(2 + c^2\*x^2) + b\*(8 + 4\*c^2\*x^2 + 3\*c^4\*x^4)))/(15\*c^6)

**Maple [A]** time = 0.009, size = 57, normalized size = 0.6

$$\frac{3bx^4c^4 + 5ac^4x^2 + 4bc^2x^2 + 10ac^2 + 8b}{15c^6} \sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)`

[Out]  $\frac{1}{15}*(c*x+1)^{(1/2)}*(c*x-1)^{(1/2)}*(3*b*c^4*x^4+5*a*c^4*x^2+4*b*c^2*x^2+10*a*c^2+8*b)/c^6$

**Maxima [A]** time = 1.39394, size = 128, normalized size = 1.24

$$\frac{\sqrt{c^2x^2-1}bx^4}{5c^2} + \frac{\sqrt{c^2x^2-1}ax^2}{3c^2} + \frac{4\sqrt{c^2x^2-1}bx^2}{15c^4} + \frac{2\sqrt{c^2x^2-1}a}{3c^4} + \frac{8\sqrt{c^2x^2-1}b}{15c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*x^3/(sqrt(c*x+1)*sqrt(c*x-1)),x,algorithm="maxima")`

[Out]  $\frac{1}{5}*\sqrt{c^2*x^2-1}*b*x^4/c^2 + \frac{1}{3}*\sqrt{c^2*x^2-1}*a*x^2/c^2 + \frac{4}{15}*\sqrt{c^2*x^2-1}*b*x^2/c^4 + \frac{2}{3}*\sqrt{c^2*x^2-1}*a/c^4 + \frac{8}{15}*\sqrt{c^2*x^2-1}*b/c^6$

**Fricas [A]** time = 0.253568, size = 333, normalized size = 3.23

$$\frac{48bc^{10}x^{10} + 20(4ac^{10} - bc^8)x^8 + 5(4ac^8 + 11bc^6)x^6 - 5(43ac^6 + 35bc^4)x^4 - 10ac^2 + 25(5ac^4 + 4bc^2)x^2 - (48bc^9x^9 - 15(16c^{11}x^5 - 20c^9x^3 + 5c^7x - (16c^{10}x^4 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*x^3/(sqrt(c*x+1)*sqrt(c*x-1)),x,algorithm="fricas")`

[Out]  $-\frac{1}{15}*(48*b*c^{10}*x^{10} + 20*(4*a*c^{10} - b*c^8)*x^8 + 5*(4*a*c^8 + 11*b*c^6)*x^6 - 5*(43*a*c^6 + 35*b*c^4)*x^4 - 10*a*c^2 + 25*(5*a*c^4 + 4*b*c^2)*x^2 - (48*b*c^9*x^9 + 3*(20*a*c^7 + 21*b*c^5)*x^5 - 35*(5*a*c^5 + 4*b*c^3)*x^3 + 10*(5*a*c^3 + 4*b*c)*x)*\sqrt{c*x+1}*\sqrt{c*x-1} - 8*b)/(16*c^{11}*x^5 - 20*c^9*x^3 + 5*c^7*x - (16*c^{10}*x^4 - 12*c^8*x^2 + c^6)*\sqrt{c*x+1}*\sqrt{c*x-1})$

**Sympy [A]** time = 89.8548, size = 216, normalized size = 2.1

$$\frac{aG_{6,6}^{6,2} \left( \begin{matrix} -\frac{5}{4}, -\frac{3}{4} & -1, -1, -\frac{1}{2}, 1 \\ -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0 \end{matrix} \middle| \frac{1}{c^2x^2} \right)}{4\pi^{\frac{3}{2}}c^4} + \frac{iaG_{6,6}^{2,6} \left( \begin{matrix} -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 1 \\ -\frac{7}{4}, -\frac{5}{4} & -2, -\frac{3}{2}, -\frac{3}{2}, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2} \right)}{4\pi^{\frac{3}{2}}c^4} + \frac{bG_{6,6}^{6,2} \left( \begin{matrix} -\frac{9}{4}, -\frac{7}{4} & -2, -2, -\frac{3}{2}, 1 \\ -\frac{5}{2}, -\frac{9}{4}, -2, -\frac{7}{4}, -\frac{3}{2}, 0 \end{matrix} \middle| \frac{1}{c^2x^2} \right)}{4\pi^{\frac{3}{2}}c^6} + \frac{ibG_{6,6}^{2,6} \left( \begin{matrix} -3, -\frac{11}{4}, -\frac{5}{2}, -\frac{9}{4}, -2, 1 \\ -\frac{11}{4}, -\frac{9}{4} & -3, -\frac{5}{2}, -\frac{5}{2}, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2} \right)}{4\pi^{\frac{3}{2}}c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(b\*x\*\*2+a)/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2),x)

[Out] a\*meijerg((( -5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*4) + I\*a\*meijerg((( -2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp\_polar(2\*I\*pi)/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*4) + b\*meijerg((( -9/4, -7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), 1/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*6) + I\*b\*meijerg((( -3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)), exp\_polar(2\*I\*pi)/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*6)

**GIAC/XCAS [A]** time = 0.21793, size = 130, normalized size = 1.26

$$\frac{(15ac^{27} + 15bc^{25} - (10ac^{27} + 20bc^{25} - (5ac^{27} + 22bc^{25} + 3((cx+1)bc^{25} - 4bc^{25})(cx+1))(cx+1))(cx+1))\sqrt{cx+1}\sqrt{cx}}{276480c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^3/(sqrt(c\*x + 1)\*sqrt(c\*x - 1)),x, algorithm="giac")

[Out] 1/276480\*(15\*a\*c^27 + 15\*b\*c^25 - (10\*a\*c^27 + 20\*b\*c^25 - (5\*a\*c^27 + 22\*b\*c^25 + 3\*((c\*x + 1)\*b\*c^25 - 4\*b\*c^25)\*(c\*x + 1))\*(c\*x + 1))\*sqrt(c\*x + 1)\*sqrt(c\*x - 1)/c

$$3.251 \quad \int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}} dx$$

**Optimal.** Leaf size=87

$$\frac{(4ac^2 + 3b) \cosh^{-1}(cx)}{8c^5} + \frac{x\sqrt{cx-1}\sqrt{cx+1}(4ac^2 + 3b)}{8c^4} + \frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}$$

[Out]  $((3*b + 4*a*c^2)*x*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x])/(8*c^4) + (b*x^3*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x])/(4*c^2) + ((3*b + 4*a*c^2)*\text{ArcCosh}[c*x])/(8*c^5)$

**Rubi [A]** time = 0.261869, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{(4ac^2 + 3b) \cosh^{-1}(cx)}{8c^5} + \frac{x\sqrt{cx-1}\sqrt{cx+1}(4ac^2 + 3b)}{8c^4} + \frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2*(a + b*x^2))/(\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x]), x]$

[Out]  $((3*b + 4*a*c^2)*x*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x])/(8*c^4) + (b*x^3*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x])/(4*c^2) + ((3*b + 4*a*c^2)*\text{ArcCosh}[c*x])/(8*c^5)$

**Rubi in Sympy [A]** time = 12.7528, size = 80, normalized size = 0.92

$$\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2} + \frac{x(4ac^2 + 3b)\sqrt{cx-1}\sqrt{cx+1}}{8c^4} + \frac{(4ac^2 + 3b)\text{acosh}(cx)}{8c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**2}*(b*x^{**2}+a)/(c*x-1)^{(1/2)/(c*x+1)^{(1/2)}, x)$

[Out]  $b*x^{**3}*\text{sqrt}(c*x - 1)*\text{sqrt}(c*x + 1)/(4*c^{**2}) + x*(4*a*c^{**2} + 3*b)*\text{sqrt}(c*x - 1)*\text{sqrt}(c*x + 1)/(8*c^{**4}) + (4*a*c^{**2} + 3*b)*\text{acosh}(c*x)/(8*c^{**5})$

**Mathematica [A]** time = 0.0936331, size = 83, normalized size = 0.95

$$\frac{cx\sqrt{cx-1}\sqrt{cx+1}(4ac^2 + b(2c^2x^2 + 3)) + (4ac^2 + 3b) \log\left(cx + \sqrt{cx-1}\sqrt{cx+1}\right)}{8c^5}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x^2*(a + b*x^2))/(\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x]), x]$

[Out]  $(c*x*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x]*(4*a*c^2 + b*(3 + 2*c^2*x^2)) + (3*b + 4*a*c^2)*\text{Log}[c*x + \text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x]])/(8*c^5)$

**Maple [C]** time = 0.028, size = 147, normalized size = 1.7

$$\frac{\text{csgn}(c)}{8c^5} \sqrt{cx-1}\sqrt{cx+1} \left( 2bx^3\sqrt{c^2x^2-1}c^3\text{csgn}(c) + 4ax\sqrt{c^2x^2-1}c^3\text{csgn}(c) + 3bx\sqrt{c^2x^2-1}\text{csgn}(c)c + 4a \ln\left(\left(\text{csgn}(c)\sqrt{cx-1}\sqrt{cx+1} + cx + \sqrt{cx-1}\sqrt{cx+1}\right)\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)`

[Out]  $\frac{1}{8} (c^2 x - 1)^{1/2} (c^2 x + 1)^{1/2} (2 b^2 x^3 (c^2 x^2 - 1)^{1/2} c^3 \operatorname{csgn}(c) + 4 a^2 x (c^2 x^2 - 1)^{1/2} c^3 \operatorname{csgn}(c) + 3 b^2 x (c^2 x^2 - 1)^{1/2} c \operatorname{csgn}(c) + c^4 a \ln((\operatorname{csgn}(c) (c^2 x^2 - 1)^{1/2} + c x) \operatorname{csgn}(c)) c^2 + 3 b \ln((\operatorname{csgn}(c) (c^2 x^2 - 1)^{1/2} + c x) \operatorname{csgn}(c))) \operatorname{csgn}(c) / (c^2 x^2 - 1)^{1/2} / c^5$

**Maxima [A]** time = 1.38053, size = 177, normalized size = 2.03

$$\frac{\sqrt{c^2 x^2 - 1} b x^3}{4 c^2} + \frac{\sqrt{c^2 x^2 - 1} a x}{2 c^2} + \frac{a \log\left(2 c^2 x + 2 \sqrt{c^2 x^2 - 1} \sqrt{c^2}\right)}{2 \sqrt{c^2} c^2} + \frac{3 \sqrt{c^2 x^2 - 1} b x}{8 c^4} + \frac{3 b \log\left(2 c^2 x + 2 \sqrt{c^2 x^2 - 1} \sqrt{c^2}\right)}{8 \sqrt{c^2} c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^2/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="maxima")`

[Out]  $\frac{1}{4} \sqrt{c^2 x^2 - 1} b^2 x^3 / c^2 + \frac{1}{2} \sqrt{c^2 x^2 - 1} a^2 x / c^2 + \frac{1}{2} a \log(2 c^2 x + 2 \sqrt{c^2 x^2 - 1} \sqrt{c^2}) / (\sqrt{c^2} c^2) + \frac{3}{8} \sqrt{c^2 x^2 - 1} b^2 x / c^4 + \frac{3}{8} b \log(2 c^2 x + 2 \sqrt{c^2 x^2 - 1} \sqrt{c^2}) / (\sqrt{c^2} c^4)$

**Fricas [A]** time = 0.243741, size = 406, normalized size = 4.67

$$\frac{16 b c^8 x^8 + 32 a c^8 x^6 - 4 (12 a c^6 + 7 b c^4) x^4 + 4 (4 a c^4 + 3 b c^2) x^2 - (16 b c^7 x^7 + 8 (4 a c^7 + b c^5) x^5 - 2 (16 a c^5 + 11 b c^3) x^3 + \dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^2/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="fricas")`

[Out]  $-\frac{1}{8} (16 b^2 c^8 x^8 + 32 a^2 c^8 x^6 - 4 (12 a^2 c^6 + 7 b^2 c^4) x^4 + 4 (4 a^2 c^4 + 3 b^2 c^2) x^2 - (16 b^2 c^7 x^7 + 8 (4 a^2 c^7 + b^2 c^5) x^5 - 2 (16 a^2 c^5 + 11 b^2 c^3) x^3 + (4 a^2 c^3 + 3 b^2 c) x) \sqrt{c^2 x + 1} \sqrt{c^2 x - 1} + (8 (4 a^2 c^6 + 3 b^2 c^4) x^4 + 4 a^2 c^2 - 8 (4 a^2 c^4 + 3 b^2 c^2) x^2 - 4 (2 (4 a^2 c^5 + 3 b^2 c^3) x^3 - (4 a^2 c^3 + 3 b^2 c) x) \sqrt{c^2 x + 1} \sqrt{c^2 x - 1} + 3 b^2) \log(-c^2 x + \sqrt{c^2 x + 1} \sqrt{c^2 x - 1})) / (8 c^9 x^4 - 8 c^7 x^2 + c^5 - 4 (2 c^8 x^3 - c^6 x) \sqrt{c^2 x + 1} \sqrt{c^2 x - 1})$

**Sympy [A]** time = 86.6118, size = 212, normalized size = 2.44

$$\frac{aG_{6,6}^{6,2} \left( \begin{matrix} -\frac{3}{4}, -\frac{1}{4} & -\frac{1}{2}, -\frac{1}{2}, 0, 1 \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{matrix} \middle| \frac{1}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^3} - \frac{iaG_{6,6}^{2,6} \left( \begin{matrix} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} & -\frac{3}{2}, -1, -1, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^3} + \frac{bG_{6,6}^{6,2} \left( \begin{matrix} -\frac{7}{4}, -\frac{5}{4} & -\frac{3}{2}, -\frac{3}{2}, -1, 1 \\ -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 0 \end{matrix} \middle| \frac{1}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^5} - \frac{ibG_{6,6}^{2,6} \left( \begin{matrix} -\frac{5}{2}, -\frac{9}{4}, -2, -\frac{7}{4}, -\frac{3}{2}, 1 \\ -\frac{9}{4}, -\frac{7}{4} & -\frac{5}{2}, -2, -2, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x\*\*2+a)/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2),x)

[Out] a\*meijerg(((−3/4, −1/4), (−1/2, −1/2, 0, 1)), ((−1, −3/4, −1/2, −1/4, 0, 0), ()), 1/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*3) − I\*a\*meijerg(((−3/2, −5/4, −1, −3/4, −1/2, 1), ()), ((−5/4, −3/4), (−3/2, −1, −1, 0)), exp\_polar(2\*I\*pi)/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*3) + b\*meijerg(((−7/4, −5/4), (−3/2, −3/2, −1, 1)), ((−2, −7/4, −3/2, −5/4, −1, 0), ()), 1/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*5) − I\*b\*meijerg(((−5/2, −9/4, −2, −7/4, −3/2, 1), ()), ((−9/4, −7/4), (−5/2, −2, −2, 0)), exp\_polar(2\*I\*pi)/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*5)

**GIAC/XCAS [A]** time = 0.249062, size = 151, normalized size = 1.74

$$\frac{(4ac^{18} + 5bc^{16} - (4ac^{18} + 9bc^{16} + 2((cx+1)bc^{16} - 3bc^{16})(cx+1))(cx+1))\sqrt{cx+1}\sqrt{cx-1} + 2(4ac^{18} + 3bc^{16})\ln\left(\left|-\frac{\dots}{114688c}\right.\right)}{114688c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^2/(sqrt(c\*x + 1)\*sqrt(c\*x - 1)),x, algorithm="giac")

[Out] −1/114688\*((4\*a\*c^18 + 5\*b\*c^16 − (4\*a\*c^18 + 9\*b\*c^16 + 2\*((c\*x + 1)\*b\*c^16 − 3\*b\*c^16)\*(c\*x + 1))\*(c\*x + 1))\*sqrt(c\*x + 1)\*sqrt(c\*x − 1) + 2\*(4\*a\*c^18 + 3\*b\*c^16)\*ln(abs(−sqrt(c\*x + 1) + sqrt(c\*x − 1))))/c

$$3.252 \quad \int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}} dx$$

**Optimal.** Leaf size=65

$$\frac{\sqrt{cx-1}\sqrt{cx+1}(3ac^2+2b)}{3c^4} + \frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}$$

[Out]  $((2*b + 3*a*c^2)*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x])/(3*c^4) + (b*x^2*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x])/(3*c^2)$

**Rubi [A]** time = 0.142211, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}(3ac^2+2b)}{3c^4} + \frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x*(a + b*x^2))/(\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x]), x]$

[Out]  $((2*b + 3*a*c^2)*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x])/(3*c^4) + (b*x^2*\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x])/(3*c^2)$

**Rubi in Sympy [A]** time = 8.28265, size = 58, normalized size = 0.89

$$\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2} + \frac{(3ac^2+2b)\sqrt{cx-1}\sqrt{cx+1}}{3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x*(b*x^2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2), x)$

[Out]  $b*x^2*\text{sqrt}(c*x - 1)*\text{sqrt}(c*x + 1)/(3*c^2) + (3*a*c^2 + 2*b)*\text{sqrt}(c*x - 1)*\text{sqrt}(c*x + 1)/(3*c^4)$

**Mathematica [A]** time = 0.0491001, size = 43, normalized size = 0.66

$$\frac{\sqrt{cx-1}\sqrt{cx+1}(3ac^2 + b(c^2x^2 + 2))}{3c^4}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x*(a + b*x^2))/(\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x]), x]$

[Out]  $(\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x]*(3*a*c^2 + b*(2 + c^2*x^2)))/(3*c^4)$

**Maple [A]** time = 0.006, size = 38, normalized size = 0.6

$$\frac{bx^2c^2 + 3ac^2 + 2b}{3c^4}\sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)`

[Out]  $1/3*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}*(b*c^2*x^2+3*a*c^2+2*b)/c^4$

**Maxima [A]** time = 1.41744, size = 73, normalized size = 1.12

$$\frac{\sqrt{c^2x^2-1}bx^2}{3c^2} + \frac{\sqrt{c^2x^2-1}a}{c^2} + \frac{2\sqrt{c^2x^2-1}b}{3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*x/(sqrt(c*x+1)*sqrt(c*x-1)),x,algorithm="maxima")`

[Out]  $1/3*\sqrt{c^2*x^2-1}*b*x^2/c^2 + \sqrt{c^2*x^2-1}*a/c^2 + 2/3*\sqrt{c^2*x^2-1}*b/c^4$

**Fricas [A]** time = 0.242697, size = 216, normalized size = 3.32

$$\frac{4bc^6x^6 + 3(4ac^6 + bc^4)x^4 + 3ac^2 - 3(5ac^4 + 3bc^2)x^2 - (4bc^5x^5 + (12ac^5 + 5bc^3)x^3 - 3(3ac^3 + 2bc)x)\sqrt{cx+1}\sqrt{cx-1}}{3(4c^7x^3 - 3c^5x - (4c^6x^2 - c^4)\sqrt{cx+1}\sqrt{cx-1})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*x/(sqrt(c*x+1)*sqrt(c*x-1)),x,algorithm="fricas")`

[Out]  $-1/3*(4*b*c^6*x^6 + 3*(4*a*c^6 + b*c^4)*x^4 + 3*a*c^2 - 3*(5*a*c^4 + 3*b*c^2)*x^2 - (4*b*c^5*x^5 + (12*a*c^5 + 5*b*c^3)*x^3 - 3*(3*a*c^3 + 2*b*c)*x)*\sqrt{c*x+1}\sqrt{c*x-1} + 2*b)/(4*c^7*x^3 - 3*c^5*x - (4*c^6*x^2 - c^4)*\sqrt{c*x+1}\sqrt{c*x-1})$

**Sympy [A]** time = 57.3893, size = 202, normalized size = 3.11

$$\frac{aG_{6,6}^{6,2}\left(-\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0\right) \left(0, 0, \frac{1}{2}, 1\right) \left|\frac{1}{c^2x^2}\right|}{4\pi^{\frac{3}{2}}c^2} + \frac{iaG_{6,6}^{2,6}\left(-1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1\right) \left(-\frac{3}{4}, -\frac{1}{4}\right) \left(-1, -\frac{1}{2}, -\frac{1}{2}, 0\right) \left|\frac{e^{2i\pi}}{c^2x^2}\right|}{4\pi^{\frac{3}{2}}c^2}$$

$$+ \frac{bG_{6,6}^{6,2}\left(-\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0\right) \left(-1, -1, -\frac{1}{2}, 1\right) \left|\frac{1}{c^2x^2}\right|}{4\pi^{\frac{3}{2}}c^4}$$

$$+ \frac{ibG_{6,6}^{2,6}\left(-2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 1\right) \left(-\frac{7}{4}, -\frac{5}{4}\right) \left(-2, -\frac{3}{2}, -\frac{3}{2}, 0\right) \left|\frac{e^{2i\pi}}{c^2x^2}\right|}{4\pi^{\frac{3}{2}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)`

[Out]  $a*\text{meijerg}((( -1/4, 1/4), (0, 0, 1/2, 1)), (( -1/2, -1/4, 0, 1/4, 1/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**2) + I*a*\text{meijerg}((( -1, -3/4, -1/2, -1/4, 0, 1), ()), (( -3/4, -1/4), (-1, -1/2, -1/2, 0)), \exp\_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**2) + b*\text{meijerg}((( -5/4, -3/4), (-1, -1, -1/2, 1)), (( -3/2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**4) + I*b*\text{meijerg}((( -2, -7/4, -3/2, -5/4, -1, 1), ()), (( -7/4, -5/4), (-2, -3/2, -3/2, 0)), \exp\_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**4)$



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**GIAC/XCAS [A]** time = 0.220132, size = 74, normalized size = 1.14

$$\frac{(3ac^{11} + 3bc^9 + ((cx + 1)bc^9 - 2bc^9)(cx + 1))\sqrt{cx + 1}\sqrt{cx - 1}}{1920c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x/(sqrt(c\*x + 1)\*sqrt(c\*x - 1)),x, algorithm="giac")

[Out] 1/1920\*(3\*a\*c^11 + 3\*b\*c^9 + ((c\*x + 1)\*b\*c^9 - 2\*b\*c^9)\*(c\*x + 1))\*sqrt(c\*x + 1)\*sqrt(c\*x - 1)/c

$$3.253 \quad \int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}} dx$$

**Optimal.** Leaf size=47

$$\frac{(2ac^2 + b) \cosh^{-1}(cx)}{2c^3} + \frac{bx\sqrt{cx-1}\sqrt{cx+1}}{2c^2}$$

[Out] (b\*x\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(2\*c^2) + ((b + 2\*a\*c^2)\*ArcCosh[c\*x])/(2\*c^3)

**Rubi [A]** time = 0.0904182, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{(2ac^2 + b) \cosh^{-1}(cx)}{2c^3} + \frac{bx\sqrt{cx-1}\sqrt{cx+1}}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] (b\*x\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(2\*c^2) + ((b + 2\*a\*c^2)\*ArcCosh[c\*x])/(2\*c^3)

**Rubi in Sympy [A]** time = 8.93401, size = 44, normalized size = 0.94

$$\frac{a \operatorname{acosh}(cx)}{c} + \frac{bx\sqrt{cx-1}\sqrt{cx+1}}{2c^2} + \frac{b \operatorname{acosh}(cx)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2), x)

[Out] a\*acosh(c\*x)/c + b\*x\*sqrt(c\*x - 1)\*sqrt(c\*x + 1)/(2\*c\*\*2) + b\*acosh(c\*x)/(2\*c\*\*3)

**Mathematica [A]** time = 0.0543974, size = 63, normalized size = 1.34

$$\frac{(2ac^2 + b) \log\left(cx + \sqrt{cx-1}\sqrt{cx+1}\right) + bcx\sqrt{cx-1}\sqrt{cx+1}}{2c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] (b\*c\*x\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x] + (b + 2\*a\*c^2)\*Log[c\*x + Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]])/(2\*c^3)

**Maple [C]** time = 0.023, size = 103, normalized size = 2.2

$$\frac{\operatorname{csgn}(c)}{2c^3} \sqrt{cx-1}\sqrt{cx+1} \left( bx\sqrt{c^2x^2-1} \operatorname{csgn}(c) c + 2a \ln\left(\left(\operatorname{csgn}(c)\sqrt{c^2x^2-1} + cx\right) \operatorname{csgn}(c)\right) c^2 + b \ln\left(\left(\operatorname{csgn}(c)\sqrt{c^2x^2-1} + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)`

[Out]  $\frac{1}{2} \cdot (c \cdot x - 1)^{1/2} \cdot (c \cdot x + 1)^{1/2} \cdot (b \cdot x \cdot (c^2 \cdot x^2 - 1)^{1/2} \cdot \text{csgn}(c) \cdot c + 2 \cdot a \cdot \ln((\text{csgn}(c) \cdot (c^2 \cdot x^2 - 1)^{1/2} + c \cdot x) \cdot \text{csgn}(c))) \cdot c^2 + b \cdot \ln((\text{csgn}(c) \cdot (c^2 \cdot x^2 - 1)^{1/2} + c \cdot x) \cdot \text{csgn}(c)))/(c^2 \cdot x^2 - 1)^{1/2} / c^3 \cdot \text{csgn}(c)$

**Maxima [A]** time = 1.4254, size = 120, normalized size = 2.55

$$\frac{a \log\left(2c^2x + 2\sqrt{c^2x^2 - 1}\sqrt{c^2}\right)}{\sqrt{c^2}} + \frac{\sqrt{c^2x^2 - 1}bx}{2c^2} + \frac{b \log\left(2c^2x + 2\sqrt{c^2x^2 - 1}\sqrt{c^2}\right)}{2\sqrt{c^2}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="maxima")`

[Out]  $a \cdot \log(2 \cdot c^2 \cdot x + 2 \cdot \sqrt{c^2 \cdot x^2 - 1} \cdot \sqrt{c^2}) / \sqrt{c^2} + 1/2 \cdot \sqrt{c^2 \cdot x^2 - 1} \cdot b \cdot x / c^2 + 1/2 \cdot b \cdot \log(2 \cdot c^2 \cdot x + 2 \cdot \sqrt{c^2 \cdot x^2 - 1} \cdot \sqrt{c^2}) / (\sqrt{c^2} \cdot c^2)$

**Fricas [A]** time = 0.242437, size = 220, normalized size = 4.68

$$\frac{2bc^4x^4 - 2bc^2x^2 - (2bc^3x^3 - bcx)\sqrt{cx+1}\sqrt{cx-1} - (2ac^2 + 2(2ac^3 + bc)\sqrt{cx+1}\sqrt{cx-1}x - 2(2ac^4 + bc^2)x^2 + b)\log\left(\frac{2c^5x^2 - 2\sqrt{cx+1}\sqrt{cx-1}c^4x - c^3}{2(2c^5x^2 - 2\sqrt{cx+1}\sqrt{cx-1}c^4x - c^3)}\right)}{2(2c^5x^2 - 2\sqrt{cx+1}\sqrt{cx-1}c^4x - c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="fricas")`

[Out]  $-1/2 \cdot (2 \cdot b \cdot c^4 \cdot x^4 - 2 \cdot b \cdot c^2 \cdot x^2 - (2 \cdot b \cdot c^3 \cdot x^3 - b \cdot c \cdot x) \cdot \sqrt{c \cdot x + 1} \cdot \sqrt{c \cdot x - 1} - (2 \cdot a \cdot c^2 + 2 \cdot (2 \cdot a \cdot c^3 + b \cdot c) \cdot \sqrt{c \cdot x + 1} \cdot \sqrt{c \cdot x - 1} \cdot x - 2 \cdot (2 \cdot a \cdot c^4 + b \cdot c^2) \cdot x^2 + b) \cdot \log(-c \cdot x + \sqrt{c \cdot x + 1} \cdot \sqrt{c \cdot x - 1})) / (2 \cdot c^5 \cdot x^2 - 2 \cdot \sqrt{c \cdot x + 1} \cdot \sqrt{c \cdot x - 1} \cdot c^4 \cdot x - c^3)$

**Sympy [A]** time = 37.0999, size = 182, normalized size = 3.87

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c} - \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 \\ -\frac{1}{4}, \frac{1}{4} \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3} - \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)`

[Out]  $a \cdot \text{meijerg}(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 1/(c^2 \cdot x^2)) / (4 \cdot \pi^{3/2} \cdot c) - I \cdot a \cdot \text{meijerg}((-1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), \text{exp\_pola}$

```
r(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c) + b*meijerg(((3/4, -1/4),
(-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(c**2
*x**2))/(4*pi**(3/2)*c**3) - I*b*meijerg(((3/2, -5/4, -1, -3/4,
-1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), exp_polar(2*I*pi
i)/(c**2*x**2))/(4*pi**(3/2)*c**3)
```

**GIAC/XCAS [A]** time = 0.246122, size = 96, normalized size = 2.04

$$\frac{((cx + 1)bc^4 - bc^4)\sqrt{cx + 1}\sqrt{cx - 1} - 2(2ac^6 + bc^4)\ln\left(|-\sqrt{cx + 1} + \sqrt{cx - 1}|\right)}{384c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="giac")
```

```
[Out] 1/384*(((c*x + 1)*b*c^4 - b*c^4)*sqrt(c*x + 1)*sqrt(c*x - 1) - 2*
(2*a*c^6 + b*c^4)*ln(abs(-sqrt(c*x + 1) + sqrt(c*x - 1))))/c
```

$$3.254 \quad \int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}} dx$$

**Optimal.** Leaf size=46

$$a \tan^{-1}(\sqrt{cx-1}\sqrt{cx+1}) + \frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2}$$

[Out] (b\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/c^2 + a\*ArcTan[Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]]

**Rubi [A]** time = 0.211028, antiderivative size = 46, normalized size of antiderivative = 1., number of rules used = 3, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$a \tan^{-1}(\sqrt{cx-1}\sqrt{cx+1}) + \frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] (b\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/c^2 + a\*ArcTan[Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]]

**Rubi in Sympy [A]** time = 9.71313, size = 41, normalized size = 0.89

$$a \operatorname{atan}(\sqrt{cx-1}\sqrt{cx+1}) + \frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2), x)

[Out] a\*atan(sqrt(c\*x - 1)\*sqrt(c\*x + 1)) + b\*sqrt(c\*x - 1)\*sqrt(c\*x + 1)/c\*\*2

**Mathematica [A]** time = 0.0651869, size = 47, normalized size = 1.02

$$\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2} - a \tan^{-1}\left(\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] (b\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/c^2 - a\*ArcTan[1/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])]

**Maple [A]** time = 0.026, size = 62, normalized size = 1.4

$$\frac{1}{c^2} \left( -a \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right) c^2 + b\sqrt{c^2x^2-1} \right) \sqrt{cx-1}\sqrt{cx+1} \frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)`

[Out]  $(-a \arctan(1/(c^2 x^2 - 1)^{1/2})) c^2 + b (c^2 x^2 - 1)^{1/2} (c x - 1)^{1/2} (c x + 1)^{1/2} / (c^2 x^2 - 1)^{1/2} / c^2$

**Maxima [A]** time = 1.70736, size = 42, normalized size = 0.91

$$-a \arcsin\left(\frac{1}{\sqrt{c^2|x|}}\right) + \frac{\sqrt{c^2x^2 - 1}b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x),x, algorithm="maxima")`

[Out]  $-a \arcsin(1/(\sqrt{c^2} \operatorname{abs}(x))) + \sqrt{c^2 x^2 - 1} b / c^2$

**Fricas [A]** time = 0.242114, size = 149, normalized size = 3.24

$$\frac{bc^2x^2 - \sqrt{cx+1}\sqrt{cx-1}bcx - 2(ac^3x - \sqrt{cx+1}\sqrt{cx-1}ac^2) \arctan(-cx + \sqrt{cx+1}\sqrt{cx-1}) - b}{c^3x - \sqrt{cx+1}\sqrt{cx-1}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x),x, algorithm="fricas")`

[Out]  $-(b c^2 x^2 - \sqrt{c x + 1} \sqrt{c x - 1} b c x - 2 (a c^3 x - \sqrt{c x + 1} \sqrt{c x - 1} a c^2) \arctan(-c x + \sqrt{c x + 1} \sqrt{c x - 1}) - b) / (c^3 x - \sqrt{c x + 1} \sqrt{c x - 1} c^2)$

**Sympy [A]** time = 36.4893, size = 162, normalized size = 3.52

$$\begin{aligned} & -\frac{a G_{6,6}^{5,3} \left( \frac{3}{4}, \frac{5}{4}, 1, 1, 1, \frac{3}{2} \middle| \frac{1}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}}} + \frac{ia G_{6,6}^{2,6} \left( 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1, 0, \frac{1}{2}, \frac{1}{2}, 0 \middle| \frac{e^{2i\pi}}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}}} \\ & + \frac{b G_{6,6}^{6,2} \left( -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \middle| \frac{1}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^2} + \frac{ib G_{6,6}^{2,6} \left( -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1, -\frac{3}{4}, -\frac{1}{4}, -1, -\frac{1}{2}, -\frac{1}{2}, 0 \middle| \frac{e^{2i\pi}}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)`

[Out]  $-a \operatorname{meijerg}(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1/(c^{**2} x^{**2}) / (4 \pi^{** (3/2)}) + I a \operatorname{meijerg}(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), \exp\_polar(2 * I * \pi) / (c^{**2} x^{**2}) / (4 \pi^{** (3/2)}) + b \operatorname{meijerg}((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), 1/(c^{**2} x^{**2}) / (4 \pi^{** (3/2)} c^{**2}) + I b \operatorname{meijerg}((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), \exp\_polar(2 * I * \pi) / (c^{**2} x^{**2}) / (4 \pi^{** (3/2)} c^{**2}))$

**GIAC/XCAS [A]** time = 0.218052, size = 61, normalized size = 1.33

$$-2 a \arctan\left(\frac{1}{2} (\sqrt{cx+1} - \sqrt{cx-1})^2\right) + \frac{\sqrt{cx+1}\sqrt{cx-1}b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + sqrt(c*x + 1)  
)*sqrt(c*x - 1)*b/c^2
```

$$3.255 \quad \int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}} dx$$

**Optimal.** Leaf size=33

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{x} + \frac{b \cosh^{-1}(cx)}{c}$$

[Out] (a\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/x + (b\*ArcCosh[c\*x])/c

**Rubi [A]** time = 0.183341, antiderivative size = 33, normalized size of antiderivative = 1., number of rules used = 2, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{x} + \frac{b \cosh^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^2\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] (a\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/x + (b\*ArcCosh[c\*x])/c

**Rubi in Sympy [A]** time = 8.5764, size = 27, normalized size = 0.82

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{x} + \frac{b \operatorname{acosh}(cx)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x\*\*2/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2), x)

[Out] a\*sqrt(c\*x - 1)\*sqrt(c\*x + 1)/x + b\*acosh(c\*x)/c

**Mathematica [A]** time = 0.0546701, size = 53, normalized size = 1.61

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{x} + \frac{b \log\left(cx + \sqrt{cx-1}\sqrt{cx+1}\right)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^2\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] (a\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/x + (b\*Log[c\*x + Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]])/c

**Maple [C]** time = 0.029, size = 77, normalized size = 2.3

$$\frac{\operatorname{csgn}(c)}{cx} \sqrt{cx-1}\sqrt{cx+1} \left( a\sqrt{c^2x^2-1}\operatorname{csgn}(c)c + b \ln\left(\left(\operatorname{csgn}(c)\sqrt{c^2x^2-1} + cx\right)\operatorname{csgn}(c)\right)x \right) \frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x^2/(c\*x-1)^(1/2)/(c\*x+1)^(1/2), x)



[Out]  $(c^2x-1)^{1/2} \cdot (c^2x+1)^{1/2} \cdot (a \cdot (c^2x^2-1)^{1/2} \cdot \text{csgn}(c) \cdot c + b \cdot \ln((\text{csgn}(c) \cdot (c^2x^2-1)^{1/2} + c^2x) \cdot \text{csgn}(c)) \cdot x) \cdot \text{csgn}(c) / (c^2x^2-1)^{1/2} / x / c$

**Maxima [A]** time = 1.69408, size = 68, normalized size = 2.06

$$\frac{b \log\left(2c^2x + 2\sqrt{c^2x^2 - 1}\sqrt{c^2}\right)}{\sqrt{c^2}} + \frac{\sqrt{c^2x^2 - 1}a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^2),x, algorithm="maxima")`

[Out]  $b \cdot \log(2 \cdot c^2 \cdot x + 2 \cdot \sqrt{c^2 \cdot x^2 - 1} \cdot \sqrt{c^2}) / \sqrt{c^2} + \sqrt{c^2 \cdot x^2 - 1} \cdot a / x$

**Fricas [A]** time = 0.239542, size = 109, normalized size = 3.3

$$\frac{ac - \left(bc^2x^2 - \sqrt{cx+1}\sqrt{cx-1}bx\right) \log\left(-cx + \sqrt{cx+1}\sqrt{cx-1}\right)}{c^2x^2 - \sqrt{cx+1}\sqrt{cx-1}cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^2),x, algorithm="fricas")`

[Out]  $(a \cdot c - (b \cdot c^2 \cdot x^2 - \sqrt{c \cdot x + 1} \cdot \sqrt{c \cdot x - 1} \cdot b \cdot x) \cdot \log(-c \cdot x + \sqrt{c \cdot x + 1} \cdot \sqrt{c \cdot x - 1})) / (c^2 \cdot x^2 - \sqrt{c \cdot x + 1} \cdot \sqrt{c \cdot x - 1} \cdot c \cdot x)$

**Sympy [A]** time = 42.4737, size = 148, normalized size = 4.48

$$\frac{acG_{6,6}^{5,3}\left(1, \frac{5}{4}, \frac{7}{4}, 1, \frac{3}{2}, \frac{3}{2}, 2 \left| \frac{1}{c^2x^2} \right.\right) - iacG_{6,6}^{2,6}\left(\frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1, \frac{3}{4}, \frac{5}{4} \left| \frac{e^{2i\pi}}{c^2x^2} \right.\right)}{4\pi^{\frac{3}{2}}} - \frac{bG_{6,6}^{6,2}\left(0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 \left| \frac{1}{c^2x^2} \right.\right) - ibG_{6,6}^{2,6}\left(-\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1, -\frac{1}{4}, \frac{1}{4} \left| \frac{e^{2i\pi}}{c^2x^2} \right.\right)}{4\pi^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x**2/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)`

[Out]  $-a \cdot c \cdot \text{meijerg}(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), 1/(c^2 \cdot x^2)) / (4 \cdot \pi^{3/2}) - I \cdot a \cdot c \cdot \text{meijerg}(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), \exp_{\text{polar}}(2 \cdot I \cdot \pi) / (c^2 \cdot x^2)) / (4 \cdot \pi^{3/2}) + b \cdot \text{meijerg}(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 1/(c^2 \cdot x^2)) / (4 \cdot \pi^{3/2} \cdot c) - I \cdot b \cdot \text{meijerg}((-1/2, -1/4, 0, 1/4, 1/2, 1), ((-1/4, 1/4), (-1/2, 0, 0, 0)), \exp_{\text{polar}}(2 \cdot I \cdot \pi) / (c^2 \cdot x^2)) / (4 \cdot \pi^{3/2} \cdot c)$

**GIAC/XCAS [A]** time = 0.223461, size = 78, normalized size = 2.36

$$\frac{\frac{16ac^2}{(\sqrt{cx+1}-\sqrt{cx-1})^4+4} - b \ln\left(\left(\sqrt{cx+1} - \sqrt{cx-1}\right)^4\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(16*a*c^2/((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4) - b*ln((sqrt(c*x + 1) - sqrt(c*x - 1))^4))/c
```

$$3.256 \quad \int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}} dx$$

**Optimal.** Leaf size=60

$$\frac{1}{2}(ac^2 + 2b) \tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right) + \frac{a\sqrt{cx-1}\sqrt{cx+1}}{2x^2}$$

[Out] (a\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(2\*x^2) + ((2\*b + a\*c^2)\*ArcTan[Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]])/2

**Rubi [A]** time = 0.215672, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{1}{2}(ac^2 + 2b) \tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right) + \frac{a\sqrt{cx-1}\sqrt{cx+1}}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^3\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] (a\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(2\*x^2) + ((2\*b + a\*c^2)\*ArcTan[Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]])/2

**Rubi in Sympy [A]** time = 9.67229, size = 49, normalized size = 0.82

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{2x^2} + \left(\frac{ac^2}{2} + b\right) \operatorname{atan}\left(\sqrt{cx-1}\sqrt{cx+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x\*\*3/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2), x)

[Out] a\*sqrt(c\*x - 1)\*sqrt(c\*x + 1)/(2\*x\*\*2) + (a\*c\*\*2/2 + b)\*atan(sqrt(c\*x - 1)\*sqrt(c\*x + 1))

**Mathematica [A]** time = 0.0891357, size = 61, normalized size = 1.02

$$\frac{1}{2}(-ac^2 - 2b) \tan^{-1}\left(\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right) + \frac{a\sqrt{cx-1}\sqrt{cx+1}}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^3\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] (a\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(2\*x^2) + ((-2\*b - a\*c^2)\*ArcTan[1/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])])/2

**Maple [A]** time = 0.028, size = 84, normalized size = 1.4

$$-\frac{1}{2x^2}\sqrt{cx-1}\sqrt{cx+1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)ac^2x^2 + 2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)bx^2 - a\sqrt{c^2x^2-1}\right)\frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)`

[Out] 
$$-1/2*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}*(\arctan(1/(c^2*x^2-1)^{(1/2)}))*a*c^2*x^2+2*\arctan(1/(c^2*x^2-1)^{(1/2)})*b*x^2-a*(c^2*x^2-1)^{(1/2)}/(c^2*x^2-1)^{(1/2)}/x^2$$

**Maxima [A]** time = 1.54408, size = 66, normalized size = 1.1

$$-\frac{1}{2}ac^2 \arcsin\left(\frac{1}{\sqrt{c^2|x|}}\right) - b \arcsin\left(\frac{1}{\sqrt{c^2|x|}}\right) + \frac{\sqrt{c^2x^2-1}a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^3),x, algorithm="maxima")`

[Out] 
$$-1/2*a*c^2*\arcsin(1/(\sqrt{c^2}*abs(x))) - b*\arcsin(1/(\sqrt{c^2})*abs(x)) + 1/2*\sqrt{c^2*x^2 - 1}*a/x^2$$

**Fricas [A]** time = 0.242027, size = 223, normalized size = 3.72

$$\frac{2ac^3x^3 - 2acx - (2ac^2x^2 - a)\sqrt{cx+1}\sqrt{cx-1} + 2\left(2(ac^3 + 2bc)\sqrt{cx+1}\sqrt{cx-1}x^3 - 2(ac^4 + 2bc^2)x^4 + (ac^2 + 2b)x^2\right)}{2\left(2c^2x^4 - 2\sqrt{cx+1}\sqrt{cx-1}cx^3 - x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^3),x, algorithm="fricas")`

[Out] 
$$-1/2*(2*a*c^3*x^3 - 2*a*c*x - (2*a*c^2*x^2 - a)*\sqrt{c*x + 1}*\sqrt{c*x - 1} + 2*(2*(a*c^3 + 2*b*c)*\sqrt{c*x + 1}*\sqrt{c*x - 1}*x^3 - 2*(a*c^4 + 2*b*c^2)*x^4 + (a*c^2 + 2*b)*x^2)*\arctan(-c*x + \sqrt{c*x + 1}*\sqrt{c*x - 1})/(2*c^2*x^4 - 2*\sqrt{c*x + 1}*\sqrt{c*x - 1}*c*x^3 - x^2)$$

**Sympy [A]** time = 56.1081, size = 141, normalized size = 2.35

$$\frac{ac^2G_{6,6}^{5,3}\left(\frac{7}{4}, \frac{9}{4}, 1, 2, 2, \frac{5}{2} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{iac^2G_{6,6}^{2,6}\left(1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1, \frac{5}{4}, \frac{7}{4} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{bG_{6,6}^{5,3}\left(\frac{3}{4}, \frac{5}{4}, 1, 1, 1, \frac{3}{2} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{ibG_{6,6}^{2,6}\left(0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1, \frac{1}{4}, \frac{3}{4} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x**3/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)`

[Out] 
$$-a*c**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*a*c**2*meijerg(((1, 5/4, 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2))$$

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**GIAC/XCAS [A]** time = 0.22855, size = 154, normalized size = 2.57

$$\frac{(ac^3 + 2bc) \arctan\left(\frac{1}{2}(\sqrt{cx+1} - \sqrt{cx-1})^2\right) + \frac{2\left(ac^3(\sqrt{cx+1}-\sqrt{cx-1})^6 - 4ac^3(\sqrt{cx+1}-\sqrt{cx-1})^2\right)}{\left((\sqrt{cx+1}-\sqrt{cx-1})^4 + 4\right)^2}}{c}$$


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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/(sqrt(c\*x + 1)\*sqrt(c\*x - 1)\*x^3),x, algorithm="giac")

[Out] -((a\*c^3 + 2\*b\*c)\*arctan(1/2\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^2) + 2\*(a\*c^3\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^6 - 4\*a\*c^3\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^2)/((sqrt(c\*x + 1) - sqrt(c\*x - 1))^4 + 4)^2)/c

$$3.257 \quad \int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}} dx$$

**Optimal.** Leaf size=62

$$\frac{\sqrt{cx-1}\sqrt{cx+1}(2ac^2+3b)}{3x} + \frac{a\sqrt{cx-1}\sqrt{cx+1}}{3x^3}$$

[Out] (a\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(3\*x^3) + ((3\*b + 2\*a\*c^2)\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(3\*x)

**Rubi [A]** time = 0.217287, antiderivative size = 62, normalized size of antiderivative = 1., number of rules used = 2, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}(2ac^2+3b)}{3x} + \frac{a\sqrt{cx-1}\sqrt{cx+1}}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^4\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] (a\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(3\*x^3) + ((3\*b + 2\*a\*c^2)\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(3\*x)

**Rubi in Sympy [A]** time = 9.31846, size = 51, normalized size = 0.82

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{3x^3} + \frac{\left(\frac{2ac^2}{3} + b\right)\sqrt{cx-1}\sqrt{cx+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x\*\*4/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2), x)

[Out] a\*sqrt(c\*x - 1)\*sqrt(c\*x + 1)/(3\*x\*\*3) + (2\*a\*c\*\*2/3 + b)\*sqrt(c\*x - 1)\*sqrt(c\*x + 1)/x

**Mathematica [A]** time = 0.0517534, size = 42, normalized size = 0.68

$$\frac{\sqrt{cx-1}\sqrt{cx+1}(2ac^2x^2+a+3bx^2)}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^4\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] (Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]\*(a + 3\*b\*x^2 + 2\*a\*c^2\*x^2))/(3\*x^3)

**Maple [A]** time = 0.008, size = 37, normalized size = 0.6

$$\frac{2ac^2x^2+3bx^2+a}{3x^3}\sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)`

[Out]  $1/3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(2*a*c^2*x^2+3*b*x^2+a)/x^3$

**Maxima [A]** time = 1.61251, size = 73, normalized size = 1.18

$$\frac{2\sqrt{c^2x^2-1}ac^2}{3x} + \frac{\sqrt{c^2x^2-1}b}{x} + \frac{\sqrt{c^2x^2-1}a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/(sqrt(c*x+1)*sqrt(c*x-1)*x^4),x,algorithm="maxima")`

[Out]  $2/3*\sqrt{c^2*x^2-1}*a*c^2/x + \sqrt{c^2*x^2-1}*b/x + 1/3*\sqrt{c^2*x^2-1}*a/x^3$

**Fricas [A]** time = 0.23031, size = 140, normalized size = 2.26

$$\frac{6bc^2x^4 + 3(ac^2 - b)x^2 - 3(2bcx^3 + acx)\sqrt{cx+1}\sqrt{cx-1} - a}{3(4c^3x^6 - 3cx^4 - (4c^2x^5 - x^3)\sqrt{cx+1}\sqrt{cx-1})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/(sqrt(c*x+1)*sqrt(c*x-1)*x^4),x,algorithm="fricas")`

[Out]  $1/3*(6*b*c^2*x^4 + 3*(a*c^2 - b)*x^2 - 3*(2*b*c*x^3 + a*c*x)*\sqrt{c*x + 1}*\sqrt{c*x - 1} - a)/(4*c^3*x^6 - 3*c*x^4 - (4*c^2*x^5 - x^3)*\sqrt{c*x + 1}*\sqrt{c*x - 1})$

**Sympy [A]** time = 84.0966, size = 146, normalized size = 2.35

$$\frac{ac^3G_{6,6}^{5,3}\left(\begin{matrix} \frac{9}{4}, \frac{11}{4}, 1 \\ 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}, 3 \end{matrix} \middle| \frac{1}{c^2x^2}\right) - iac^3G_{6,6}^{2,6}\left(\begin{matrix} \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}, 1 \\ \frac{7}{4}, \frac{9}{4} \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{bcG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 \\ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 \end{matrix} \middle| \frac{1}{c^2x^2}\right) - ibcG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1 \\ \frac{3}{4}, \frac{5}{4} \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x**4/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)`

[Out]  $-a*c**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 1/4, 3), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*a*c**3*meijerg(((3/2, 7/4, 2, 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) - b*c*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*b*c*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2))$

**GIAC/XCAS [A]** time = 0.224913, size = 157, normalized size = 2.53

$$\frac{8 \left( 3 b c^2 \left( \sqrt{c x + 1} - \sqrt{c x - 1} \right)^8 + 24 a c^4 \left( \sqrt{c x + 1} - \sqrt{c x - 1} \right)^4 + 24 b c^2 \left( \sqrt{c x + 1} - \sqrt{c x - 1} \right)^4 + 32 a c^4 + 48 b c^2 \right)}{3 \left( \left( \sqrt{c x + 1} - \sqrt{c x - 1} \right)^4 + 4 \right)^3 c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/(sqrt(c\*x + 1)\*sqrt(c\*x - 1)\*x^4),x, algorithm="giac")

[Out] 8/3\*(3\*b\*c^2\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^8 + 24\*a\*c^4\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^4 + 24\*b\*c^2\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^4 + 32\*a\*c^4 + 48\*b\*c^2)/(((sqrt(c\*x + 1) - sqrt(c\*x - 1))^4 + 4)^3\*c)



$$3.258 \quad \int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}} dx$$

**Optimal.** Leaf size=99

$$\frac{\sqrt{cx-1}\sqrt{cx+1}(3ac^2+4b)}{8x^2} + \frac{1}{8}c^2(3ac^2+4b)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right) + \frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}$$

[Out] (a\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(4\*x^4) + ((4\*b + 3\*a\*c^2)\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(8\*x^2) + (c^2\*(4\*b + 3\*a\*c^2)\*ArcTan[Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]])/8

**Rubi [A]** time = 0.285726, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{\sqrt{cx-1}\sqrt{cx+1}(3ac^2+4b)}{8x^2} + \frac{1}{8}c^2(3ac^2+4b)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right) + \frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^5\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] (a\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(4\*x^4) + ((4\*b + 3\*a\*c^2)\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(8\*x^2) + (c^2\*(4\*b + 3\*a\*c^2)\*ArcTan[Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]])/8

**Rubi in Sympy [A]** time = 13.7745, size = 88, normalized size = 0.89

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4} + \frac{c^2\left(\frac{3ac^2}{4} + b\right)\operatorname{atan}\left(\sqrt{cx-1}\sqrt{cx+1}\right)}{2} + \frac{\left(\frac{3ac^2}{8} + \frac{b}{2}\right)\sqrt{cx-1}\sqrt{cx+1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x\*\*5/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2), x)

[Out] a\*sqrt(c\*x - 1)\*sqrt(c\*x + 1)/(4\*x\*\*4) + c\*\*2\*(3\*a\*c\*\*2/4 + b)\*atan(sqrt(c\*x - 1)\*sqrt(c\*x + 1))/2 + (3\*a\*c\*\*2/8 + b/2)\*sqrt(c\*x - 1)\*sqrt(c\*x + 1)/x\*\*2

**Mathematica [A]** time = 0.132844, size = 80, normalized size = 0.81

$$\frac{1}{8}\left(\frac{\sqrt{cx-1}\sqrt{cx+1}(x^2(3ac^2+4b)+2a)}{x^4} - c^2(3ac^2+4b)\tan^{-1}\left(\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^5\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] ((Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]\*(2\*a + (4\*b + 3\*a\*c^2)\*x^2))/x^4 - c^2\*(4\*b + 3\*a\*c^2)\*ArcTan[1/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])])/8

**Maple [A]** time = 0.029, size = 125, normalized size = 1.3

$$-\frac{1}{8x^4}\sqrt{cx-1}\sqrt{cx+1}\left(3c^4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)ax^4 + 4c^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)bx^4 - 3\sqrt{c^2x^2-1}ac^2x^2 - 4\sqrt{c^2x^2-1}bx^2 - \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)`

[Out] 
$$-1/8*(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}*(3*c^4*\arctan(1/(c^2*x^2-1))^{(1/2)})*a*x^4+4*c^2*\arctan(1/(c^2*x^2-1))^{(1/2)}*b*x^4-3*(c^2*x^2-1)^{(1/2)}*a*c^2*x^2-4*(c^2*x^2-1)^{(1/2)}*b*x^2-2*a*(c^2*x^2-1)^{(1/2)})/(c^2*x^2-1)^{(1/2)}/x^4$$

**Maxima [A]** time = 1.58707, size = 120, normalized size = 1.21

$$-\frac{3}{8}ac^4 \arcsin\left(\frac{1}{\sqrt{c^2|x|}}\right) - \frac{1}{2}bc^2 \arcsin\left(\frac{1}{\sqrt{c^2|x|}}\right) + \frac{3\sqrt{c^2x^2-1}ac^2}{8x^2} + \frac{\sqrt{c^2x^2-1}b}{2x^2} + \frac{\sqrt{c^2x^2-1}a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^5),x, algorithm="maxima")`

[Out] 
$$-3/8*a*c^4*\arcsin(1/(\sqrt{c^2}*abs(x))) - 1/2*b*c^2*\arcsin(1/(\sqrt{c^2}*abs(x))) + 3/8*\sqrt{c^2*x^2 - 1}*a*c^2/x^2 + 1/2*\sqrt{c^2*x^2 - 1}*b/x^2 + 1/4*\sqrt{c^2*x^2 - 1}*a/x^4$$

**Fricas [A]** time = 0.244998, size = 423, normalized size = 4.27

$$8(3ac^7 + 4bc^5)x^7 - 4(5ac^5 + 12bc^3)x^5 - 4(3ac^3 - 4bc)x^3 + 8acx - (8(3ac^6 + 4bc^4)x^6 - 8(ac^4 + 4bc^2)x^4 - (13ac^2 - 4b^2)x^2 + a^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^5),x, algorithm="fricas")`

[Out] 
$$-1/8*(8*(3*a*c^7 + 4*b*c^5)*x^7 - 4*(5*a*c^5 + 12*b*c^3)*x^5 - 4*(3*a*c^3 - 4*b*c)*x^3 + 8*a*c*x - (8*(3*a*c^6 + 4*b*c^4)*x^6 - 8*(a*c^4 + 4*b*c^2)*x^4 - (13*a*c^2 - 4*b^2)*x^2 + 2*a)*\sqrt{c*x + 1}*\sqrt{c*x - 1} - 2*(8*(3*a*c^8 + 4*b*c^6)*x^8 - 8*(3*a*c^6 + 4*b*c^4)*x^6 + (3*a*c^4 + 4*b*c^2)*x^4 - 4*(2*(3*a*c^7 + 4*b*c^5)*x^7 - (3*a*c^5 + 4*b*c^3)*x^5)*\sqrt{c*x + 1}*\sqrt{c*x - 1})*\arctan(-c*x + \sqrt{c*x + 1}*\sqrt{c*x - 1})/(8*c^4*x^8 - 8*c^2*x^6 + x^4 - 4*(2*c^3*x^7 - c*x^5)*\sqrt{c*x + 1}*\sqrt{c*x - 1})$$

**Sympy [A]** time = 124.583, size = 148, normalized size = 1.49

$$\frac{ac^4 G_{6,6}^{5,3}\left(\frac{11}{4}, \frac{13}{4}, 1, 3, 3, \frac{7}{2} \middle| \frac{1}{c^2 x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{iac^4 G_{6,6}^{2,6}\left(2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}, 3, 1, \frac{9}{4}, \frac{11}{4} \middle| \frac{e^{2i\pi}}{c^2 x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{bc^2 G_{6,6}^{5,3}\left(\frac{7}{4}, \frac{9}{4}, 1, 2, 2, \frac{5}{2} \middle| \frac{1}{c^2 x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{ibc^2 G_{6,6}^{2,6}\left(1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1, \frac{5}{4}, \frac{7}{4} \middle| \frac{e^{2i\pi}}{c^2 x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x**5/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)`

[Out] 
$$-a*c**4*meijerg(((11/4, 13/4, 1), (3, 3, 7/2)), ((5/2, 11/4, 3, 13/4, 7/2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*a*c**4*meijerg$$

```
((2, 9/4, 5/2, 11/4, 3, 1), ()), ((9/4, 11/4), (2, 5/2, 5/2, 0))
, exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) - b*c**2*meijerg((
(7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,)), 1/(
c**2*x**2))/(4*pi**(3/2)) + I*b*c**2*meijerg(((1, 5/4, 3/2, 7/4,
2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi)/(c*
**2*x**2))/(4*pi**(3/2))
```

**GIAC/XCAS [A]** time = 0.229307, size = 362, normalized size = 3.66

$$(3ac^5 + 4bc^3) \arctan\left(\frac{1}{2}(\sqrt{cx+1} - \sqrt{cx-1})\right) + \frac{2\left(3ac^5(\sqrt{cx+1}-\sqrt{cx-1})^{14} + 4bc^3(\sqrt{cx+1}-\sqrt{cx-1})^{14} + 44ac^5(\sqrt{cx+1}-\sqrt{cx-1})^{10} + 16bc^3(\sqrt{cx+1}-\sqrt{cx-1})^{10}\right)}{4c}$$

4c

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/(sqrt(c\*x + 1)\*sqrt(c\*x - 1)\*x^5),x, algorithm="giac")

[Out] -1/4\*((3\*a\*c^5 + 4\*b\*c^3)\*arctan(1/2\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^2) + 2\*(3\*a\*c^5\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^14 + 4\*b\*c^3\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^14 + 44\*a\*c^5\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^10 + 16\*b\*c^3\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^10 - 176\*a\*c^5\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^6 - 64\*b\*c^3\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^6 - 192\*a\*c^5\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^2 - 256\*b\*c^3\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^2)/((sqrt(c\*x + 1) - sqrt(c\*x - 1))^4 + 4)^4)/c

$$3.259 \quad \int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}} dx$$

**Optimal.** Leaf size=164

$$\frac{c^2x\sqrt{dx-c}\sqrt{c+dx}(6ad^2+5bc^2)}{16d^6} + \frac{x^3\sqrt{dx-c}\sqrt{c+dx}(6ad^2+5bc^2)}{24d^4} \\ + \frac{c^4(6ad^2+5bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^7} + \frac{bx^5\sqrt{dx-c}\sqrt{c+dx}}{6d^2}$$

[Out] (c^2\*(5\*b\*c^2 + 6\*a\*d^2)\*x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(16\*d^6) + ((5\*b\*c^2 + 6\*a\*d^2)\*x^3\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(24\*d^4) + (b\*x^5\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(6\*d^2) + (c^4\*(5\*b\*c^2 + 6\*a\*d^2)\*ArcTanh[Sqrt[-c + d\*x]/Sqrt[c + d\*x]])/(8\*d^7)

**Rubi [A]** time = 0.422232, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$

$$\frac{c^2x\sqrt{dx-c}\sqrt{c+dx}(6ad^2+5bc^2)}{16d^6} + \frac{x^3\sqrt{dx-c}\sqrt{c+dx}(6ad^2+5bc^2)}{24d^4} \\ + \frac{c^4(6ad^2+5bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^7} + \frac{bx^5\sqrt{dx-c}\sqrt{c+dx}}{6d^2}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(a + b\*x^2))/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out] (c^2\*(5\*b\*c^2 + 6\*a\*d^2)\*x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(16\*d^6) + ((5\*b\*c^2 + 6\*a\*d^2)\*x^3\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(24\*d^4) + (b\*x^5\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(6\*d^2) + (c^4\*(5\*b\*c^2 + 6\*a\*d^2)\*ArcTanh[Sqrt[-c + d\*x]/Sqrt[c + d\*x]])/(8\*d^7)

**Rubi in Sympy [A]** time = 26.1904, size = 146, normalized size = 0.89

$$\frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} + \frac{c^4(6ad^2+5bc^2)\operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{8d^7} \\ + \frac{c^2x\sqrt{-c+dx}\sqrt{c+dx}(6ad^2+5bc^2)}{16d^6} + \frac{x^3\sqrt{-c+dx}\sqrt{c+dx}(6ad^2+5bc^2)}{24d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(b\*x\*\*2+a)/(d\*x-c)\*\*(1/2)/(d\*x+c)\*\*(1/2),x)

[Out] b\*x\*\*5\*sqrt(-c + d\*x)\*sqrt(c + d\*x)/(6\*d\*\*2) + c\*\*4\*(6\*a\*d\*\*2 + 5\*b\*c\*\*2)\*atanh(sqrt(c + d\*x)/sqrt(-c + d\*x))/(8\*d\*\*7) + c\*\*2\*x\*sqrt(-c + d\*x)\*sqrt(c + d\*x)\*(6\*a\*d\*\*2 + 5\*b\*c\*\*2)/(16\*d\*\*6) + x\*\*3\*sqrt(-c + d\*x)\*sqrt(c + d\*x)\*(6\*a\*d\*\*2 + 5\*b\*c\*\*2)/(24\*d\*\*4)

**Mathematica [A]** time = 0.170729, size = 123, normalized size = 0.75

$$\frac{3(6ac^4d^2 + 5bc^6)\log\left(\sqrt{dx-c}\sqrt{c+dx} + dx\right) + dx\sqrt{dx-c}\sqrt{c+dx}(6ad^2(3c^2 + 2d^2x^2) + b(15c^4 + 10c^2d^2x^2 + 8d^4x^4))}{48d^7}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*(a + b\*x^2))/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out] (d\*x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(6\*a\*d^2\*(3\*c^2 + 2\*d^2\*x^2) + b\*(15\*c^4 + 10\*c^2\*d^2\*x^2 + 8\*d^4\*x^4)) + 3\*(5\*b\*c^6 + 6\*a\*c^4\*d^2)\*Log[d\*x + Sqrt[-c + d\*x]\*Sqrt[c + d\*x]])/(48\*d^7)

**Maple [C]** time = 0.04, size = 240, normalized size = 1.5

$$\frac{\operatorname{csgn}(d)}{48 d^7} \sqrt{dx - c} \sqrt{dx + c} \left( 8 \operatorname{csgn}(d) x^5 b d^5 \sqrt{d^2 x^2 - c^2} + 12 \operatorname{csgn}(d) x^3 a d^5 \sqrt{d^2 x^2 - c^2} + 10 \operatorname{csgn}(d) x^3 b c^2 d^3 \sqrt{d^2 x^2 - c^2} + 18 a \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x)

[Out] 1/48\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)\*(8\*csgn(d)\*x^5\*b\*d^5\*(d^2\*x^2-c^2)^(1/2)+12\*csgn(d)\*x^3\*a\*d^5\*(d^2\*x^2-c^2)^(1/2)+10\*csgn(d)\*x^3\*b\*c^2\*d^3\*(d^2\*x^2-c^2)^(1/2)+18\*a\*c^2\*x\*(d^2\*x^2-c^2)^(1/2)\*d^3\*csgn(d)+15\*b\*c^4\*x\*(d^2\*x^2-c^2)^(1/2)\*csgn(d)\*d+18\*a\*c^4\*ln((csgn(d)\*(d^2\*x^2-c^2)^(1/2)+d\*x)\*csgn(d))\*d^2+15\*b\*c^6\*ln((csgn(d)\*(d^2\*x^2-c^2)^(1/2)+d\*x)\*csgn(d)))\*csgn(d)/(d^2\*x^2-c^2)^(1/2)/d^7

**Maxima [A]** time = 1.39756, size = 289, normalized size = 1.76

$$\frac{\sqrt{d^2 x^2 - c^2} b x^5}{6 d^2} + \frac{5 \sqrt{d^2 x^2 - c^2} b c^2 x^3}{24 d^4} + \frac{\sqrt{d^2 x^2 - c^2} a x^3}{4 d^2} + \frac{5 b c^6 \log\left(2 d^2 x + 2 \sqrt{d^2 x^2 - c^2} \sqrt{d^2}\right)}{16 \sqrt{d^2} d^6} + \frac{3 a c^4 \log\left(2 d^2 x + 2 \sqrt{d^2 x^2 - c^2} \sqrt{d^2}\right)}{8 \sqrt{d^2} d^4} + \frac{5 \sqrt{d^2 x^2 - c^2} b c^4 x}{16 d^6} + \frac{3 \sqrt{d^2 x^2 - c^2} a c^2 x}{8 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^4/(sqrt(d\*x + c)\*sqrt(d\*x - c)),x, algorithm="maxima")

[Out] 1/6\*sqrt(d^2\*x^2 - c^2)\*b\*x^5/d^2 + 5/24\*sqrt(d^2\*x^2 - c^2)\*b\*c^2\*x^3/d^4 + 1/4\*sqrt(d^2\*x^2 - c^2)\*a\*x^3/d^2 + 5/16\*b\*c^6\*log(2\*d^2\*x + 2\*sqrt(d^2\*x^2 - c^2)\*sqrt(d^2))/(sqrt(d^2)\*d^6) + 3/8\*a\*c^4\*log(2\*d^2\*x + 2\*sqrt(d^2\*x^2 - c^2)\*sqrt(d^2))/(sqrt(d^2)\*d^4) + 5/16\*sqrt(d^2\*x^2 - c^2)\*b\*c^4\*x/d^6 + 3/8\*sqrt(d^2\*x^2 - c^2)\*a\*c^2\*x/d^4

**Fricas [A]** time = 0.361266, size = 734, normalized size = 4.48

$$256 b d^{12} x^{12} - 192 (b c^2 d^{10} - 2 a d^{12}) x^{10} + 48 (3 b c^4 d^8 - 4 a c^2 d^{10}) x^8 - 4 (157 b c^6 d^6 + 174 a c^4 d^8) x^6 + 102 (5 b c^8 d^4 + 6 a c^6 d^6)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^4/(sqrt(d\*x + c)\*sqrt(d\*x - c)),x, algorithm="fricas")

[Out] -1/48\*(256\*b\*d^12\*x^12 - 192\*(b\*c^2\*d^10 - 2\*a\*d^12)\*x^10 + 48\*(3\*b\*c^4\*d^8 - 4\*a\*c^2\*d^10)\*x^8 - 4\*(157\*b\*c^6\*d^6 + 174\*a\*c^4\*d^8)\*x^6 + 102\*(5\*b\*c^8\*d^4 + 6\*a\*c^6\*d^6)\*x^4 - 18\*(5\*b\*c^10\*d^2 + 6\*a\*c^8\*d^4)\*x^2 - (256\*b\*d^11\*x^11 + 144\*b\*c^4\*d^7\*x^7 - 64\*(b\*c^2\*d^9 - 6\*a\*d^11)\*x^9 - 4\*(137\*b\*c^6\*d^5 + 162\*a\*c^4\*d^7)\*x^5 + 52\*(5\*b\*c^8\*d^3 + 6\*a\*c^6\*d^5)\*x^3 - 3\*(5\*b\*c^10\*d + 6\*a\*c^8\*d^3)\*x)\*sqrt(d\*x + c)\*sqrt(d\*x - c) - 3\*(5\*b\*c^12 + 6\*a\*c^10\*d^2 - 32

$$\frac{(5b^5c^6d^6 + 6a^5c^4d^8)x^6 + 48(5b^5c^8d^4 + 6a^5c^6d^6)x^4 - 18(5b^5c^{10}d^2 + 6a^5c^8d^4)x^2 + 2(16(5b^5c^6d^5 + 6a^5c^4d^7)x^5 - 16(5b^5c^8d^3 + 6a^5c^6d^5)x^3 + 3(5b^5c^{10}d + 6a^5c^8d^3)x)\sqrt{dx+c}\sqrt{dx-c}\log(-dx+\sqrt{dx+c}\sqrt{dx-c})}{(32d^{13}x^6 - 48c^2d^{11}x^4 + 18c^4d^9x^2 - c^6d^7 - 2(16d^{12}x^5 - 16c^2d^{10}x^3 + 3c^4d^8x)\sqrt{dx+c}\sqrt{dx-c})}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(b\*x\*\*2+a)/(d\*x-c)\*\*(1/2)/(d\*x+c)\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.258263, size = 246, normalized size = 1.5

$$\frac{(33bc^5d^{36} + 30ac^3d^{38} - (85bc^4d^{36} + 54ac^2d^{38} - 2(55bc^3d^{36} + 18acd^{38} - (45bc^2d^{36} + 6ad^{38} + 4((dx+c)bd^{36} - 5bcd^{36})))\sqrt{dx+c}\sqrt{dx-c} + 6(5b^5c^6d^{36} + 6a^5c^4d^{38})\ln(\text{abs}(-\sqrt{dx+c} + \sqrt{dx-c})))}{34603008d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^4/(sqrt(d\*x + c)\*sqrt(d\*x - c)),x, algorithm="giac")

[Out] -1/34603008\*((33\*b\*c^5\*d^36 + 30\*a\*c^3\*d^38 - (85\*b\*c^4\*d^36 + 54\*a\*c^2\*d^38 - 2\*(55\*b\*c^3\*d^36 + 18\*a\*c\*d^38 - (45\*b\*c^2\*d^36 + 6\*a\*d^38 + 4\*((d\*x + c)\*b\*d^36 - 5\*b\*c\*d^36)\*(d\*x + c))\*(d\*x + c))\*sqrt(d\*x + c)\*sqrt(d\*x - c) + 6\*(5\*b\*c^6\*d^36 + 6\*a\*c^4\*d^38)\*ln(abs(-sqrt(d\*x + c) + sqrt(d\*x - c))))/d

$$3.260 \quad \int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}} dx$$

**Optimal.** Leaf size=118

$$\frac{2c^2\sqrt{dx-c}\sqrt{c+dx}(5ad^2+4bc^2)}{15d^6} + \frac{x^2\sqrt{dx-c}\sqrt{c+dx}(5ad^2+4bc^2)}{15d^4} + \frac{bx^4\sqrt{dx-c}\sqrt{c+dx}}{5d^2}$$

[Out] (2\*c^2\*(4\*b\*c^2 + 5\*a\*d^2)\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(15\*d^6) + ((4\*b\*c^2 + 5\*a\*d^2)\*x^2\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(15\*d^4) + (b\*x^4\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(5\*d^2)

**Rubi [A]** time = 0.305834, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$\frac{2c^2\sqrt{dx-c}\sqrt{c+dx}(5ad^2+4bc^2)}{15d^6} + \frac{x^2\sqrt{dx-c}\sqrt{c+dx}(5ad^2+4bc^2)}{15d^4} + \frac{bx^4\sqrt{dx-c}\sqrt{c+dx}}{5d^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*x^2))/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]), x]

[Out] (2\*c^2\*(4\*b\*c^2 + 5\*a\*d^2)\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(15\*d^6) + ((4\*b\*c^2 + 5\*a\*d^2)\*x^2\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(15\*d^4) + (b\*x^4\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(5\*d^2)

**Rubi in Sympy [A]** time = 17.574, size = 105, normalized size = 0.89

$$\frac{bx^4\sqrt{-c+dx}\sqrt{c+dx}}{5d^2} + \frac{2c^2\sqrt{-c+dx}\sqrt{c+dx}(5ad^2+4bc^2)}{15d^6} + \frac{x^2\sqrt{-c+dx}\sqrt{c+dx}(5ad^2+4bc^2)}{15d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(b\*x\*\*2+a)/(d\*x-c)\*\*(1/2)/(d\*x+c)\*\*(1/2), x)

[Out] b\*x\*\*4\*sqrt(-c + d\*x)\*sqrt(c + d\*x)/(5\*d\*\*2) + 2\*c\*\*2\*sqrt(-c + d\*x)\*sqrt(c + d\*x)\*(5\*a\*d\*\*2 + 4\*b\*c\*\*2)/(15\*d\*\*6) + x\*\*2\*sqrt(-c + d\*x)\*sqrt(c + d\*x)\*(5\*a\*d\*\*2 + 4\*b\*c\*\*2)/(15\*d\*\*4)

**Mathematica [A]** time = 0.079503, size = 74, normalized size = 0.63

$$\frac{\sqrt{dx-c}\sqrt{c+dx}(5ad^2(2c^2+d^2x^2)+b(8c^4+4c^2d^2x^2+3d^4x^4))}{15d^6}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*x^2))/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]), x]

[Out] (Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(5\*a\*d^2\*(2\*c^2 + d^2\*x^2) + b\*(8\*c^4 + 4\*c^2\*d^2\*x^2 + 3\*d^4\*x^4)))/(15\*d^6)

**Maple [A]** time = 0.009, size = 68, normalized size = 0.6

$$\frac{3bd^4x^4 + 5ad^4x^2 + 4bc^2d^2x^2 + 10ac^2d^2 + 8bc^4}{15d^6}\sqrt{dx+c}\sqrt{dx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)`

[Out]  $\frac{1}{15} \cdot (d \cdot x + c)^{\frac{1}{2}} \cdot (3 \cdot b \cdot d^4 \cdot x^4 + 5 \cdot a \cdot d^4 \cdot x^2 + 4 \cdot b \cdot c^2 \cdot d^2 \cdot x^2 + 10 \cdot a \cdot c^2 \cdot d^2 + 8 \cdot b \cdot c^4) / d^6 \cdot (d \cdot x - c)^{\frac{1}{2}}$

**Maxima [A]** time = 1.37497, size = 167, normalized size = 1.42

$$\frac{\sqrt{d^2x^2 - c^2}bx^4}{5d^2} + \frac{4\sqrt{d^2x^2 - c^2}bc^2x^2}{15d^4} + \frac{\sqrt{d^2x^2 - c^2}ax^2}{3d^2} + \frac{8\sqrt{d^2x^2 - c^2}bc^4}{15d^6} + \frac{2\sqrt{d^2x^2 - c^2}ac^2}{3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^3/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="maxima")`

[Out]  $\frac{1}{5} \cdot \sqrt{d^2 \cdot x^2 - c^2} \cdot b \cdot x^4 / d^2 + \frac{4}{15} \cdot \sqrt{d^2 \cdot x^2 - c^2} \cdot b \cdot c^4 \cdot x^2 / d^4 + \frac{1}{3} \cdot \sqrt{d^2 \cdot x^2 - c^2} \cdot a \cdot x^2 / d^2 + \frac{8}{15} \cdot \sqrt{d^2 \cdot x^2 - c^2} \cdot b \cdot c^4 / d^6 + \frac{2}{3} \cdot \sqrt{d^2 \cdot x^2 - c^2} \cdot a \cdot c^2 / d^4$

**Fricas [A]** time = 0.2754, size = 420, normalized size = 3.56

$$\frac{48bd^{10}x^{10} - 8bc^{10} - 10ac^8d^2 - 20(bc^2d^8 - 4ad^{10})x^8 + 5(11bc^4d^6 + 4ac^2d^8)x^6 - 5(35bc^6d^4 + 43ac^4d^6)x^4 + 25(4bc^8d^2 + 11bc^6d^4 + 43ac^4d^6)x^2 - 25(4bc^8d^2 + 11bc^6d^4 + 43ac^4d^6)}{15(16d^{11}x^5 - 20c^2d^9x^3 + 5c^4d^7x - 5c^6d^5 + 5c^8d^3 - 5c^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^3/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="fricas")`

[Out]  $-\frac{1}{15} \cdot (48 \cdot b \cdot d^{10} \cdot x^{10} - 8 \cdot b \cdot c^{10} - 10 \cdot a \cdot c^8 \cdot d^2 - 20 \cdot (b \cdot c^2 \cdot d^8 - 4 \cdot a \cdot d^{10}) \cdot x^8 + 5 \cdot (11 \cdot b \cdot c^4 \cdot d^6 + 4 \cdot a \cdot c^2 \cdot d^8) \cdot x^6 - 5 \cdot (35 \cdot b \cdot c^6 \cdot d^4 + 43 \cdot a \cdot c^4 \cdot d^6) \cdot x^4 + 25 \cdot (4 \cdot b \cdot c^8 \cdot d^2 + 5 \cdot a \cdot c^6 \cdot d^4) \cdot x^2 - (48 \cdot b \cdot d^9 \cdot x^9 + 4 \cdot (b \cdot c^2 \cdot d^7 + 20 \cdot a \cdot d^9) \cdot x^7 + 3 \cdot (21 \cdot b \cdot c^4 \cdot d^5 + 20 \cdot a \cdot c^2 \cdot d^7) \cdot x^5 - 35 \cdot (4 \cdot b \cdot c^6 \cdot d^3 + 5 \cdot a \cdot c^4 \cdot d^5) \cdot x^3 + 10 \cdot (4 \cdot b \cdot c^8 \cdot d + 5 \cdot a \cdot c^6 \cdot d^3) \cdot x) \cdot \sqrt{d \cdot x + c} \cdot \sqrt{d \cdot x - c}) / (16 \cdot d^{11} \cdot x^5 - 20 \cdot c^2 \cdot d^9 \cdot x^3 + 5 \cdot c^4 \cdot d^7 \cdot x - (16 \cdot d^{10} \cdot x^4 - 12 \cdot c^2 \cdot d^8 \cdot x^2 + c^4 \cdot d^6) \cdot \sqrt{d \cdot x + c} \cdot \sqrt{d \cdot x - c})$

**Sympy [A]** time = 136.219, size = 240, normalized size = 2.03

$$\frac{ac^3 G_{6,6}^{6,2} \left( \begin{matrix} -\frac{5}{4}, -\frac{3}{4} & -1, -1, -\frac{1}{2}, 1 \\ -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^4} + \frac{iac^3 G_{6,6}^{2,6} \left( \begin{matrix} -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 1 \\ -\frac{7}{4}, -\frac{5}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^4} + \frac{bc^5 G_{6,6}^{6,2} \left( \begin{matrix} -\frac{9}{4}, -\frac{7}{4} & -2, -2, -\frac{3}{2}, 1 \\ -\frac{5}{2}, -\frac{9}{4}, -2, -\frac{7}{4}, -\frac{3}{2}, 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^6} + \frac{ibc^5 G_{6,6}^{2,6} \left( \begin{matrix} -3, -\frac{11}{4}, -\frac{5}{2}, -\frac{9}{4}, -2, 1 \\ -\frac{11}{4}, -\frac{9}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^6}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x\*\*3\*(b\*x\*\*2+a)/(d\*x-c)\*\*(1/2)/(d\*x+c)\*\*(1/2),x)

[Out] a\*c\*\*3\*meijerg((( -5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), c\*\*2/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d\*\*4) + I\*a\*c\*\*3\*meijerg((( -2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4, -2, -3/2, -3/2, 0)), c\*\*2\*exp\_polar(2\*I\*pi)/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d\*\*4) + b\*c\*\*5\*meijerg((( -9/4, -7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), c\*\*2/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d\*\*6) + I\*b\*c\*\*5\*meijerg((( -3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)), c\*\*2\*exp\_polar(2\*I\*pi)/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d\*\*6)

**GIAC/XCAS [A]** time = 0.221716, size = 151, normalized size = 1.28

$$\frac{(15bc^4d^{25} + 15ac^2d^{27} - (20bc^3d^{25} + 10acd^{27} - (22bc^2d^{25} + 5ad^{27} + 3((dx+c)bd^{25} - 4bcd^{25}))(dx+c))(dx+c)(dx+c))}{276480d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^3/(sqrt(d\*x + c)\*sqrt(d\*x - c)),x, algorithm="giac")

[Out] 1/276480\*(15\*b\*c^4\*d^25 + 15\*a\*c^2\*d^27 - (20\*b\*c^3\*d^25 + 10\*a\*c\*d^27 - (22\*b\*c^2\*d^25 + 5\*a\*d^27 + 3\*((d\*x + c)\*b\*d^25 - 4\*b\*c\*d^25)\*(d\*x + c))\*(d\*x + c))\*(d\*x + c))\*sqrt(d\*x + c)\*sqrt(d\*x - c)/d

$$3.261 \quad \int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}} dx$$

**Optimal.** Leaf size=118

$$\frac{c^2(4ad^2 + 3bc^2) \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^5} + \frac{x\sqrt{dx-c}\sqrt{c+dx}(4ad^2 + 3bc^2)}{8d^4} + \frac{bx^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}$$

[Out]  $((3*b*c^2 + 4*a*d^2)*x*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/(8*d^4) + (b*x^3*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/(4*d^2) + (c^2*(3*b*c^2 + 4*a*d^2)*\text{ArcTanh}[\text{Sqrt}[-c + d*x]/\text{Sqrt}[c + d*x]])/(4*d^5)$

**Rubi [A]** time = 0.317815, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$

$$\frac{c^2(4ad^2 + 3bc^2) \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^5} + \frac{x\sqrt{dx-c}\sqrt{c+dx}(4ad^2 + 3bc^2)}{8d^4} + \frac{bx^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2*(a + b*x^2))/(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]), x]$

[Out]  $((3*b*c^2 + 4*a*d^2)*x*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/(8*d^4) + (b*x^3*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/(4*d^2) + (c^2*(3*b*c^2 + 4*a*d^2)*\text{ArcTanh}[\text{Sqrt}[-c + d*x]/\text{Sqrt}[c + d*x]])/(4*d^5)$

**Rubi in Sympy [A]** time = 17.9971, size = 104, normalized size = 0.88

$$\frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2} + \frac{c^2(4ad^2 + 3bc^2) \operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{4d^5} + \frac{x\sqrt{-c+dx}\sqrt{c+dx}(4ad^2 + 3bc^2)}{8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**2}*(b*x^{**2}+a)/(d*x-c)^{(1/2)/(d*x+c)^{(1/2)}, x)$

[Out]  $b*x^{**3}*\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)/(4*d^{**2}) + c^{**2}*(4*a*d^{**2} + 3*b*c^{**2})*\text{atanh}(\text{sqrt}(c + d*x)/\text{sqrt}(-c + d*x))/(4*d^{**5}) + x*\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)*(4*a*d^{**2} + 3*b*c^{**2})/(8*d^{**4})$

**Mathematica [A]** time = 0.106383, size = 96, normalized size = 0.81

$$\frac{dx\sqrt{dx-c}\sqrt{c+dx}(4ad^2 + 3bc^2 + 2bd^2x^2) + (4ac^2d^2 + 3bc^4) \log\left(\sqrt{dx-c}\sqrt{c+dx} + dx\right)}{8d^5}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x^2*(a + b*x^2))/(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]), x]$

[Out]  $(d*x*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]*(3*b*c^2 + 4*a*d^2 + 2*b*d^2*x^2) + (3*b*c^4 + 4*a*c^2*d^2)*\text{Log}[d*x + \text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]])/(8*d^5)$

**Maple [C]** time = 0.03, size = 182, normalized size = 1.5

$$\frac{\operatorname{csgn}(d)}{8d^5} \sqrt{dx-c}\sqrt{dx+c} \left( 2 \operatorname{csgn}(d) x^3 b d^3 \sqrt{d^2 x^2 - c^2} + 4 a x \sqrt{d^2 x^2 - c^2} d^3 \operatorname{csgn}(d) + 3 b c^2 x \sqrt{d^2 x^2 - c^2} \operatorname{csgn}(d) d + 4 a c^2 \ln \left( \left( \frac{\sqrt{d^2 x^2 - c^2} b x^3}{4 d^2} + \frac{3 b c^4 \log \left( 2 d^2 x + 2 \sqrt{d^2 x^2 - c^2} \sqrt{d^2} \right)}{8 \sqrt{d^2} d^4} + \frac{a c^2 \log \left( 2 d^2 x + 2 \sqrt{d^2 x^2 - c^2} \sqrt{d^2} \right)}{2 \sqrt{d^2} d^2} + \frac{3 \sqrt{d^2 x^2 - c^2} b c^2 x}{8 d^4} + \frac{\sqrt{d^2 x^2 - c^2} a x}{2 d^2} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x)

[Out] 1/8\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)\*(2\*csgn(d)\*x^3\*b\*d^3\*(d^2\*x^2-c^2)^(1/2)+4\*a\*x\*(d^2\*x^2-c^2)^(1/2)\*d^3\*csgn(d)+3\*b\*c^2\*x\*(d^2\*x^2-c^2)^(1/2)\*csgn(d)\*d+4\*a\*c^2\*ln((csgn(d)\*(d^2\*x^2-c^2)^(1/2)+d\*x)\*csgn(d))\*d^2+3\*b\*c^4\*ln((csgn(d)\*(d^2\*x^2-c^2)^(1/2)+d\*x)\*csgn(d)))\*csgn(d)/(d^2\*x^2-c^2)^(1/2)/d^5

**Maxima [A]** time = 1.38641, size = 216, normalized size = 1.83

$$\frac{\sqrt{d^2 x^2 - c^2} b x^3}{4 d^2} + \frac{3 b c^4 \log \left( 2 d^2 x + 2 \sqrt{d^2 x^2 - c^2} \sqrt{d^2} \right)}{8 \sqrt{d^2} d^4} + \frac{a c^2 \log \left( 2 d^2 x + 2 \sqrt{d^2 x^2 - c^2} \sqrt{d^2} \right)}{2 \sqrt{d^2} d^2} + \frac{3 \sqrt{d^2 x^2 - c^2} b c^2 x}{8 d^4} + \frac{\sqrt{d^2 x^2 - c^2} a x}{2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^2/(sqrt(d\*x + c)\*sqrt(d\*x - c)),x, algorithm="maxima")

[Out] 1/4\*sqrt(d^2\*x^2 - c^2)\*b\*x^3/d^2 + 3/8\*b\*c^4\*log(2\*d^2\*x + 2\*sqrt(d^2\*x^2 - c^2)\*sqrt(d^2))/(sqrt(d^2)\*d^4) + 1/2\*a\*c^2\*log(2\*d^2\*x + 2\*sqrt(d^2\*x^2 - c^2)\*sqrt(d^2))/(sqrt(d^2)\*d^2) + 3/8\*sqrt(d^2\*x^2 - c^2)\*b\*c^2\*x/d^4 + 1/2\*sqrt(d^2\*x^2 - c^2)\*a\*x/d^2

**Fricas [A]** time = 0.252791, size = 508, normalized size = 4.31

$$\frac{16 b d^8 x^8 + 32 a d^8 x^6 - 4 (7 b c^4 d^4 + 12 a c^2 d^6) x^4 + 4 (3 b c^6 d^2 + 4 a c^4 d^4) x^2 - (16 b d^7 x^7 + 8 (b c^2 d^5 + 4 a d^7) x^5 - 2 (11 b c^4 d^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^2/(sqrt(d\*x + c)\*sqrt(d\*x - c)),x, algorithm="fricas")

[Out] -1/8\*(16\*b\*d^8\*x^8 + 32\*a\*d^8\*x^6 - 4\*(7\*b\*c^4\*d^4 + 12\*a\*c^2\*d^6)\*x^4 + 4\*(3\*b\*c^6\*d^2 + 4\*a\*c^4\*d^4)\*x^2 - (16\*b\*d^7\*x^7 + 8\*(b\*c^2\*d^5 + 4\*a\*d^7)\*x^5 - 2\*(11\*b\*c^4\*d^3 + 16\*a\*c^2\*d^5)\*x^3 + (3\*b\*c^6\*d + 4\*a\*c^4\*d^3)\*x)\*sqrt(d\*x + c)\*sqrt(d\*x - c) + (3\*b\*c^8 + 4\*a\*c^6\*d^2 + 8\*(3\*b\*c^4\*d^4 + 4\*a\*c^2\*d^6)\*x^4 - 8\*(3\*b\*c^6\*d^2 + 4\*a\*c^4\*d^4)\*x^2 - 4\*(2\*(3\*b\*c^4\*d^3 + 4\*a\*c^2\*d^5)\*x^3 - (3\*b\*c^6\*d + 4\*a\*c^4\*d^3)\*x)\*sqrt(d\*x + c)\*sqrt(d\*x - c))\*log(-d\*x + sqrt(d\*x + c)\*sqrt(d\*x - c))/(8\*d^9\*x^4 - 8\*c^2\*d^7\*x^2 + c^4\*d^5 - 4\*(2\*d^8\*x^3 - c^2\*d^6\*x)\*sqrt(d\*x + c)\*sqrt(d\*x - c))

**Sympy [A]** time = 117.765, size = 236, normalized size = 2.

$$\frac{ac^2 G_{6,6}^{6,2} \left( \begin{matrix} -\frac{3}{4}, -\frac{1}{4} & -\frac{1}{2}, -\frac{1}{2}, 0, 1 \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3} - \frac{iac^2 G_{6,6}^{2,6} \left( \begin{matrix} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} & -\frac{3}{2}, -1, -1, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3} + \frac{bc^4 G_{6,6}^{6,2} \left( \begin{matrix} -\frac{7}{4}, -\frac{5}{4} & -\frac{3}{2}, -\frac{3}{2}, -1, 1 \\ -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^5} + \frac{ibc^4 G_{6,6}^{2,6} \left( \begin{matrix} -\frac{5}{2}, -\frac{9}{4}, -2, -\frac{7}{4}, -\frac{3}{2}, 1 \\ -\frac{9}{4}, -\frac{7}{4} & -\frac{5}{2}, -2, -2, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x\*\*2+a)/(d\*x-c)\*\*(1/2)/(d\*x+c)\*\*(1/2),x)

[Out] a\*c\*\*2\*meijerg((( -3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), c\*\*2/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d\*\*3) - I\*a\*c\*\*2\*meijerg((( -3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4, -3/2, -1, -1, 0)), c\*\*2\*exp\_polar(2\*I\*pi)/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d\*\*3) + b\*c\*\*4\*meijerg((( -7/4, -5/4), (-3/2, -3/2, -1, 1)), ((-2, -7/4, -3/2, -5/4, -1, 0), ()), c\*\*2/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d\*\*5) - I\*b\*c\*\*4\*meijerg((( -5/2, -9/4, -2, -7/4, -3/2, 1), ()), ((-9/4, -7/4), (-5/2, -2, -2, 0)), c\*\*2\*exp\_polar(2\*I\*pi)/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d\*\*5)

**GIAC/XCAS [A]** time = 0.255064, size = 176, normalized size = 1.49

$$\frac{(5bc^3d^{16} + 4acd^{18} - (9bc^2d^{16} + 4ad^{18} + 2((dx+c)bd^{16} - 3bcd^{16})(dx+c))(dx+c)\sqrt{dx+c}\sqrt{dx-c} + 2(3bc^4d^{16} + 4acd^{18}))}{114688d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^2/(sqrt(d\*x + c)\*sqrt(d\*x - c)),x, algorithm="giac")

[Out] -1/114688\*((5\*b\*c^3\*d^16 + 4\*a\*c\*d^18 - (9\*b\*c^2\*d^16 + 4\*a\*d^18 + 2\*((d\*x + c)\*b\*d^16 - 3\*b\*c\*d^16)\*(d\*x + c))\*(d\*x + c)\*sqrt(d\*x + c)\*sqrt(d\*x - c) + 2\*(3\*b\*c^4\*d^16 + 4\*a\*c^2\*d^18)\*ln(abs(-sqrt(d\*x + c) + sqrt(d\*x - c))))/d

$$3.262 \quad \int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}} dx$$

**Optimal.** Leaf size=72

$$\frac{\sqrt{dx-c}\sqrt{c+dx}(3ad^2+2bc^2)}{3d^4} + \frac{bx^2\sqrt{dx-c}\sqrt{c+dx}}{3d^2}$$

[Out]  $((2*b*c^2 + 3*a*d^2)*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/(3*d^4) + (b*x^2*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/(3*d^2)$

**Rubi [A]** time = 0.158959, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{\sqrt{dx-c}\sqrt{c+dx}(3ad^2+2bc^2)}{3d^4} + \frac{bx^2\sqrt{dx-c}\sqrt{c+dx}}{3d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x*(a + b*x^2))/(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]), x]$

[Out]  $((2*b*c^2 + 3*a*d^2)*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/(3*d^4) + (b*x^2*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/(3*d^2)$

**Rubi in Sympy [A]** time = 9.97023, size = 61, normalized size = 0.85

$$\frac{bx^2\sqrt{-c+dx}\sqrt{c+dx}}{3d^2} + \frac{\sqrt{-c+dx}\sqrt{c+dx}(3ad^2+2bc^2)}{3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x*(b*x^2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2), x)$

[Out]  $b*x^2*\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)/(3*d^2) + \text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)*(3*a*d^2 + 2*b*c^2)/(3*d^4)$

**Mathematica [A]** time = 0.0556434, size = 48, normalized size = 0.67

$$\frac{\sqrt{dx-c}\sqrt{c+dx}(3ad^2+2bc^2+bd^2x^2)}{3d^4}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x*(a + b*x^2))/(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]), x]$

[Out]  $(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]*(2*b*c^2 + 3*a*d^2 + b*d^2*x^2))/(3*d^4)$

**Maple [A]** time = 0.006, size = 43, normalized size = 0.6

$$\frac{bd^2x^2 + 3ad^2 + 2bc^2}{3d^4} \sqrt{dx+c} \sqrt{dx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)`

[Out]  $1/3*(d*x+c)^(1/2)*(b*d^2*x^2+3*a*d^2+2*b*c^2)/d^4*(d*x-c)^(1/2)$

**Maxima [A]** time = 1.38412, size = 93, normalized size = 1.29

$$\frac{\sqrt{d^2x^2 - c^2}bx^2}{3d^2} + \frac{2\sqrt{d^2x^2 - c^2}bc^2}{3d^4} + \frac{\sqrt{d^2x^2 - c^2}a}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="maxima")`

[Out]  $1/3*\sqrt{d^2*x^2 - c^2}*b*x^2/d^2 + 2/3*\sqrt{d^2*x^2 - c^2}*b*c^2/d^4 + \sqrt{d^2*x^2 - c^2}*a/d^2$

**Fricas [A]** time = 0.240768, size = 262, normalized size = 3.64

$$\frac{4bd^6x^6 + 2bc^6 + 3ac^4d^2 + 3(bc^2d^4 + 4ad^6)x^4 - 3(3bc^4d^2 + 5ac^2d^4)x^2 - (4bd^5x^5 + (5bc^2d^3 + 12ad^5)x^3 - 3(2bc^4d + 3(4d^7x^3 - 3c^2d^5x - (4d^6x^2 - c^2d^4)\sqrt{dx+c}\sqrt{dx-c}))}{3(4d^7x^3 - 3c^2d^5x - (4d^6x^2 - c^2d^4)\sqrt{dx+c}\sqrt{dx-c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="fricas")`

[Out]  $-1/3*(4*b*d^6*x^6 + 2*b*c^6 + 3*a*c^4*d^2 + 3*(b*c^2*d^4 + 4*a*d^6)*x^4 - 3*(3*b*c^4*d^2 + 5*a*c^2*d^4)*x^2 - (4*b*d^5*x^5 + (5*b*c^2*d^3 + 12*a*d^5)*x^3 - 3*(2*b*c^4*d + 3*a*c^2*d^3)*x)*\sqrt{d*x + c}*\sqrt{d*x - c})/(4*d^7*x^3 - 3*c^2*d^5*x - (4*d^6*x^2 - c^2*d^4)*\sqrt{d*x + c}*\sqrt{d*x - c})$

**Sympy [A]** time = 76.0226, size = 223, normalized size = 3.1

$$\frac{acG_{6,6}^{6,2}\left(-\frac{1}{4}, \frac{1}{4}, 0, 0, \frac{1}{2}, 1 \left| \frac{c^2}{d^2x^2} \right.\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{iacG_{6,6}^{2,6}\left(-1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \left| \frac{c^2e^{2i\pi}}{d^2x^2} \right.\right)}{4\pi^{\frac{3}{2}}d^2}$$

$$+ \frac{bc^3G_{6,6}^{6,2}\left(-\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0 \left| \frac{c^2}{d^2x^2} \right.\right)}{4\pi^{\frac{3}{2}}d^4}$$

$$+ \frac{ibc^3G_{6,6}^{2,6}\left(-2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 1 \left| \frac{c^2e^{2i\pi}}{d^2x^2} \right.\right)}{4\pi^{\frac{3}{2}}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)`

[Out]  $a*c*\text{meijerg}((( -1/4, 1/4), (0, 0, 1/2, 1)), (( -1/2, -1/4, 0, 1/4, 1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*a*c*\text{meijerg}((( -1, -3/4, -1/2, -1/4, 0, 1), ()), (( -3/4, -1/4), (-1, -1/2, -1/2, 0)), c**2*\text{exp\_polar}(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2) + b*c**3*\text{meijerg}((( -5/4, -3/4), (-1, -1, -1/2, 1)), (( -3/2, -5/4, -1, -3/4, -1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**4) + I*b*c**3*\text{meijerg}((( -2, -7/4, -3/2, -5/4, -1, 1), ()), (( -7/4, -5/4), (-2, -3/2, -3/2, 0)), c**2*\text{exp\_polar}(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**4)$

$\pi^{**} (3/2) * d^{**} 4)$

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**GIAC/XCAS [A]** time = 0.217631, size = 82, normalized size = 1.14

$$\frac{(3bc^2d^9 + 3ad^{11} + ((dx + c)bd^9 - 2bcd^9)(dx + c))\sqrt{dx + c}\sqrt{dx - c}}{1920d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x/(sqrt(d\*x + c)\*sqrt(d\*x - c)),x, algorithm="giac")

[Out] 1/1920\*(3\*b\*c^2\*d^9 + 3\*a\*d^11 + ((d\*x + c)\*b\*d^9 - 2\*b\*c\*d^9)\*(d\*x + c))\*sqrt(d\*x + c)\*sqrt(d\*x - c)/d

$$3.263 \quad \int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}} dx$$

**Optimal.** Leaf size=68

$$\frac{(2ad^2 + bc^2) \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3} + \frac{bx\sqrt{dx-c}\sqrt{c+dx}}{2d^2}$$

[Out] (b\*x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(2\*d^2) + ((b\*c^2 + 2\*a\*d^2)\*ArcTanh[Sqrt[-c + d\*x]/Sqrt[c + d\*x]])/d^3

**Rubi [A]** time = 0.115793, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$

$$\frac{(2ad^2 + bc^2) \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3} + \frac{bx\sqrt{dx-c}\sqrt{c+dx}}{2d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]), x]

[Out] (b\*x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(2\*d^2) + ((b\*c^2 + 2\*a\*d^2)\*ArcTanh[Sqrt[-c + d\*x]/Sqrt[c + d\*x]])/d^3

**Rubi in Sympy [A]** time = 13.259, size = 75, normalized size = 1.1

$$\frac{2a \operatorname{atanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d} + \frac{bc^2 \operatorname{atanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3} + \frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/(d\*x-c)\*\*(1/2)/(d\*x+c)\*\*(1/2), x)

[Out] 2\*a\*atanh(sqrt(-c + d\*x)/sqrt(c + d\*x))/d + b\*c\*\*2\*atanh(sqrt(-c + d\*x)/sqrt(c + d\*x))/d\*\*3 + b\*x\*sqrt(-c + d\*x)\*sqrt(c + d\*x)/(2\*d\*\*2)

**Mathematica [A]** time = 0.0665225, size = 71, normalized size = 1.04

$$\frac{(2ad^2 + bc^2) \log\left(\sqrt{dx-c}\sqrt{c+dx} + dx\right) + bdx\sqrt{dx-c}\sqrt{c+dx}}{2d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]), x]

[Out] (b\*d\*x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x] + (b\*c^2 + 2\*a\*d^2)\*Log[d\*x + Sqrt[-c + d\*x]\*Sqrt[c + d\*x]])/(2\*d^3)

**Maple [C]** time = 0.025, size = 124, normalized size = 1.8

$$\frac{\operatorname{csgn}(d)}{2d^3} \sqrt{dx-c}\sqrt{c+dx} \left( bx\sqrt{d^2x^2 - c^2} \operatorname{csgn}(d) d + bc^2 \ln\left(\left(\operatorname{csgn}(d)\sqrt{d^2x^2 - c^2} + dx\right) \operatorname{csgn}(d)\right) \right) + 2 \ln\left(\left(\operatorname{csgn}(d)\sqrt{d^2x^2 - c^2} + dx\right) \operatorname{csgn}(d)\right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)`

[Out]  $\frac{1}{2} (d^2 x - c)^{1/2} (d^2 x + c)^{1/2} (b^2 x^2 (d^2 x^2 - c^2)^{1/2} \operatorname{csgn}(d) + d + b^2 c^2 \ln(\operatorname{csgn}(d) (d^2 x^2 - c^2)^{1/2} + d^2 x) \operatorname{csgn}(d) + 2 \ln(\operatorname{csgn}(d) (d^2 x^2 - c^2)^{1/2} + d^2 x) \operatorname{csgn}(d) a^2 d^2) / (d^2 x^2 - c^2)^{1/2} / d^3 \operatorname{csgn}(d)$

**Maxima [A]** time = 1.39622, size = 140, normalized size = 2.06

$$\frac{a \log\left(2 d^2 x + 2 \sqrt{d^2 x^2 - c^2} \sqrt{d^2}\right)}{\sqrt{d^2}} + \frac{b c^2 \log\left(2 d^2 x + 2 \sqrt{d^2 x^2 - c^2} \sqrt{d^2}\right)}{2 \sqrt{d^2} d^2} + \frac{\sqrt{d^2 x^2 - c^2} b x}{2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="maxima")`

[Out]  $a \log(2 d^2 x + 2 \sqrt{d^2 x^2 - c^2}) \sqrt{d^2} / \sqrt{d^2} + 1/2 b^2 c^2 \log(2 d^2 x + 2 \sqrt{d^2 x^2 - c^2}) \sqrt{d^2} / (\sqrt{d^2})^2 d^2 + 1/2 \sqrt{d^2 x^2 - c^2} b^2 x / d^2$

**Fricas [A]** time = 0.237688, size = 261, normalized size = 3.84

$$\frac{2 b d^4 x^4 - 2 b c^2 d^2 x^2 - (2 b d^3 x^3 - b c^2 d x) \sqrt{d x + c} \sqrt{d x - c} - (b c^4 + 2 a c^2 d^2 + 2 (b c^2 d + 2 a d^3) \sqrt{d x + c} \sqrt{d x - c} x - 2 (b c^2 d^2 + 2 a d^3) \sqrt{d x + c} \sqrt{d x - c})}{2 (2 d^5 x^2 - 2 \sqrt{d x + c} \sqrt{d x - c} d^4 x - c^2 d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="fricas")`

[Out]  $-1/2 (2 b^2 d^4 x^4 - 2 b^2 c^2 d^2 x^2 - (2 b^2 d^3 x^3 - b^2 c^2 d x) \sqrt{d x + c} \sqrt{d x - c} - (b^2 c^4 + 2 a^2 c^2 d^2 + 2 (b^2 c^2 d + 2 a^2 d^3) \sqrt{d x + c} \sqrt{d x - c}) x - 2 (b^2 c^2 d^2 + 2 a^2 d^3) \sqrt{d x + c} \sqrt{d x - c}) / (2 d^5 x^2 - 2 \sqrt{d x + c} \sqrt{d x - c} d^4 x - c^2 d^3)$

**Sympy [A]** time = 43.6661, size = 199, normalized size = 2.93

$$\frac{a G_{6,6}^{6,2} \left( \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right) - i a G_{6,6}^{2,6} \left( \begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 \\ -\frac{1}{4}, \frac{1}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{4 \pi^{\frac{3}{2}} d} + \frac{b c^2 G_{6,6}^{6,2} \left( \begin{matrix} -\frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{4 \pi^{\frac{3}{2}} d^3} - \frac{i b c^2 G_{6,6}^{2,6} \left( \begin{matrix} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{4 \pi^{\frac{3}{2}} d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)`

```
[Out] a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1,
0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d) - I*a*meijerg(((1/2,
-1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), c**2*
exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d) + b*c**2*meijerg((
(-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0),
()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*b*c**2*meijerg(((
3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1,
0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

**GIAC/XCAS [A]** time = 0.255644, size = 107, normalized size = 1.57

$$\frac{((dx+c)bd^4 - bcd^4)\sqrt{dx+c}\sqrt{dx-c} - 2(bc^2d^4 + 2ad^6)\ln\left(\left|-\sqrt{dx+c} + \sqrt{dx-c}\right|\right)}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="giac")
```

```
[Out] 1/384*(((d*x + c)*b*d^4 - b*c*d^4)*sqrt(d*x + c)*sqrt(d*x - c) -
2*(b*c^2*d^4 + 2*a*d^6)*ln(abs(-sqrt(d*x + c) + sqrt(d*x - c))))/
d
```

$$3.264 \quad \int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}} dx$$

**Optimal.** Leaf size=56

$$\frac{a \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c} + \frac{b\sqrt{dx-c}\sqrt{c+dx}}{d^2}$$

[Out] (b\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/d^2 + (a\*ArcTan[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/c])/c

**Rubi [A]** time = 0.241805, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\frac{a \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c} + \frac{b\sqrt{dx-c}\sqrt{c+dx}}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]), x]

[Out] (b\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/d^2 + (a\*ArcTan[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/c])/c

**Rubi in Sympy [A]** time = 12.8825, size = 44, normalized size = 0.79

$$\frac{a \operatorname{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{c} + \frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x/(d\*x-c)\*\*(1/2)/(d\*x+c)\*\*(1/2), x)

[Out] a\*atan(sqrt(-c + d\*x)\*sqrt(c + d\*x)/c)/c + b\*sqrt(-c + d\*x)\*sqrt(c + d\*x)/d\*\*2

**Mathematica [A]** time = 0.0996939, size = 55, normalized size = 0.98

$$\frac{b\sqrt{dx-c}\sqrt{c+dx}}{d^2} - \frac{a \tan^{-1}\left(\frac{c}{\sqrt{dx-c}\sqrt{c+dx}}\right)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]), x]

[Out] (b\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/d^2 - (a\*ArcTan[c/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])])/c

**Maple [B]** time = 0.026, size = 108, normalized size = 1.9

$$\frac{1}{d^2} \left( -\ln \left( -2 \frac{c^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2}}{x} \right) a d^2 + b \sqrt{-c^2} \sqrt{d^2 x^2 - c^2} \right) \sqrt{dx-c} \sqrt{dx+c} \frac{1}{\sqrt{-c^2}} \frac{1}{\sqrt{d^2 x^2 - c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)`

[Out]  $(-\ln(-2*(c^2-(-c^2)^{1/2}*(d^2*x^2-c^2)^{1/2}))/x)*a*d^2+b*(-c^2)^{1/2}*(d^2*x^2-c^2)^{1/2}*(d*x-c)^{1/2}*(d*x+c)^{1/2}/(d^2*x^2-c^2)^{1/2}/(-c^2)^{1/2}/d^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/(sqrt(d*x+c)*sqrt(d*x-c)*x),x,algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.240987, size = 176, normalized size = 3.14

$$\frac{bcd^2x^2 - \sqrt{dx+c}\sqrt{dx-c}bcdx - bc^3 - 2(ad^3x - \sqrt{dx+c}\sqrt{dx-c}ad^2) \arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)}{cd^3x - \sqrt{dx+c}\sqrt{dx-c}cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/(sqrt(d*x+c)*sqrt(d*x-c)*x),x,algorithm="fricas")`

[Out]  $-(b*c*d^2*x^2 - \sqrt{d*x+c}*\sqrt{d*x-c}*b*c*d*x - b*c^3 - 2*(a*d^3*x - \sqrt{d*x+c}*\sqrt{d*x-c}*a*d^2)*\arctan(-(d*x - \sqrt{d*x+c}*\sqrt{d*x-c}))/c)/(c*d^3*x - \sqrt{d*x+c}*\sqrt{d*x-c}*c*d^2)$

**Sympy [A]** time = 44.7243, size = 178, normalized size = 3.18

$$\frac{aG_{6,6}^{5,3}\left(\frac{3}{4}, \frac{5}{4}, 1, 1, 1, \frac{3}{2} \left| \frac{c^2}{d^2x^2} \right.\right)}{4\pi^{\frac{3}{2}}c} + \frac{iaG_{6,6}^{2,6}\left(0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1, \frac{1}{4}, \frac{3}{4} \left| \frac{c^2e^{2i\pi}}{d^2x^2} \right.\right)}{4\pi^{\frac{3}{2}}c} + \frac{bcG_{6,6}^{6,2}\left(-\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0, 0, 0, \frac{1}{2}, 1 \left| \frac{c^2}{d^2x^2} \right.\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{ibcG_{6,6}^{2,6}\left(-1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1, -\frac{3}{4}, -\frac{1}{4} \left| \frac{c^2e^{2i\pi}}{d^2x^2} \right.\right)}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)`

[Out]  $-a*\text{meijerg}(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c) + I*a*\text{meijerg}(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), c**2*\exp\_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c) + b*c*\text{meijerg}((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*b*c*\text{meijerg}((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), c**2*\exp\_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)$

**GIAC/XCAS [A]** time = 0.220305, size = 74, normalized size = 1.32

$$-\frac{2 a \arctan\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2c}\right)}{c} + \frac{\sqrt{dx+c}\sqrt{dx-c}b}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/(sqrt(d\*x + c)\*sqrt(d\*x - c)\*x),x, algorithm="giac")

[Out] -2\*a\*arctan(1/2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2/c)/c + sqrt(d\*x + c)\*sqrt(d\*x - c)\*b/d^2

$$3.265 \quad \int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}} dx$$

**Optimal.** Leaf size=57

$$\frac{a\sqrt{dx-c}\sqrt{c+dx}}{c^2x} + \frac{2b \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}$$

[Out] (a\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(c^2\*x) + (2\*b\*ArcTanh[Sqrt[-c + d\*x]/Sqrt[c + d\*x]])/d

**Rubi [A]** time = 0.227326, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\frac{a\sqrt{dx-c}\sqrt{c+dx}}{c^2x} + \frac{2b \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^2\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]), x]

[Out] (a\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(c^2\*x) + (2\*b\*ArcTanh[Sqrt[-c + d\*x]/Sqrt[c + d\*x]])/d

**Rubi in Sympy [A]** time = 11.5265, size = 46, normalized size = 0.81

$$\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x} + \frac{2b \operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x\*\*2/(d\*x-c)\*\*(1/2)/(d\*x+c)\*\*(1/2), x)

[Out] a\*sqrt(-c + d\*x)\*sqrt(c + d\*x)/(c\*\*2\*x) + 2\*b\*atanh(sqrt(c + d\*x)/sqrt(-c + d\*x))/d

**Mathematica [A]** time = 0.0726016, size = 60, normalized size = 1.05

$$\frac{a\sqrt{dx-c}\sqrt{c+dx}}{c^2x} + \frac{b \log\left(\sqrt{dx-c}\sqrt{c+dx} + dx\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^2\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]), x]

[Out] (a\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(c^2\*x) + (b\*Log[d\*x + Sqrt[-c + d\*x]\*Sqrt[c + d\*x]])/d

**Maple [C]** time = 0.029, size = 97, normalized size = 1.7

$$\frac{\operatorname{csgn}(d)}{c^2xd} \sqrt{dx-c}\sqrt{dx+c} \left( \ln\left(\left(\operatorname{csgn}(d)\sqrt{d^2x^2-c^2} + dx\right) \operatorname{csgn}(d)\right) xbc^2 + a\sqrt{d^2x^2-c^2}\operatorname{csgn}(d)d \right) \frac{1}{\sqrt{d^2x^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)`

[Out]  $(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/c^2*(\ln((\text{csgn}(d)*(d^2*x^2-c^2))^{(1/2)+d*x}*\text{csgn}(d))*x*b*c^2+a*(d^2*x^2-c^2)^{(1/2)}*\text{csgn}(d)*d)*\text{csgn}(d)/(d^2*x^2-c^2)^{(1/2)}/x/d$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.237346, size = 117, normalized size = 2.05

$$\frac{ad - (bdx^2 - \sqrt{dx+c}\sqrt{dx-c}bx) \log(-dx + \sqrt{dx+c}\sqrt{dx-c})}{d^2x^2 - \sqrt{dx+c}\sqrt{dx-c}cdx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x^2),x, algorithm="fricas")`

[Out]  $(a*d - (b*d*x^2 - \sqrt{d*x+c}*\sqrt{d*x-c}*b*x)*\log(-d*x + \sqrt{d*x+c}*\sqrt{d*x-c}))/((d^2*x^2 - \sqrt{d*x+c}*\sqrt{d*x-c})*d*x)$

**Sympy [A]** time = 51.5059, size = 165, normalized size = 2.89

$$\frac{adG_{6,6}^{5,3}\left(1, \frac{5}{4}, \frac{7}{4}, 1, \frac{3}{2}, \frac{3}{2}, 2 \middle| \frac{c^2}{d^2x^2}\right) - iadG_{6,6}^{2,6}\left(\frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1, \frac{1}{2}, 1, 1, 0 \middle| \frac{c^2 e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2} + \frac{bG_{6,6}^{6,2}\left(0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0, \frac{1}{2}, \frac{1}{2}, 1, 1 \middle| \frac{c^2}{d^2x^2}\right) - ibG_{6,6}^{2,6}\left(-\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1, -\frac{1}{4}, \frac{1}{4}, -\frac{1}{2}, 0, 0, 0 \middle| \frac{c^2 e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x**2/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)`

[Out]  $-a*d*\text{meijerg}(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**2) - I*a*d*\text{meijerg}((1/2, 3/4, 1, 5/4, 3/2, 1), (), ((3/4, 5/4), (1/2, 1, 1, 0)), c**2*\text{exp\_polar}(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c**2) + b*\text{meijerg}((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*\text{meijerg}((-1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), c**2*\text{exp\_polar}(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d)$

**GIAC/XCAS [A]** time = 0.224909, size = 89, normalized size = 1.56

$$\frac{\frac{16ad^2}{(\sqrt{dx+c}-\sqrt{dx-c})^4+4c^2} - b\ln\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(16*a*d^2/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2) - b*ln(
(sqrt(d*x + c) - sqrt(d*x - c))^4))/d
```



$$3.266 \quad \int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}} dx$$

**Optimal.** Leaf size=76

$$\frac{(ad^2 + 2bc^2) \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^3} + \frac{a\sqrt{dx-c}\sqrt{c+dx}}{2c^2x^2}$$

[Out] (a\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(2\*c^2\*x^2) + ((2\*b\*c^2 + a\*d^2)\*ArcTan[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/c])/(2\*c^3)

**Rubi [A]** time = 0.260603, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\frac{(ad^2 + 2bc^2) \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^3} + \frac{a\sqrt{dx-c}\sqrt{c+dx}}{2c^2x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^3\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]), x]

[Out] (a\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(2\*c^2\*x^2) + ((2\*b\*c^2 + a\*d^2)\*ArcTan[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/c])/(2\*c^3)

**Rubi in Sympy [A]** time = 13.3616, size = 63, normalized size = 0.83

$$\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{2c^2x^2} + \frac{(ad^2 + 2bc^2) \operatorname{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x\*\*3/(d\*x-c)\*\*(1/2)/(d\*x+c)\*\*(1/2), x)

[Out] a\*sqrt(-c + d\*x)\*sqrt(c + d\*x)/(2\*c\*\*2\*x\*\*2) + (a\*d\*\*2 + 2\*b\*c\*\*2)\*atan(sqrt(-c + d\*x)\*sqrt(c + d\*x)/c)/(2\*c\*\*3)

**Mathematica [C]** time = 0.139132, size = 103, normalized size = 1.36

$$\frac{ac\sqrt{dx-c}\sqrt{c+dx} - ix^2(ad^2 + 2bc^2) \log\left(\frac{4c^2(\sqrt{dx-c}\sqrt{c+dx}-ic)}{x(ad^2+2bc^2)}\right)}{2c^3x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^3\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]), x]

[Out] (a\*c\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x] - I\*(2\*b\*c^2 + a\*d^2)\*x^2\*Log[(4\*c^2\*((-I)\*c + Sqrt[-c + d\*x]\*Sqrt[c + d\*x]))/((2\*b\*c^2 + a\*d^2)\*x)])/(2\*c^3\*x^2)

**Maple [B]** time = 0.029, size = 158, normalized size = 2.1

$$-\frac{1}{2c^2x^2}\sqrt{dx-c}\sqrt{c+dx}\left(\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^2ad^2+2\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^2bc^2-a\sqrt{d^2x^2-c^2}\sqrt{-c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2), x)`

[Out] 
$$-1/2*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/c^2*(\ln(-2*(c^2-(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)})/x)*x^2*a*d^2+2*\ln(-2*(c^2-(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)})/x)*x^2*b*c^2-a*(d^2*x^2-c^2)^{(1/2)}*(-c^2)^{(1/2)})/(d^2*x^2-c^2)^{(1/2)}/(-c^2)^{(1/2)}/x^2$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x^3), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.245142, size = 279, normalized size = 3.67

$$\frac{2acd^3x^3 - 2ac^3dx - (2acd^2x^2 - ac^3)\sqrt{dx+c}\sqrt{dx-c} + 2\left(2(bc^2d + ad^3)\sqrt{dx+c}\sqrt{dx-c}x^3 - 2(bc^2d^2 + ad^4)x^4 + (2c^3d^2x^4 - 2\sqrt{dx+c}\sqrt{dx-c}cc^3dx^3 - c^5x^2)\right)}{2\left(2c^3d^2x^4 - 2\sqrt{dx+c}\sqrt{dx-c}cc^3dx^3 - c^5x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x^3), x, algorithm="fricas")`

[Out] 
$$-1/2*(2*a*c*d^3*x^3 - 2*a*c^3*d*x - (2*a*c*d^2*x^2 - a*c^3)*\sqrt{d*x + c}*\sqrt{d*x - c} + 2*(2*(2*b*c^2*d + a*d^3)*\sqrt{d*x + c}*\sqrt{d*x - c}*x^3 - 2*(2*b*c^2*d^2 + a*d^4)*x^4 + (2*b*c^4 + a*c^2*d^2)*x^2)*\arctan(-(d*x - \sqrt{d*x + c})*\sqrt{d*x - c})/c)/(2*c^3*d^2*x^4 - 2*\sqrt{d*x + c}*\sqrt{d*x - c}*c^3*d*x^3 - c^5*x^2)$$

**Sympy [A]** time = 68.5575, size = 162, normalized size = 2.13

$$\frac{ad^2G_{6,6}^{5,3}\left(\frac{7}{4}, \frac{9}{4}, 1, 2, 2, \frac{5}{2} \middle| \frac{c^2}{d^2x^2}\right) + iad^2G_{6,6}^{2,6}\left(1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1, \frac{5}{4}, \frac{7}{4} \middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3} + \frac{bG_{6,6}^{5,3}\left(\frac{3}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 0 \middle| \frac{c^2}{d^2x^2}\right) + ibG_{6,6}^{2,6}\left(0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1, \frac{1}{4}, \frac{3}{4} \middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x**3/(d*x-c)**(1/2)/(d*x+c)**(1/2), x)`

[Out] 
$$-a*d^{**2}*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,)), c^{**2}/(d^{**2}*x^{**2}))/ (4*pi^{**3/2}*c^{**3}) + I*a*d^{**2}*meijerg(((1, 5/4, 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), c^{**2}*exp_polar(2*I*pi)/(d^{**2}*x^{**2}))/ (4*pi^{**3/2}*c^{**3}) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), c^{**2}/(d^{**2}*x^{**2}))/ (4*pi^{**3/2}*c) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), c^{**2}*exp_polar(2*I*pi)/(d^{**2}*x^{**2}))/ (4*pi^{**3/2}*c)$$

---

**GIAC/XCAS [A]** time = 0.229743, size = 190, normalized size = 2.5

$$\frac{(2bc^2d+ad^3) \arctan\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2c}\right)}{c^3} + \frac{2\left(ad^3(\sqrt{dx+c}-\sqrt{dx-c})^6 - 4ac^2d^3(\sqrt{dx+c}-\sqrt{dx-c})^2\right)}{\left((\sqrt{dx+c}-\sqrt{dx-c})^4 + 4c^2\right)^2 c^2}$$


---

$d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/(sqrt(d\*x + c)\*sqrt(d\*x - c)\*x^3),x, algorithm="giac")

[Out] -((2\*b\*c^2\*d + a\*d^3)\*arctan(1/2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2/c)/c^3 + 2\*(a\*d^3\*(sqrt(d\*x + c) - sqrt(d\*x - c))^6 - 4\*a\*c^2\*d^3\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2)/(((sqrt(d\*x + c) - sqrt(d\*x - c))^4 + 4\*c^2)^2\*c^2)/d

$$3.267 \quad \int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}} dx$$

**Optimal.** Leaf size=75

$$\frac{\sqrt{dx-c}\sqrt{c+dx}(2ad^2+3bc^2)}{3c^4x} + \frac{a\sqrt{dx-c}\sqrt{c+dx}}{3c^2x^3}$$

[Out] (a\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(3\*c^2\*x^3) + ((3\*b\*c^2 + 2\*a\*d^2)\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(3\*c^4\*x)

**Rubi [A]** time = 0.254355, antiderivative size = 75, normalized size of antiderivative = 1., number of rules used = 2, number of rules used = 2, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{\sqrt{dx-c}\sqrt{c+dx}(2ad^2+3bc^2)}{3c^4x} + \frac{a\sqrt{dx-c}\sqrt{c+dx}}{3c^2x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^4\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]), x]

[Out] (a\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(3\*c^2\*x^3) + ((3\*b\*c^2 + 2\*a\*d^2)\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(3\*c^4\*x)

**Rubi in Sympy [A]** time = 11.9889, size = 63, normalized size = 0.84

$$\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{3c^2x^3} + \frac{\sqrt{-c+dx}\sqrt{c+dx}(2ad^2+3bc^2)}{3c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x\*\*4/(d\*x-c)\*\*(1/2)/(d\*x+c)\*\*(1/2), x)

[Out] a\*sqrt(-c + d\*x)\*sqrt(c + d\*x)/(3\*c\*\*2\*x\*\*3) + sqrt(-c + d\*x)\*sqrt(c + d\*x)\*(2\*a\*d\*\*2 + 3\*b\*c\*\*2)/(3\*c\*\*4\*x)

**Mathematica [A]** time = 0.0741589, size = 56, normalized size = 0.75

$$\sqrt{dx-c}\sqrt{c+dx} \left( \frac{2ad^2+3bc^2}{3c^4x} + \frac{a}{3c^2x^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^4\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]), x]

[Out] (a/(3\*c^2\*x^3) + (3\*b\*c^2 + 2\*a\*d^2)/(3\*c^4\*x))\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]

**Maple [A]** time = 0.008, size = 49, normalized size = 0.7

$$\frac{2ad^2x^2 + 3bc^2x^2 + ac^2}{3x^3c^4} \sqrt{dx+c} \sqrt{dx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)`

[Out]  $1/3*(d*x+c)^{1/2}*(2*a*d^2*x^2+3*b*c^2*x^2+a*c^2)/x^3/c^4*(d*x-c)^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/(sqrt(d*x+c)*sqrt(d*x-c)*x^4),x,algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.237519, size = 162, normalized size = 2.16

$$\frac{6bd^2x^4 - ac^2 - 3(bc^2 - ad^2)x^2 - 3(2bdx^3 + adx)\sqrt{dx+c}\sqrt{dx-c}}{3(4d^3x^6 - 3c^2dx^4 - (4d^2x^5 - c^2x^3)\sqrt{dx+c}\sqrt{dx-c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/(sqrt(d*x+c)*sqrt(d*x-c)*x^4),x,algorithm="fricas")`

[Out]  $1/3*(6*b*d^2*x^4 - a*c^2 - 3*(b*c^2 - a*d^2)*x^2 - 3*(2*b*d*x^3 + a*d*x)*sqrt(d*x+c)*sqrt(d*x-c))/(4*d^3*x^6 - 3*c^2*d*x^4 - (4*d^2*x^5 - c^2*x^3)*sqrt(d*x+c)*sqrt(d*x-c))$

**Sympy [A]** time = 108.996, size = 170, normalized size = 2.27

$$\frac{ad^3G_{6,6}^{5,3}\left(\begin{matrix} \frac{9}{4}, \frac{11}{4}, 1 \\ 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}, 3 \end{matrix} \middle| \frac{c^2}{d^2x^2}\right) - iad^3G_{6,6}^{2,6}\left(\begin{matrix} \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}, 1 \\ \frac{7}{4}, \frac{9}{4} \end{matrix} \middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4} - \frac{bdG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 \\ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 \end{matrix} \middle| \frac{c^2}{d^2x^2}\right) - ibdG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1 \\ \frac{3}{4}, \frac{5}{4} \end{matrix} \middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x**4/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)`

[Out]  $-a*d^3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 1/4, 3), (0,)), c^2/(d^2*x^2))/(4*pi**(3/2)*c^4) - I*a*d^3*meijerg(((3/2, 7/4, 2, 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), c^2*exp_polar(2*I*pi)/(d^2*x^2))/(4*pi**(3/2)*c^4) - b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), c^2/(d^2*x^2))/(4*pi**(3/2)*c^2) - I*b*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), c^2*exp_polar(2*I*pi)/(d^2*x^2))/(4*pi**(3/2)*c^2)$

**GIAC/XCAS [A]** time = 0.234151, size = 185, normalized size = 2.47

$$\frac{8 \left( 3 b d^2 \left( \sqrt{d x + c} - \sqrt{d x - c} \right)^8 + 24 b c^2 d^2 \left( \sqrt{d x + c} - \sqrt{d x - c} \right)^4 + 24 a d^4 \left( \sqrt{d x + c} - \sqrt{d x - c} \right)^4 + 48 b c^4 d^2 + 32 a c^2 d^4 \right)}{3 \left( \left( \sqrt{d x + c} - \sqrt{d x - c} \right)^4 + 4 c^2 \right)^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/(sqrt(d\*x + c)\*sqrt(d\*x - c)\*x^4),x, algorithm="giac")

[Out] 8/3\*(3\*b\*d^2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^8 + 24\*b\*c^2\*d^2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^4 + 24\*a\*d^4\*(sqrt(d\*x + c) - sqrt(d\*x - c))^4 + 48\*b\*c^4\*d^2 + 32\*a\*c^2\*d^4)/(((sqrt(d\*x + c) - sqrt(d\*x - c))^4 + 4\*c^2)^3\*d)

$$3.268 \quad \int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}} dx$$

**Optimal.** Leaf size=123

$$\frac{d^2(3ad^2 + 4bc^2) \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^5} + \frac{\sqrt{dx-c}\sqrt{c+dx}(3ad^2 + 4bc^2)}{8c^4x^2} + \frac{a\sqrt{dx-c}\sqrt{c+dx}}{4c^2x^4}$$

[Out] (a\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(4\*c^2\*x^4) + ((4\*b\*c^2 + 3\*a\*d^2)\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(8\*c^4\*x^2) + (d^2\*(4\*b\*c^2 + 3\*a\*d^2)\*ArcTan[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/c])/(8\*c^5)

**Rubi [A]** time = 0.342501, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$

$$\frac{d^2(3ad^2 + 4bc^2) \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^5} + \frac{\sqrt{dx-c}\sqrt{c+dx}(3ad^2 + 4bc^2)}{8c^4x^2} + \frac{a\sqrt{dx-c}\sqrt{c+dx}}{4c^2x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^5\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]), x]

[Out] (a\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(4\*c^2\*x^4) + ((4\*b\*c^2 + 3\*a\*d^2)\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(8\*c^4\*x^2) + (d^2\*(4\*b\*c^2 + 3\*a\*d^2)\*ArcTan[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/c])/(8\*c^5)

**Rubi in Sympy [A]** time = 19.7361, size = 107, normalized size = 0.87

$$\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4} + \frac{\sqrt{-c+dx}\sqrt{c+dx}(3ad^2 + 4bc^2)}{8c^4x^2} + \frac{d^2(3ad^2 + 4bc^2) \operatorname{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{8c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x\*\*5/(d\*x-c)\*\*(1/2)/(d\*x+c)\*\*(1/2), x)

[Out] a\*sqrt(-c + d\*x)\*sqrt(c + d\*x)/(4\*c\*\*2\*x\*\*4) + sqrt(-c + d\*x)\*sqrt(c + d\*x)\*(3\*a\*d\*\*2 + 4\*b\*c\*\*2)/(8\*c\*\*4\*x\*\*2) + d\*\*2\*(3\*a\*d\*\*2 + 4\*b\*c\*\*2)\*atan(sqrt(-c + d\*x)\*sqrt(c + d\*x)/c)/(8\*c\*\*5)

**Mathematica [C]** time = 0.19576, size = 135, normalized size = 1.1

$$\frac{c\sqrt{dx-c}\sqrt{c+dx}(2ac^2 + 3ad^2x^2 + 4bc^2x^2) - id^2x^4(3ad^2 + 4bc^2) \log\left(\frac{16c^4(\sqrt{dx-c}\sqrt{c+dx}-ic)}{d^2x(3ad^2+4bc^2)}\right)}{8c^5x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^5\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]), x]

[Out] (c\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(2\*a\*c^2 + 4\*b\*c^2\*x^2 + 3\*a\*d^2\*x^2) - I\*d^2\*(4\*b\*c^2 + 3\*a\*d^2)\*x^4\*Log[(16\*c^4\*((-I)\*c + Sqrt[-c + d\*x]\*Sqrt[c + d\*x]))/(d^2\*(4\*b\*c^2 + 3\*a\*d^2)\*x)])/(8\*c^5\*x^4)

**Maple [B]** time = 0.03, size = 227, normalized size = 1.9

$$-\frac{1}{8c^4x^4}\sqrt{dx-c}\sqrt{dx+c}\left(3\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^4ad^4+4\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^4bc^2d^2-3ad^2\sqrt{d^2x^2-c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x^5/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x)

[Out] 
$$-1/8*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/c^4*(3*\ln(-2*(c^2-(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)})/x)*x^4*a*d^4+4*\ln(-2*(c^2-(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)})/x)*x^4*b*c^2*d^2-3*a*d^2*(d^2*x^2-c^2)^{(1/2)}*x^2*(-c^2)^{(1/2)}-4*b*(d^2*x^2-c^2)^{(1/2)}*c^2*x^2*(-c^2)^{(1/2)}-2*a*(d^2*x^2-c^2)^{(1/2)}*c^2*(-c^2)^{(1/2)})/(d^2*x^2-c^2)^{(1/2)}/x^4/(-c^2)^{(1/2)}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/(sqrt(d\*x + c)\*sqrt(d\*x - c)\*x^5),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.272273, size = 544, normalized size = 4.42

$$8ac^7dx + 8(4bc^3d^5 + 3acd^7)x^7 - 4(12bc^5d^3 + 5ac^3d^5)x^5 + 4(4bc^7d - 3ac^5d^3)x^3 - (2ac^7 + 8(4bc^3d^4 + 3acd^6))x^6 - 8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/(sqrt(d\*x + c)\*sqrt(d\*x - c)\*x^5),x, algorithm="fricas")

[Out] 
$$-1/8*(8*a*c^7*d*x + 8*(4*b*c^3*d^5 + 3*a*c*d^7)*x^7 - 4*(12*b*c^5*d^3 + 5*a*c^3*d^5)*x^5 + 4*(4*b*c^7*d - 3*a*c^5*d^3)*x^3 - (2*a*c^7 + 8*(4*b*c^3*d^4 + 3*a*c*d^6))*x^6 - 8*(4*b*c^5*d^3 + 5*a*c^3*d^5)*x^5 + 4*(4*b*c^7*d - 13*a*c^5*d^3)*x^2)*sqrt(d*x + c)*sqrt(d*x - c) - 2*(8*(4*b*c^2*d^6 + 3*a*d^8)*x^8 - 8*(4*b*c^4*d^4 + 3*a*c^2*d^6)*x^6 + (4*b*c^6*d^2 + 3*a*c^4*d^4)*x^4 - 4*(2*(4*b*c^2*d^5 + 3*a*d^7)*x^7 - (4*b*c^4*d^3 + 3*a*c^2*d^5)*x^5))*sqrt(d*x + c)*sqrt(d*x - c))*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c)/(8*c^5*d^4*x^8 - 8*c^7*d^2*x^6 + c^9*x^4 - 4*(2*c^5*d^3*x^7 - c^7*d*x^5))*sqrt(d*x + c)*sqrt(d*x - c))$$

**Sympy [A]** time = 157.614, size = 172, normalized size = 1.4

$$\frac{ad^4G_{6,6}^{5,3}\left(\frac{11}{4}, \frac{13}{4}, 1, 3, 3, \frac{7}{2} \mid \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^5} + \frac{iad^4G_{6,6}^{2,6}\left(2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}, 3, 1, \frac{9}{4}, \frac{11}{4} \mid \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^5} \\ - \frac{bd^2G_{6,6}^{5,3}\left(\frac{7}{4}, \frac{9}{4}, 1, 2, 2, \frac{5}{2} \mid \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3} + \frac{ibd^2G_{6,6}^{2,6}\left(1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1, \frac{5}{4}, \frac{7}{4} \mid \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((b\*x\*\*2+a)/x\*\*5/(d\*x-c)\*\*(1/2)/(d\*x+c)\*\*(1/2),x)

[Out] -a\*d\*\*4\*meijerg(((11/4, 13/4, 1), (3, 3, 7/2)), ((5/2, 11/4, 3, 13/4, 7/2), (0,)), c\*\*2/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*5) + I\*a\*d\*\*4\*meijerg(((2, 9/4, 5/2, 11/4, 3, 1), ()), ((9/4, 11/4), (2, 5/2, 5/2, 0)), c\*\*2\*exp\_polar(2\*I\*pi)/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*5) - b\*d\*\*2\*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,)), c\*\*2/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*3) + I\*b\*d\*\*2\*meijerg(((1, 5/4, 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), c\*\*2\*exp\_polar(2\*I\*pi)/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*3)

**GIAC/XCAS [A]** time = 0.239979, size = 439, normalized size = 3.57

$$\frac{(4bc^2d^3+3ad^5) \arctan\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2c}\right)}{c^5} + \frac{2\left(4bc^2d^3(\sqrt{dx+c}-\sqrt{dx-c})^{14} + 3ad^5(\sqrt{dx+c}-\sqrt{dx-c})^{14} + 16bc^4d^3(\sqrt{dx+c}-\sqrt{dx-c})^{10} + 44ac^2d^5(\sqrt{dx+c}-\sqrt{dx-c})^{10}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/(sqrt(d\*x + c)\*sqrt(d\*x - c)\*x^5),x, algorithm="giac")

[Out] -1/4\*((4\*b\*c^2\*d^3 + 3\*a\*d^5)\*arctan(1/2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2/c)/c^5 + 2\*(4\*b\*c^2\*d^3\*(sqrt(d\*x + c) - sqrt(d\*x - c))^14 + 3\*a\*d^5\*(sqrt(d\*x + c) - sqrt(d\*x - c))^14 + 16\*b\*c^4\*d^3\*(sqrt(d\*x + c) - sqrt(d\*x - c))^10 + 44\*a\*c^2\*d^5\*(sqrt(d\*x + c) - sqrt(d\*x - c))^10 - 64\*b\*c^4\*d^3\*(sqrt(d\*x + c) - sqrt(d\*x - c))^6 - 176\*a\*c^4\*d^5\*(sqrt(d\*x + c) - sqrt(d\*x - c))^6 - 256\*b\*c^8\*d^3\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2 - 192\*a\*c^6\*d^5\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2)/((sqrt(d\*x + c) - sqrt(d\*x - c))^4 + 4\*c^2)^4\*c^4)/d

$$3.269 \quad \int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=161

$$\frac{3c^2(4ad^2+5bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^7} + \frac{3x\sqrt{dx-c}\sqrt{c+dx}(4ad^2+5bc^2)}{8d^6} - \frac{x^3(4ad^2+5bc^2)}{4d^4\sqrt{dx-c}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

[Out]  $-\left(\frac{5b^2c^2+4a^2d^2}{4d^4}\right)x^3/\left(\sqrt{-c+dx}\sqrt{c+dx}\right) + \left(\frac{bx^5}{4d^2}\right)/\left(\sqrt{-c+dx}\sqrt{c+dx}\right) + \frac{3(5b^2c^2+4a^2d^2)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^6} + \frac{3c^2(5b^2c^2+4a^2d^2)\operatorname{ArcTanh}\left[\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right]}{4d^7}$

**Rubi [A]** time = 0.417564, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$

$$\frac{3c^2(4ad^2+5bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^7} + \frac{3x\sqrt{dx-c}\sqrt{c+dx}(4ad^2+5bc^2)}{8d^6} - \frac{x^3(4ad^2+5bc^2)}{4d^4\sqrt{dx-c}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}, x\right]$

[Out]  $-\left(\frac{5b^2c^2+4a^2d^2}{4d^4}\right)x^3/\left(\sqrt{-c+dx}\sqrt{c+dx}\right) + \left(\frac{bx^5}{4d^2}\right)/\left(\sqrt{-c+dx}\sqrt{c+dx}\right) + \frac{3(5b^2c^2+4a^2d^2)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^6} + \frac{3c^2(5b^2c^2+4a^2d^2)\operatorname{ArcTanh}\left[\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right]}{4d^7}$

**Rubi in Sympy [A]** time = 26.8087, size = 146, normalized size = 0.91

$$\frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{3c^2(4ad^2+5bc^2)\operatorname{atanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^7} - \frac{x^3(4ad^2+5bc^2)}{4d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{3x\sqrt{-c+dx}\sqrt{c+dx}(4ad^2+5bc^2)}{8d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}(x^{**4}(b*x^{**2}+a)/(d*x-c)^{(3/2)/(d*x+c)^{(3/2)}, x)$

[Out]  $b*x^{**5}/(4*d^{**2}\sqrt{-c+dx}\sqrt{c+dx}) + 3*c^{**2}(4*a*d^{**2} + 5*b*c^{**2})\operatorname{atanh}(\sqrt{-c+dx}/\sqrt{c+dx})/(4*d^{**7}) - x^{**3}(4*a*d^{**2} + 5*b*c^{**2})/(4*d^{**4}\sqrt{-c+dx}\sqrt{c+dx}) + 3*x^s\sqrt{-c+dx}\sqrt{c+dx}(4*a*d^{**2} + 5*b*c^{**2})/(8*d^{**6})$

**Mathematica [A]** time = 0.196303, size = 150, normalized size = 0.93

$$\frac{dx\sqrt{dx-c}\sqrt{c+dx}(4ad^2(d^2x^2-3c^2)+b(-15c^4+5c^2d^2x^2+2d^4x^4))-3c^2(c^2-d^2x^2)(4ad^2+5bc^2)\log\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{dx-c}\right)}{8d^7(dx-c)(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*(a + b\*x^2))/((-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)),x]

[Out] (d\*x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(4\*a\*d^2\*(-3\*c^2 + d^2\*x^2) + b\*(-15\*c^4 + 5\*c^2\*d^2\*x^2 + 2\*d^4\*x^4)) - 3\*c^2\*(5\*b\*c^2 + 4\*a\*d^2)\*(c^2 - d^2\*x^2)\*Log[d\*x + Sqrt[-c + d\*x]\*Sqrt[c + d\*x]])/(8\*d^7\*(-c + d\*x)\*(c + d\*x))

**Maple [C]** time = 0.047, size = 316, normalized size = 2.

$$\frac{\operatorname{csgn}(d)}{8d^7} \left( 2 \operatorname{csgn}(d) x^5 b d^5 \sqrt{d^2 x^2 - c^2} + 4 \operatorname{csgn}(d) x^3 a d^5 \sqrt{d^2 x^2 - c^2} + 5 \operatorname{csgn}(d) x^3 b c^2 d^3 \sqrt{d^2 x^2 - c^2} + 12 \ln \left( \left( \operatorname{csgn}(d) \sqrt{d^2 x^2} \right. \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(b\*x^2+a)/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x)

[Out] 1/8\*(2\*csgn(d)\*x^5\*b\*d^5\*(d^2\*x^2-c^2)^(1/2)+4\*csgn(d)\*x^3\*a\*d^5\*(d^2\*x^2-c^2)^(1/2)+5\*csgn(d)\*x^3\*b\*c^2\*d^3\*(d^2\*x^2-c^2)^(1/2)+12\*ln((csgn(d)\*(d^2\*x^2-c^2)^(1/2)+d\*x)\*csgn(d))\*x^2\*a\*c^2\*d^4+15\*ln((csgn(d)\*(d^2\*x^2-c^2)^(1/2)+d\*x)\*csgn(d))\*x^2\*b\*c^4\*d^2-12\*a\*c^2\*x\*(d^2\*x^2-c^2)^(1/2)\*d^3\*csgn(d)-15\*b\*c^4\*x\*(d^2\*x^2-c^2)^(1/2)\*csgn(d)\*d-12\*a\*c^4\*ln((csgn(d)\*(d^2\*x^2-c^2)^(1/2)+d\*x)\*csgn(d))\*d^2-15\*b\*c^4\*ln((csgn(d)\*(d^2\*x^2-c^2)^(1/2)+d\*x)\*csgn(d))\*csgn(d)/(d^2\*x^2-c^2)^(1/2)/d^7/(d\*x+c)^(1/2)/(d\*x-c)^(1/2)

**Maxima [A]** time = 1.43494, size = 289, normalized size = 1.8

$$\frac{bx^5}{4\sqrt{d^2x^2 - c^2}d^2} + \frac{5bc^2x^3}{8\sqrt{d^2x^2 - c^2}d^4} + \frac{ax^3}{2\sqrt{d^2x^2 - c^2}d^2} - \frac{15bc^4x}{8\sqrt{d^2x^2 - c^2}d^6} - \frac{3ac^2x}{2\sqrt{d^2x^2 - c^2}d^4} + \frac{15bc^4 \log\left(2d^2x + 2\sqrt{d^2x^2 - c^2}\sqrt{d^2}\right)}{8\sqrt{d^2}d^6} + \frac{3ac^2 \log\left(2d^2x + 2\sqrt{d^2x^2 - c^2}\sqrt{d^2}\right)}{2\sqrt{d^2}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^4/((d\*x + c)^(3/2)\*(d\*x - c)^(3/2)),x, algorithm="maxima")

[Out] 1/4\*b\*x^5/(sqrt(d^2\*x^2 - c^2)\*d^2) + 5/8\*b\*c^2\*x^3/(sqrt(d^2\*x^2 - c^2)\*d^4) + 1/2\*a\*x^3/(sqrt(d^2\*x^2 - c^2)\*d^2) - 15/8\*b\*c^4\*x/(sqrt(d^2\*x^2 - c^2)\*d^6) - 3/2\*a\*c^2\*x/(sqrt(d^2\*x^2 - c^2)\*d^4) + 15/8\*b\*c^4\*log(2\*d^2\*x + 2\*sqrt(d^2\*x^2 - c^2)\*sqrt(d^2))/(sqrt(d^2)\*d^6) + 3/2\*a\*c^2\*log(2\*d^2\*x + 2\*sqrt(d^2\*x^2 - c^2)\*sqrt(d^2))/(sqrt(d^2)\*d^4)

**Fricas [A]** time = 0.244879, size = 702, normalized size = 4.36

$$\frac{32bd^{10}x^{10} - 8bc^{10} - 8ac^8d^2 + 8(5bc^2d^8 + 8ad^{10})x^8 - 2(101bc^4d^6 + 72ac^2d^8)x^6 + (101bc^6d^4 + 36ac^4d^6)x^4 + (29bc^8d^2 - 8ac^6d^4)x^2 - 2(85b^2c^4d^5 + 5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^4/((d\*x + c)^(3/2)\*(d\*x - c)^(3/2)),x, algorithm="fricas")

[Out] -1/8\*(32\*b\*d^10\*x^10 - 8\*b\*c^10 - 8\*a\*c^8\*d^2 + 8\*(5\*b\*c^2\*d^8 + 8\*a\*d^10)\*x^8 - 2\*(101\*b\*c^4\*d^6 + 72\*a\*c^2\*d^8)\*x^6 + (101\*b\*c^6\*d^4 + 36\*a\*c^4\*d^6)\*x^4 + (29\*b\*c^8\*d^2 + 44\*a\*c^6\*d^4)\*x^2 - (32\*b\*d^9\*x^9 + 8\*(7\*b\*c^2\*d^7 + 8\*a\*d^9)\*x^7 - 2\*(85\*b\*c^4\*d^5 + 5

$$6*a*c^2*d^7)*x^5 + (25*b*c^6*d^3 - 12*a*c^4*d^5)*x^3 + (25*b*c^8*d + 28*a*c^6*d^3)*x)*\sqrt{d*x + c}*\sqrt{d*x - c} - 3*(5*b*c^10 + 4*a*c^8*d^2 - 16*(5*b*c^4*d^6 + 4*a*c^2*d^8)*x^6 + 28*(5*b*c^6*d^4 + 4*a*c^4*d^6)*x^4 - 13*(5*b*c^8*d^2 + 4*a*c^6*d^4)*x^2 + (16*(5*b*c^4*d^5 + 4*a*c^2*d^7)*x^5 - 20*(5*b*c^6*d^3 + 4*a*c^4*d^5)*x^3 + 5*(5*b*c^8*d + 4*a*c^6*d^3)*x)*\sqrt{d*x + c}*\sqrt{d*x - c))*\log(-d*x + \sqrt{d*x + c}*\sqrt{d*x - c}))/((16*d^13*x^6 - 28*c^2*d^11*x^4 + 13*c^4*d^9*x^2 - c^6*d^7 - (16*d^12*x^5 - 20*c^2*d^10*x^3 + 5*c^4*d^8*x)*\sqrt{d*x + c}*\sqrt{d*x - c}))$$

**Sympy [A]** time = 102.515, size = 233, normalized size = 1.45

$$a \left( \frac{c^2 G_{6,6}^{6,2} \left( \begin{matrix} -\frac{5}{4}, -\frac{3}{4} & -\frac{3}{2}, -\frac{1}{2}, 0, 1 \\ -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0, 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} d^5} \right. \\ \left. + \frac{ic^2 G_{6,6}^{2,6} \left( \begin{matrix} -\frac{5}{2}, -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, 1 \\ -\frac{7}{4}, -\frac{5}{4} & -\frac{5}{2}, -2, -1, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} d^5} \right) \\ + b \left( \frac{c^4 G_{6,6}^{6,2} \left( \begin{matrix} -\frac{9}{4}, -\frac{7}{4} & -\frac{5}{2}, -\frac{3}{2}, -1, 1 \\ -\frac{9}{4}, -2, -\frac{7}{4}, -\frac{3}{2}, -1, 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} d^7} \right. \\ \left. + \frac{ic^4 G_{6,6}^{2,6} \left( \begin{matrix} -\frac{7}{2}, -3, -\frac{11}{4}, -\frac{5}{2}, -\frac{9}{4}, 1 \\ -\frac{11}{4}, -\frac{9}{4} & -\frac{7}{2}, -3, -2, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} d^7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(b\*x\*\*2+a)/(d\*x-c)\*\*(3/2)/(d\*x+c)\*\*(3/2),x)

[Out] a\*(c\*\*2\*meijerg((( -5/4, -3/4), (-3/2, -1/2, 0, 1)), (( -5/4, -1, -3/4, -1/2, 0, 0), ()), c\*\*2/(d\*\*2\*x\*\*2))/(2\*pi\*\*(3/2)\*d\*\*5) + I\*c\*\*2\*meijerg((( -5/2, -2, -7/4, -3/2, -5/4, 1), ()), (( -7/4, -5/4), (-5/2, -2, -1, 0)), c\*\*2\*exp\_polar(2\*I\*pi)/(d\*\*2\*x\*\*2))/(2\*pi\*\*(3/2)\*d\*\*5) + b\*(c\*\*4\*meijerg((( -9/4, -7/4), (-5/2, -3/2, -1, 1)), (( -9/4, -2, -7/4, -3/2, -1, 0), ()), c\*\*2/(d\*\*2\*x\*\*2))/(2\*pi\*\*(3/2)\*d\*\*7) + I\*c\*\*4\*meijerg((( -7/2, -3, -11/4, -5/2, -9/4, 1), ()), (( -11/4, -9/4), (-7/2, -3, -2, 0)), c\*\*2\*exp\_polar(2\*I\*pi)/(d\*\*2\*x\*\*2))/(2\*pi\*\*(3/2)\*d\*\*7)

**GIAC/XCAS [A]** time = 0.243284, size = 290, normalized size = 1.8

$$-\frac{1}{688128} (5bc^3d^{35} + 4acd^{37}) \ln \left( \left( \sqrt{dx+c} - \sqrt{dx-c} \right)^2 \right) \\ \frac{\left( \left( \left( 2 \left( 5bd^{35} - \frac{(dx+c)bd^{35}}{c} \right) (dx+c) - \frac{25bc^2d^{35}+4ad^{37}}{c} \right) (dx+c) + \frac{35bc^3d^{35}+12acd^{37}}{c} \right) (dx+c) - \frac{2(7bc^4d^{35}+2ac^2d^{37})}{c} \right) \sqrt{dx+c}}{2064384 \sqrt{dx-c}} \\ - \frac{2(bc^5 + ac^3d^2)}{\left( \left( \sqrt{dx+c} - \sqrt{dx-c} \right)^2 + 2c \right) d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^4/((d\*x + c)^(3/2)\*(d\*x - c)^(3/2)),x, algorithm="giac")

[Out] 
$$-1/688128*(5*b*c^3*d^35 + 4*a*c*d^37)*\ln((\sqrt{d*x + c} - \sqrt{d*x - c})^2) - 1/2064384*(((2*(5*b*d^35 - (d*x + c)*b*d^35/c)*(d*x + c) - (25*b*c^2*d^35 + 4*a*d^37)/c)*(d*x + c) + (35*b*c^3*d^35 + 12*a*c*d^37)/c)*(d*x + c) - 2*(7*b*c^4*d^35 + 2*a*c^2*d^37)/c)*\sqrt{d*x + c}/\sqrt{d*x - c} - 2*(b*c^5 + a*c^3*d^2)/(((\sqrt{d*x + c} - \sqrt{d*x - c})^2 + 2*c)*d^7)$$

$$3.270 \quad \int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=115

$$\frac{2\sqrt{dx-c}\sqrt{c+dx}(3ad^2+4bc^2)}{3d^6} - \frac{x^2(3ad^2+4bc^2)}{3d^4\sqrt{dx-c}\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

[Out]  $-\left(\frac{4b^2c^2+3a^2d^2}{3d^4}\sqrt{-c+dx}\sqrt{c+dx}\right) + \frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{2(4b^2c^2+3a^2d^2)\sqrt{-c+dx}\sqrt{c+dx}}{3d^6}$

**Rubi [A]** time = 0.316393, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$\frac{2\sqrt{dx-c}\sqrt{c+dx}(3ad^2+4bc^2)}{3d^6} - \frac{x^2(3ad^2+4bc^2)}{3d^4\sqrt{dx-c}\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a+b\*x^2))/((-c+d\*x)^(3/2)\*(c+d\*x)^(3/2)),x]

[Out]  $-\left(\frac{4b^2c^2+3a^2d^2}{3d^4}\sqrt{-c+dx}\sqrt{c+dx}\right) + \frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{2(4b^2c^2+3a^2d^2)\sqrt{-c+dx}\sqrt{c+dx}}{3d^6}$

**Rubi in Sympy [A]** time = 18.6131, size = 102, normalized size = 0.89

$$\frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{x^2(3ad^2+4bc^2)}{3d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{2\sqrt{-c+dx}\sqrt{c+dx}(3ad^2+4bc^2)}{3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(b\*x\*\*2+a)/(d\*x-c)\*\*(3/2)/(d\*x+c)\*\*(3/2),x)

[Out]  $\frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{x^2(3ad^2+4bc^2)}{3d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{2\sqrt{-c+dx}\sqrt{c+dx}(3ad^2+4bc^2)}{3d^6}$

**Mathematica [A]** time = 0.0896928, size = 72, normalized size = 0.63

$$\frac{-6ac^2d^2+3ad^4x^2-8bc^4+4bc^2d^2x^2+bd^4x^4}{3d^6\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a+b\*x^2))/((-c+d\*x)^(3/2)\*(c+d\*x)^(3/2)),x]

[Out]  $\frac{-8b^2c^4-6a^2c^2d^2+4b^2c^2d^2x^2+3a^2d^4x^2+bd^4x^4}{3d^6\sqrt{-c+dx}\sqrt{c+dx}}$

**Maple [A]** time = 0.008, size = 68, normalized size = 0.6

$$\frac{-bd^4x^4-3ad^4x^2-4bc^2d^2x^2+6ac^2d^2+8bc^4}{3d^6} \frac{1}{\sqrt{dx+c}} \frac{1}{\sqrt{dx-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)`

[Out] 
$$-1/3*(-b*d^4*x^4-3*a*d^4*x^2-4*b*c^2*d^2*x^2+6*a*c^2*d^2+8*b*c^4)/(d*x+c)^(1/2)/d^6/(d*x-c)^(1/2)$$

**Maxima [A]** time = 1.40378, size = 166, normalized size = 1.44

$$\frac{bx^4}{3\sqrt{d^2x^2-c^2d^2}} + \frac{4bc^2x^2}{3\sqrt{d^2x^2-c^2d^4}} + \frac{ax^2}{\sqrt{d^2x^2-c^2d^2}} - \frac{8bc^4}{3\sqrt{d^2x^2-c^2d^6}} - \frac{2ac^2}{\sqrt{d^2x^2-c^2d^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*x^3/((d*x+c)^(3/2)*(d*x-c)^(3/2)),x,algorithm="maxima")`

[Out] 
$$1/3*b*x^4/(\sqrt{d^2*x^2-c^2}*d^2) + 4/3*b*c^2*x^2/(\sqrt{d^2*x^2-c^2}*d^4) + a*x^2/(\sqrt{d^2*x^2-c^2}*d^2) - 8/3*b*c^4/(\sqrt{d^2*x^2-c^2}*d^6) - 2*a*c^2/(\sqrt{d^2*x^2-c^2}*d^4)$$

**Fricas [A]** time = 0.254866, size = 354, normalized size = 3.08

$$\frac{8bd^8x^8 - 8bc^8 - 6ac^6d^2 + 24(bc^2d^6 + ad^8)x^6 - (95bc^4d^4 + 72ac^2d^6)x^4 + 17(4bc^6d^2 + 3ac^4d^4)x^2 - 4(2bd^7x^7 + (7bc^7d^5 + 7b^2c^5d^3)x^5 + (7b^2c^5d^3 + 7b^2c^5d^3)x^3 + 7b^2c^5d^3)x}{3(8d^{11}x^5 - 12c^2d^9x^3 + 4c^4d^7x - (8d^{10}x^4 - 8c^2d^8x^2 + c^4d^6))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*x^3/((d*x+c)^(3/2)*(d*x-c)^(3/2)),x,algorithm="fricas")`

[Out] 
$$-1/3*(8*b*d^8*x^8 - 8*b*c^8 - 6*a*c^6*d^2 + 24*(b*c^2*d^6 + a*d^8)*x^6 - (95*b*c^4*d^4 + 72*a*c^2*d^6)*x^4 + 17*(4*b*c^6*d^2 + 3*a*c^4*d^4)*x^2 - 4*(2*b*d^7*x^7 + (7*b*c^2*d^5 + 6*a*d^7)*x^5 - 5*(4*b*c^4*d^3 + 3*a*c^2*d^5)*x^3 + 2*(4*b*c^6*d + 3*a*c^4*d^3)*x)*\sqrt{d*x+c}*\sqrt{d*x-c})/(8*d^{11}*x^5 - 12*c^2*d^9*x^3 + 4*c^4*d^7*x - (8*d^{10}*x^4 - 8*c^2*d^8*x^2 + c^4*d^6)*\sqrt{d*x+c}*\sqrt{d*x-c})$$

**Sympy [A]** time = 82.2935, size = 226, normalized size = 1.97

$$\begin{aligned}
 & a \left( \frac{c G_{6,6}^{6,2} \left( \begin{array}{c} -\frac{3}{4}, -\frac{1}{4} \\ -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{2}, 0 \end{array} \right) \left| \frac{c^2}{d^2 x^2} \right.}{2\pi^{\frac{3}{2}} d^4} \right. \\
 & \quad \left. - \frac{ic G_{6,6}^{2,6} \left( \begin{array}{c} -2, -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{array} \right) \left| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right.}{2\pi^{\frac{3}{2}} d^4} \right) \\
 & + b \left( \frac{c^3 G_{6,6}^{6,2} \left( \begin{array}{c} -\frac{7}{4}, -\frac{5}{4} \\ -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{1}{2}, 0 \end{array} \right) \left| \frac{c^2}{d^2 x^2} \right.}{2\pi^{\frac{3}{2}} d^6} \right. \\
 & \quad \left. - \frac{ic^3 G_{6,6}^{2,6} \left( \begin{array}{c} -3, -\frac{5}{2}, -\frac{9}{4}, -2, -\frac{7}{4}, 1 \\ -\frac{9}{4}, -\frac{7}{4} \end{array} \right) \left| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right.}{2\pi^{\frac{3}{2}} d^6} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)`

[Out] `a*(c*meijerg(((3/4, 1/4), (-1, 0, 1/2, 1)), ((3/4, 1/2, -1/4, 0, 1/2, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**4) - I*c*meijerg(((2, -3/2, -5/4, -1, -3/4, 1), ()), ((5/4, -3/4), (-2, -3/2, -1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**4) + b*(c**3*meijerg(((7/4, 5/4), (-2, -1, -1/2, 1)), ((7/4, -3/2, -5/4, -1, -1/2, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**6) - I*c**3*meijerg(((3, 5/2, 9/4, -2, 7/4, 1), ()), ((9/4, -7/4), (-3, -5/2, -3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**6)`

**GIAC/XCAS [A]** time = 0.236065, size = 194, normalized size = 1.69

$$\begin{aligned}
 & \frac{\left( 2 \left( \left( 4bd^{24} - \frac{(dx+c)bd^{24}}{c} \right) (dx+c) - \frac{10bc^2d^{24}+3ad^{26}}{c} \right) (dx+c) + \frac{3(9bc^3d^{24}+5acd^{26})}{c} \right) \sqrt{dx+c}}{23040 \sqrt{dx-c}} \\
 & + \frac{2(bc^4 + ac^2d^2)}{\left( (\sqrt{dx+c} - \sqrt{dx-c})^2 + 2c \right) d^6}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^3/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="giac")`

[Out] `-1/23040*(2*((4*b*d^24 - (d*x + c)*b*d^24/c)*(d*x + c) - (10*b*c^2*d^24 + 3*a*d^26)/c)*(d*x + c) + 3*(9*b*c^3*d^24 + 5*a*c*d^26)/c)*sqrt(d*x + c)/sqrt(d*x - c) + 2*(b*c^4 + a*c^2*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*d^6)`



$$3.271 \quad \int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=152

$$-\frac{\sqrt{dx-c}(2ad^2+3bc^2)}{2d^5\sqrt{c+dx}} - \frac{c(2ad^2+3bc^2)}{2d^5\sqrt{dx-c}\sqrt{c+dx}} + \frac{(2ad^2+3bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^5} + \frac{bx^3}{2d^2\sqrt{dx-c}\sqrt{c+dx}}$$

[Out]  $-(c*(3*b*c^2 + 2*a*d^2))/(2*d^5*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) + (b*x^3)/(2*d^2*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) - ((3*b*c^2 + 2*a*d^2)*\text{Sqrt}[-c + d*x])/(2*d^5*\text{Sqrt}[c + d*x]) + ((3*b*c^2 + 2*a*d^2)*\text{ArcTanh}[\text{Sqrt}[-c + d*x]/\text{Sqrt}[c + d*x]])/d^5$

**Rubi [A]** time = 0.380678, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$

$$-\frac{\sqrt{dx-c}(2ad^2+3bc^2)}{2d^5\sqrt{c+dx}} - \frac{c(2ad^2+3bc^2)}{2d^5\sqrt{dx-c}\sqrt{c+dx}} + \frac{(2ad^2+3bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^5} + \frac{bx^3}{2d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)), x]$

[Out]  $-(c*(3*b*c^2 + 2*a*d^2))/(2*d^5*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) + (b*x^3)/(2*d^2*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) - ((3*b*c^2 + 2*a*d^2)*\text{Sqrt}[-c + d*x])/(2*d^5*\text{Sqrt}[c + d*x]) + ((3*b*c^2 + 2*a*d^2)*\text{ArcTanh}[\text{Sqrt}[-c + d*x]/\text{Sqrt}[c + d*x]])/d^5$

**Rubi in Sympy [A]** time = 24.6745, size = 134, normalized size = 0.88

$$\frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{c(2ad^2+3bc^2)}{2d^5\sqrt{-c+dx}\sqrt{c+dx}} - \frac{\sqrt{-c+dx}(2ad^2+3bc^2)}{2d^5\sqrt{c+dx}} + \frac{(2ad^2+3bc^2)\text{atanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**2}*(b*x^{**2}+a)/(d*x-c)^{(3/2)/(d*x+c)^{(3/2)}, x)$

[Out]  $b*x^{**3}/(2*d^{**2}*\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)) - c*(2*a*d^{**2} + 3*b*c^{**2})/(2*d^{**5}*\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)) - \text{sqrt}(-c + d*x)*(2*a*d^{**2} + 3*b*c^{**2})/(2*d^{**5}*\text{sqrt}(c + d*x)) + (2*a*d^{**2} + 3*b*c^{**2})*\text{atanh}(\text{sqrt}(-c + d*x)/\text{sqrt}(c + d*x))/d^{**5}$

**Mathematica [A]** time = 0.16081, size = 121, normalized size = 0.8

$$\frac{dx\sqrt{dx-c}\sqrt{c+dx}(-2ad^2-3bc^2+bd^2x^2) - (c^2-d^2x^2)(2ad^2+3bc^2)\log(\sqrt{dx-c}\sqrt{c+dx}+dx)}{2d^5(dx-c)(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x^2*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)), x]$

[Out]  $(d*x*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]*(-3*b*c^2 - 2*a*d^2 + b*d^2*x^2) - (3*b*c^2 + 2*a*d^2)*(c^2 - d^2*x^2)*\text{Log}[d*x + \text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]])/(2*d^5*(-c + d*x)*(c + d*x))$

**Maple [C]** time = 0.033, size = 254, normalized size = 1.7

$$\frac{\operatorname{csgn}(d)}{2d^5} \left( \operatorname{csgn}(d) x^3 b d^3 \sqrt{d^2 x^2 - c^2} + 2 \ln \left( \left( \operatorname{csgn}(d) \sqrt{d^2 x^2 - c^2} + dx \right) \operatorname{csgn}(d) \right) x^2 a d^4 + 3 \ln \left( \left( \operatorname{csgn}(d) \sqrt{d^2 x^2 - c^2} + dx \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(b\*x^2+a)/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x)

[Out] 1/2\*(csgn(d)\*x^3\*b\*d^3\*(d^2\*x^2-c^2)^(1/2)+2\*ln((csgn(d)\*(d^2\*x^2-c^2)^(1/2)+d\*x)\*csgn(d))\*x^2\*a\*d^4+3\*ln((csgn(d)\*(d^2\*x^2-c^2)^(1/2)+d\*x)\*csgn(d))\*x^2\*b\*c^2\*d^2-2\*a\*x\*(d^2\*x^2-c^2)^(1/2)\*d^3\*csgn(d)-3\*b\*c^2\*x\*(d^2\*x^2-c^2)^(1/2)\*csgn(d)\*d-2\*a\*c^2\*ln((csgn(d)\*(d^2\*x^2-c^2)^(1/2)+d\*x)\*csgn(d))\*d^2-3\*b\*c^4\*ln((csgn(d)\*(d^2\*x^2-c^2)^(1/2)+d\*x)\*csgn(d)))\*csgn(d)/(d^2\*x^2-c^2)^(1/2)/d^5/(d\*x+c)^(1/2)/(d\*x-c)^(1/2)

**Maxima [A]** time = 1.41408, size = 211, normalized size = 1.39

$$\frac{bx^3}{2\sqrt{d^2x^2 - c^2}d^2} - \frac{3bc^2x}{2\sqrt{d^2x^2 - c^2}d^4} - \frac{ax}{\sqrt{d^2x^2 - c^2}d^2} + \frac{3bc^2 \log\left(2d^2x + 2\sqrt{d^2x^2 - c^2}\sqrt{d^2}\right)}{2\sqrt{d^2}d^4} + \frac{a \log\left(2d^2x + 2\sqrt{d^2x^2 - c^2}\sqrt{d^2}\right)}{\sqrt{d^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^2/((d\*x + c)^(3/2)\*(d\*x - c)^(3/2)),x, algorithm="maxima")

[Out] 1/2\*b\*x^3/(sqrt(d^2\*x^2 - c^2)\*d^2) - 3/2\*b\*c^2\*x/(sqrt(d^2\*x^2 - c^2)\*d^4) - a\*x/(sqrt(d^2\*x^2 - c^2)\*d^2) + 3/2\*b\*c^2\*log(2\*d^2\*x + 2\*sqrt(d^2\*x^2 - c^2)\*sqrt(d^2))/(sqrt(d^2)\*d^4) + a\*log(2\*d^2\*x + 2\*sqrt(d^2\*x^2 - c^2)\*sqrt(d^2))/(sqrt(d^2)\*d^2)

**Fricas [A]** time = 0.246244, size = 448, normalized size = 2.95

$$\frac{4bd^6x^6 - 7bc^2d^4x^4 + 2bc^6 + 2ac^4d^2 - (bc^4d^2 + 4ac^2d^4)x^2 - (4bd^5x^5 - 5bc^2d^3x^3 - (3bc^4d + 4ac^2d^3)x)\sqrt{dx + c}\sqrt{dx - c}}{2(4d^9x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^2/((d\*x + c)^(3/2)\*(d\*x - c)^(3/2)),x, algorithm="fricas")

[Out] -1/2\*(4\*b\*d^6\*x^6 - 7\*b\*c^2\*d^4\*x^4 + 2\*b\*c^6 + 2\*a\*c^4\*d^2 - (b\*c^4\*d^2 + 4\*a\*c^2\*d^4)\*x^2 - (4\*b\*d^5\*x^5 - 5\*b\*c^2\*d^3\*x^3 - (3\*b\*c^4\*d + 4\*a\*c^2\*d^3)\*x)\*sqrt(d\*x + c)\*sqrt(d\*x - c) + (3\*b\*c^6 + 2\*a\*c^4\*d^2 + 4\*(3\*b\*c^2\*d^4 + 2\*a\*d^6)\*x^4 - 5\*(3\*b\*c^4\*d^2 + 2\*a\*c^2\*d^4)\*x^2 - (4\*(3\*b\*c^2\*d^4 + 2\*a\*d^6)\*x^3 - 3\*(3\*b\*c^4\*d + 2\*a\*c^2\*d^3)\*x)\*sqrt(d\*x + c)\*sqrt(d\*x - c))\*log(-d\*x + sqrt(d\*x + c)\*sqrt(d\*x - c)))/(4\*d^9\*x^4 - 5\*c^2\*d^7\*x^2 + c^4\*d^5 - (4\*d^8\*x^3 - 3\*c^2\*d^6\*x)\*sqrt(d\*x + c)\*sqrt(d\*x - c))

**Sympy [A]** time = 80.9724, size = 212, normalized size = 1.39

$$\begin{aligned}
 & a \left( \frac{G_{6,6}^{6,2} \left( \begin{matrix} -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, \frac{1}{2}, 1, 1 \\ -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1, 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} d^3} + \frac{iG_{6,6}^{2,6} \left( \begin{matrix} -\frac{3}{2}, -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 1 \\ -\frac{3}{4}, -\frac{1}{4} & -\frac{3}{2}, -1, 0, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} d^3} \right) \\
 & + b \left( \frac{c^2 G_{6,6}^{6,2} \left( \begin{matrix} -\frac{5}{4}, -\frac{3}{4} & -\frac{3}{2}, -\frac{1}{2}, 0, 1 \\ -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0, 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} d^5} \right. \\
 & \left. + \frac{ic^2 G_{6,6}^{2,6} \left( \begin{matrix} -\frac{5}{2}, -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, 1 \\ -\frac{7}{4}, -\frac{5}{4} & -\frac{5}{2}, -2, -1, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} d^5} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x\*\*2+a)/(d\*x-c)\*\*(3/2)/(d\*x+c)\*\*(3/2),x)

[Out] a\*(meijerg(((−1/4, 1/4), (−1/2, 1/2, 1, 1)), ((−1/4, 0, 1/4, 1/2, 1, 0), ()), c\*\*2/(d\*\*2\*x\*\*2))/(2\*pi\*\*(3/2)\*d\*\*3) + I\*meijerg(((−3/2, −1, −3/4, −1/2, −1/4, 1), ()), ((−3/4, −1/4), (−3/2, −1, 0, 0)), c\*\*2\*exp\_polar(2\*I\*pi)/(d\*\*2\*x\*\*2))/(2\*pi\*\*(3/2)\*d\*\*3) + b\*(c\*\*2\*meijerg(((−5/4, −3/4), (−3/2, −1/2, 0, 1)), ((−5/4, −1, −3/4, −1/2, 0, 0), ()), c\*\*2/(d\*\*2\*x\*\*2))/(2\*pi\*\*(3/2)\*d\*\*5) + I\*c\*\*2\*meijerg(((−5/2, −2, −7/4, −3/2, −5/4, 1), ()), ((−7/4, −5/4), (−5/2, −2, −1, 0)), c\*\*2\*exp\_polar(2\*I\*pi)/(d\*\*2\*x\*\*2))/(2\*pi\*\*(3/2)\*d\*\*5))

**GIAC/XCAS [A]** time = 0.241076, size = 208, normalized size = 1.37

$$\begin{aligned}
 & \frac{\left( \left( 3bd^{15} - \frac{(dx+c)bd^{15}}{c} \right) (dx+c) - \frac{bc^2d^{15}-ad^{17}}{c} \right) \sqrt{dx+c}}{768\sqrt{dx-c}} \\
 & - \frac{(3bc^2d^{15} + 2ad^{17}) \ln\left( \left( \sqrt{dx+c} - \sqrt{dx-c} \right)^2 \right)}{768c} - \frac{2(bc^3 + acd^2)}{\left( \left( \sqrt{dx+c} - \sqrt{dx-c} \right)^2 + 2c \right) d^5}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^2/((d\*x + c)^(3/2)\*(d\*x - c)^(3/2)),x, algorithm="giac")

[Out] -1/768\*((3\*b\*d^15 - (d\*x + c)\*b\*d^15/c)\*(d\*x + c) - (b\*c^2\*d^15 - a\*d^17)/c)\*sqrt(d\*x + c)/sqrt(d\*x - c) - 1/768\*(3\*b\*c^2\*d^15 + 2\*a\*d^17)\*ln((sqrt(d\*x + c) - sqrt(d\*x - c))^2)/c - 2\*(b\*c^3 + a\*c\*d^2)/(((sqrt(d\*x + c) - sqrt(d\*x - c))^2 + 2\*c)\*d^5)

$$3.272 \quad \int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=76

$$\frac{\sqrt{dx-c}\sqrt{c+dx}(ad^2+2bc^2)}{c^2d^4} - \frac{x^2\left(\frac{a}{c^2} + \frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

[Out]  $-\left(\left(\frac{a}{c^2} + \frac{b}{d^2}\right)x^2\right)/\left(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]\right) + \left(\left(2*b*c^2 + a*d^2\right)*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]\right)/\left(c^2*d^4\right)$

**Rubi [A]** time = 0.181657, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{\sqrt{dx-c}\sqrt{c+dx}(ad^2+2bc^2)}{c^2d^4} - \frac{x^2\left(\frac{a}{c^2} + \frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}\left[\frac{x*(a + b*x^2)}{((-c + d*x)^{(3/2)}*(c + d*x)^{(3/2))}, x\right]$

[Out]  $-\left(\left(\frac{a}{c^2} + \frac{b}{d^2}\right)x^2\right)/\left(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]\right) + \left(\left(2*b*c^2 + a*d^2\right)*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]\right)/\left(c^2*d^4\right)$

**Rubi in Sympy [A]** time = 11.3348, size = 65, normalized size = 0.86

$$-\frac{x^2\left(\frac{a}{c^2} + \frac{b}{d^2}\right)}{\sqrt{-c + dx}\sqrt{c + dx}} + \frac{\sqrt{-c + dx}\sqrt{c + dx}(ad^2 + 2bc^2)}{c^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x*(b*x^2+a)/(d*x-c)^{(3/2)/(d*x+c)^{(3/2)}, x)$

[Out]  $-x^2*(a/c^2 + b/d^2)/(\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)) + \text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)*(a*d^2 + 2*b*c^2)/(c^2*d^4)$

**Mathematica [A]** time = 0.07601, size = 45, normalized size = 0.59

$$\frac{-ad^2 - 2bc^2 + bd^2x^2}{d^4\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}\left[\frac{x*(a + b*x^2)}{((-c + d*x)^{(3/2)}*(c + d*x)^{(3/2))}, x\right]$

[Out]  $(-2*b*c^2 - a*d^2 + b*d^2*x^2)/(d^4*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])$

**Maple [A]** time = 0.005, size = 43, normalized size = 0.6

$$-\frac{-bd^2x^2 + ad^2 + 2bc^2}{d^4} \frac{1}{\sqrt{dx+c}} \frac{1}{\sqrt{dx-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)`

[Out]  $-(b*d^2*x^2+a*d^2+2*b*c^2)/(d*x+c)^(1/2)/d^4/(d*x-c)^(1/2)$

**Maxima [A]** time = 1.38135, size = 93, normalized size = 1.22

$$\frac{bx^2}{\sqrt{d^2x^2 - c^2d^2}} - \frac{2bc^2}{\sqrt{d^2x^2 - c^2d^4}} - \frac{a}{\sqrt{d^2x^2 - c^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="maxima")`

[Out]  $b*x^2/(\sqrt{d^2*x^2 - c^2}*d^2) - 2*b*c^2/(\sqrt{d^2*x^2 - c^2}*d^4) - a/(\sqrt{d^2*x^2 - c^2}*d^2)$

**Fricas [A]** time = 0.247176, size = 196, normalized size = 2.58

$$\frac{2bd^4x^4 + 2bc^4 + ac^2d^2 - (5bc^2d^2 + 2ad^4)x^2 - 2(bd^3x^3 - (2bc^2d + ad^3)x)\sqrt{dx + c}\sqrt{dx - c}}{2d^7x^3 - 2c^2d^5x - (2d^6x^2 - c^2d^4)\sqrt{dx + c}\sqrt{dx - c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="fricas")`

[Out]  $-(2*b*d^4*x^4 + 2*b*c^4 + a*c^2*d^2 - (5*b*c^2*d^2 + 2*a*d^4)*x^2 - 2*(b*d^3*x^3 - (2*b*c^2*d + a*d^3)*x)*\sqrt{d*x + c}*\sqrt{d*x - c})/(2*d^7*x^3 - 2*c^2*d^5*x - (2*d^6*x^2 - c^2*d^4)*\sqrt{d*x + c}*\sqrt{d*x - c})$

**Sympy [A]** time = 71.8296, size = 201, normalized size = 2.64

$$a \left( \frac{G_{6,6}^{5,3} \left( \begin{matrix} \frac{1}{4}, \frac{3}{4}, 1 \\ \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2} \end{matrix} \middle| \frac{c^2}{d^2x^2} \right) - iG_{6,6}^{2,6} \left( \begin{matrix} -1, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, 1 \\ -\frac{1}{4}, \frac{1}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2x^2} \right)}{2\pi^{\frac{3}{2}}cd^2} - \frac{iG_{6,6}^{2,6} \left( \begin{matrix} -1, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, 1 \\ -\frac{1}{4}, \frac{1}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2x^2} \right)}{2\pi^{\frac{3}{2}}cd^2} \right) + b \left( \frac{cG_{6,6}^{6,2} \left( \begin{matrix} -\frac{3}{4}, -\frac{1}{4} \\ -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{2}, 0 \end{matrix} \middle| \frac{c^2}{d^2x^2} \right) - iG_{6,6}^{2,6} \left( \begin{matrix} -2, -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2x^2} \right)}{2\pi^{\frac{3}{2}}d^4} - \frac{iG_{6,6}^{2,6} \left( \begin{matrix} -2, -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2x^2} \right)}{2\pi^{\frac{3}{2}}d^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)`

[Out]  $a*(-\text{meijerg}(((1/4, 3/4, 1), (0, 1, 3/2)), ((1/4, 1/2, 3/4, 1, 3/2), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c*d**2) - I*\text{meijerg}((-1,$

```
, -1/2, -1/4, 0, 1/4, 1), ()), ((-1/4, 1/4), (-1, -1/2, 1/2, 0)),
c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c*d**2)) + b*(c
*meijerg((-3/4, -1/4), (-1, 0, 1/2, 1)), ((-3/4, -1/2, -1/4, 0,
1/2, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**4) - I*c*meijerg(
((-2, -3/2, -5/4, -1, -3/4, 1), ()), ((-5/4, -3/4), (-2, -3/2, -1
/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**4))
```

**GIAC/XCAS [A]** time = 0.22925, size = 127, normalized size = 1.67

$$\frac{\left(2(dx+c)bd^8 - \frac{5bc^2d^8+ad^{10}}{c}\right)\sqrt{dx+c}}{32\sqrt{dx-c}} + \frac{2(bc^2+ad^2)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2c\right)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="giac")
```

```
[Out] 1/32*(2*(d*x + c)*b*d^8 - (5*b*c^2*d^8 + a*d^10)/c)*sqrt(d*x + c)
/sqrt(d*x - c) + 2*(b*c^2 + a*d^2)/(((sqrt(d*x + c) - sqrt(d*x -
c))^2 + 2*c)*d^4)
```

$$3.273 \quad \int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=63

$$\frac{2b \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3} - \frac{x\left(\frac{a}{c^2} + \frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

[Out] -(((a/c^2 + b/d^2)\*x)/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])) + (2\*b\*ArcTanh[Sqrt[-c + d\*x]/Sqrt[c + d\*x]])/d^3

**Rubi [A]** time = 0.114977, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$

$$\frac{2b \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3} - \frac{x\left(\frac{a}{c^2} + \frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/((-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)), x]

[Out] -(((a/c^2 + b/d^2)\*x)/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])) + (2\*b\*ArcTanh[Sqrt[-c + d\*x]/Sqrt[c + d\*x]])/d^3

**Rubi in Sympy [A]** time = 21.0818, size = 94, normalized size = 1.49

$$-\frac{ax}{c^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{bc}{d^3\sqrt{-c+dx}\sqrt{c+dx}} - \frac{b\sqrt{-c+dx}}{d^3\sqrt{c+dx}} + \frac{2b \operatorname{atanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/(d\*x-c)\*\*(3/2)/(d\*x+c)\*\*(3/2), x)

[Out] -a\*x/(c\*\*2\*sqrt(-c + d\*x)\*sqrt(c + d\*x)) - b\*c/(d\*\*3\*sqrt(-c + d\*x)\*sqrt(c + d\*x)) - b\*sqrt(-c + d\*x)/(d\*\*3\*sqrt(c + d\*x)) + 2\*b\*a\*tanh(sqrt(-c + d\*x)/sqrt(c + d\*x))/d\*\*3

**Mathematica [A]** time = 0.0934242, size = 100, normalized size = 1.59

$$\frac{dx\sqrt{dx-c}\sqrt{c+dx}(ad^2+bc^2)+bc^2(c^2-d^2x^2)\log\left(\sqrt{dx-c}\sqrt{c+dx}+dx\right)}{c^2d^3(c-dx)(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/((-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)), x]

[Out] (d\*(b\*c^2 + a\*d^2)\*x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x] + b\*c^2\*(c^2 - d^2\*x^2)\*Log[d\*x + Sqrt[-c + d\*x]\*Sqrt[c + d\*x]])/(c^2\*d^3\*(c - d\*x)\*(c + d\*x))

**Maple [C]** time = 0.027, size = 160, normalized size = 2.5

$$\frac{\operatorname{csgn}(d)}{c^2d^3} \left( \ln \left( \left( \operatorname{csgn}(d) \sqrt{d^2x^2 - c^2} + dx \right) \operatorname{csgn}(d) \right) x^2bc^2d^2 - ax\sqrt{d^2x^2 - c^2}d^3 \operatorname{csgn}(d) - bc^2x\sqrt{d^2x^2 - c^2} \operatorname{csgn}(d) d - bc^4 \ln \left( \right) \right)$$





```
rg((-1/4, 1/4), (-1/2, 1/2, 1, 1)), ((-1/4, 0, 1/4, 1/2, 1, 0),
()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**3) + I*meijerg((-3/2, -1,
-3/4, -1/2, -1/4, 1), ()), ((-3/4, -1/4), (-3/2, -1, 0, 0)), c**
2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**3))
```

**GIAC/XCAS [A]** time = 0.231704, size = 153, normalized size = 2.43

$$-\frac{b \ln\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{d^3} - \frac{2(bc^2+ad^2)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2c\right)cd^3} - \frac{(bc^2d^3+ad^5)\sqrt{dx+c}}{2\sqrt{dx-c}c^2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="giac")
```

```
[Out] -b*ln((sqrt(d*x + c) - sqrt(d*x - c))^2)/d^3 - 2*(b*c^2 + a*d^2)/
(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c*d^3) - 1/2*(b*c^2*d^
3 + a*d^5)*sqrt(d*x + c)/(sqrt(d*x - c)*c^2*d^6)
```

$$3.274 \quad \int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=65

$$-\frac{\frac{a}{c^2} + \frac{b}{d^2}}{\sqrt{dx-c}\sqrt{c+dx}} - \frac{a \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c^3}$$

[Out]  $-\left(\frac{a/c^2 + b/d^2}{\text{Sqrt}[-c + d*x] * \text{Sqrt}[c + d*x]}\right) - \left(\frac{a * \text{ArcTan}[\text{Sqrt}[-c + d*x] * \text{Sqrt}[c + d*x]/c]}{c^3}\right)$

**Rubi [A]** time = 0.266444, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$-\frac{\frac{a}{c^2} + \frac{b}{d^2}}{\sqrt{dx-c}\sqrt{c+dx}} - \frac{a \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)), x]

[Out]  $-\left(\frac{a/c^2 + b/d^2}{\text{Sqrt}[-c + d*x] * \text{Sqrt}[c + d*x]}\right) - \left(\frac{a * \text{ArcTan}[\text{Sqrt}[-c + d*x] * \text{Sqrt}[c + d*x]/c]}{c^3}\right)$

**Rubi in Sympy [A]** time = 14.2202, size = 53, normalized size = 0.82

$$-\frac{a \operatorname{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{c^3} - \frac{\frac{a}{c^2} + \frac{b}{d^2}}{\sqrt{-c+dx}\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x/(d\*x-c)\*\*(3/2)/(d\*x+c)\*\*(3/2), x)

[Out]  $-a * \operatorname{atan}(\operatorname{sqrt}(-c + d*x) * \operatorname{sqrt}(c + d*x)/c)/c**3 - (a/c**2 + b/d**2)/(\operatorname{sqrt}(-c + d*x) * \operatorname{sqrt}(c + d*x))$

**Mathematica [A]** time = 0.15873, size = 96, normalized size = 1.48

$$\frac{c\sqrt{dx-c}\sqrt{c+dx}(ad^2+bc^2)+ad^2(c^2-d^2x^2)\tan^{-1}\left(\frac{c}{\sqrt{dx-c}\sqrt{c+dx}}\right)}{c^3d^2(c-dx)(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)), x]

[Out]  $(c*(b*c^2 + a*d^2)*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x] + a*d^2*(c^2 - d^2*x^2)*\text{ArcTan}[c/(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])])/(c^3*d^2*(c - d*x)*(c + d*x))$

**Maple [B]** time = 0.034, size = 188, normalized size = 2.9

$$\frac{1}{c^2d^2} \left( \ln \left( -2 \frac{c^2 - \sqrt{-c^2}\sqrt{d^2x^2 - c^2}}{x} \right) x^2 ad^4 - ac^2 \ln \left( -2 \frac{c^2 - \sqrt{-c^2}\sqrt{d^2x^2 - c^2}}{x} \right) d^2 - a\sqrt{d^2x^2 - c^2}d^2\sqrt{-c^2} - bc^2\sqrt{-c^2}\sqrt{d^2x^2 - c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)`

[Out]  $\frac{1}{c^2} \ln(-2 \cdot (c^2 - (-c^2)^{1/2}) \cdot (d^2 x^2 - c^2)^{1/2}) / x \cdot x^2 \cdot a \cdot d^4 - a \cdot c^2 \cdot \ln(-2 \cdot (c^2 - (-c^2)^{1/2}) \cdot (d^2 x^2 - c^2)^{1/2}) / x \cdot d^4 - a \cdot (d^2 x^2 - c^2)^{1/2} \cdot d^2 \cdot (-c^2)^{1/2} - b \cdot c^2 \cdot (-c^2)^{1/2} \cdot (d^2 x^2 - c^2)^{1/2} / (-c^2)^{1/2} / (d^2 x^2 - c^2)^{1/2} / d^2 / (d \cdot x + c)^{1/2} / (d \cdot x - c)^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.242387, size = 223, normalized size = 3.43

$$\frac{(bc^3 + acd^2)\sqrt{dx+c}\sqrt{dx-c} - (bc^3d + acd^3)x + 2(ad^4x^2 - \sqrt{dx+c}\sqrt{dx-c}cad^3x - ac^2d^2) \arctan\left(\frac{-dx - \sqrt{dx+c}\sqrt{dx-c}}{c}\right)}{c^3d^4x^2 - \sqrt{dx+c}\sqrt{dx-c}c^3d^3x - c^5d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x),x, algorithm="fricas")`

[Out]  $-\left((b \cdot c^3 + a \cdot c \cdot d^2) \cdot \sqrt{d \cdot x + c} \cdot \sqrt{d \cdot x - c} - (b \cdot c^3 \cdot d + a \cdot c \cdot d^3) \cdot x + 2 \cdot (a \cdot d^4 \cdot x^2 - \sqrt{d \cdot x + c} \cdot \sqrt{d \cdot x - c} \cdot a \cdot d^3 \cdot x - a \cdot c \cdot d^2 \cdot d^2) \cdot \arctan\left(\frac{-d \cdot x - \sqrt{d \cdot x + c} \cdot \sqrt{d \cdot x - c}}{c}\right)\right) / (c^3 \cdot d^4 \cdot x^2 - \sqrt{d \cdot x + c} \cdot \sqrt{d \cdot x - c} \cdot c^3 \cdot d^3 \cdot x - c^5 \cdot d^2)$

**Sympy [A]** time = 93.7092, size = 172, normalized size = 2.65

$$a \left( \frac{G_{6,6}^{5,3} \left( \begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 \\ \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, \frac{5}{2} \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right) - i G_{6,6}^{2,6} \left( \begin{matrix} 0, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, 1 \\ \frac{3}{4}, \frac{5}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} c^3} \right) + b \left( \frac{G_{6,6}^{5,3} \left( \begin{matrix} \frac{1}{4}, \frac{3}{4}, 1 \\ \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2} \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right) - i G_{6,6}^{2,6} \left( \begin{matrix} -1, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, 1 \\ -\frac{1}{4}, \frac{1}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} c d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)`

[Out]  $a \cdot (-\text{meijerg}(((5/4, 7/4, 1), (1, 2, 5/2)), ((5/4, 3/2, 7/4, 2, 5/2), (0,)), c^{**2}/(d^{**2} \cdot x^{**2}))/ (2 \cdot \pi^{**}(3/2) \cdot c^{**3}) - \Gamma \cdot \text{meijerg}(((0, 1/2, 3/4, 1, 5/4, 1), ()), ((3/4, 5/4), (0, 1/2, 3/2, 0)), c^{**2} \cdot \exp_{\text{polar}}(2 \cdot \Gamma \cdot \pi) / (d^{**2} \cdot x^{**2})) / (2 \cdot \pi^{**}(3/2) \cdot c^{**3})) + b \cdot (-\text{meijerg}(((1/4, 3/4, 1), (0, 1, 3/2)), ((1/4, 1/2, 3/4, 1, 3/2), (0,)), c^{**2}$

$$\frac{1}{(d^2 x^2)} \frac{1}{(2\pi)^{3/2} c d^2} - I^* \text{meijerg}(\left(\left(-1, -1/2, -1/4, 0, 1/4, 1\right), \left(\right), \left(-1/4, 1/4\right), \left(-1, -1/2, 1/2, 0\right)\right), c^2 \exp_{\text{polar}}(2I\pi) / (d^2 x^2)) / (2\pi)^{3/2} c d^2)$$

**GIAC/XCAS [A]** time = 0.252711, size = 155, normalized size = 2.38

$$\frac{2a \arctan\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2c}\right)}{c^3} - \frac{(bc^2 + ad^2)\sqrt{dx+c}}{2\sqrt{dx-c}c^3d^2} + \frac{2(bc^2 + ad^2)}{\left((\sqrt{dx+c}-\sqrt{dx-c})^2 + 2c\right)c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/((d\*x + c)^(3/2)\*(d\*x - c)^(3/2)\*x),x, algorithm="giac")

[Out]  $2*a*\arctan(1/2*(\sqrt{d*x + c} - \sqrt{d*x - c})^2/c)/c^3 - 1/2*(b*c^2 + a*d^2)*\sqrt{d*x + c}/(\sqrt{d*x - c}*c^3*d^2) + 2*(b*c^2 + a*d^2)/(((\sqrt{d*x + c} - \sqrt{d*x - c})^2 + 2*c)*c^2*d^2)$

$$3.275 \quad \int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=67

$$\frac{a}{c^2x\sqrt{dx-c}\sqrt{c+dx}} - \frac{x(2ad^2+bc^2)}{c^4\sqrt{dx-c}\sqrt{c+dx}}$$

[Out]  $a/(c^2*x*\text{Sqrt}[-c+d*x]*\text{Sqrt}[c+d*x]) - ((b*c^2+2*a*d^2)*x)/(c^4*\text{Sqrt}[-c+d*x]*\text{Sqrt}[c+d*x])$

**Rubi [A]** time = 0.239412, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{a}{c^2x\sqrt{dx-c}\sqrt{c+dx}} - \frac{x(2ad^2+bc^2)}{c^4\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a+b*x^2)/(x^2*(-c+d*x)^{(3/2)}*(c+d*x)^{(3/2)}),x]$

[Out]  $a/(c^2*x*\text{Sqrt}[-c+d*x]*\text{Sqrt}[c+d*x]) - ((b*c^2+2*a*d^2)*x)/(c^4*\text{Sqrt}[-c+d*x]*\text{Sqrt}[c+d*x])$

**Rubi in Sympy [A]** time = 11.3995, size = 56, normalized size = 0.84

$$\frac{a}{c^2x\sqrt{-c+dx}\sqrt{c+dx}} - \frac{x(2ad^2+bc^2)}{c^4\sqrt{-c+dx}\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x**2+a)/x**2/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)$

[Out]  $a/(c**2*x*\text{sqrt}(-c+d*x)*\text{sqrt}(c+d*x)) - x*(2*a*d**2+b*c**2)/(c**4*\text{sqrt}(-c+d*x)*\text{sqrt}(c+d*x))$

**Mathematica [A]** time = 0.0849212, size = 51, normalized size = 0.76

$$\frac{a(c^2-2d^2x^2)-bc^2x^2}{c^4x\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a+b*x^2)/(x^2*(-c+d*x)^{(3/2)}*(c+d*x)^{(3/2)}),x]$

[Out]  $(-(b*c^2*x^2)+a*(c^2-2*d^2*x^2))/(c^4*x*\text{Sqrt}[-c+d*x]*\text{Sqrt}[c+d*x])$

**Maple [A]** time = 0.007, size = 48, normalized size = 0.7

$$\frac{-2ad^2x^2-bc^2x^2+ac^2}{xc^4} \frac{1}{\sqrt{dx+c}} \frac{1}{\sqrt{dx-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)`

[Out] `(-2*a*d^2*x^2-b*c^2*x^2+a*c^2)/(d*x+c)^(1/2)/x/c^4/(d*x-c)^(1/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/((d*x + c)^(3/2) * (d*x - c)^(3/2) * x^2),x, algorithm="maxima`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.237032, size = 117, normalized size = 1.75

$$\frac{bdx^2 - \sqrt{dx + c}\sqrt{dx - c}bx + ad}{2d^4x^4 - 2c^2d^2x^2 - (2d^3x^3 - c^2dx)\sqrt{dx + c}\sqrt{dx - c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/((d*x + c)^(3/2) * (d*x - c)^(3/2) * x^2),x, algorithm="fricas`

[Out] `(b*d*x^2 - sqrt(d*x + c)*sqrt(d*x - c)*b*x + a*d)/(2*d^4*x^4 - 2*c^2*d^2*x^2 - (2*d^3*x^3 - c^2*d*x)*sqrt(d*x + c)*sqrt(d*x - c))`

**Sympy [A]** time = 152.285, size = 165, normalized size = 2.46

$$a \left( \frac{{}_2F_6^{5,3} \left( \begin{matrix} \frac{7}{4}, \frac{9}{4}, 1 \\ \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}, 3 \end{matrix} \middle| \frac{c^2}{d^2x^2} \right)}{2\pi^{\frac{3}{2}}c^4} + \frac{i {}_2F_6^{2,6} \left( \begin{matrix} \frac{1}{2}, 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 1 \\ \frac{5}{4}, \frac{7}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2x^2} \right)}{2\pi^{\frac{3}{2}}c^4} \right) + b \left( \frac{{}_2F_6^{5,3} \left( \begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 \\ \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 2 \end{matrix} \middle| \frac{c^2}{d^2x^2} \right)}{2\pi^{\frac{3}{2}}c^2d} + \frac{i {}_2F_6^{2,6} \left( \begin{matrix} -\frac{1}{2}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1 \\ \frac{1}{4}, \frac{3}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2x^2} \right)}{2\pi^{\frac{3}{2}}c^2d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x**2/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)`

[Out] `a*(-d*meijerg(((7/4, 9/4, 1), (3/2, 5/2, 3)), ((7/4, 2, 9/4, 5/2, 3), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c**4) + I*d*meijerg(((1/2, 1, 5/4, 3/2, 7/4, 1), ()), ((5/4, 7/4), (1/2, 1, 2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c**4) + b*(-meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c**2*d) + I*meijerg((-1/2, 0, 1/4, 1/2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c**2*d))`

**GIAC/XCAS [A]** time = 0.277904, size = 296, normalized size = 4.42

$$\frac{(bc^2 + ad^2)\sqrt{dx + c}}{2\sqrt{dx - c}c^4d} \frac{2\left(bc^2(\sqrt{dx + c} - \sqrt{dx - c})^4 + ad^2(\sqrt{dx + c} - \sqrt{dx - c})^4 + 4acd^2(\sqrt{dx + c} - \sqrt{dx - c})^2 + 4bc^4 + 12ac^2d^2\right)}{\left((\sqrt{dx + c} - \sqrt{dx - c})^6 + 2c(\sqrt{dx + c} - \sqrt{dx - c})^4 + 4c^2(\sqrt{dx + c} - \sqrt{dx - c})^2 + 8c^3\right)c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/((d\*x + c)^(3/2)\*(d\*x - c)^(3/2)\*x^2),x, algorithm="giac")

[Out] -1/2\*(b\*c^2 + a\*d^2)\*sqrt(d\*x + c)/(sqrt(d\*x - c)\*c^4\*d) - 2\*(b\*c^2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^4 + a\*d^2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^4 + 4\*a\*c\*d^2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2 + 4\*b\*c^4 + 12\*a\*c^2\*d^2)/(((sqrt(d\*x + c) - sqrt(d\*x - c))^6 + 2\*c\*(sqrt(d\*x + c) - sqrt(d\*x - c))^4 + 4\*c^2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2 + 8\*c^3)\*c^3\*d)

$$3.276 \quad \int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=117

$$-\frac{(3ad^2 + 2bc^2) \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^5} - \frac{3ad^2 + 2bc^2}{2c^4\sqrt{dx-c}\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}$$

[Out]  $-(2*b*c^2 + 3*a*d^2)/(2*c^4*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) + a/(2*c^2*x^2*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) - ((2*b*c^2 + 3*a*d^2)*\text{ArcTan}[(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/c])/(2*c^5)$

**Rubi [A]** time = 0.347629, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$

$$-\frac{(3ad^2 + 2bc^2) \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^5} - \frac{3ad^2 + 2bc^2}{2c^4\sqrt{dx-c}\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^3\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)), x]

[Out]  $-(2*b*c^2 + 3*a*d^2)/(2*c^4*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) + a/(2*c^2*x^2*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) - ((2*b*c^2 + 3*a*d^2)*\text{ArcTan}[(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/c])/(2*c^5)$

**Rubi in Sympy [A]** time = 21.4516, size = 100, normalized size = 0.85

$$\frac{a}{2c^2x^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{3ad^2 + 2bc^2}{2c^4\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(3ad^2 + 2bc^2) \operatorname{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x\*\*3/(d\*x-c)\*\*(3/2)/(d\*x+c)\*\*(3/2), x)

[Out]  $a/(2*c**2*x**2*\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)) - (3*a*d**2 + 2*b*c**2)/(2*c**4*\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)) - (3*a*d**2 + 2*b*c**2)*\text{atan}(\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)/c)/(2*c**5)$

**Mathematica [C]** time = 0.289895, size = 126, normalized size = 1.08

$$\frac{a(c^3-3cd^2x^2)-2bc^3x^2}{x^2\sqrt{dx-c}\sqrt{c+dx}} + i(3ad^2 + 2bc^2) \log\left(\frac{-4c^4\sqrt{dx-c}\sqrt{c+dx}+4ic^5}{3ad^2x+2bc^2x}\right)$$

$$2c^5$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^3\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)), x]

[Out]  $((-2*b*c^3*x^2 + a*(c^3 - 3*c*d^2*x^2))/(x^2*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) + I*(2*b*c^2 + 3*a*d^2)*\text{Log}[(4*I)*c^5 - 4*c^4*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/(2*b*c^2*x + 3*a*d^2*x)]/(2*c^5)$





**GIAC/XCAS [A]** time = 0.295925, size = 285, normalized size = 2.44

$$\frac{(2bc^2 + 3ad^2) \arctan\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2c}\right)}{c^5} - \frac{(bc^2 + ad^2)\sqrt{dx+c}}{2\sqrt{dx-c}c^5} + \frac{2(bc^2 + ad^2)}{\left((\sqrt{dx+c}-\sqrt{dx-c})^2 + 2c\right)c^4}$$

$$+ \frac{2\left(ad^2(\sqrt{dx+c}-\sqrt{dx-c})^6 - 4ac^2d^2(\sqrt{dx+c}-\sqrt{dx-c})^2\right)}{\left((\sqrt{dx+c}-\sqrt{dx-c})^4 + 4c^2\right)^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/((d\*x + c)^(3/2)\*(d\*x - c)^(3/2)\*x^3),x, algorithm="giac")

[Out] (2\*b\*c^2 + 3\*a\*d^2)\*arctan(1/2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2/c)/c^5 - 1/2\*(b\*c^2 + a\*d^2)\*sqrt(d\*x + c)/(sqrt(d\*x - c)\*c^5) + 2\*(b\*c^2 + a\*d^2)/(((sqrt(d\*x + c) - sqrt(d\*x - c))^2 + 2\*c)\*c^4) + 2\*(a\*d^2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^6 - 4\*a\*c^2\*d^2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2)/(((sqrt(d\*x + c) - sqrt(d\*x - c))^4 + 4\*c^2)^2\*c^4)

$$3.277 \quad \int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=119

$$-\frac{2d^2x(4ad^2+3bc^2)}{3c^6\sqrt{dx-c}\sqrt{c+dx}} + \frac{4ad^2+3bc^2}{3c^4x\sqrt{dx-c}\sqrt{c+dx}} + \frac{a}{3c^2x^3\sqrt{dx-c}\sqrt{c+dx}}$$

[Out]  $a/(3*c^2*x^3*\text{Sqrt}[-c+d*x]*\text{Sqrt}[c+d*x]) + (3*b*c^2 + 4*a*d^2)/(3*c^4*x*\text{Sqrt}[-c+d*x]*\text{Sqrt}[c+d*x]) - (2*d^2*(3*b*c^2 + 4*a*d^2)*x)/(3*c^6*\text{Sqrt}[-c+d*x]*\text{Sqrt}[c+d*x])$

**Rubi [A]** time = 0.323861, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$-\frac{2d^2x(4ad^2+3bc^2)}{3c^6\sqrt{dx-c}\sqrt{c+dx}} + \frac{4ad^2+3bc^2}{3c^4x\sqrt{dx-c}\sqrt{c+dx}} + \frac{a}{3c^2x^3\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^4\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)), x]

[Out]  $a/(3*c^2*x^3*\text{Sqrt}[-c+d*x]*\text{Sqrt}[c+d*x]) + (3*b*c^2 + 4*a*d^2)/(3*c^4*x*\text{Sqrt}[-c+d*x]*\text{Sqrt}[c+d*x]) - (2*d^2*(3*b*c^2 + 4*a*d^2)*x)/(3*c^6*\text{Sqrt}[-c+d*x]*\text{Sqrt}[c+d*x])$

**Rubi in Sympy [A]** time = 17.9077, size = 105, normalized size = 0.88

$$\frac{a}{3c^2x^3\sqrt{-c+dx}\sqrt{c+dx}} + \frac{4ad^2+3bc^2}{3c^4x\sqrt{-c+dx}\sqrt{c+dx}} - \frac{2d^2x(4ad^2+3bc^2)}{3c^6\sqrt{-c+dx}\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x\*\*4/(d\*x-c)\*\*(3/2)/(d\*x+c)\*\*(3/2), x)

[Out]  $a/(3*c**2*x**3*\text{sqrt}(-c+d*x)*\text{sqrt}(c+d*x)) + (4*a*d**2 + 3*b*c**2)/(3*c**4*x*\text{sqrt}(-c+d*x)*\text{sqrt}(c+d*x)) - 2*d**2*x*(4*a*d**2 + 3*b*c**2)/(3*c**6*\text{sqrt}(-c+d*x)*\text{sqrt}(c+d*x))$

**Mathematica [A]** time = 0.107597, size = 77, normalized size = 0.65

$$\frac{a(c^4 + 4c^2d^2x^2 - 8d^4x^4) + 3bc^2x^2(c^2 - 2d^2x^2)}{3c^6x^3\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^4\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)), x]

[Out]  $(3*b*c^2*x^2*(c^2 - 2*d^2*x^2) + a*(c^4 + 4*c^2*d^2*x^2 - 8*d^4*x^4))/(3*c^6*x^3*\text{Sqrt}[-c+d*x]*\text{Sqrt}[c+d*x])$

**Maple [A]** time = 0.008, size = 73, normalized size = 0.6

$$\frac{-8ad^4x^4 - 6bc^2d^2x^4 + 4ac^2d^2x^2 + 3bc^4x^2 + ac^4}{3x^3c^6} \frac{1}{\sqrt{dx+c}} \frac{1}{\sqrt{dx-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2), x)`

[Out]  $\frac{1}{3} \cdot (-8 \cdot a \cdot d^4 \cdot x^4 - 6 \cdot b \cdot c^2 \cdot d^2 \cdot x^4 + 4 \cdot a \cdot c^2 \cdot d^2 \cdot x^2 + 3 \cdot b \cdot c^4 \cdot x^2 + a \cdot c^4) / (d \cdot x + c)^{(1/2)} / x^3 / c^6 / (d \cdot x - c)^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x^4), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.235783, size = 193, normalized size = 1.62

$$\frac{6bd^2x^4 - ac^2 - (3bc^2 - 4ad^2)x^2 - 2(3bdx^3 + 2adx)\sqrt{dx+c}\sqrt{dx-c}}{3(8d^5x^8 - 12c^2d^3x^6 + 4c^4dx^4 - (8d^4x^7 - 8c^2d^2x^5 + c^4x^3)\sqrt{dx+c}\sqrt{dx-c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x^4), x, algorithm="fricas")`

[Out]  $\frac{1}{3} \cdot (6 \cdot b \cdot d^2 \cdot x^4 - a \cdot c^2 - (3 \cdot b \cdot c^2 - 4 \cdot a \cdot d^2) \cdot x^2 - 2 \cdot (3 \cdot b \cdot d \cdot x^3 + 2 \cdot a \cdot d \cdot x) \cdot \sqrt{d \cdot x + c} \cdot \sqrt{d \cdot x - c}) / (8 \cdot d^5 \cdot x^8 - 12 \cdot c^2 \cdot d^3 \cdot x^6 + 4 \cdot c^4 \cdot d \cdot x^4 - (8 \cdot d^4 \cdot x^7 - 8 \cdot c^2 \cdot d^2 \cdot x^5 + c^4 \cdot x^3) \cdot \sqrt{d \cdot x + c} \cdot \sqrt{d \cdot x - c})$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x**4/(d*x-c)**(3/2)/(d*x+c)**(3/2), x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.313585, size = 327, normalized size = 2.75

$$\frac{(bc^2d + ad^3)\sqrt{dx+c}}{2\sqrt{dx-cc^6}} - \frac{2(bc^2d + ad^3)}{\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^2 + 2c\right)c^5}$$

$$\frac{8\left(3bc^2d\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^8 + 3ad^3\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^8 + 24bc^4d\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^4 + 48ac^2d^3\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^4\right)}{3\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^4 + 4c^2\right)^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/((d\*x + c)^(3/2)\*(d\*x - c)^(3/2)\*x^4),x, algorithm="giac")

[Out] 
$$-1/2*(b*c^2*d + a*d^3)*\sqrt{d*x + c}/(\sqrt{d*x - c}*c^6) - 2*(b*c^2*d + a*d^3)/(((\sqrt{d*x + c} - \sqrt{d*x - c})^2 + 2*c)*c^5) - 8/3*(3*b*c^2*d*(\sqrt{d*x + c} - \sqrt{d*x - c})^8 + 3*a*d^3*(\sqrt{d*x + c} - \sqrt{d*x - c})^8 + 24*b*c^4*d*(\sqrt{d*x + c} - \sqrt{d*x - c})^4 + 48*a*c^2*d^3*(\sqrt{d*x + c} - \sqrt{d*x - c})^4 + 48*b*c^6*d + 80*a*c^4*d^3)/(((\sqrt{d*x + c} - \sqrt{d*x - c})^4 + 4*c^2)^3*c^4)$$

$$3.278 \quad \int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=166

$$\begin{aligned} & -\frac{3d^2(5ad^2+4bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^7} - \frac{3d^2(5ad^2+4bc^2)}{8c^6\sqrt{dx-c}\sqrt{c+dx}} \\ & + \frac{5ad^2+4bc^2}{8c^4x^2\sqrt{dx-c}\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{dx-c}\sqrt{c+dx}} \end{aligned}$$

[Out]  $(-3*d^2*(4*b*c^2 + 5*a*d^2))/(8*c^6*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) + a/(4*c^4*x^4*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) + (4*b*c^2 + 5*a*d^2)/(8*c^4*x^2*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) - (3*d^2*(4*b*c^2 + 5*a*d^2)*\text{ArcTan}[(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/c])/(8*c^7)$

**Rubi [A]** time = 0.459333, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$

$$\begin{aligned} & -\frac{3d^2(5ad^2+4bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^7} - \frac{3d^2(5ad^2+4bc^2)}{8c^6\sqrt{dx-c}\sqrt{c+dx}} \\ & + \frac{5ad^2+4bc^2}{8c^4x^2\sqrt{dx-c}\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{dx-c}\sqrt{c+dx}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)/(x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2)), x]$

[Out]  $(-3*d^2*(4*b*c^2 + 5*a*d^2))/(8*c^6*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) + a/(4*c^4*x^4*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) + (4*b*c^2 + 5*a*d^2)/(8*c^4*x^2*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) - (3*d^2*(4*b*c^2 + 5*a*d^2)*\text{ArcTan}[(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/c])/(8*c^7)$

**Rubi in Sympy [A]** time = 30.9584, size = 150, normalized size = 0.9

$$\begin{aligned} & \frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{5ad^2+4bc^2}{8c^4x^2\sqrt{-c+dx}\sqrt{c+dx}} \\ & - \frac{3d^2(5ad^2+4bc^2)}{8c^6\sqrt{-c+dx}\sqrt{c+dx}} - \frac{3d^2(5ad^2+4bc^2)\text{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{8c^7} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x**2+a)/x**5/(d*x-c)**(3/2)/(d*x+c)**(3/2), x)$

[Out]  $a/(4*c**2*x**4*\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)) + (5*a*d**2 + 4*b*c**2)/(8*c**4*x**2*\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)) - 3*d**2*(5*a*d**2 + 4*b*c**2)/(8*c**6*\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)) - 3*d**2*(5*a*d**2 + 4*b*c**2)*\text{atan}(\text{sqrt}(-c + d*x)*\text{sqrt}(c + d*x)/c)/(8*c**7)$

**Mathematica [C]** time = 0.38189, size = 157, normalized size = 0.95

$$\frac{a(2c^5+5c^3d^2x^2-15cd^4x^4)+4bc^3x^2(c^2-3d^2x^2)}{x^4\sqrt{dx-c}\sqrt{c+dx}} + 3i(5ad^4+4bc^2d^2)\log\left(\frac{-16c^6\sqrt{dx-c}\sqrt{c+dx}+16ic^7}{15ad^4x+12bc^2d^2x}\right)}{8c^7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^5\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)),x]

[Out] ((4\*b\*c^3\*x^2\*(c^2 - 3\*d^2\*x^2) + a\*(2\*c^5 + 5\*c^3\*d^2\*x^2 - 15\*c\*d^4\*x^4))/(x^4\*sqrt[-c + d\*x]\*sqrt[c + d\*x]) + (3\*I)\*(4\*b\*c^2\*d^2 + 5\*a\*d^4)\*Log[((16\*I)\*c^7 - 16\*c^6\*sqrt[-c + d\*x]\*sqrt[c + d\*x])]/(12\*b\*c^2\*d^2\*x + 15\*a\*d^4\*x)]/(8\*c^7)

**Maple [B]** time = 0.038, size = 387, normalized size = 2.3

$$\frac{1}{8c^6x^4} \left( 15 \ln \left( -2 \frac{c^2 - \sqrt{-c^2} \sqrt{d^2x^2 - c^2}}{x} \right) x^6 ad^6 + 12 \ln \left( -2 \frac{c^2 - \sqrt{-c^2} \sqrt{d^2x^2 - c^2}}{x} \right) x^6 bc^2 d^4 - 15 \ln \left( -2 \frac{c^2 - \sqrt{-c^2} \sqrt{d^2x^2 - c^2}}{x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x^5/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x)

[Out] 1/8/c^6\*(15\*ln(-2\*(c^2-(-c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2))/x)\*x^6\*a\*d^6+12\*ln(-2\*(c^2-(-c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2))/x)\*x^6\*b\*c^2\*d^4-15\*ln(-2\*(c^2-(-c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2))/x)\*x^4\*a\*c^2\*d^4-12\*ln(-2\*(c^2-(-c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2))/x)\*x^4\*b\*c^4\*d^2-15\*x^4\*a\*d^4\*(-c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2)-12\*x^4\*b\*c^2\*d^2\*(-c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2)+5\*x^2\*a\*c^2\*d^2\*(-c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2)+4\*x^2\*b\*c^4\*(-c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2)+2\*a\*c^4\*(-c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2))/(-c^2)^(1/2)/x^4/(d^2\*x^2-c^2)^(1/2)/(d\*x+c)^(1/2)/(d\*x-c)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/((d\*x + c)^(3/2)\*(d\*x - c)^(3/2)\*x^5),x, algorithm="maxima)

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.304714, size = 707, normalized size = 4.26

$$10ac^9dx - 48(4bc^3d^7 + 5acd^9)x^9 + 76(4bc^5d^5 + 5ac^3d^7)x^7 - (140bc^7d^3 + 143ac^5d^5)x^5 + 5(4bc^9d - 3ac^7d^3)x^3 - (2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/((d\*x + c)^(3/2)\*(d\*x - c)^(3/2)\*x^5),x, algorithm="fricas)

[Out] -1/8\*(10\*a\*c^9\*d\*x - 48\*(4\*b\*c^3\*d^7 + 5\*a\*c\*d^9)\*x^9 + 76\*(4\*b\*c^5\*d^5 + 5\*a\*c^3\*d^7)\*x^7 - (140\*b\*c^7\*d^3 + 143\*a\*c^5\*d^5)\*x^5 + 5\*(4\*b\*c^9\*d - 3\*a\*c^7\*d^3)\*x^3 - (2\*a\*c^9 - 48\*(4\*b\*c^3\*d^7 + 5\*a\*c\*d^9)\*x^6 - (60\*b\*c^7\*d^2 + 43\*a\*c^5\*d^4)\*x^4 + (4\*b\*c^9 - 19\*a\*c^7\*d^2)\*x^2)\*sqrt(d\*x + c)\*sqrt(d\*x - c) + 6\*(16\*(4\*b\*c^2\*d^8 + 5\*a\*d^10)\*x^10 - 28\*(4\*b\*c^4\*d^6 + 5\*a\*c^2\*d^8)\*x^8 + 13\*(4\*b\*c^6\*d^4 + 5\*a\*c^4\*d^6)\*x^6 - (4\*b\*c^8\*d^2 + 5\*a\*c^6\*d^4)\*x^4 - (16\*(4\*b\*c^2\*d^7 + 5\*a\*d^9)\*x^9 - 20\*(4\*b\*c^4\*d^5 + 5\*a\*c^2\*d^7)\*x^7 + 5\*(4\*b\*c^6\*d^3 + 5\*a\*c^4\*d^5)\*x^5)\*sqrt(d\*x + c)\*sqrt(d\*x - c))/((16\*c^7\*d^6\*x^10 - 28\*c^9\*d^4\*x^8 + 13\*c^11\*d^2\*x^6 - c^13\*x^4 - (16\*c^7\*d^5\*x^9 - 20\*c^9\*d^3\*x^7 + 5\*c^11\*

$d^5 x \sqrt{dx+c} \sqrt{dx-c}$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/x\*\*5/(d\*x-c)\*\*(3/2)/(d\*x+c)\*\*(3/2),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.345015, size = 543, normalized size = 3.27

$$\frac{3(4bc^2d^2 + 5ad^4) \arctan\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2c}\right)}{4c^7} - \frac{(bc^2d^2 + ad^4)\sqrt{dx+c}}{2\sqrt{dx-c}c^7} + \frac{2(bc^2d^2 + ad^4)}{\left((\sqrt{dx+c}-\sqrt{dx-c})^2 + 2c\right)c^6} + \frac{4bc^2d^2(\sqrt{dx+c}-\sqrt{dx-c})^{14} + 7ad^4(\sqrt{dx+c}-\sqrt{dx-c})^{14} + 16bc^4d^2(\sqrt{dx+c}-\sqrt{dx-c})^{10} + 60ac^2d^4(\sqrt{dx+c}-\sqrt{dx-c})^6}{2\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2 + 2c\right)c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/((d\*x + c)^(3/2) \* (d\*x - c)^(3/2) \* x^5), x, algorithm="giac")

[Out]  $\frac{3}{4} \cdot (4b^2c^2d^2 + 5a^2d^4) \cdot \arctan\left(\frac{1}{2} \cdot (\sqrt{dx+c} - \sqrt{dx-c})^2 / c\right) / c^7 - \frac{1}{2} \cdot (b^2c^2d^2 + a^2d^4) \cdot \sqrt{dx+c} / (\sqrt{dx-c} \cdot c^7) + \frac{2 \cdot (b^2c^2d^2 + a^2d^4)}{\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^2 + 2c\right) \cdot c^6} + \frac{1}{2} \cdot (4b^2c^2d^2 + 5a^2d^4) \cdot (\sqrt{dx+c} - \sqrt{dx-c})^{14} + 7a^2d^4 \cdot (\sqrt{dx+c} - \sqrt{dx-c})^{14} + 16b^2c^4d^2 \cdot (\sqrt{dx+c} - \sqrt{dx-c})^{10} + 60a^2c^2d^4 \cdot (\sqrt{dx+c} - \sqrt{dx-c})^6 - 64b^2c^6d^2 \cdot (\sqrt{dx+c} - \sqrt{dx-c})^6 - 240a^2c^4d^4 \cdot (\sqrt{dx+c} - \sqrt{dx-c})^6 - 256b^2c^8d^2 \cdot (\sqrt{dx+c} - \sqrt{dx-c})^2 - 448a^2c^6d^4 \cdot (\sqrt{dx+c} - \sqrt{dx-c})^2}{\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^2 + 2c\right) \cdot c^6}$



$$3.279 \quad \int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}} dx$$

**Optimal.** Leaf size=40

$$\sqrt{cx-1}\sqrt{cx+1} + \tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)$$

[Out] Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x] + ArcTan[Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]]

**Rubi [A]** time = 0.198525, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\sqrt{cx-1}\sqrt{cx+1} + \tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + c^2\*x^2)/(x\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x] + ArcTan[Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]]

**Rubi in Sympy [A]** time = 9.2897, size = 34, normalized size = 0.85

$$\sqrt{cx-1}\sqrt{cx+1} + \text{atan}\left(\sqrt{cx-1}\sqrt{cx+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*\*2\*x\*\*2+1)/x/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2), x)

[Out] sqrt(c\*x - 1)\*sqrt(c\*x + 1) + atan(sqrt(c\*x - 1)\*sqrt(c\*x + 1))

**Mathematica [A]** time = 0.0540141, size = 42, normalized size = 1.05

$$\sqrt{cx-1}\sqrt{cx+1} - \tan^{-1}\left(\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + c^2\*x^2)/(x\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x] - ArcTan[1/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])]

**Maple [A]** time = 0.024, size = 53, normalized size = 1.3

$$1\sqrt{cx-1}\sqrt{cx+1}\left(\sqrt{c^2x^2-1} - \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\right)\frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^2\*x^2+1)/x/(c\*x-1)^(1/2)/(c\*x+1)^(1/2), x)

[Out]  $(c^2x-1)^{1/2} (c^2x+1)^{1/2} / (c^2x^2-1)^{1/2} * ((c^2x^2-1)^{1/2} - \arctan(1/(c^2x^2-1)^{1/2}))$

**Maxima [A]** time = 1.52933, size = 34, normalized size = 0.85

$$\sqrt{c^2x^2 - 1} - \arcsin\left(\frac{1}{\sqrt{c^2|x|}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c^2*x^2 + 1)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x),x, algorithm="maxima")`

[Out] `sqrt(c^2*x^2 - 1) - arcsin(1/(sqrt(c^2)*abs(x)))`

**Fricas [A]** time = 0.241092, size = 127, normalized size = 3.18

$$\frac{c^2x^2 - \sqrt{cx+1}\sqrt{cx-1}cx - 2\left(cx - \sqrt{cx+1}\sqrt{cx-1}\right)\arctan\left(-cx + \sqrt{cx+1}\sqrt{cx-1}\right) - 1}{cx - \sqrt{cx+1}\sqrt{cx-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c^2*x^2 + 1)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x),x, algorithm="fricas")`

[Out] `-(c^2*x^2 - sqrt(c*x + 1)*sqrt(c*x - 1)*c*x - 2*(c*x - sqrt(c*x + 1)*sqrt(c*x - 1))*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) - 1)/(c*x - sqrt(c*x + 1)*sqrt(c*x - 1))`

**Sympy [A]** time = 31.06, size = 148, normalized size = 3.7

$$\frac{G_{6,6}^{6,2}\left(-\frac{1}{4}, \frac{1}{4}, 0, 0, \frac{1}{2}, 1 \middle| \frac{1}{c^2x^2}\right) - G_{6,6}^{5,3}\left(\frac{3}{4}, \frac{5}{4}, 1, 1, 1, \frac{3}{2} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{iG_{6,6}^{2,6}\left(-1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \middle| \frac{e^{2i\pi}}{c^2x^2}\right) + iG_{6,6}^{2,6}\left(0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1 \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c**2*x**2+1)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)`

[Out] `meijerg((( -1/4, 1/4), (0, 0, 1/2, 1)), (( -1/2, -1/4, 0, 1/4, 1/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)) - meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*meijerg((( -1, -3/4, -1/2, -1/4, 0, 1), ()), (( -3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) + I*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2))`

**GIAC/XCAS [A]** time = 0.219661, size = 54, normalized size = 1.35

$$\sqrt{cx+1}\sqrt{cx-1} - 2 \arctan\left(\frac{1}{2}\left(\sqrt{cx+1} - \sqrt{cx-1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2 + 1)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x),x, algorithm="giac")
```

```
[Out] sqrt(c*x + 1)*sqrt(c*x - 1) - 2*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2)
```

$$3.280 \quad \int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}} (c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}} dx$$

**Optimal.** Leaf size=53

$$\sqrt{bx-a}\sqrt{a+bx} \left( \frac{c}{a^2} + \frac{d}{b^2} \right) x^{-\frac{b^2c}{a^2d+b^2c}}$$

[Out] ((c/a^2 + d/b^2)\*Sqrt[-a + b\*x]\*Sqrt[a + b\*x])/x^((b^2\*c)/(b^2\*c + a^2\*d))

**Rubi [A]** time = 0.297772, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 57,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.018$

$$\sqrt{bx-a}\sqrt{a+bx} \left( \frac{c}{a^2} + \frac{d}{b^2} \right) x^{-\frac{b^2c}{a^2d+b^2c}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^2)/(x^((2\*b^2\*c + a^2\*d)/(b^2\*c + a^2\*d))\*Sqrt[-a + b\*x]\*Sqrt[a + b\*x]), x]

[Out] ((c/a^2 + d/b^2)\*Sqrt[-a + b\*x]\*Sqrt[a + b\*x])/x^((b^2\*c)/(b^2\*c + a^2\*d))

**Rubi in Sympy [A]** time = 10.0874, size = 42, normalized size = 0.79

$$x^{-\frac{b^2c}{a^2d+b^2c}} \sqrt{-a+bx}\sqrt{a+bx} \left( \frac{d}{b^2} + \frac{c}{a^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((d\*x\*\*2+c)/(x\*\*((a\*\*2\*d+2\*b\*\*2\*c)/(a\*\*2\*d+b\*\*2\*c)))/(b\*x-a)\*\*(1/2)/(b\*x+a)\*\*(1/2), x)

[Out] x\*\*(-b\*\*2\*c/(a\*\*2\*d + b\*\*2\*c))\*sqrt(-a + b\*x)\*sqrt(a + b\*x)\*(d/b\*\*2 + c/a\*\*2)

**Mathematica [C]** time = 4.26321, size = 1424, normalized size = 26.87

$$d(da^2 + b^2c) x^{-\frac{b^2c}{da^2+b^2c}} \left( \frac{d(a-bx)^2 \sqrt{\frac{bx}{a}} {}_1F_1\left(-\frac{b^2c}{da^2+b^2c}; -\frac{1}{2}, \frac{1}{2}; \frac{a^2d}{da^2+b^2c}; \frac{bx}{a}, -\frac{bx}{a}\right) a^3}{c \sqrt{1-\frac{bx}{a}} \left( 2a^3 {}_2F_1\left(-\frac{b^2c}{da^2+b^2c}; -\frac{1}{2}, \frac{1}{2}; \frac{a^2d}{da^2+b^2c}; \frac{bx}{a}, -\frac{bx}{a}\right) - b(da^2+b^2c)x \left( F_1\left(\frac{a^2d}{da^2+b^2c}; -\frac{1}{2}, \frac{3}{2}; \frac{2da^2+b^2c}{da^2+b^2c}; \frac{bx}{a}, -\frac{bx}{a}\right) + {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{a^2d}{da^2+b^2c}; \frac{bx}{a}, -\frac{bx}{a}\right) \right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^2)/(x^((2\*b^2\*c + a^2\*d)/(b^2\*c + a^2\*d))\*Sqrt[-a + b\*x]\*Sqrt[a + b\*x]), x]

[Out] -((d\*(b^2\*c + a^2\*d)\*(-(((a - b\*x)\*(a + b\*x)\*Hypergeometric2F1[-1/2, -(b^2\*c)/(2\*(b^2\*c + a^2\*d)), 1 - (b^2\*c)/(2\*(b^2\*c + a^2\*d)), (b^2\*x^2)/a^2])/c) + (a\*b^2\*(a - b\*x)^2\*Sqrt[1 + (b\*x)/a]\*Appell1F1[-((b^2\*c)/(b^2\*c + a^2\*d)), -1/2, 1/2, (a^2\*d)/(b^2\*c + a^2\*d), (b\*x)/a, -((b\*x)/a)]/(Sqrt[1 - (b\*x)/a]\*(2\*a^3\*d\*Appell1F1[-((

$$\frac{b^2c}{b^2c + a^2d}, -1/2, 1/2, (a^2d)/(b^2c + a^2d), (bx)/a, -((bx)/a)] - b^*(b^2c + a^2d)*x*(\text{AppellF1}[(a^2d)/(b^2c + a^2d), -1/2, 3/2, (b^2c + 2*a^2d)/(b^2c + a^2d), (bx)/a, -((bx)/a)] + \text{HypergeometricPFQ}[\{1/2, (a^2d)/(2*(b^2c + a^2d))\}, \{(b^2c)/(b^2c + a^2d) + (3*a^2d)/(2*(b^2c + a^2d))\}, (b^2*x^2)/a^2])) + (a^3*d*(a - bx)^2*\text{Sqrt}[1 + (bx)/a]*\text{AppellF1}[-((b^2c)/(b^2c + a^2d)), -1/2, 1/2, (a^2d)/(b^2c + a^2d), (bx)/a, -((bx)/a)])/(c*\text{Sqrt}[1 - (bx)/a]*(2*a^3*d*\text{AppellF1}[-((b^2c)/(b^2c + a^2d)), -1/2, 1/2, (a^2d)/(b^2c + a^2d), (bx)/a, -((bx)/a)] - b^*(b^2c + a^2d)*x*(\text{AppellF1}[(a^2d)/(b^2c + a^2d), -1/2, 3/2, (b^2c + 2*a^2d)/(b^2c + a^2d), (bx)/a, -((bx)/a)] + \text{HypergeometricPFQ}[\{1/2, (a^2d)/(2*(b^2c + a^2d))\}, \{(b^2c)/(b^2c + a^2d) + (3*a^2d)/(2*(b^2c + a^2d))\}, (b^2*x^2)/a^2])) + (a*b^2*(a + bx)^2*\text{Sqrt}[1 - (bx)/a]*\text{AppellF1}[-((b^2c)/(b^2c + a^2d)), 1/2, -1/2, (a^2d)/(b^2c + a^2d), (bx)/a, -((bx)/a)])/(Sqrt[1 + (bx)/a]*(2*a^3*d*\text{AppellF1}[-((b^2c)/(b^2c + a^2d)), 1/2, -1/2, (a^2d)/(b^2c + a^2d), (bx)/a, -((bx)/a)] + b^*(b^2c + a^2d)*x*(\text{AppellF1}[(a^2d)/(b^2c + a^2d), 3/2, -1/2, (b^2c + 2*a^2d)/(b^2c + a^2d), (bx)/a, -((bx)/a)] + \text{HypergeometricPFQ}[\{1/2, (a^2d)/(2*(b^2c + a^2d))\}, \{(b^2c)/(b^2c + a^2d) + (3*a^2d)/(2*(b^2c + a^2d))\}, (b^2*x^2)/a^2])) + (a^3*d*(a + bx)^2*\text{Sqrt}[1 - (bx)/a]*\text{AppellF1}[-((b^2c)/(b^2c + a^2d)), 1/2, -1/2, (a^2d)/(b^2c + a^2d), (bx)/a, -((bx)/a)])/(c*\text{Sqrt}[1 + (bx)/a]*(2*a^3*d*\text{AppellF1}[-((b^2c)/(b^2c + a^2d)), 1/2, -1/2, (a^2d)/(b^2c + a^2d), (bx)/a, -((bx)/a)] + b^*(b^2c + a^2d)*x*(\text{AppellF1}[(a^2d)/(b^2c + a^2d), 3/2, -1/2, (b^2c + 2*a^2d)/(b^2c + a^2d), (bx)/a, -((bx)/a)] + \text{HypergeometricPFQ}[\{1/2, (a^2d)/(2*(b^2c + a^2d))\}, \{(b^2c)/(b^2c + a^2d) + (3*a^2d)/(2*(b^2c + a^2d))\}, (b^2*x^2)/a^2])))))/(b^4*x^(b^2c/(b^2c + a^2d))*\text{Sqrt}[-a + bx]*\text{Sqrt}[a + bx]*\text{Sqrt}[1 - (b^2*x^2)/a^2]))$$

**Maple [A]** time = 0.014, size = 66, normalized size = 1.3

$$\frac{x(a^2d + b^2c)}{a^2b^2} \sqrt{bx + a} \sqrt{bx - a} \left( x \frac{a^2d + 2b^2c}{a^2d + b^2c} \right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^2+c)/(x^((a^2\*d+2\*b^2\*c)/(a^2\*d+b^2\*c)))/(b\*x-a)^(1/2)/(b\*x+a)^(1/2),

[Out] x\*(a^2\*d+b^2\*c)\*(b\*x+a)^(1/2)/a^2/b^2/(x^((a^2\*d+2\*b^2\*c)/(a^2\*d+b^2\*c)))\*(b\*x-a)^(1/2)

**Maxima [A]** time = 1.85853, size = 107, normalized size = 2.02

$$\frac{(b^2c + a^2d) \sqrt{bx + a} \sqrt{bx - a} x e^{\left( \frac{2b^2c \log(x)}{b^2c + a^2d} - \frac{a^2d \log(x)}{b^2c + a^2d} \right)}}{a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)/(sqrt(b\*x + a)\*sqrt(b\*x - a)\*x^((2\*b^2\*c + a^2\*d)/(b^2\*c + a^2\*d)),

[Out] (b^2\*c + a^2\*d)\*sqrt(b\*x + a)\*sqrt(b\*x - a)\*x\*e^(-2\*b^2\*c\*log(x)/(b^2\*c + a^2\*d) - a^2\*d\*log(x)/(b^2\*c + a^2\*d))/(a^2\*b^2)

**Fricas [A]** time = 0.262433, size = 88, normalized size = 1.66

$$\frac{(b^2c + a^2d) \sqrt{bx + a} \sqrt{bx - ax}}{a^2b^2 x \frac{2b^2c + a^2d}{b^2c + a^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)/(sqrt(b\*x + a)\*sqrt(b\*x - a)\*x^((2\*b^2\*c + a^2\*d)/(b^2\*c + a^2\*d))))

[Out] (b^2\*c + a^2\*d)\*sqrt(b\*x + a)\*sqrt(b\*x - a)\*x/(a^2\*b^2\*x^((2\*b^2\*c + a^2\*d)/(b^2\*c + a^2\*d)))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*2+c)/(x\*\*((a\*\*2\*d+2\*b\*\*2\*c)/(a\*\*2\*d+b\*\*2\*c)))/(b\*x-a)\*\*(1/2)/(b\*x+a)\*\*(1/2), x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\sqrt{bx + a}\sqrt{bx - a}x^{\frac{2b^2c + a^2d}{b^2c + a^2d}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)/(sqrt(b\*x + a)\*sqrt(b\*x - a)\*x^((2\*b^2\*c + a^2\*d)/(b^2\*c + a^2\*d))))

[Out] integrate((d\*x^2 + c)/(sqrt(b\*x + a)\*sqrt(b\*x - a)\*x^((2\*b^2\*c + a^2\*d)/(b^2\*c + a^2\*d))), x)

$$3.281 \quad \int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}} dx$$

**Optimal.** Leaf size=36

$$\frac{\sqrt{1-x} \sin^{-1}(x)}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}}$$

[Out] (Sqrt[1 - x]\*ArcSin[x])/(Sqrt[-1 - Sqrt[x]]\*Sqrt[-1 + Sqrt[x]])

**Rubi [A]** time = 0.0708708, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$

$$\frac{\sqrt{1-x} \sin^{-1}(x)}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-1 - Sqrt[x]]\*Sqrt[-1 + Sqrt[x]]\*Sqrt[1 + x]), x]

[Out] (Sqrt[1 - x]\*ArcSin[x])/(Sqrt[-1 - Sqrt[x]]\*Sqrt[-1 + Sqrt[x]])

**Rubi in Sympy [A]** time = 8.37875, size = 31, normalized size = 0.86

$$\frac{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1} \operatorname{asin}(x)}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(1+x)\*\*(1/2)/(-1-x\*\*(1/2))\*\*(1/2)/(-1+x\*\*(1/2))\*\*(1/2), x)

[Out] sqrt(-sqrt(x) - 1)\*sqrt(sqrt(x) - 1)\*asin(x)/sqrt(-x + 1)

**Mathematica [A]** time = 0.947749, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(Sqrt[-1 - Sqrt[x]]\*Sqrt[-1 + Sqrt[x]]\*Sqrt[1 + x]), x]

[Out] Integrate[1/(Sqrt[-1 - Sqrt[x]]\*Sqrt[-1 + Sqrt[x]]\*Sqrt[1 + x]), x]

**Maple [F]** time = 0.062, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{1+x}} \frac{1}{\sqrt{-1-\sqrt{x}}} \frac{1}{\sqrt{-1+\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2), x)

[Out] `int(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)), x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)), x)`

**Fricas [A]** time = 0.236123, size = 93, normalized size = 2.58

$$-i \log\left(\frac{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1} + ix - 1}{x}\right) + i \log\left(\frac{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1} - ix - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)), x, algorithm="fricas")`

[Out] `-I*log((sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1) + I*x - 1)/x) + I*log((sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1) - I*x - 1)/x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}\sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x)**(1/2)/(-1-x**(1/2))**(1/2)/(-1+x**(1/2))**(1/2), x)`

[Out] `Integral(1/(sqrt(-sqrt(x) - 1)*sqrt(sqrt(x) - 1)*sqrt(x + 1)), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)), x, algorithm="giac")`

[Out] Exception raised: TypeError



$$3.282 \quad \int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}} dx$$

**Optimal.** Leaf size=75

$$\frac{2\sqrt{a^2 - b^2x} \tan^{-1}\left(\frac{\sqrt{a^2 - b^2x}}{\sqrt{a^2 + b^2x}}\right)}{b^2\sqrt{a - b\sqrt{x}}\sqrt{a + b\sqrt{x}}}$$

[Out] (-2\*Sqrt[a^2 - b^2\*x]\*ArcTan[Sqrt[a^2 - b^2\*x]/Sqrt[a^2 + b^2\*x]])/(b^2\*Sqrt[a - b\*Sqrt[x]]\*Sqrt[a + b\*Sqrt[x]])

**Rubi [A]** time = 0.146345, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$

$$\frac{2\sqrt{a^2 - b^2x} \tan^{-1}\left(\frac{\sqrt{a^2 - b^2x}}{\sqrt{a^2 + b^2x}}\right)}{b^2\sqrt{a - b\sqrt{x}}\sqrt{a + b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a - b\*Sqrt[x]]\*Sqrt[a + b\*Sqrt[x]]\*Sqrt[a^2 + b^2\*x]), x]

[Out] (-2\*Sqrt[a^2 - b^2\*x]\*ArcTan[Sqrt[a^2 - b^2\*x]/Sqrt[a^2 + b^2\*x]])/(b^2\*Sqrt[a - b\*Sqrt[x]]\*Sqrt[a + b\*Sqrt[x]])

**Rubi in Sympy [A]** time = 18.2102, size = 65, normalized size = 0.87

$$\frac{2\sqrt{a - b\sqrt{x}}\sqrt{a + b\sqrt{x}} \operatorname{atan}\left(\frac{\sqrt{a^2 + b^2x}}{\sqrt{a^2 - b^2x}}\right)}{b^2\sqrt{a^2 - b^2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*\*2\*x+a\*\*2)\*\*(1/2)/(a-b\*x\*\*(1/2))\*\*(1/2)/(a+b\*x\*\*(1/2))\*\*(1/2))

[Out] 2\*sqrt(a - b\*sqrt(x))\*sqrt(a + b\*sqrt(x))\*atan(sqrt(a\*\*2 + b\*\*2\*x)/sqrt(a\*\*2 - b\*\*2\*x))/(b\*\*2\*sqrt(a\*\*2 - b\*\*2\*x))

**Mathematica [A]** time = 1.96338, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a - b\sqrt{x}}\sqrt{a + b\sqrt{x}}\sqrt{a^2 + b^2x}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(Sqrt[a - b\*Sqrt[x]]\*Sqrt[a + b\*Sqrt[x]]\*Sqrt[a^2 + b^2\*x]), x]

[Out] Integrate[1/(Sqrt[a - b\*Sqrt[x]]\*Sqrt[a + b\*Sqrt[x]]\*Sqrt[a^2 + b^2\*x]), x]

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x + a^2}} \frac{1}{\sqrt{a - b\sqrt{x}}} \frac{1}{\sqrt{a + b\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x)`

[Out] `int(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x + a^2}\sqrt{b\sqrt{x} + a}\sqrt{-b\sqrt{x} + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)),x, algo`

[Out] `integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)), x)`

**Fricas [A]** time = 0.242236, size = 68, normalized size = 0.91

$$-\frac{2 \arctan\left(-\frac{a^2 - \sqrt{b^2x + a^2}\sqrt{b\sqrt{x} + a}\sqrt{-b\sqrt{x} + a}}{b^2x}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)),x, algo`

[Out] `-2*arctan(-(a^2 - sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a))/(b^2*x))/b^2`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a - b\sqrt{x}}\sqrt{a + b\sqrt{x}}\sqrt{a^2 + b^2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b**2*x+a**2)**(1/2)/(a-b*x**(1/2))**(1/2)/(a+b*x**(1/2))**(1/2),x)`

[Out] `Integral(1/(sqrt(a - b*sqrt(x))*sqrt(a + b*sqrt(x))*sqrt(a**2 + b**2*x)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x + a^2}\sqrt{b\sqrt{x} + a}\sqrt{-b\sqrt{x} + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)),x, algo`

[Out] `integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)), x)`

$$3.283 \quad \int (a - bx^n)^p (a + bx^n)^p (c + dx^{2n})^q dx$$

**Optimal.** Leaf size=113

$$x (a - bx^n)^p (a + bx^n)^p \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{-p} (c + dx^{2n})^q \left(\frac{dx^{2n}}{c} + 1\right)^{-q} F_1\left(\frac{1}{2n}; -p, -q; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2 x^{2n}}{a^2}, -\frac{dx^{2n}}{c}\right)$$

[Out] (x\*(a - b\*x^n)^p\*(a + b\*x^n)^p\*(c + d\*x^(2\*n))^q\*AppellF1[1/(2\*n), -p, -q, (2 + n^(-1))/2, (b^2\*x^(2\*n))/a^2, -((d\*x^(2\*n))/c)])/(1 - (b^2\*x^(2\*n))/a^2)^p\*(1 + (d\*x^(2\*n))/c)^q

**Rubi [A]** time = 0.23465, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$x (a - bx^n)^p (a + bx^n)^p \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{-p} (c + dx^{2n})^q \left(\frac{dx^{2n}}{c} + 1\right)^{-q} F_1\left(\frac{1}{2n}; -p, -q; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2 x^{2n}}{a^2}, -\frac{dx^{2n}}{c}\right)$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^n)^p\*(a + b\*x^n)^p\*(c + d\*x^(2\*n))^q, x]

[Out] (x\*(a - b\*x^n)^p\*(a + b\*x^n)^p\*(c + d\*x^(2\*n))^q\*AppellF1[1/(2\*n), -p, -q, (2 + n^(-1))/2, (b^2\*x^(2\*n))/a^2, -((d\*x^(2\*n))/c)])/(1 - (b^2\*x^(2\*n))/a^2)^p\*(1 + (d\*x^(2\*n))/c)^q

**Rubi in Sympy [A]** time = 37.8593, size = 88, normalized size = 0.78

$$x \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{-p} \left(1 + \frac{dx^{2n}}{c}\right)^{-q} (a - bx^n)^p (a + bx^n)^p (c + dx^{2n})^q \text{appellf}_1\left(\frac{1}{2n}, -p, -q, \frac{n + \frac{1}{2}}{n}, \frac{b^2 x^{2n}}{a^2}, -\frac{dx^{2n}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a-b\*x\*\*n)\*\*p\*(a+b\*x\*\*n)\*\*p\*(c+d\*x\*\*(2\*n))\*\*q, x)

[Out] x\*(1 - b\*\*2\*x\*\*(2\*n)/a\*\*2)\*\*(-p)\*(1 + d\*x\*\*(2\*n)/c)\*\*(-q)\*(a - b\*x\*\*n)\*\*p\*(a + b\*x\*\*n)\*\*p\*(c + d\*x\*\*(2\*n))\*\*q\*appellf1(1/(2\*n), -p, -q, (n + 1/2)/n, b\*\*2\*x\*\*(2\*n)/a\*\*2, -d\*x\*\*(2\*n)/c)

**Mathematica [A]** time = 0.370362, size = 0, normalized size = 0.

$$\int (a - bx^n)^p (a + bx^n)^p (c + dx^{2n})^q dx$$

Verification is Not applicable to the result.

[In] Integrate[(a - b\*x^n)^p\*(a + b\*x^n)^p\*(c + d\*x^(2\*n))^q, x]

[Out] Integrate[(a - b\*x^n)^p\*(a + b\*x^n)^p\*(c + d\*x^(2\*n))^q, x]

**Maple [F]** time = 0.813, size = 0, normalized size = 0.

$$\int (a - bx^n)^p (a + bx^n)^p (c + dx^{2n})^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x)`

[Out] `int((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^{2n} + c)^q (bx^n + a)^p (-bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="maxima")`

[Out] `integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx^{2n} + c)^q (bx^n + a)^p (-bx^n + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="fricas")`

[Out] `integral((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a-b*x**n)**p*(a+b*x**n)**p*(c+d*x**(2*n))**q,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^{2n} + c)^q (bx^n + a)^p (-bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="giac")`

[Out] `integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)`

$$3.284 \quad \int (a - bx^n)^p (a + bx^n)^p (a^2 + b^2 x^{2n})^p dx$$

**Optimal.** Leaf size=87

$$x (a - bx^n)^p (a + bx^n)^p (a^2 + b^2 x^{2n})^p \left(1 - \frac{b^4 x^{4n}}{a^4}\right)^{-p} {}_2F_1\left(\frac{1}{4n}, -p; \frac{1}{4}\left(4 + \frac{1}{n}\right); \frac{b^4 x^{4n}}{a^4}\right)$$

[Out] (x\*(a - b\*x^n)^p\*(a + b\*x^n)^p\*(a^2 + b^2\*x^(2\*n))^p\*Hypergeometric2F1[1/(4\*n), -p, (4 + n^(-1))/4, (b^4\*x^(4\*n))/a^4])/(1 - (b^4\*x^(4\*n))/a^4)^p

**Rubi [A]** time = 0.150388, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$

$$x (a - bx^n)^p (a + bx^n)^p (a^2 + b^2 x^{2n})^p \left(1 - \frac{b^4 x^{4n}}{a^4}\right)^{-p} {}_2F_1\left(\frac{1}{4n}, -p; \frac{1}{4}\left(4 + \frac{1}{n}\right); \frac{b^4 x^{4n}}{a^4}\right)$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^n)^p\*(a + b\*x^n)^p\*(a^2 + b^2\*x^(2\*n))^p, x]

[Out] (x\*(a - b\*x^n)^p\*(a + b\*x^n)^p\*(a^2 + b^2\*x^(2\*n))^p\*Hypergeometric2F1[1/(4\*n), -p, (4 + n^(-1))/4, (b^4\*x^(4\*n))/a^4])/(1 - (b^4\*x^(4\*n))/a^4)^p

**Rubi in Sympy [A]** time = 32.5251, size = 70, normalized size = 0.8

$$x \left(1 - \frac{b^4 x^{4n}}{a^4}\right)^{-p} (a - bx^n)^p (a + bx^n)^p (a^2 + b^2 x^{2n})^p {}_2F_1\left(\frac{-p, \frac{1}{4n}}{\frac{n+\frac{1}{4}}{n}} \middle| \frac{b^4 x^{4n}}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a-b\*x\*\*n)\*\*p\*(a+b\*x\*\*n)\*\*p\*(a\*\*2+b\*\*2\*x\*\*(2\*n))\*\*p, x)

[Out] x\*(1 - b\*\*4\*x\*\*(4\*n)/a\*\*4)\*\*(-p)\*(a - b\*x\*\*n)\*\*p\*(a + b\*x\*\*n)\*\*p\*(a\*\*2 + b\*\*2\*x\*\*(2\*n))\*\*p\*hyper((-p, 1/(4\*n)), ((n + 1/4)/n), b\*\*4\*x\*\*(4\*n)/a\*\*4)

**Mathematica [A]** time = 0.278254, size = 0, normalized size = 0.

$$\int (a - bx^n)^p (a + bx^n)^p (a^2 + b^2 x^{2n})^p dx$$

Verification is Not applicable to the result.

[In] Integrate[(a - b\*x^n)^p\*(a + b\*x^n)^p\*(a^2 + b^2\*x^(2\*n))^p, x]

[Out] Integrate[(a - b\*x^n)^p\*(a + b\*x^n)^p\*(a^2 + b^2\*x^(2\*n))^p, x]

**Maple [F]** time = 0.806, size = 0, normalized size = 0.

$$\int (a - bx^n)^p (a + bx^n)^p (a^2 + b^2 x^{2n})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x)`

[Out] `int((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b^2x^{2n} + a^2)^P (bx^n + a)^P (-bx^n + a)^P dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="maxima")`

[Out] `integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b^2x^{2n} + a^2)^P (bx^n + a)^P (-bx^n + a)^P, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="fricas")`

[Out] `integral((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a-b*x**n)**p*(a+b*x**n)**p*(a**2+b**2*x**(2*n))**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (b^2x^{2n} + a^2)^P (bx^n + a)^P (-bx^n + a)^P dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="giac")`

[Out] `integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)`

$$3.285 \quad \int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)} dx$$

**Optimal.** Leaf size=76

$$\frac{x(c+dx^{2n})^p \left(\frac{dx^{2n}}{c} + 1\right)^{-p} F_1\left(\frac{1}{2n}; 1, -p; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2x^{2n}}{a^2}, -\frac{dx^{2n}}{c}\right)}{a^2}$$

[Out]  $(x*(c + d*x^(2*n)))^p*AppellF1[1/(2*n), 1, -p, (2 + n^(-1))/2, (b^2*x^(2*n))/a^2, -((d*x^(2*n))/c)]/(a^2*(1 + (d*x^(2*n))/c)^p)$

**Rubi [A]** time = 0.168164, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\frac{x(c+dx^{2n})^p \left(\frac{dx^{2n}}{c} + 1\right)^{-p} F_1\left(\frac{1}{2n}; 1, -p; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2x^{2n}}{a^2}, -\frac{dx^{2n}}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^(2\*n))^p/((a - b\*x^n)\*(a + b\*x^n)), x]

[Out]  $(x*(c + d*x^(2*n)))^p*AppellF1[1/(2*n), 1, -p, (2 + n^(-1))/2, (b^2*x^(2*n))/a^2, -((d*x^(2*n))/c)]/(a^2*(1 + (d*x^(2*n))/c)^p)$

**Rubi in Sympy [A]** time = 35.3518, size = 58, normalized size = 0.76

$$\frac{x\left(1 + \frac{dx^{2n}}{c}\right)^{-p} (c + dx^{2n})^p \text{appellf1}\left(\frac{1}{2n}, 1, -p, \frac{n+1}{n}, \frac{b^2x^{2n}}{a^2}, -\frac{dx^{2n}}{c}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c+d\*x\*\*(2\*n))\*\*p/(a-b\*x\*\*n)/(a+b\*x\*\*n), x)

[Out]  $x*(1 + d*x**(2*n)/c)**(-p)*(c + d*x**(2*n))**p*appellf1(1/(2*n), 1, -p, (n + 1/2)/n, b**2*x**(2*n)/a**2, -d*x**(2*n)/c)/a**2$

**Mathematica [B]** time = 0.492794, size = 258, normalized size = 3.39

$$\frac{a^2c(2n+1)x(c+dx^{2n})^p F_1\left(\frac{1}{2n}; -p, 1; 1 + \frac{1}{2n}; -\frac{dx^{2n}}{c}, \frac{b^2x^{2n}}{a^2}\right)}{(a^2 - b^2x^{2n})\left(2a^2dnp x^{2n} F_1\left(1 + \frac{1}{2n}; 1 - p, 1; 2 + \frac{1}{2n}; -\frac{dx^{2n}}{c}, \frac{b^2x^{2n}}{a^2}\right) + 2b^2cnx^{2n} F_1\left(1 + \frac{1}{2n}; -p, 2; 2 + \frac{1}{2n}; -\frac{dx^{2n}}{c}, \frac{b^2x^{2n}}{a^2}\right) + a^2c\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^(2\*n))^p/((a - b\*x^n)\*(a + b\*x^n)), x]

[Out]  $(a^2*c*(1 + 2*n)*x*(c + d*x^(2*n))^p*AppellF1[1/(2*n), -p, 1, 1 + 1/(2*n), -((d*x^(2*n))/c), (b^2*x^(2*n))/a^2])/((a^2 - b^2*x^(2*n))*(2*a^2*d*n*p*x^(2*n)*AppellF1[1 + 1/(2*n), 1 - p, 1, 2 + 1/(2*n), -((d*x^(2*n))/c), (b^2*x^(2*n))/a^2] + 2*b^2*c*n*x^(2*n)*AppellF1[1 + 1/(2*n), -p, 2, 2 + 1/(2*n), -((d*x^(2*n))/c), (b^2*x^(2*n))/a^2] + a^2*c*(1 + 2*n)*AppellF1[1/(2*n), -p, 1, 1 + 1/(2*n), -((d*x^(2*n))/c), (b^2*x^(2*n))/a^2]))$

**Maple [F]** time = 0.139, size = 0, normalized size = 0.

$$\int \frac{(c + dx^{2n})^p}{(a - bx^n)(a + bx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d\*x^(2\*n))^p/(a-b\*x^n)/(a+b\*x^n), x)

[Out] int((c+d\*x^(2\*n))^p/(a-b\*x^n)/(a+b\*x^n), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{(dx^{2n} + c)^p}{(bx^n + a)(bx^n - a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(d\*x^(2\*n) + c)^p/((b\*x^n + a)\*(b\*x^n - a)), x, algorithm="maxima")

[Out] -integrate((d\*x^(2\*n) + c)^p/((b\*x^n + a)\*(b\*x^n - a)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(dx^{2n} + c)^p}{b^2x^{2n} - a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(d\*x^(2\*n) + c)^p/((b\*x^n + a)\*(b\*x^n - a)), x, algorithm="fricas")

[Out] integral(-(d\*x^(2\*n) + c)^p/(b^2\*x^(2\*n) - a^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*(2\*n))\*\*p/(a-b\*x\*\*n)/(a+b\*x\*\*n), x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{(dx^{2n} + c)^p}{(bx^n + a)(bx^n - a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(d\*x^(2\*n) + c)^p/((b\*x^n + a)\*(b\*x^n - a)), x, algorithm="giac")

[Out] integrate(-(d\*x^(2\*n) + c)^p/((b\*x^n + a)\*(b\*x^n - a)), x)



$$3.286 \quad \int (a - bx^{n/2})^p (a + bx^{n/2})^p \left( \frac{a^2 d(1+p)}{b^2 \left(1 + \frac{-1-2n-np}{n}\right)} + dx^n \right)^{\frac{-1-2n-np}{n}} dx$$

**Optimal.** Leaf size=96

$$\frac{b^2 x(np+n+1) (a - bx^{n/2})^{p+1} (a + bx^{n/2})^{p+1} \left( dx^n - \frac{a^2 dn(p+1)}{b^2(np+n+1)} \right)^{-\frac{np+n+1}{n}}}{a^4 dn(p+1)}$$

[Out]  $-\left(\left(b^2(1+n+n^*p)\right)^*x^*(a-b*x^{(n/2)})^{(1+p)}(a+b*x^{(n/2)})^{(1+p)}\right)/\left(a^4*d*n*(1+p)*\left(-\left(\left(a^2*d*n*(1+p)\right)/\left(b^2*(1+n+n^*p)\right)\right)+d*x^n\right)^{\left((1+n+n^*p)/n\right)}$

**Rubi [A]** time = 0.361072, antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, integrand size = 76,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$

$$\frac{b^2 x(np+n+1) (a^2 - b^2 x^n) (a - bx^{n/2})^p (a + bx^{n/2})^p \left( dx^n - \frac{a^2 dn(p+1)}{b^2(np+n+1)} \right)^{-\frac{np+n+1}{n}}}{a^4 dn(p+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}\left[\left(a - b*x^{(n/2)}\right)^p \left(a + b*x^{(n/2)}\right)^p \left(\left(a^2*d*(1+p)\right)/\left(b^2*(1+(-1-2*n-n*p)/n)\right) + d*x^n\right)^{\left((-1-2*n-n*p)/n\right)}, x\right]$

[Out]  $-\left(\left(b^2(1+n+n^*p)\right)^*x^*(a-b*x^{(n/2)})^p(a+b*x^{(n/2)})^p(a^2-b^2*x^n)\right)/\left(a^4*d*n*(1+p)*\left(-\left(\left(a^2*d*n*(1+p)\right)/\left(b^2*(1+n+n^*p)\right)\right)+d*x^n\right)^{\left((1+n+n^*p)/n\right)}$

**Rubi in Sympy [A]** time = 51.4377, size = 102, normalized size = 1.06

$$\frac{b^2 x \left(a - bx^{\frac{n}{2}}\right)^p \left(a + bx^{\frac{n}{2}}\right)^p (a^2 - b^2 x^n)^{-p} (a^2 - b^2 x^n)^{p+1} \left(-\frac{a^2 dn(p+1)}{b^2(np+n+1)} + dx^n\right)^{-p-1-\frac{1}{n}} (np+n+1)}{a^4 dn(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}\left(\left(a-b*x^{(1/2*n)}\right)^*p*\left(a+b*x^{(1/2*n)}\right)^*p*\left(a^2*d*(1+p)/b^2/(1+n*p-2*n-1)/n+d*x^n\right)^{\left((-n*p-2*n-1)/n\right)}, x\right]$

[Out]  $-b^2*x*(a-b*x^{(n/2)})^p(a+b*x^{(n/2)})^p(a^2-b^2*x^n)^{-p}*(a^2-b^2*x^n)^{p+1}\left(-\frac{a^2*d*n*(p+1)}{b^2*(np+n+1)}+dx^n\right)^{-p-1-\frac{1}{n}}(np+n+1)+d*x^n)^{\left((1+n+n^*p)/n\right)}$

**Mathematica [A]** time = 0.92852, size = 0, normalized size = 0.

$$\int (a - bx^{n/2})^p (a + bx^{n/2})^p \left( \frac{a^2 d(1+p)}{b^2 \left(1 + \frac{-1-2n-np}{n}\right)} + dx^n \right)^{\frac{-1-2n-np}{n}} dx$$

Verification is Not applicable to the result.

[In]  $\text{Integrate}\left[\left(a - b*x^{(n/2)}\right)^p \left(a + b*x^{(n/2)}\right)^p \left(\left(a^2*d*(1+p)\right)/\left(b^2*(1+(-1-2*n-n*p)/n)\right) + d*x^n\right)^{\left((-1-2*n-n*p)/n\right)}, x\right]$

[Out] Integrate[(a - b\*x^(n/2))^p\*(a + b\*x^(n/2))^p\*((a^2\*d\*(1 + p))/(b^2\*(1 + (-1 - 2\*n - n\*p)/n)) + d\*x^n)^((-1 - 2\*n - n\*p)/n), x]

**Maple [F]** time = 1.044, size = 0, normalized size = 0.

$$\int \left( a - bx^{\frac{n}{2}} \right)^p \left( a + bx^{\frac{n}{2}} \right)^p \left( \frac{a^2 d (1 + p)}{b^2} \left( 1 + \frac{-np - 2n - 1}{n} \right)^{-1} + dx^n \right)^{\frac{-np - 2n - 1}{n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-b\*x^(1/2\*n))^p\*(a+b\*x^(1/2\*n))^p\*(a^2\*d\*(1+p)/b^2/(1+(-n\*p-2\*n-1)/n)+d\*x^n)^((-n\*p-2\*n-1)/n), x)

[Out] int((a-b\*x^(1/2\*n))^p\*(a+b\*x^(1/2\*n))^p\*(a^2\*d\*(1+p)/b^2/(1+(-n\*p-2\*n-1)/n)+d\*x^n)^((-n\*p-2\*n-1)/n), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( bx^{\frac{1}{2}n} + a \right)^p \left( -bx^{\frac{1}{2}n} + a \right)^p \left( dx^n - \frac{a^2 d (p + 1)}{b^2 \left( \frac{np + 2n + 1}{n} - 1 \right)} \right)^{-\frac{np + 2n + 1}{n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^(1/2\*n) + a)^p\*(-b\*x^(1/2\*n) + a)^p\*(d\*x^n - a^2\*d\*(p + 1)/(b^2\*(n\*p + 2\*n + 1)/n), x, algorithm="maxima")

[Out] integrate((b\*x^(1/2\*n) + a)^p\*(-b\*x^(1/2\*n) + a)^p\*(d\*x^n - a^2\*d\*(p + 1)/(b^2\*((n\*p + 2\*n + 1)/n - 1)))^(-(n\*p + 2\*n + 1)/n), x)

**Fricas [A]** time = 0.28891, size = 243, normalized size = 2.53

$$\frac{\left( (b^4 np + b^4 n + b^4) x x^{2n} - (2 a^2 b^2 np + 2 a^2 b^2 n + a^2 b^2) x x^n + (a^4 np + a^4 n) x \right) \left( bx^{\frac{1}{2}n} + a \right)^p \left( -bx^{\frac{1}{2}n} + a \right)^p}{(a^4 np + a^4 n) \left( -\frac{a^2 d np + a^2 d n - (b^2 d np + b^2 d n + b^2 d) x^n}{b^2 np + b^2 n + b^2} \right)^{\frac{np + 2n + 1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^(1/2\*n) + a)^p\*(-b\*x^(1/2\*n) + a)^p/(d\*x^n - a^2\*d\*(p + 1)/(b^2\*(n\*p + 2\*n + 1)/n)), x)

[Out] ((b^4\*n\*p + b^4\*n + b^4)\*x\*x^(2\*n) - (2\*a^2\*b^2\*n\*p + 2\*a^2\*b^2\*n + a^2\*b^2)\*x\*x^n + (a^4\*n\*p + a^4\*n)\*x)\*(b\*x^(1/2\*n) + a)^p\*(-b\*x^(1/2\*n) + a)^p/((a^4\*n\*p + a^4\*n)\*(-(a^2\*d\*n\*p + a^2\*d\*n - (b^2\*d\*n\*p + b^2\*d\*n + b^2\*d)\*x^n)/(b^2\*n\*p + b^2\*n + b^2))^((n\*p + 2\*n + 1)/n))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x\*\*(1/2\*n))\*\*p\*(a+b\*x\*\*(1/2\*n))\*\*p\*(a\*\*2\*d\*(1+p)/b\*\*2/(1+(-n\*p-2\*n-1)/n)+d\*x\*\*n)\*\*((-n\*p-2\*n-1)/n), x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^{\frac{1}{2}n} + a)^p (-bx^{\frac{1}{2}n} + a)^p}{\left(dx^n - \frac{a^2 d(p+1)}{b^2 \left(\frac{np+2n+1}{n} - 1\right)}\right)^{\frac{np+2n+1}{n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^(1/2\*n) + a)^p\*(-b\*x^(1/2\*n) + a)^p/(d\*x^n - a^2\*d\*(p + 1)/(b^2\*(n\*p + 2\*n + 1)/n - 1)), x)

[Out] integrate((b\*x^(1/2\*n) + a)^p\*(-b\*x^(1/2\*n) + a)^p/(d\*x^n - a^2\*d\*(p + 1)/(b^2\*((n\*p + 2\*n + 1)/n - 1))), x)

## 4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result, optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result, optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_, optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result, Complex] || Not[FreeQ[optimal, Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result, Integrate] && FreeQ[result, Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```



```

# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);
    #This check below actually is not needed, since I only call this grading only for
    #passed integrals. i.e. I check for "F" before calling this.

    if not type(result,freeof('int')) then
        return "F";
    end if;

    if ExpnType_result<=ExpnType_optimal then
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                #both result and optimal complex
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A";
                else
                    return "B";
                end if
            else #result contains complex but optimal is not
                return "C";
            end if
        else # result do not contain complex
            # this assumes optimal do not as well
            if leaf_count_result<=2*leaf_count_optimal then
                return "A";
            else
                return "B";
            end if
        end if
    else #ExpnType(result) > ExpnType(optimal)
        return "C";
    end if
end proc:

```

```

# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'``^``') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
        max(2,ExpnType(op(1,expn))) end if else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'``+``') or type(expn,'``*``') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

```

```

ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```